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# A MEASUREMENT OF THE PION RADIUS

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ABSTRACT

We propose a wire spark chamber experiment to measure the pion electromagnetic radius accurate to ±0.03f by measuring the scattering cross section for 50--80 GeV pions from electrons in a liquid hydrogen target. The data will distinguish between the ρ dominance prediction of 0.64f and the "proton-like" radius to 0.81f.

## II. PHYSICS JUSTIFICATION

The electromagnetic dimensions of the various particles are of fundamental interest. The charge radius of both the pion and proton are predicted to be 0.64f from rho-dominance, yet the proton radius measured from electron-proton scattering is 0.81f. A measurement of the pion radius is crucial in order to understand whether this difference is due to some peculiarity of the nucleon or to a breakdown of vector dominance. In a larger sense, this is one place where theory has far out-stripped experiment. The pion radius is one of the fundamental numbers of physics and has inspired a wide variety of theoretical predictions only loosely subjected to experimental test. Many of these predictions fail to differentiate between "proton-like" (0.81f) and "rho-dominance" (0.64f) radii. Recent speculation[1] has suggested that the pion radius may be even smaller if higher mass particles couple to the photon. This experiment will differentiate among these values for the radius by measuring cross sections to a precision of one per cent. This accuracy distinguishes between the two values of 0.64f and 0.81f by six standard deviations and rejects a point-like pion by about fifteen standard deviations.

We will measure the cross section differential in the final state electron energy. This is given in terms of the point cross section by

$$\frac{d\sigma}{dE} = \left(\frac{d\sigma}{dE}\right)_{point} f_\pi^{\,2}$$

where $f_\pi$ is the pion form factor. Since the momentum transfer is small in pi-e collisions, $f_\pi$ depends only on the mean radius, $<r_\pi>$ and, $q^2$, the four momentum-transfer squared:

$$f_\pi \approx 1 + \frac{1}{6} q^2 <r_\pi>^2$$

Direct pi-e scattering has been measured in several experiments. The most accurate completed experiment[2] quotes $<r_\pi> < 3 \times 10^{-13}$cm. A Dubna group led by E. Tsyganov is scheduled to carry out an experiment whose aim is to measure the radius with a 50 GeV/c $\pi^-$ beam and a wire-spark-chamber spectrometer. Even if the Serpukhov experiment is successful, the added intensity and improved techniques of this proposed experiment will provide a more accurate result with less systematic error. A group from Harvard led by Richard Wilson has proposed to do the experiment at the AGS with a pion beam of about 25 GeV/c. The effect to be measured in the present experiment is at least a factor of three larger. Table I compares the effect expected in the cross section for three incident pion beam energies representative of the three experiments. The AGS experiment would run at 25 GeV, the Serpukhov experiment at 50, and this experiment at both 50 and 80 GeV. We use the previous equations, $q^2 = 2m_e^2 - 2m_eE_e$ and $q^2_{max} = -\frac{4m_e p_\pi^2}{m_e^2 + m_\pi^2 + 2m_eE_\pi}$ where $p_\pi$, $E_\pi$ refer to the initial pion; $m_e$ and $m_\pi$ are the electron and pion mass; and $E_e$ is the final state electron energy.

| pion beam energy | range of accepted recoil electron energy | % deviation from the point cross section | |
|---|---|---|---|
| | | $\langle r_\pi \rangle = 0.81$f | $\langle r_\pi \rangle = 0.64$f |
| 80 GeV | 40 -- 64 GeV | 23 -- 36 | 14.5 -- 23 |
| 50 | 25 -- 37.5 | 13.5--21.5 | 8.5 -- 13.5 |
| 25 | 8 -- 14 | 4.7-- 8.0 | 2.9 -- 5.2 |

Three other methods have been used to measure the pion form factor. Berkelman et al.[3] and Mistretta et al.[4] isolated the one-pion exchange diagram in $\pi^+$ electroproduction and measured its contribution as a function of $q^2$ to extract $\langle r_\pi \rangle$. They find $\langle r_\pi \rangle \approx 0.8 \pm 0.1$f but the result is uncertain theoretically because of the difficulty in estimating the contribution of other terms to the cross section. Block et al.[5] use $\pi^\pm$ - $He^4$ elastic scattering to find the pion form factor via an interference effect. They find $\langle r_\pi \rangle < 1 \times 10^{-13}$cm. This method also suffers from significant uncertainty due to the contribution of terms other than Coulomb scattering. The third method is via colliding beams. These elegant experiments measure the form factor in the time-like region so that a measurement of the form-factor in the space-like region provides an opportunity to test the ability to extrapolate to short distances. At the present time, if the electroproduction experiments are correct, this extrapolation fails[6] as can be seen in Fig. 1.

We choose 50 and 80 GeV to perform this experiment because

1) The effect in the cross section is larger than at lower energies.
2) Backgrounds leading to systematic errors can be suppressed via longitudinal momentum balance at these energies. This might not be possible at higher energies. ±0.1% resolution in incoming and

outgoing momenta permit balance to ±120 MeV at 80 GeV, sufficient to reject backgrounds in the final state from strong interaction by two standard deviations.

3) Counting rates decrease with energy but are still more than adequate.

4) Systematic effects in efficiencies are energy independent and their effect is less pronounced than at lower energies.

## III. EXPERIMENTAL DETAILS

### Introduction

A diagram of the experimental layout is shown in Fig. 2. The apparatus is situated in the medium-energy high-resolution charged beam (MEHR). A flux of $3 \times 10^6$ charged pions/pulse is incident on a 50cm liquid hydrogen target. The momentum and scattering angles of the recoiling pi-e pair are measured with a combination of proportional chambers, magnetostrictive-wire chambers and a spectrometer magnet. Differentiation between pions and electrons is accomplished with total absorption shower counters. The trigger consists of two charged particles within a 10 msr-scattering cone, defined by scintillation counter $S_1$, which also pass into appropriate momentum intervals defined by scintillation counters $S_2$ and $S_4$. The particle momenta accepted by the apparatus at 50 and 80 GeV/c are shown below.

| Incident Momentum | Pion Range | Electron Range |
|---|---|---|
| 50 GeV/c | 12.5 -- 25 GeV/c | 25 -- 37.5 GeV/c |
| 80 GeV/c | 16 -- 40 GeV/c | 40 -- 64 GeV/c |

Charged particle background from the hydrogen target is rejected in the trigger by the anti-coincidence counter A1. Muon triggers are rejected by means of a bank of anti-coincidence counters behind a steel wall.

We will run the experiment with negatively and positively charged pions at 50 GeV/c and with negative pions at 80 GeV/c. Positive pions will be used to test for charge dependent systematic effects, while the 80 GeV/c data will determine energy dependent systematic effects. With an incident $\pi^-$ beam, a single spectrometer arm behind the magnet is necessary. In Figure 2, this consists of wire chambers SC 1-4, scintillation counters S2 and S4, shower counters S3 and S5, and the muon telescope. The magnet polarity is set appropriately to bend negatively charged particles into this arm. With an incident $\pi^+$ beam, the magnet polarity is reversed such that negatively charged particles are bent in the opposite direction and a double arm spectrometer is used. The additional arm now detects the recoil electron and consists of wire chambers SC 5-8, and scintillator S2 with shower counter S3 taken from the original arm. S5 is also removed from the original arm which now detects only $\pi^+$ and $\mu^+$.

Resolution

Adequate momentum resolution is obtained by spacing the chambers before and after the spectrometer magnet over a 10 meter interval and by requiring the field integral for the spectrometer magnet to be 100 kgauss-meters. The momentum resolution is then given approximately by

$$\frac{\Delta p}{p} = 1.1 \times 10^{-4} \; (.045\; p^2 + 31)^{1/2}$$

where $p^2$ is in $(\mathrm{GeV/c})^2$.

The momentum dependent term is based upon a 0.5 mm spark resolution, the momentum independent term comes from multiple scattering. The terms are approximately equal at 25 GeV/c where the resolution is ±0.1%. Multiple scattering diminishes in importance at higher energies. At 50 GeV/c the momentum balance can be done at a precision such that $\Delta E_f = \pm 50$ MeV.

The horizontal aperture of the magnet should be 48" in order to accept the wide momentum range of the final state. A 10" gap is adequate for a 10 msr acceptance in the vertical direction. A hodoscope placed at the momentum slit of the incident pion beam will serve to define the incident pion energy to ±0.1% so that at 50 GeV/c, $\Delta E_i = 50$ MeV. The longitudinal momentum balance will be ±80 MeV at 50 GeV/c and ±120 MeV at 80 GeV.

Event Rate

Our estimates of running time are based on the following considerations. The Bhabha cross section is given by

$$\frac{d\sigma}{dE} = \frac{2\, m_c r_o^2}{E^2} \left( 1 - \frac{E}{E_m} \right)$$

where E = energy of outgoing $e^-$ and $E_m$ is its maximum value

Electrons are accepted between 25 and 37.5 GeV for the 50 GeV case, thus

$$\sigma = 25.5 \times 10^{-29} \int_{25}^{37.5} \frac{d\sigma}{dE}\, dE$$

$$= 0.645 \mu b$$

For 100% geometric efficiency and a 50cm hydrogen target, the yield is

$$Y = 1.4 \times 10^{-6} \text{ per incident pion.}$$

At 50 GeV we will run the beam at $3 \times 10^6$ negative pions per pulse, the event trigger rate being

$$T_e = 1.4 \times 10^{-6} \times 3 \times 10^6 \approx 4 \text{ per pulse}$$

The $\pi^-$ flux at 80 GeV will be lower, we expect only $10^6$ per pulse. The 50 GeV $\pi^+$ beam contains 60% protons so we will limit its intensity to $10^6$ $\pi^+$ per pulse. Running time would be proportioned as follows:

| | |
|---|---|
| 50 hours | checkout |
| 40 hours (100,000 counts) | 50 GeV $\pi^-$ |
| 60 hours ( 50,000 counts) | 80 GeV $\pi^-$ |
| 60 hours ( 50,000 counts) | 50 GeV $\pi^+$ |
| 210 hours | |

In order to accept $3 \times 10^6$ negative pions/pulse at 50 GeV/c we will use proportional chambers to define the trajectories of the incident pion and the recoiling pi-e pair. The proportional chambers upstream of the hydrogen target will also be used as beam counters. In the rear of the magnet, where beam loading can be avoided and where an increased area is involved, magnetostrictive-wire chambers will be used.

Backgrounds

We consider two background processes at 50 GeV for the $\pi^- e^-$ detection configuration only. The first is

(1) $$\pi^- + p \rightarrow \pi^- + p + \pi^0$$
$$\pi^0 \rightarrow \gamma + \gamma, \quad \gamma \rightarrow e^+ e^-$$

Here a $\gamma$ ray from the $\pi^0$ converts to an $e^+e^-$ pair, the $\pi^-$ and $e^-$ simulating a scattering event. The yield from this reaction has been calculated by the Dubna group[7] by assuming that the yield of secondaries from incident pions is the same as that from incident protons. The very small solid angle of the $\pi$-e process is of great importance in rejecting these strong interactions backgrounds. The probability per incident pion that the $\pi^-$ and $\pi^0$ be produced in the appropriate solid angle and momentum interval is

$$P = P_{\pi^-} \cdot P_{\pi^0}$$

where $P_{\pi^0}$ and $P_{\pi^-}$ are the separate probabilities for the $\pi^0$ and $\pi^-$. They find the probability to be

$$P = 10^{-3} \times 10^{-3} = 10^{-6} \text{ per incident pion}$$

The probability of conversion of a $\gamma$ ray with an $e^-$ energy in the triggering range is $10^{-2}$. The anti-coincidence counter around the target should further reduce this background at least a factor of $10^{-1}$ as estimated by a Monte Carlo calculation performed at UCLA. The expected yield is $10^{-6} \times 10^{-2} \times 10^{-1} = 10^{-9}$ per incident pion. Coplanarity and opening angle cuts reduce this background in analysis by another factor of $10^{-2}$. With an event yield of $1.4 \times 10^{-6}$, this background ratio is

$$B = \frac{10^{-9} \times 10^{-2}}{Y} = \frac{10^{-11}}{1.4 \times 10^{-6}}$$

and is negligible. The expected trigger rate for this background in a beam of $3 \times 10^6$ pions is

$$T_b = 10^{-9} \times 3 \times 10^6 = 3 \times 10^{-3} \text{ per pulse}$$

The second background process that we have considered is

(2) $$\pi^- + p \rightarrow \pi^- + \pi^- + \pi^+ + p$$

Here one $\pi^-$ is incorrectly interpreted as an $e^-$. The $\pi^-\pi^-$ yield calculated by the Dubna group into the solid angle and momentum acceptance is expected to be

$$P = 3 \times 10^{-4} \quad \text{per incident pion}$$

Because of this high yield we have performed a Monte Carlo calculation for this reaction. The program was based upon

1) peripheral $\Delta^{++}$ production,

2) phase space for the $\pi^-\pi^-$ pair,

3) the $\pi^+p$ (treated as a Breit-Wigner resonance) of mass 1238 MeV.

We find that the anti-coincidence counter (either the $\pi^+$ or the proton were required to traverse more than 2" of hydrogen) around the target is extremely effective. Of 100,000 events thrown, $63 \times 10^3$ would normally have triggered the apparatus, but only 37 survived the anti coincidence. Considering counter inefficiencies, we assume $10^{-2}$ rejection for this anti-coincidence. Furthermore, coplanarity, opening angle, and longitudinal momentum balance cuts reduce these 630 events further. Thus only five candidates are left after analysis. Shower counter identification of the electron should reduce this by an additional factor of $10^{-2}$. We find for the background ratio

$$B = 3 \times 10^{-4} \times \frac{5}{63 \times 10^3} \times 10^{-2} \times \frac{1}{Y}$$

$$= 1.7 \times 10^{-4}$$

and for the trigger rate (we assume no rejection by the shower counter in the trigger)

$$T_b = 3 \times 10^{-4} \times 10^{-2} \times 3 \times 10^6$$

$$= 9 \text{ per pulse}$$

We estimate all other backgrounds to be small. Proton and kaon scatters cannot be confused with π-e scattering events because the kinematics are completely different. Electron contamination in the beam can be determined by looking above the kinematic energy allowed by pi-e kinematics or by inserting Pb in the beam to alter the pi to e ratio; μ-e scatters are eliminated by muon identification. One could use these muon events for normalization in the experiment although the statistics will be limited (≈ 2,000 events).

Radiative corrections

The radiative corrections to this experiment must be accurate because absolute cross sections will be measured to ±1% accuracy. Such corrections depend upon the details of the experiment and we have not yet calculated them precisely. In principle, they are, exactly calculable if the experimental resolution is well-known; so that no fundamental problems are expected. We can estimate the size of these corrections using the notation $\sigma = \sigma_{exp}(1 + \Sigma\delta_i)$ where the $\delta_i$ are found as follows.

For a 0.2% momentum resolution of the outgoing particle, the bremsstrahlung correction for 25cm of hydrogen is

$$\delta_B = \tau \int_{E_{min}}^{E_{max}} \frac{dK}{K}$$

where K is the photon energy and
τ is the radiator thickness

$$\approx 0.025 \ln \frac{35\ \text{GeV}}{35\ \text{MeV}}$$

$$= 0.0155$$

The Landau loss for the particles in the target is small because the momentum balance is uncertain to ≈ 50 MeV

$$\delta_L = \frac{\xi \ell}{\Delta E} \approx 0.005$$

where $\xi \approx 10^{-2}$

$\ell = 25$cm

$\Delta E = 50$ MeV

Radiative corrections have been estimated by Bardin et al.[8] and by Kahane.[9] There are three contributions. $\delta_1$ is the elastic one-photon exchange contribution.

$$\delta_1 = \frac{\alpha}{\pi} \left[\frac{13}{6} \ln \frac{q^2}{m_\pi^2} + \frac{2(K' + 1)}{a'} \ln \frac{a' + K'}{a' - K'} - \frac{46}{9}\right]$$

where

$$K' = \frac{q^2}{2m_\pi^2}$$

$$a' = (K'^2 + 2K')^{1/2}$$

At 50 GeV and maximum momentum transfer, this term is

$$\delta_1 = -0.55$$

$\delta_2$ is the elastic two-photon exchange contribution. Since it is small, we ignore its contribution for this discussion.

$$\delta_2 \approx 0$$

The third contribution $\delta_3$ is from diagrams with external photon lines. This correction depends on the experimental resolution. We approximate this result by considering only those terms dependent upon momentum resolution and by ignoring the angular measurements. The resulting expression is lengthy but gives approximately

$$\delta_3 = 0.15$$

for our experimental conditions.

The total correction is

$$\delta = \delta_B + \delta_L + \delta_1 + \delta_2 + \delta_3$$

$$\delta \approx 0.255$$

The uncertainty in the correction depends in principle only on the knowledge of our experimental resolution and this will be studied in detail in the course of the experiment.

## IV. APPARATUS

The major components of the apparatus shown in Fig. 2 are as follows:

1) A liquid-hydrogen target 50cm long.
2) An analyzing magnet with a field integral of 100 kgauss-meters.
3) Seven proportional chambers and four magnetostrictive chambers.
4) Two total absorption Cherenkov shower counters for particle identification.
5) A muon wall for muon rejection.
6) Scintillation counters for triggering purposes and for anti-coincidence around the liquid-hydrogen target.
7) A differential Cherenkov counter for tagging beam pions.

We will require that NAL supply the analyzing magnet, the liquid hydrogen target, and the differential Cherenkov counter. The magnet should be approximately five meters in length with a peak field of ≈20 Kg. The downstream limiting aperture should be 48" horizontally by 10" vertically. It would be acceptable to break the magnet into two separate magnets. In this case the aperture of the upstream magnet could be reduced to about 8" x 24". The liquid hydrogen target should be 3cm in diameter and 50cm long. It must have an accurately known density and length.

We will supply our own on-line computation in the form of a Hewlett-Packard 2116B computer. However, we would like to tie this into a larger computer for an additional floating-point facility if one is available. The apparatus requires no scanning facilities.

We will also require from NAL the fast electronics to form a trigger. UCLA will supply the remainder of the apparatus including the electronics for the proportional chambers. They will have an active volume of 30cm by 30cm. Since the downstream chambers are large (1m x 2m), they will be "conventional" wire chambers with magnetostrictive readout. In order to use them effectively the area of the incident pion beam will be deadened.

All equipment will be ready in June, 1972. We will require of NAL the usual support facilities involved in the setup and running of an experiment.

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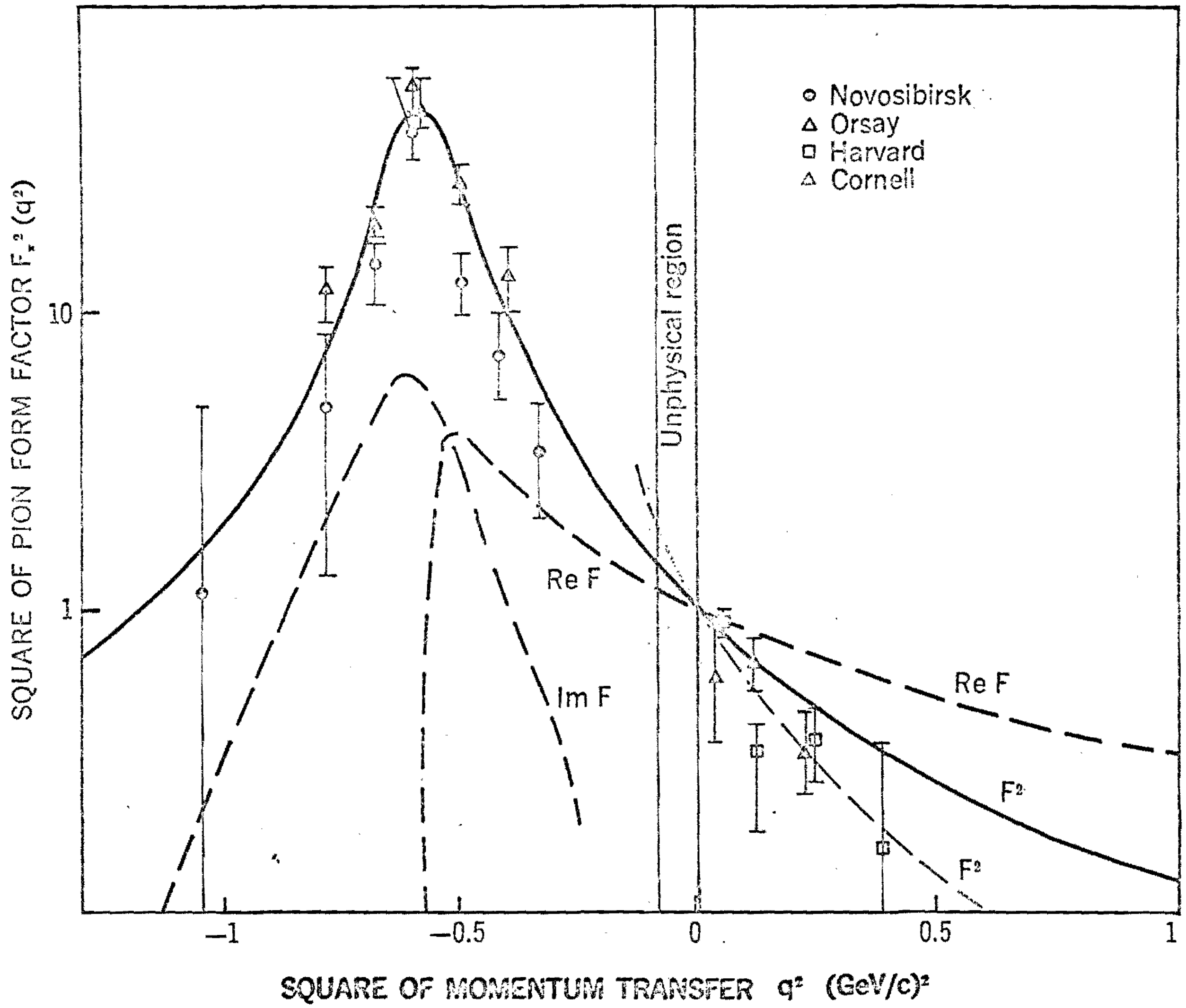


**PION FORM FACTOR** (squared) in both time-like and space-like regions. Data from four laboratories is compared to the vector-dominance model (black lines; solid line is $F^2$, dashed lines are Re $F$ and Im $F$). Colored line is proportional to the nucleon form factors. Vector-dominance model has a single resonance and has small adjustments to satisfy analyticity requirements.

FIGURE 1

# EXPERIMENTAL LAY-OUT FOR PION-ELECTRON SCATTERING

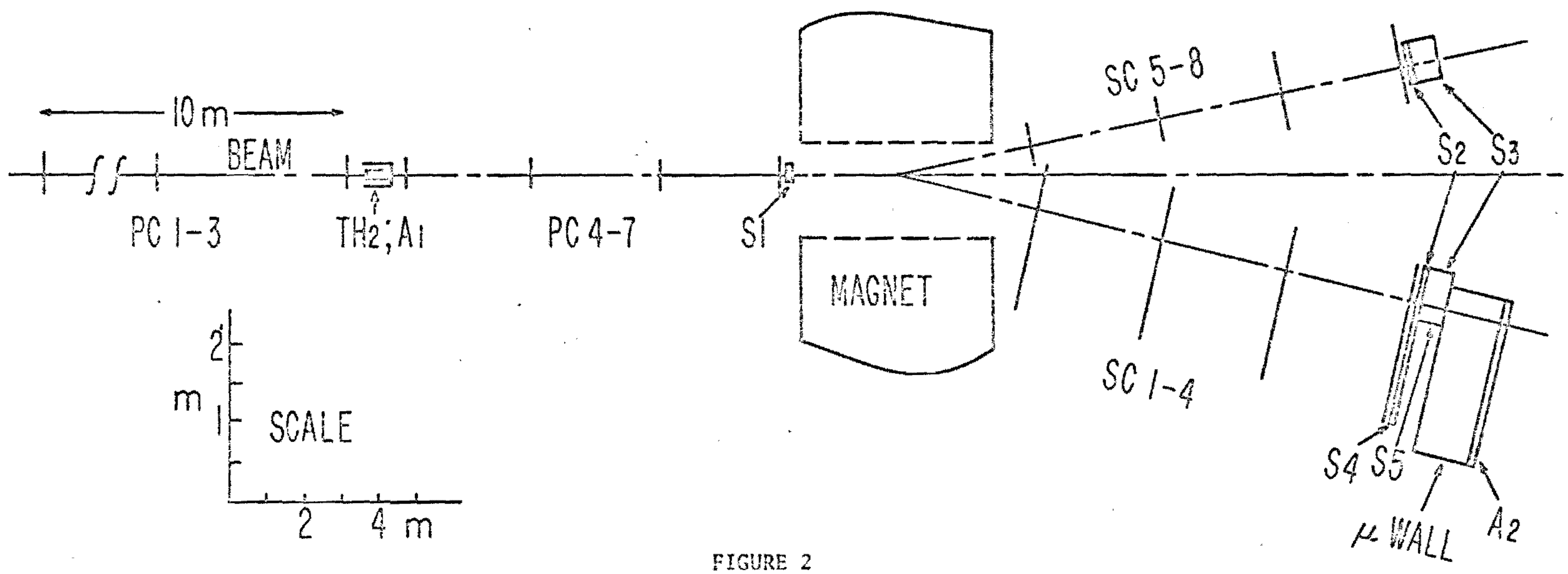


FIGURE 2

PC 1-7 are proportional chambers; SC 1-8 are magnetostrictive wire chambers; $TH_2$ is the liquid-hydrogen target; $A_1$ is the anti-coincidence counter for the target; $S_1$ is a scintillator which defines the acceptance; $S_2$ and $S_4$ are scintillators which detect the pion and electron; $S_3$ and $S_5$ are shower counters; $A_2$ is an anti-coincidence counter for muon rejection.