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DECAYS OF NEUTRAL PSEUDOSCALAR MESONS INTO LEPTON PAIRS

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ABSTRACT

We calculate the branching ratio $\Gamma(P \to \ell^+\ell^-)/\Gamma(P \to \gamma\gamma)$ to lowest contributing order in quantum electrodynamics, with a vector meson model for the pseudoscalar meson form factor. We treat the processes $\eta \to \mu^+\mu^-$; $\eta \to e^+e^-$; $K_2^0 \to \mu^+\mu^-$; $K_2^0 \to e^+e^-$; $\pi^0 \to e^+e^-$. Our results are compared with those of previous calculations.

I. INTRODUCTION

The decays of neutral pseudoscalar mesons into lepton pairs are of interest because the observation of high branching ratios to these modes may indicate the existence of neutral lepton currents. A reliable estimate of the branching ratios due to conventional mechanisms is desirable, to give meaning to the notion of "high branching ratios". A lower limit for the branching ratio $\Gamma(P \to \ell^+\ell^-)/\Gamma(P \to \gamma\gamma)$ has been given by Geffen and B.-L. Young.[1] This lower bound (sometimes called the unitarity limit) is model-independent and depends only on the assumption that the two-photon intermediate state dominates the unitarity sum for the absorptive part of the amplitude for $P \to \ell^+\ell^-$ (see Fig. 1). The scale for the branching ratio is set by this unitarity limit ($\sim 10^{-5}$ for $\eta \to \mu^+\mu^-$). But the actual partial decay rate into lepton pairs may be an order of magnitude or more larger, depending on the size of the real part of the amplitude. Previous calculations by Drell,[2] Berman and Geffen,[3] Sehgal,[4] and B.-L. Young[5] have, in fact, given some values very much larger than the unitarity limit, depending on the cut-off parameters and other details of the models.

Because of the interest by experimenters in a plausible theoretical estimate of the branching ratio $\Gamma(P \to \ell^+\ell^-)/\Gamma(P \to \gamma\gamma)$, and because of the wide range in the previous theoretical estimates, we present yet another calculation, based on a vector-dominance model of electromagnetic couplings. In the main we assume that there are no direct $P\gamma\gamma$ or $PV\gamma$ couplings. All photon couplings then occur via the intermediary of vector mesons, as is shown for the relevant processes in Fig. 1.

The form factor for the transition of a pseudoscalar meson of mass M into two virtual photons, k_1 and k_2, $F(k_1^2, k_2^2; M^2)$, is therefore proportional to $[(m_1^2 + k_1^2)(m_2^2 + k_2^2)]^{-1}$, where m_1 and m_2 are vector meson masses. In the process $P \to \ell^+\ell^-$ such a form factor gives a rapidly convergent loop integral. In advance of the detailed computation we may anticipate that our result should correspond roughly to those of Drell[2] and Berman and Geffen[3], provided their cut-off parameters are taken around the vector meson mass. For comparison we also evaluate the branching ratio with a single vector meson propagator, corresponding to the existence of a $PV\gamma$ coupling. This is the same calculation as was done by Sehgal,[4] repeated here because Sehgal gave no formulas and only numerical values for K_2^0 decay for three choices of cut-off mass.

We compute the branching ratio $\Gamma(\eta \to \mu^+\mu^-)/\Gamma(\eta \to \gamma\gamma)$ as a function of vector meson mass. The model is also applied to the electronic decays of η, K_2^0, and π^0; and to the decay $K_2^0 \to \mu^+\mu^-$. The resulting branching ratios are somewhat smaller than those obtained by previous authors.[2-5] To indicate the relative importance of the processes here considered, we include in Appendix C the predicted branching ratios for the competing Dalitz pair and double Dalitz pair decays.

Part of our aim in this paper is frankly pedagogical. We have included an appendix on our conventions for the calculation of Feynman amplitudes and on the evaluation of loop integrals over internal four-momenta. In a second appendix we give some of the details of the present calculation.

II. MODEL AND CALCULATION

We assume that electromagnetic decays of η proceed through intermediate states of two identical vector mesons (V). The Feynman diagrams for the processes $\eta \to VV \to \gamma\gamma$ and $\eta \to VV \to \ell^+\ell^-$ are shown in Fig. 1. A form factor for the η is needed in the first place to circumvent the logarithmic divergence in the amplitude for $\eta \to \ell^+\ell^-$, which occurs in the limit of a point interaction. The use of identical vector mesons is inspired by the SU(3) Hamiltonian for the ηVV vertex. The Hamiltonian $\mathcal{H}_{VVP} = \mathrm{Tr}(V_9 V_9 P)$ contains the piece $(\rho^0\rho^0 + \omega\omega - 2\varphi\varphi)\eta$.

A. Radiative Decay

The Feynman amplitude for the process $\eta \to \gamma\gamma$ is*

$$\mathcal{M}_{\gamma\gamma} = \frac{f}{\mu} G^2 \frac{\epsilon_{\lambda\mu\nu\sigma} k_{1\lambda} k_{2\mu} \epsilon_{1\nu} \epsilon_{2\sigma}}{i[k_1^2 + \mu^2][k_2^2 + \mu^2]} , \tag{1}$$

where f/μ is the ηVV coupling constant, G is the $V\gamma$ coupling constant, μ is the mass of the vector meson, ϵ_i is the polarization vector for the ith photon and k_i its momentum. Therefore the radiative decay rate is

$$\Gamma_{\gamma\gamma} = \frac{1}{16\pi} \frac{f^2 G^4 M^3}{\mu^{10}} , \tag{2}$$

where M is the η meson mass.

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* We use the Pauli metric. A complete discussion of our conventions is given in Appendix A.

B. Leptonic Decay

For the process $\eta \to \ell^+\ell^-$, The Feynman amplitude is

$$\mathcal{M}_{\ell^+\ell^-} = \frac{f}{\mu} G^2 e^2 \bar{u}(q_1) \mathcal{O} v(q_2) \quad , \tag{3}$$

where

$$\mathcal{O} = \frac{1}{(2\pi)^4} \int \frac{d^4k \, \epsilon_{\lambda\mu\nu\sigma} \, k_\lambda (p - k)_\mu \, \gamma_\sigma [m - i\gamma \cdot (k - q_2)] \gamma_\nu}{k^2(k^2 + \mu^2)[(p - k)^2 + \mu^2](p - k)^2[(k - q_2)^2 + m^2]} \, . \tag{4}$$

Here e is the lepton charge, u and v are respectively positive- and negative-energy Dirac spinors, m is the lepton mass, and γ_α a Dirac matrix.

The evaluation of (3) and (4) is straightforward; the standard techniques of quantum electrodynamics can be brought to bear. The manipulations are given in Appendix B.

C. Branching Ratio and Unitarity Limit

The branching ratio for $P \to \ell^+\ell^-$ to $P \to \gamma\gamma$ can be written as

$$\frac{\Gamma_{\ell^+\ell^-}}{\Gamma_{\gamma\gamma}} = 2\alpha^2 \frac{m^2}{M^2} \sqrt{1 - \frac{4m^2}{M^2}} \, [X^2 + Y^2] \quad , \tag{5}$$

where α is the fine structure constant. The quantities X and Y, defined in Appendix B, are proportional to the dispersive and absorptive parts of the matrix element, (3). The absorptive part Y is independent of the model chosen for the η form factor, depending only upon the on-mass-shell amplitudes for $\eta \to \gamma\gamma$ and $\gamma\gamma \to \ell^+\ell^-$. Hence neglecting X in Eq. (5) gives us an almost rigorous lower bound on the branching ratio, as first observed by Geffen and B.-L. Young.[1] The value of Y is (see Appendix B)

$$Y = -\frac{1}{\sqrt{1 - \frac{4m^2}{M^2}}} \ell n \left(\frac{M + \sqrt{M^2 - 4m^2}}{2m} \right) . \tag{6}$$

The unitarity limit for the branching ratio is thus

$$\frac{\Gamma_{\ell^+\ell^-}}{\Gamma_{\gamma\gamma}} \geqslant 2\alpha^2 \frac{m^2}{M^2} \frac{1}{\sqrt{1 - \frac{4m^2}{M^2}}} \left[\ell n \left(\frac{M + \sqrt{M^2 - 4m^2}}{2m} \right) \right]^2 . \tag{7}$$

We do not have an equally compact expression for X; it is necessary to perform numerically the final one-dimensional integration (over an auxiliary Feynman parameter). These last integrals are written down explicitly in Appendix B. The results are presented in Section III.

D. Another Model

Another possible model for the form factor is a single vector meson propagator, corresponding to a direct $\eta V \gamma$ coupling. This model provides a somewhat "harder" form factor and comparison of the results of the two models will give some indication of the sensitivity of the branching ratio to the details of the assumptions about the vertex. The two calculations are very analogous, the second one involving one less propagator in the denominator of (4). Details again are given in Appendix B and the results in Section III. Sehgal[4] calculated the branching ratio $\Gamma(K_2^0 \to \mu^+\mu^-)/\Gamma(K_2^0 \to \gamma\gamma)$ using this model. But his paper only sketches the calculation and gives numerical values for just three choices of cut-off (vector meson) mass.

III. RESULTS OF THIS CALCULATION; COMPARISON WITH PREVIOUS CALCULATIONS

A. $\eta \to \mu^+\mu^-$

The lower bound on the branching ratio is given by Eq. (7) as 1.07×10^{-5}. Our results are shown as a function of vector meson mass in Fig. 2. The results of Drell[2] and of Berman and Geffen[3] for the decay $\pi^0 \to e^+e^-$ can be converted to $\eta \to \mu^+\mu^-$; these are also shown in Fig. 2.

1. Behavior as a function of μ; Comparisons with other results

We first consider the general behavior of the branching ratio as a function of vector meson mass. For the two models employed here the real part of the amplitude has a zero for $\mu/M \gtrsim 1$. This is visible in Fig. 2 for the $\eta V\gamma$ model, but occurs at such a small value of $(\mu/M - 1)$ for the ηVV model that it cannot be seen on the scale of Fig. 2. For large values of μ/M there is a divergence of the amplitude as $\ell n\, \mu$, corresponding to the logarithmic divergence which occurs for point coupling of $\eta\gamma\gamma$. Explicitly, the asymptotic branching ratio for both models is

$$\lim_{\frac{\mu}{M} \gg 1} \frac{\Gamma_{\ell^+\ell^-}}{\Gamma_{\gamma\gamma}} = \frac{18\alpha^2}{\pi^2} \frac{m^2}{M^2} \left[\ell n\left(\frac{\mu}{m}\right)\right]^2 . \tag{8}$$

Berman and Geffen[3] used the form factor

$$F[k_1^2, k_2^2] = \frac{\mu^2}{\mu^2 + k_1^2 + k_2^2} \quad , \tag{9}$$

whence

$$\frac{\Gamma_{\ell^+\ell^-}}{\Gamma_{\gamma\gamma}} = 2\left(\frac{\alpha m}{\pi M}\right)^2 |N|^2 \quad ;$$

$$N = [(\ell n\, M/m)^2 - 3\ell n\, M/m - 3/2\, \ell n\, \mu^2/2M^2 + \pi^2/12 - 9/8 + O(M^2/\mu^2)]$$
$$-i[\pi\, \ell n\, M/m + O(m^2/M^2)] \quad . \tag{10}$$

It is perhaps not surprising that the Berman-Geffen result gives numerical values lying between those of our two models, as shown in Fig. 2, since their form factor has characteristics intermediate between the ηVV and $\eta V\gamma$ form factors. The limiting form of their branching ratio can be seen from (10) to be the same as (8).

Drell[2] considered a dispersion relation (in the square of the pseudoscalar meson mass) for the form factor describing the decay $P \to \ell^+\ell^-$. The imaginary part of the form factor is proportional to our Eq. (6), times a form factor $G(Q^2)$ which describes the decay of a pseudoscalar meson of mass $\sqrt{-Q^2}$ into two real photons $[G(Q^2) = F(0, 0; -Q^2)$, where $F(k_1^2, k_2^2; M^2)$ is our form factor].

Drell chose

$$G(Q^2) = \begin{cases} 1, & -Q^2 < \mu^2 \\ 0, & -Q^2 > \mu^2 \end{cases} \quad , \tag{11}$$

where μ is now a cutoff paramter. This gives for the branching ratio

$$\frac{\Gamma_{\ell^+\ell^-}}{\Gamma_{\gamma\gamma}} = \frac{1}{2}\left(\frac{\alpha m}{\pi M}\right)^2 \left\{\left[\ell n\left(\frac{\mu^2}{M^2}\right) \ell n\left(\frac{M\mu}{m^2}\right)\right]^2 + \left[\pi \ \ell n\left(\frac{M^2}{m^2}\right)\right]^2\right\}. \qquad (12)$$

Drell's result diverges much more rapidly as a function of μ than Eq. (8). We remark here that there is not a clear physical interpretation for the cutoff parameter μ in Drell's (or even in Berman and Geffen's) calculation. In particular, there is no obvious correspondence between the cutoff and our vector meson mass. Consequently one should not take too literally the graphs which give all results as a function of the same mass parameter.

2. <u>Branching ratio for realistic vector meson mass values</u>

For the physical vector meson masses the numerical values of the branching ratio for the ηVV model of the form factor are

$$\frac{\Gamma_{\mu^+\mu^-}}{\Gamma_{\gamma\gamma}} = \begin{Bmatrix} 1.13 \\ 1.17 \\ 1.29 \end{Bmatrix} \times 10^{-5} \quad \text{for} \quad \mu = \begin{Bmatrix} m_\rho \\ m_\omega \\ m_\varphi \end{Bmatrix}, \qquad (13)$$

compared with the lower limit of 1.07×10^{-5}. We note that the real part of the amplitude contributes only 10 to 20 percent in the rate.

The spread in the above values may be taken as an indication of the variation expected from the breaking of SU(3) symmetry. But it is of interest to consider the symmetry breaking from a somewhat more basic point of view. The nonet model of PVV coupling gives a Lagrangian density proportional to $(\rho^0\rho^0 + \omega\omega - 2\varphi\varphi)\eta$, where the space-time structure has been suppressed. For the present purposes we assume that the couplings are of this form for the physical particles. We assume that the photon transforms as $\gamma \sim \rho^0 + \frac{1}{\sqrt{3}}\omega_8 = \rho^0 + \frac{1}{3}\omega - \frac{\sqrt{2}}{3}\varphi$, and that the vector meson-photon coupling constants are of the "universal" form, $G_i = em_i^2/\gamma_V$. Then the $X(\mu)$ of Eq. (5) and (B.14) is replaced by

$$X \rightarrow \frac{3}{2}[X(m_\rho) + \frac{1}{9}X(m_\omega) - \frac{4}{9}X(m_\varphi)] \quad . \tag{14}$$

This gives a branching ratio,

$$\frac{\Gamma_{\mu^+\mu^-}}{\Gamma_{\gamma\gamma}} = 1.08 \times 10^{-5} \quad , \tag{15}$$

even closer to the unitarity limit than the value found with $\mu = m_\rho$ in (13).

The estimate just made included symmetry breaking in a very special way (hadronic couplings unbroken, photon-vector-meson coupling of universal form, etc.). Clearly there are a myriad of other ways to break the symmetry, each one giving a different branching ratio. But if SU(3) symmetry is good to, say, 50% accuracy, it is difficult to imagine the branching ratio lying outside the interval of from one to two times the unitarity bound, at least in our vector dominance model.

Similar considerations about symmetry breaking can be made for the $\eta V\gamma$ model of the form factor. It is clear from Fig. 2 that the same conclusion will be reached, and that a result more than three times the unitarity limit would be surprising, unless there are other mechanisms at work.

3. <u>Discussion of B.-L. Young's results</u>

Extensive estimates of the branching ratio have been made by B.-L. Young.[5] As a model for the η form factor, Young has a cutoff function times a vertex function which is a linear combination of $\eta\gamma\gamma$, $\eta V\gamma$, and ηVV contributions. The form factor is schematically illustrated in Fig. 3. He uses physical masses of vector mesons and SU(3) and empirical estimates for the coupling constants. Young has several models for the cutoff function, but the results are not sensitive to these variations, provided different models are compared at equivalent effective values of the cutoff parameter Λ. In Fig. 4 we have plotted the boundaries of Young's various curves which he calculated with different values of f_i, g_{ij}, and Γ_0 (c.f. Fig. 3).

The range of values for the branching ratio is, at first glance, almost ununderstandably large. As a first remark we observe that, while the $\eta V\gamma$ and ηVV parts of the amplitude need no cut-off, the point coupling $\eta\gamma\gamma$ does. Thus Young's results diverge logarithmically with his cut-off parameter (which has nothing to do with the mass of a vector meson) provided $\lim\limits_{k_1^2, k_2^2 \to \infty} \Gamma(k_1^2, k_2^2) \neq 0$, apart from the cut-off function, i.e., his $L_\infty \neq 0$. The second point is that the detailed behavior of $\Gamma(k_1^2, k_2^2)$ for small or moderate k_i^2 depends on the magnitudes and relative signs of the contributions from $\eta\gamma\gamma$, $\eta V\gamma$, and ηVV, and this behavior affects the magnitude of the branching ratio. The largest values come from (a) the smallest values of Γ_0 (obtained from $\Gamma_0^{(\eta)}/\Gamma_0^{(\pi^0)} = \sqrt{\frac{1}{3}}$ and the π^0 lifetime); (b) choices of signs of f_i and g_{ij} which make $\Gamma(k_1^2, k_2^2)$ increase with k_1^2, $k_2^2 \neq 0$ until eventually damped by the cutoff function. His "dipole model" has two cutoff parameters, one fixed and one variable, and the behavior of the result is governed mainly by the fixed, relatively small cutoff. This produces the lower, flat curve in our Fig. 4.

B. $K_2^0 \to \mu^+\mu^-$

For this process the unitarity bound is 1.17×10^{-5}. In this case, the motivation for our model is less clear since the decay $K^0 \to \gamma\gamma$ involves both weak and electromagnetic interactions. But if the electromagnetic part is dominated by vector mesons the model should provide a fair estimate of the real part of the amplitude.

In Fig. 5 we display our results for the branching ratio, along with those of Drell and of Berman and Geffen for this process as a function of vector meson mass. Sehgal's three values,[4] for $\mu/m_K = 1$, 2, and 4 are 1.6, 2.0 and 3.5, respectively, in units of 10^{-5}. The first value is considerably larger than our result of 1.26×10^{-5} at $\mu/m_K = 1$, but the other two values are in agreement with our curve for the $KV\gamma$ model. The relations among the various calculations as a function of vector meson mass or cut-off are quite similar to those found for η decay, although details such as the zero in the real part of the amplitude at some value of μ are different because of the somewhat different kinematics.

Bég[6] considered a specific model for decays involving both weak and electromagnetic interactions. Although phrased in dispersion relation language, the model is effectively equivalent to a current-current Hamiltonian for the hadronic part of the weak interactions with $\Delta S = 0, \pm 1$ neutral currents. In particular, the $\Delta S = 0$ vector current has a contribution from the ρ^0-meson field and the axial vector current from the divergence of the π^0 field. The decay $K_2^0 \to \ell^+\ell^-$ would then proceed mainly as $K_2^0 \to \pi^0$ via the $\Delta S = 1$ neutral hadronic current, and $\pi^0 \to \gamma\gamma$, $\gamma\gamma \to \ell^+\ell^-$ by one of the models discussed here. Bég uses Drell's model with $\mu = 2m_N$, and an upper limit for the matrix element of $K_2^0 \to \pi^0$, to give an approximate absolute upper limit of $\Gamma(K_2^0 \to \mu^+\mu^-) < 0.7\ \mathrm{sec}^{-1}$. Evidently Bég's value for the branching ratio $\Gamma_{\ell^+\ell^-}/\Gamma_{\gamma\gamma}$ is just that of the Drell model. Although not strictly relevant for the present considerations, it is perhaps of interest to examine the experimental data on $K_2^0 \to \gamma\gamma$ so that Bég's absolute rate can be converted into a branching ratio. The most recent and apparently most accurate value for the rate of $K_2^0 \to \gamma\gamma$ is that of Banner et al.[7] They find $\Gamma(K_2^0 \to \gamma\gamma)/\Gamma(K_2^0 \to \mathrm{all}) = (4.68 \pm 0.64) \times 10^{-4}$, giving an absolute rate of $\Gamma(K_2^0 \to \gamma\gamma) = (8.9 \pm 1.3) \times 10^3\ \mathrm{sec}^{-1}$. Bég's upper limit then becomes an upper limit on the branching ratio of roughly

8×10^{-5}. From Fig. 5 we see that Drell's model gives 2.8×10^{-5} for $\mu = 2m_N (\mu/m_K = 3.8)$. Within the framework of his model, this means that Bég's estimate for the $K_2^0 \to \pi^0$ matrix element was too large by a factor of $\sqrt{8/2.8} \simeq 1.7$, remarkably close considering that it was called a "generous upper limit"!

C. $\eta \to e^+e^-$ and $K_2^0 \to e^+e^-$

For these extremely rare decay modes, the branching ratios are again close to the unitarity bound, for reasonable masses of the vector mesons. We therefore state only the lower bounds:

$$\frac{\Gamma(\eta \to e^+e^-)}{\Gamma(\eta \to \gamma\gamma)} \geqslant 4.5 \times 10^{-9} \ ;$$

$$\frac{\Gamma(K_2^0 \to e^+e^-)}{\Gamma(K_2^0 \to \gamma\gamma)} \geqslant 5.3 \times 10^{-9}.$$

D. $\pi^0 \to e^+e^-$

The direct decay of the neutral pion into an electron-positron pair was the process originally studied by Drell,[2] and by Berman and Geffen.[3] The predictions of Berman and Geffen, and of our calculation are, as before, rather insensitive to the value taken for the cutoff or vector meson mass, while Drell's expression is quite sensitive to the cutoff. The zero in the real part of the amplitude occurs in this case for a rather large value of the cutoff, both for our models and for that of Berman and Geffen. Consequently, over the range of cutoff masses corresponding to intermediate states ρ, ω, φ, the branching ratio is decreasing. The predictions for this process are summarized in Table I. Only our values for the πVV model are quoted.

IV. SUMMARY

Our calculations indicate that for the vector dominance model the branching ratios for the decays $(P \to \ell^+\ell^-)$ to $(P \to \gamma\gamma)$ are not much larger than the lower bounds given by unitarity. For the decay $\eta \to \mu^+\mu^-$, we therefore expect that

$$\frac{\Gamma(\eta \to \mu^+\mu^-)}{\Gamma(\eta \to \text{all})} \sim (0.4\text{-}1.0) \times 10^{-5} \quad .$$

Detailed numerical values are given in Section IIIA. For K_2^0 decays the branching ratio $\Gamma_{\ell^+\ell^-}/\Gamma_{\gamma\gamma}$ is of the same magnitude as for η decay, but because of the small fraction of decays $K_2^0 \to \gamma\gamma$,[7] the process $K_2^0 \to \mu^+\mu^-$ will be much less common:

$$\frac{\Gamma(K_2^0 \to \mu^+\mu^-)}{\Gamma(K_2^0 \to \text{all})} \sim (0.5\text{-}1.0) \times 10^{-8} \quad .$$

In both cases the decays to electron pairs are suppressed by an additional factor of about 4×10^{-4}. Because of the insensitivity of our results to vector meson mass, near the physical masses of vector mesons, we believe our predictions for the total branching ratios reliable within a factor of two.

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Table I

Branching Ratios for $\pi^0 \to e^+e^-$

Source	Cutoff or Vector Meson Mass (Units of pion mass)	$\Gamma(\pi^0 \to e^+e^-)/\Gamma(\pi^0 \to \gamma\gamma)$
Unitarity	--	4.7×10^{-8}
Drell	1.0	3×10^{-8}
Drell	6.95	12×10^{-8}
Drell	13.90	22×10^{-8}
Berman and Geffen	3.16	6.7×10^{-8}
Berman and Geffen	9.8	5.7×10^{-8}
This Calculation	5.7 (ρ)	6.4×10^{-8}
This Calculation	7.6 (φ)	6.1×10^{-8}
This Calculation	10	4.9×10^{-8}

FOOTNOTES AND REFERENCES

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APPENDIX A: CONVENTIONS, NOTATION, BASIC FORMULAS AND FEYNMAN INTEGRALS

1. Metric and Dirac Matrices

The notation for 4-vectors is $A_\mu = (\underset{\sim}{A}, A_4 = iA_0)$, so that scalar products are $A \cdot B = \underset{\sim}{A} \cdot \underset{\sim}{B} - A_0 B_0$. The spinor notation is that of Pauli's Handbuch article, with Hermitean γ-matrices and γ_4 diagonal. Explicitly,

$$\underset{\sim}{\gamma} = \begin{pmatrix} 0 & -i\underset{\sim}{\sigma} \\ i\sigma & 0 \end{pmatrix} ; \quad \gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \quad \gamma_5 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} .$$

The spin tensor is $\sigma_{\mu\nu} = \frac{1}{2i} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$. The spinors are normalized according to $(\bar{u}u) = 2m$, $(\bar{v}v) = -2m$. They satisfy the free-particle equations, $(i\gamma \cdot p + m)\, u(p) = 0$ and $(i\gamma \cdot p - m)\, v(p) = 0$. For an antiparticle of momentum $\underset{\sim}{p}$ and helicity λ it is sometimes convenient to use $v_\lambda(\underset{\sim}{p}) = (-1)^{\lambda - 1/2}\, \gamma_5 u_{-\lambda}(\underset{\sim}{p})$.

2. S-Matrix Formulas

The invariant amplitude $\mathcal{M}$ is related to the S-matrix through the relation,

$$S_{\beta\alpha} = \delta_{\beta\alpha} - i(2\pi)^4 \delta^{(4)} (p_\beta - p_\alpha) \mathcal{M}_{\beta\alpha} / \sqrt{\prod_i (2E_i)} \qquad (A.1)$$

where α and β are the initial and final state labels and the product of factors $(2E_i)$ is over both initial and final states. For a decay process $\alpha \to (1,2,\cdots,n)$ the transition probability is

$$dw_{\beta\alpha} = \frac{(2\pi)^4}{2E_\alpha} |\mathcal{M}_{\beta\alpha}|^2 \prod_{i=1}^{n} \frac{d^3p_i}{(2\pi)^3(2E_i)} \delta^{(4)}(p_1+\cdots+p_n - p_\alpha) . \tag{A.2}$$

For a two-particle final state,

$$dw_{\beta\alpha} = \frac{1}{32\pi^2} |\mathcal{M}_{\beta\alpha}|^2 \frac{p_{CM} d\Omega_{CM}}{m_\alpha^2} , \tag{A.3}$$

where

$$\frac{p_{CM}^2}{m_\alpha^2} = \frac{1}{4}\left[1 - \left(\frac{m_1 + m_2}{m_\alpha}\right)^2\right]\left[1 - \left(\frac{m_1 - m_2}{m_\alpha}\right)^2\right] .$$

3. The Evaluation of Feynman Integrals[8]

In general, the integral over the undetermined loop momentum k in a Feynman diagram takes the form

$$I = \frac{1}{(2\pi)^4} \int \frac{d^4k\, F(k;\, p_i;\, m_j)}{a_1\, a_2 \cdots a_n} \tag{A.4}$$

where

$a_i = (k - s_i)^2 + m_i^2$

s_i is a linear combination of external momenta p_i

m_j are the (internal and external) masses in the problem, and

F is a polynomial in the components of k.

To evaluate such an integral, it is convenient to introduce auxiliary parameters,[8] through one of the following identities.

$$\xi_n \equiv \frac{1}{a_1\, a_2 \cdots a_n} = (n-1)! \int_0^1 dz_1 \cdots \int_0^1 dz_n$$

$$\times \frac{\delta\left(\sum\limits_{i=1}^{n} z_i - 1\right)}{\left[\sum\limits_{i=1}^{n} a_i z_i\right]^n} . \qquad \text{(A.5)}$$

$$\xi_n = (n-1)! \int_0^1 dz_1 \int_0^{z_1} dz_2 \cdots \int_0^{z_{n-2}} dz_{n-1}$$

$$\times [z_{n-1}(a_n - a_{n-1}) + z_{n-2}(a_{n-1} - a_{n-2}) + \cdots + z_1(a_2 - a_1) + a_1]^{-n}$$

(A.6)

$$\xi_n = (n-1)! \int_0^1 z_1^{n-2}\, dz_1 \int_0^1 z_2^{n-3}\, dz_2 \cdots \int_0^1 dz_{n-1}$$

$$\times [z_1\, z_2 \cdots z_{n-1}(a_1 - a_2) + z_1\, z_2\, z_{n-2}(a_2 - a_3) + \cdots + a_n]^{-n} .$$

(A.7)

Some other useful relations are

$$\frac{1}{nA^n B} = \int_0^1 \frac{x^{n-1}\, dx}{[Ax + B(1-x)]^{n+1}} \quad ; \qquad \text{(A.8)}$$

$$\frac{1}{A^n} - \frac{1}{B^n} = -\int_0^1 dx \frac{n(A-B)}{[(A - B)x + B]^{n+1}} \quad . \tag{A.9}$$

An integral of the form (A.4) can always be brought to the schematic form

$$I = \frac{(n - 1)!}{(2\pi)^4} \underbrace{\int \cdots\cdots \int}_{\substack{\text{(one-dimensional} \\ \text{integrals)}}} \int \frac{d^4k\ F(k;\ p_i;\ m_j)}{[(k - R)^2 + a^2]^n} \quad . \tag{A.10}$$

The exact form will depend on which of the above identities one chooses to employ. If the k-space integral is at worst logarithmically divergent, we can make a change of variable,

$$k' = k - R \quad , \tag{A.11}$$

without changing the value of the integral (nor adding any finite number for the case of a logarithmic divergence). Hence we can always bring the k-space integral to the form

$$\int \frac{d^4k\ F(k + R;\ p_i;\ m_j)}{[k^2 + a^2]^n} \quad . \tag{A.12}$$

Because of the symmetry of the range of integration, the odd powers of k_μ in F do not contribute. To get to the final, usable form we must average over k_μ, which amounts to the substitutions

$$k_\mu k_\nu = \frac{1}{4}\delta_{\mu\nu}k^2$$

$$k_\mu k_\nu k_\rho k_\sigma = \frac{1}{24}[k^2]^2 \, [\delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho}] \; .$$

etc.

Therefore we need only evaluate integrals of the form

$$\mathcal{I}_{mn} = \int \frac{d^4k \; (k^2)^{m-2}}{[k^2 + a^2]^n} = \frac{i\pi^2}{(a^2)^{n-m}} \; \frac{(m - 1)! \; (n - m - 1)!}{(n - 1)!} \; , \tag{A.13}$$

which exist, provided $n > m > 0$.

Quite clearly the major task in the evaluation of Feynman integrals is the computation of the integrals over the auxiliary parameters.

APPENDIX B: DETAILS OF THE CALCULATION

We begin with Eqs. (3) and (4) of Section II. Use of the Dirac equation for the leptons and various identities allows us to write the effective value of $\mathcal{O}$ in the form,

$$\mathcal{O} = \frac{1}{(2\pi)^4} \int d^4k \; \frac{1}{D} (mA + iB\gamma\cdot k)\gamma_5 \quad , \tag{B.1}$$

where

$$A = 4k^2 - 2k\cdot p, \qquad B = M^2 + 2k\cdot p \tag{B.2}$$

and

$$D = k^2(k^2 + \mu^2)(p - k)^2[(p - k)^2 + \mu^2][(k - q_2)^2 + m^2]. \tag{B.3}$$

The evaluation of $\bar{u}(q_1)\mathcal{O}v(q_2)$ in the helicity representation leads to a matrix element,

$$\mathcal{M} = \frac{f\,G^2e^2}{\mu} (-1)^{\lambda_2 - \frac{1}{2}} \frac{m}{M} \; I \; \delta_{\lambda_1\lambda_2} \quad ,$$

where

$$I = \frac{1}{(2\pi)^4} \int d^4k \frac{(M^2A + 2k\cdot pB)}{D} = \frac{4}{(2\pi)^4} \int d^4k \frac{[M^2k^2 + (k\cdot p)^2]}{D} \quad . \tag{B.4}$$

The quantities X and Y, appearing in Eq. (5) of Section II for the branching ratio, are related to I by

$$Z \equiv X + iY = i\, 8\pi\, M^2 \left(\frac{\mu}{M}\right)^4 \quad I \quad . \tag{B.5}$$

The evaluation if I is straightforward, but the presence of five denominators necessitates some manipulation. As a preliminary we remark that the change of variables, $k \to p - k$, leaves the numerator in (B.4) invariant and leaves D unchanged except for $q_2 \to q_1$. Furthermore, in the frame where $\vec{p} = 0$, the transformation $\vec{k} \to -\vec{k}$ causes $(k - q_1)^2 \to (k - q_2)^2$, while leaving k^2 and $(p - k)^2$ invariant. These two changes of variable can be used to simplify the integrand in (B.4), as follows:

$$\frac{4}{D}[M^2k^2 + (k\cdot p)^2] = \frac{1}{D}\{4M^2k^2 + [M^2 + (p - k)^2 - k^2]^2\}$$

$$= \frac{1}{D}\{M^4 + 4M^2k^2 + 2M^2[(p - k)^2 - k^2] + [(p - k)^2 - k^2]^2\} \; .$$

The third term in the curly bracket gives zero contribution to the integral, as can be seen by the above changes of variable. The last term can be written

$$\frac{[(p - k)^2 - k^2]^2}{D} = \left(\frac{1}{k^2} - \frac{1}{(p - k)^2}\right)\left(\frac{1}{k^2 + \mu^2} - \frac{1}{(p - k)^2 + \mu^2}\right) \times$$

$$\times \frac{1}{[(k - q_2)^2 + m^2]} \to \frac{2}{k^2}\left(\frac{1}{k^2 + \mu^2} - \frac{1}{(p - k)^2 + \mu^2}\right) \frac{1}{[(k - q_2)^2 + m^2]} \quad .$$

Similar use of partial fractions and the above changes of variables can be used to reduce I to a sum of terms involving only three denominators. The result can be written as

$$\frac{Z}{8\pi M^2} = i\left(\frac{\mu}{M}\right)^4 I = J(0, 0) - 2\left(\frac{\mu^2}{M^2} - 1\right)^2 J(0, \mu)$$

$$- \left(\frac{4\mu^2}{M^2} - 1\right) J(\mu, \mu) - \frac{1}{8\pi^2 M^2} \cdot \frac{\mu^2}{M^2} L \ , \tag{B.6}$$

where

$$J(m_1, m_2) = \frac{i}{(2\pi)^4} \int d^4k \{(k^2 + m_1^2)[(p - k)^2 + m_2^2][(k - q)^2 + m^2]\}^{-1} \tag{B.7}$$

and

$$L = \frac{\mu^2}{\pi^2 i} \int d^4k \{k^2(k^2 + \mu^2)[(k - q)^2 + m^2]\}^{-1} \ . \tag{B.8}$$

The simpler integral L can be evaluated immediately using Feynman parameterization. The result is

$$L = \frac{\mu^2}{m^2} \ln\left(\frac{\mu}{m}\right) - \frac{\mu^2}{2m^2} \sqrt{1 - \frac{4m^2}{\mu^2}} \ln\left[\frac{1 + \sqrt{1 - \frac{4m^2}{\mu^2}}}{1 - \sqrt{1 - \frac{4m^2}{\mu^2}}}\right] . \tag{B.9}$$

In passing we note that for large vector meson mass $(\mu/m \gg 1)$, L has the asymptotic value,

$$L \to \ln\left(\frac{\mu^2}{m^2}\right) + O(1) \ . \tag{B.10}$$

The remaining integral $J(m_1, m_2)$ can, by means of the Feynman parameterization, be expressed as a double integral, the first of which can be performed in terms of elementary functions. The resultant is

$$J(m_1,m_2) = \frac{1}{16\pi^2} \int_0^1 dx \frac{1}{\sqrt{\Delta}} \ell n \left[\frac{(m_1^2 - m_2^2)^2 - (\sqrt{\Delta} + M^2x)^2}{(m_1^2 - m_2^2)^2 - (\sqrt{\Delta} - M^2x)^2} \right] , \qquad (B.11)$$

where

$$\Delta = (m_1^2 + m_2^2 - M^2x)^2 - 4m_1^2m_2^2 - 4M^2m^2(1 - x)^2 . \qquad (B.12)$$

The remaining integral over x is most conveniently done numerically for the specific (m_1, m_2) values needed in (B.6).

Before displaying the final forms suitable for numerical computation it is of interest to consider the question of the unitarity limit [Eq. (7)]. This bound comes from the existence of a model-independent absorptive part from physically allowed two-photon intermediate states in Fig. 1(b). This absorptive part can be calculated directly from unitarity equations and the physical amplitudes for $\eta \to \gamma\gamma$ and $\gamma\gamma \to \ell^+\ell^-$, as in Ref. 1, or by replacing the propagators by delta functions, as discussed by Sehgal.[4] Alternatively, it must emerge directly from any model calculation. If it is assumed that the vector meson mass is large enough that neither γV nor VV intermediate states are physical, then in the expression (B.6), only $J(0, 0)$ can give rise to an absorptive (imaginary) part. This is because $J(m_1, m_2)$ corresponds to a simple spinless triangle graph of the form of Fig. 1(b) with the diagonal internal legs having masses m_1 and m_2. To see explicitly how the imaginary part emerges, consider (B.11) and (B.12) with $m_1 = m_2 = 0$. We have

$$J(0,0) = \frac{1}{8\pi^2}\int_0^1 dx \frac{1}{M^2\sqrt{x^2 - \frac{4m^2}{M^2}(1-x)^2}}\, \ell n\left[\frac{\sqrt{x^2 - \frac{4m^2}{M^2}(1-x)^2} + x}{\sqrt{x^2 - \frac{4m^2}{M^2}(1-x)^2} - x}\right], \tag{B.13}$$

where, for the moment, the sign of the argument of the logarithm should be considered as not yet certain because of the ambiguity in $\ell n(f^2) = 2\ell n(\pm f)$. To ascertain the proper sign we note that for $M^2 < 0$ there can be no physically allowed intermediate state and hence no absorptive part. By inspection of (B.13) as it stands it is easily verified that $J(0,0)$ is real for $M^2 < 0$. Now we can consider $M^2 > 0$. The square root in (B.13) is now real and less than x for $\left(\frac{M}{2m} + 1\right)^{-1} < x < 1$ and imaginary for $0 < x < \left(\frac{M}{2m} + 1\right)^{-1}$. This means that the integral receives a real contribution over the whole range of integration and an imaginary contribution for x on the range, $\left(\frac{M}{2m} + 1\right)^{-1} < x < 1$:

$$8\pi M^2 \, \mathrm{Im}\, J(0,0) = -\int_{\frac{1}{\frac{M}{2m}+1}}^{1} dx \frac{1}{\sqrt{x^2 - \frac{4m^2}{M^2}(1-x)^2}} .$$

Evaluation of this integral leads directly to the expression Y in Eq. (6) of Section II.

The reader who finds the explicit evaluation of the imaginary part of $J(0,0)$ too specific can consider the analytic properties of the triangle graph represented by (B.7) or (B.11), using techniques developed for arbitrary Feynman diagrams.[13]

We now return to the task of exhibiting the final forms of $J(m_1,m_2)$ needed in (B.6). The features of the integrand of $J(0,0)$, noted above, imply that the logarithm becomes an arctangent over part of the range of integration. For $J(0, \mu)$ the log form holds over the whole range, while for $J(\mu, \mu)$ the arctangent. The complete expression for $X = \mathrm{Re}\, Z$ from (B.6) is

$$X = A_1 + 2\left(\frac{\mu^2}{M^2} - 1\right)^2 A_2 - \left(\frac{4\mu^2}{M^2} - 1\right) A_3 - \frac{\mu^2}{M^2} A_4 , \qquad \text{(B.14)}$$

where

$$A_1 = \frac{2}{\pi}\int_0^a dx \frac{1}{\xi\sqrt{(a-x)(b+x)}}\left[\arctan\left(\frac{\xi\sqrt{(a-x)(b+x)}}{x}\right) - \frac{\pi}{2}\right]$$

$$+ \frac{1}{\pi}\int_a^1 dx \frac{1}{\xi\sqrt{(x-a)(x+b)}}\ \ell n\left[\frac{x + \xi\sqrt{(x-a)(x+b)}}{x - \xi\sqrt{(x-a)(x+b)}}\right]$$

$$A_2 = \frac{1}{2\pi}\int_0^1 dx \frac{1}{\xi\sqrt{(c-x)(d-x)}}\ \ell n\left[\frac{\frac{\mu^4}{M^4} - \left(x - \xi\sqrt{(c-x)(d-x)}\right)^2}{\frac{\mu^4}{M^4} - \left(x + \xi\sqrt{(c-x)(d-x)}\right)^2}\right]$$

$$A_3 = \frac{2}{\pi}\int_0^1 dx \frac{1}{\xi\sqrt{(e-x)(f+x)}}\left[\arctan\left(\frac{\xi\sqrt{(e-x)(f+x)}}{x}\right) - \frac{\pi}{2}\right]$$

$$A_4 = \frac{1}{\pi}\frac{\mu^2}{m^2}\left\{\ell n\left(\frac{\mu}{m}\right) - \frac{1}{2}\sqrt{1 - \frac{4m^2}{\mu^2}}\ \ell n\left[\frac{1 + \sqrt{1 - \frac{4m^2}{\mu^2}}}{1 - \sqrt{1 - \frac{4m^2}{\mu^2}}}\right]\right\}$$

and

$$\xi = \sqrt{1 - \frac{4m^2}{M^2}}$$

$$a = \left(\frac{M}{2m} + 1\right)^{-1} \qquad b = \left(\frac{M}{2m} - 1\right)^{-1}$$

$$c = \frac{\left(\frac{\mu^2}{2Mm} + 1\right)}{\left(\frac{M}{2m} + 1\right)} \qquad d = \frac{\left(\frac{\mu^2}{2Mm} - 1\right)}{\left(\frac{M}{2m} - 1\right)}$$

$$e = \frac{2}{M^2 - 4m^2}\left[\sqrt{m^2M^2 - 4m^2\mu^2 + \mu^4} + \mu^2 - 2m^2\right]$$

$$f = \frac{2}{M^2 - 4m^2}\left[\sqrt{m^2M^2 - 4m^2\mu^2 + \mu^4} - \mu^2 + 2m^2\right].$$

In A_1 and A_3 the arctangents are to be chosen on the interval $(0, \frac{\pi}{2})$.

The calculation for the second model, with a single vector meson propagator ($\eta V\gamma$ coupling, instead of ηVV) closely parallels the previous one. The denominator (B.3) is replaced as follows:

$$\frac{1}{D} \rightarrow \frac{1}{2\mu^2}\left[\frac{k^2 + \mu^2}{D} + \frac{(p - k)^2 + \mu^2}{D}\right] .$$

Here, for convenience, we have written a form explicitly symmetric in k^2 and $(p - k)^2$ and have included a factor of μ^{-2}. Use of the same transformation of variables as discussed below (B.5) yields an integrand to replace that in (B.4) of the form:

$$\frac{4[M^2k^2 + (k\cdot p)^2]}{D} \to \frac{1}{k^2[(k - q)^2 + m^2]} \left\{ \frac{M^4}{\mu^4} \cdot \frac{1}{(p - k)^2} - \frac{\left(\frac{M^2}{\mu^2} - 1\right)^2}{(p - k)^2 + \mu^2} \right.$$

$$\left. + \frac{\left(\frac{M^2}{\mu^2} + 1\right)}{k^2 + \mu^2} - \frac{2p\cdot k}{\mu^2(k^2 + \mu^2)} \right\} .$$

Only the last term gives a new integral, not present in the first model. The expression replacing (B.6) is therefore

$$\frac{Z'}{8\pi M^2} = J(0,\,0) - \left(\frac{\mu^2}{M^2} - 1\right)^2 J(0,\,\mu) - \frac{1}{16\pi^2 M^2} \cdot \frac{\mu^2}{M^2} L\left(1 + \frac{M^2}{\mu^2}\right)$$

$$- \frac{1}{8\pi^2 M^2} \cdot \frac{\mu^2}{M^2} L' \quad , \tag{B.15}$$

where

$$L' = \frac{1}{\pi^2 i} \int d^4k \frac{(-p\cdot k)}{k^2(k^2 + \mu^2)[(k - q)^2 + m^2]} \quad . \tag{B.16}$$

Explicitly we have

$$L' = \frac{M^2}{4m^2}\left[L - \ell n\left(\frac{\mu^2}{m^2}\right) - 1 \right] \quad . \tag{B.17}$$

The final result for X in this model is

$$X' = A_1 + \left(\frac{\mu^2}{M^2} - 1\right)^2 A_2 - \frac{\mu^2}{2M^2} A_4 \left(1 + \frac{M^2}{\mu^2} + \frac{M^2}{2m^2}\right)$$

$$+ \frac{1}{4\pi} \frac{\mu^2}{m^2} \left[\ell n\left(\frac{\mu^2}{m^2}\right) + 1\right]$$

where the integrals A_i are defined below (B.14).

APPENDIX C: RATES FOR COMPETING PROCESSES

The branching ratios for the Dalitz pair and double Dalitz pair decay modes of the η meson have been calculated by Jarlskog and Pilkuhn,[9] using standard methods of QED. Applying their results to the decays of η, K_2^0, and π^0, we obtain the following branching ratios.

$$\frac{(\eta \rightarrow \mu^+\mu^-\mu^+\mu^-)}{(\eta \rightarrow \gamma\gamma)} \approx 6 \times 10^{-8}$$

$$\frac{(\eta \rightarrow \mu^+\mu^-e^+e^-)}{(\eta \rightarrow \gamma\gamma)} = 4 \times 10^{-6}$$

$$\frac{(\eta \rightarrow e^+e^-\gamma)}{(\eta \rightarrow \gamma\gamma)} = 1.6 \times 10^{-2}$$

$$\frac{(\eta \rightarrow e^+e^-e^+e^-)}{(\eta \rightarrow \gamma\gamma)} = 6.6 \times 10^{-5}$$

$$\frac{(K_2^0 \rightarrow \mu^+\mu^-\mu^+\mu^-)}{(K_2^0 \rightarrow \gamma\gamma)} \approx 6 \times 10^{-8}$$

$$\frac{(K_2^0 \rightarrow \mu^+\mu^-e^+e^-)}{(K_2^0 \rightarrow \gamma\gamma)} = 4 \times 10^{-6}$$

$$\frac{(K_2^0 \rightarrow e^+e^-\gamma)}{(K_2^0 \rightarrow \gamma\gamma)} = 1.6 \times 10^{-2}$$

$$\frac{(K_2^0 \to e^+e^-e^+e^-)}{(K_2^0 \to \gamma\gamma)} = 6.3 \times 10^{-5}$$

$$\frac{(\pi^0 \to e^+e^-\gamma)}{(\pi^0 \to \gamma\gamma)} = 1.19 \times 10^{-2};$$

Experiment:[10] $1.17 \pm 0.04 \times 10^{-2}$

$$\frac{(\pi^0 \to e^+e^-e^+e^-)}{(\pi^0 \to \gamma\gamma)} = 3.5 \times 10^{-5};$$

Experiment:[11] $3.18 \pm 0.30 \times 10^{-5}$

The single Dalitz pair formation is perhaps of most interest because of its possible presence as a background for the $\mu^+\mu^-$ decay mode. We calculate the branching ratio $\rho = \Gamma(\eta \to \mu^+\mu^-\gamma)/\Gamma(\eta \to \gamma\gamma)$, using our model for the η form factor. The process in the numerator is Dalitz pair production $\eta \to \gamma\gamma \to \mu^+\mu^-\gamma$; we ignore contributions from inner Bremsstrahlung $\eta \to \mu^+\mu^- \to \mu^+\mu^-\gamma$. The latter process is suppressed by a factor of about α compared with $\eta \to \mu^+\mu^-$. We find

$$\rho = \frac{2\alpha}{3\pi} \int_{4m^2}^{M^2} \frac{ds}{s^2}(s + 2m^2)(1 - 4m^2/s)^{\frac{1}{2}}(1 - s/M^2)^3 \frac{\mu^4}{(\mu^2 - s)^2}, \tag{C.5}$$

where s is the effective-mass-squared of the lepton pair, and other symbols have been defined previously. This is Eq. (13) of Kroll and Wada,[12] with the additional factor $\mu^4/(\mu^2 - s)^2$ in the integrand.

The spectrum in s is strongly peaked towards small s, corresponding to "almost real" intermediate photons converting into the lepton pair. Hence the presence or absence of the vector meson propagator is of little consequence.

Using a ρ-meson intermediate state, we find for η-decay, $\rho = 7.8 \times 10^{-4}$; and for K_2^0 decay $\rho = 5.6 \times 10^{-4}$. These branching ratios are ~ 50 times the branching ratio for the direct decay $\eta \to \mu^+\mu^-$ of experimental interest. However, high effective masses of the $\mu^+\mu^-$ system are strongly suppressed, so that an experiment with reasonable mass resolution can minimize the contamination. To show this quantitatively, we plot in Fig. 6 the fraction of Dalitz pairs with effective mass-squared greater than minimum accepted values of mass-squared. For example, an experiment with resolution of $0.3(M^2 - 4m^2) \sim 0.08\ \text{GeV}^2$ in the effective mass squared, would accept about 1 Dalitz pair for every 2 directly-produced pairs.

Finally we note that "inner Bremsstrahlung" gives rise to a tail on the mass-square distribution of the directly-produced pairs, which can easily be treated separately.[14]

FIGURE CAPTIONS

Fig. 1. Feynman diagrams for the decay processes.
(a): $\eta \to \gamma\gamma$; (b): $\eta \to \ell^+\ell^-$.

Fig. 2. Branching ratio $\Gamma_{\mu^+\mu^-}/\Gamma_{\gamma\gamma}$ for eta decay as a function of vector meson or cutoff mass. Dotted line: Drell; Dashed line: Berman and Geffen; Solid line: Present ηVV model; long dashes: Present $\eta V\gamma$ model; Dot-dashed line: Lower bound from unitarity.

Fig. 3. Schematic representation of Young's form factor.

Fig. 4. Range of branching ratios obtained by Young versus his cutoff parameter.

Fig. 5. Branching ratio $\Gamma_{\mu^+\mu^-}/\Gamma_{\gamma\gamma}$ for K_2^0 decay as a function of vector meson or cutoff mass. (Same labels as Fig. 2.)

Fig. 6. Fraction of Dalitz pairs in $\eta \to \gamma\mu^+\mu^-$ with effective mass-squared > lower limit accepted by experiment, s_0. Multiply right-hand scale by 0.72 for K_2^0 decay.

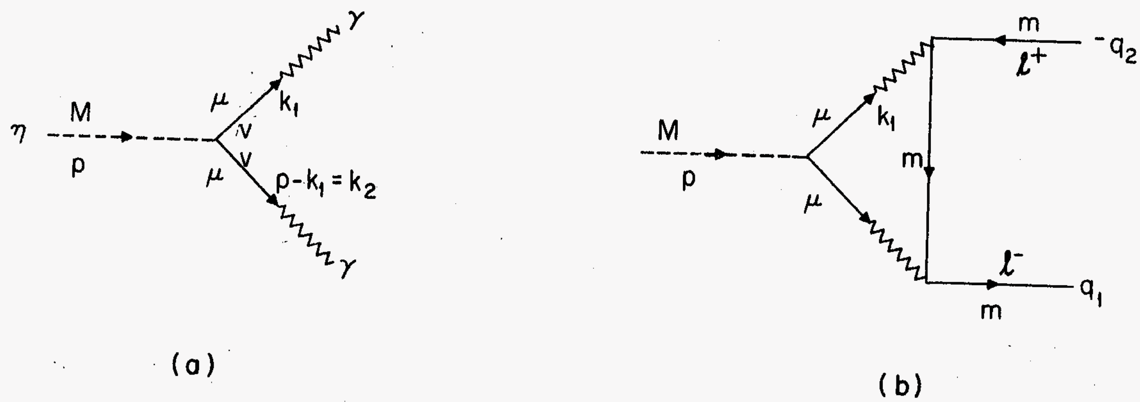

Fig. 1.   XBL689-6809

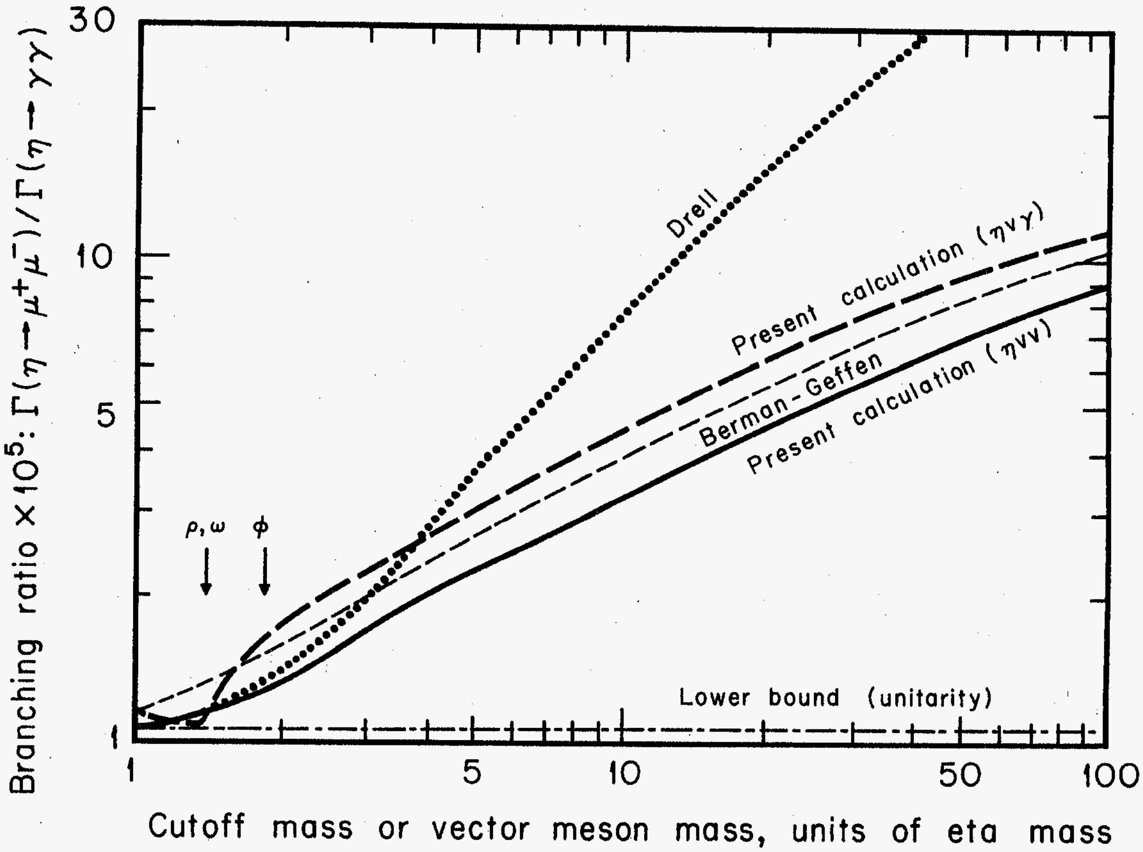

Fig. 2. XBL689-6810

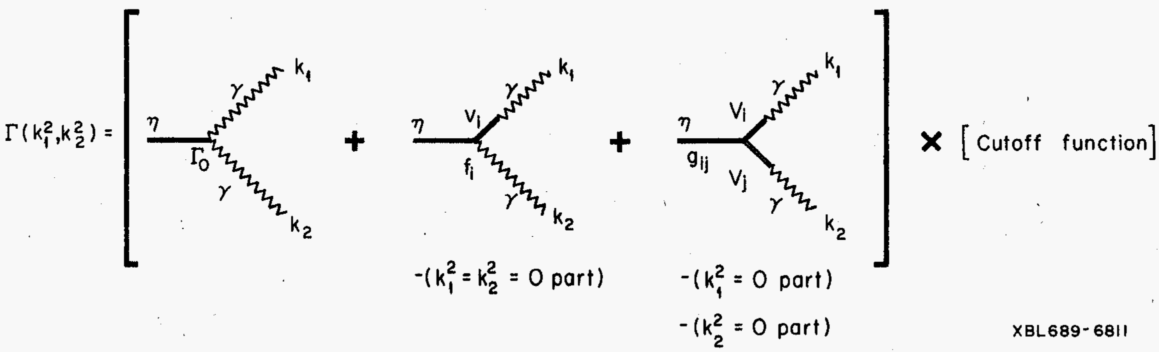
$\Gamma(k_1^2,k_2^2) =$
η
Γ_0
γ
k_1
k_2
+
V_i
f_i
$-(k_1^2 = k_2^2 = 0$ part)
g_{ij}
V_j
$-(k_1^2 = 0$ part)
$-(k_2^2 = 0$ part)
× [Cutoff function]
XBL689-6811

Fig. 3.

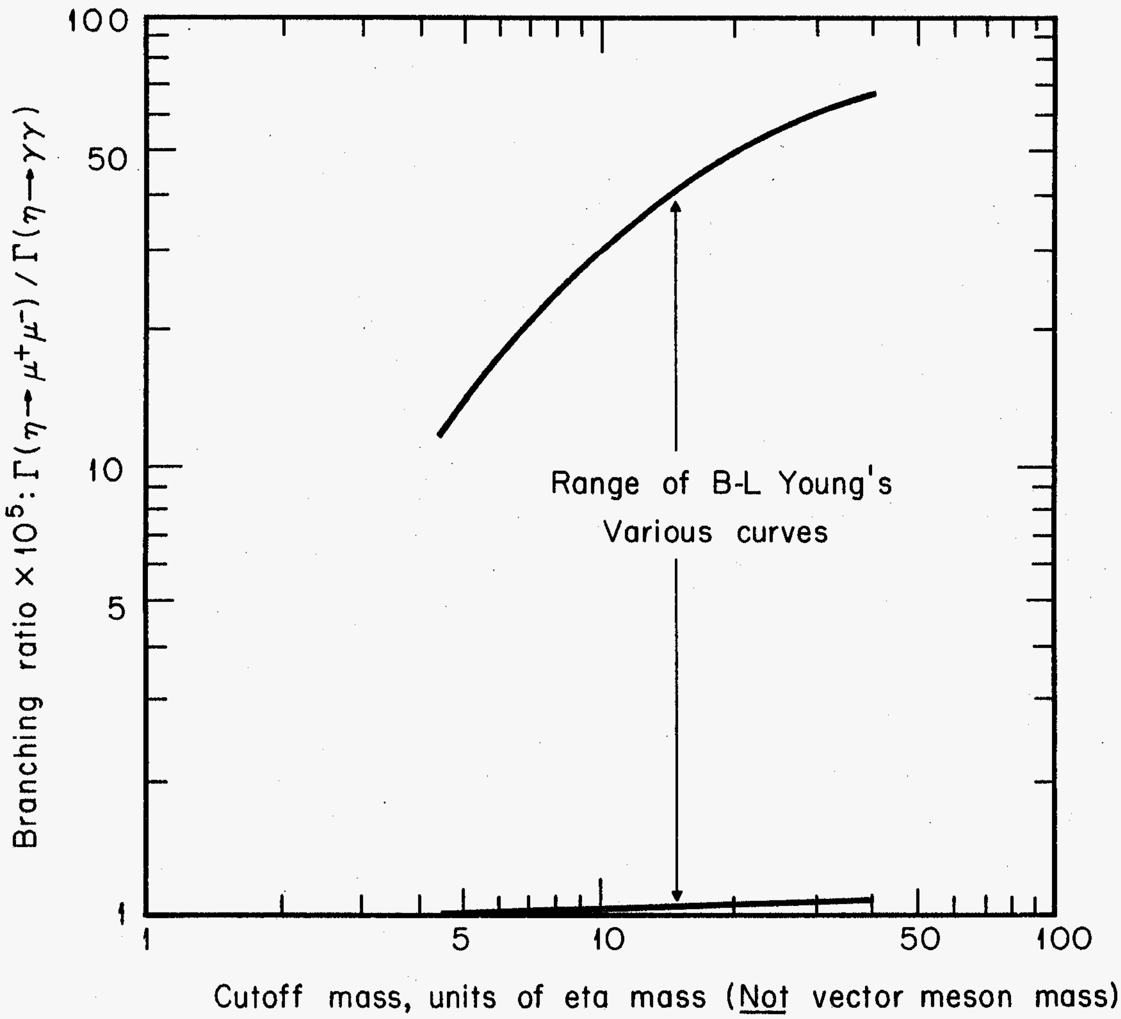

Fig. 4.

XBL689-6812

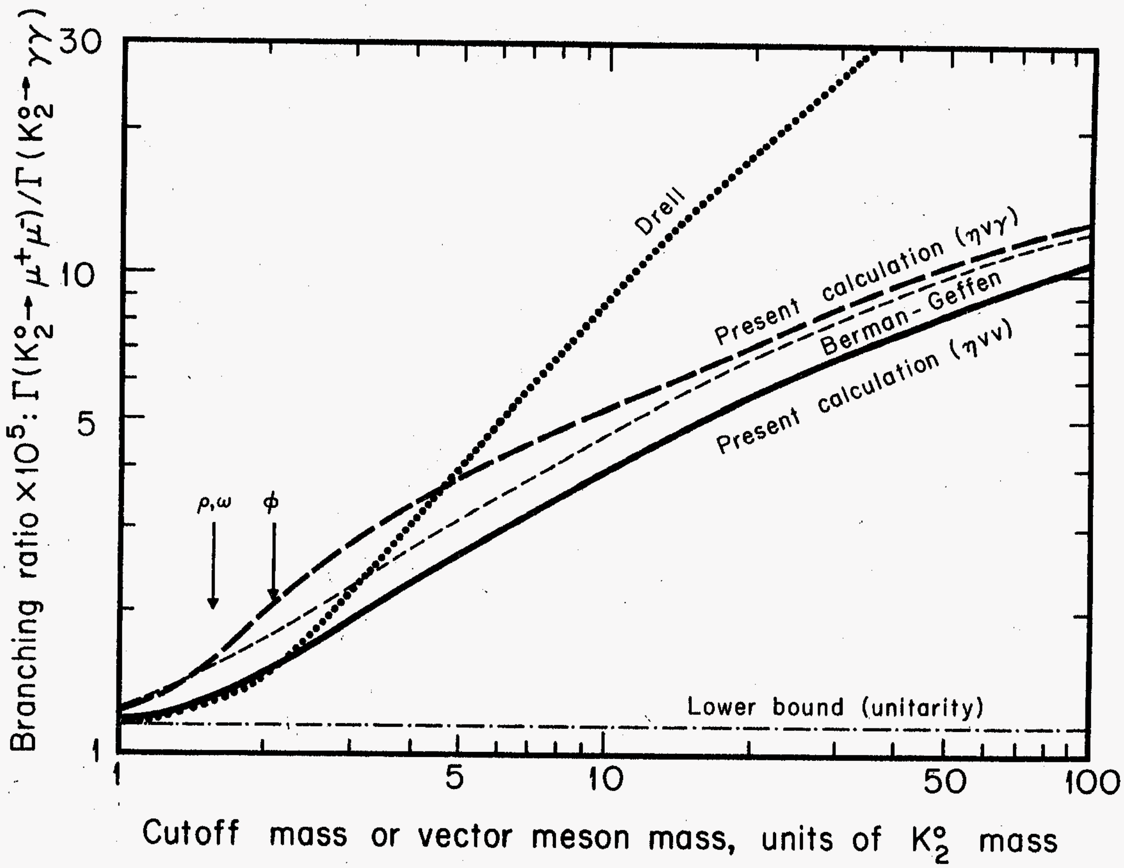

Fig. 5.

XBL689-6813

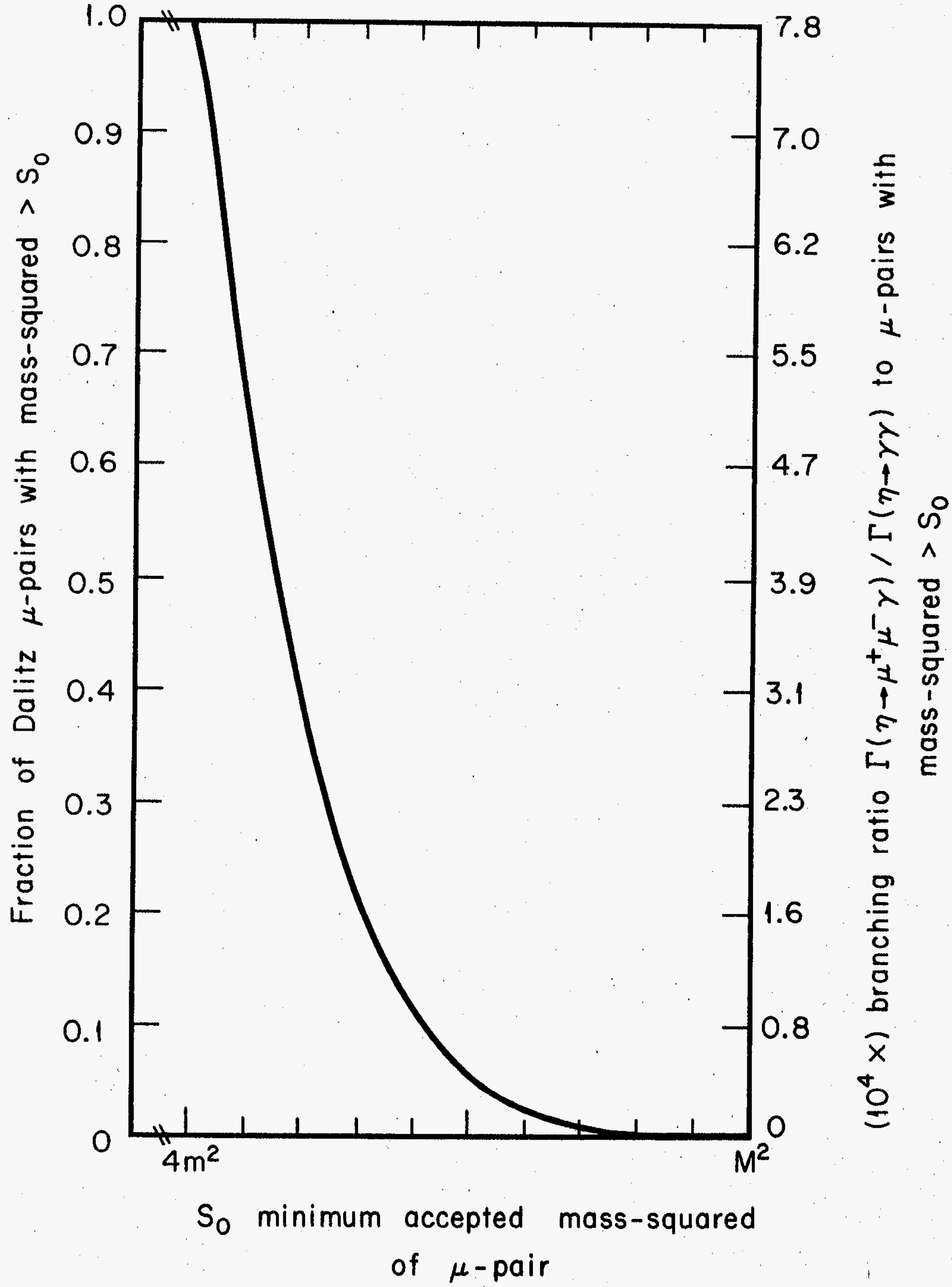

Fig. 6.   XBL689-6814