NAL PROPOSAL No.   43

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PROPOSAL TO STUDY SINGLE MESON PRODUCTION

IN MESON NUCLEON INTERACTIONS

AT 50 AND 100 GeV/c

Purdue High Energy Physics Group

June 10, 1970

Proposal to Study Single Meson Production

in Meson Nucleon Interactions

at 50 and 100 GeV/c.

Submitted by:

Purdue High Energy Physics Group

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Date: June 10, 1970

Proposal to Study Single Meson Production in Meson

Nucleon Interactions at 50 and 100 GeV/c.

ABSTRACT

It is proposed to study the general type of reaction meson + nucleon $\rightarrow$ meson + meson + nucleon. In particular, we would first propose analyzing the specific t channel reactions

$$\pi^- p \rightarrow n + \pi^+ + \pi^- \tag{1}$$

$$\pi^- p \rightarrow n + K^+ + K^- \tag{2}$$

$$\pi^+ p \rightarrow n + \pi^+ + \pi^+ \tag{3}$$

$$K^- p \rightarrow n + K^- + \pi^+ \tag{4}$$

$$K^+ p \rightarrow n + K^+ + \pi^+ \tag{5}$$

with a spectrometer or hybrid spectrometer.

The physics to be obtained would be:

1) Study of high mass resonant states to obtain widths, positions, coupling constants, spin-parities and G parities.

2) Study of the one pion exchange contribution for the above reactions at high energies.

3) Study of the $\pi\pi$ scattering amplitudes in the T = 2, 1 and 0 isospin states.

4) Study of the diffractive single meson production process at the nucleon vertex and its interference with the one particle exchange single meson production process.

5) Obtain information on the properties of the Pomeron trajectory (diffraction scattering) from the $\pi^+\pi^+$, angular distribution.

6) We would also study such reactions as $\pi^+ p \rightarrow \Sigma^+ K^+$ which have essentially the same requirements. (See Appendix A.)

Names of Experimenters

The Purdue High Energy Group consists of the following members:

Professors:	V. E. Barnes	Messrs:	A. C. Ammann
	D. D. Carmony		R. C. Badewitz
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	K. Rangan		
	J. H. Scharenguivel		

In the coming year at least one and probably two people with experience in counter work will join our group to aid our effort at N.A.L.

Date: June 10, 1970

This proposal is submitted by D. H. Miller and L. J. Gutay (Purdue High Energy Physics Group).
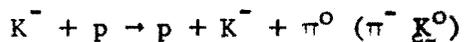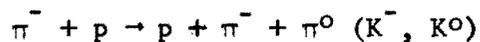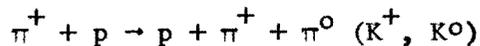
Proposal to study Single Meson Production in Meson Nucleon

Interactions at 50 and 100 GeV/c.

States to be analyzed

We propose to analyze the following reactions in the t channel.

$$\pi^- + p \to n + \pi^+ + \pi^- \qquad (1)$$

$$\pi^- + p \to n + K^+ + K^- \qquad (2)$$

$$\pi^+ + p \to n + \pi^+ + \pi^+ \qquad (3)$$

$$K^- + p \to n + K^- + \pi^+ \qquad (4)$$

$$K^+ + p \to n + K^+ + \pi^+ \qquad (5)$$

with $1000 \to 5000$ events/per μb. These form a subset of the more general three body final states such as:

$$\pi^+ + p \to p + \pi^+ + \pi^0 \ (K^+, K^0)$$

$$\pi^- + p \to p + \pi^- + \pi^0 \ (K^-, K^0)$$

$$K^- + p \to p + K^- + \pi^0 \ (\pi^- \underline{K}^0)$$

$$K^+ + p \to p + K^+ + \pi^0 \ (\pi^+ K^0)$$

or the states with nucleon-antinucleon pairs instead of mesons. In addition, one can study all these reactions in the u channel. The reactions (1) $\to$ (5) have been chosen because they appear to offer the best trigger and reasonable event rates. All the reactions outlined above, however, concern the physics we are interested in and we hope eventually to collaborate with other groups in a systematic program to study them all.

Physics Justifications

In the following we will deal primarily with reaction 1, 2, and 3. The same physics will apply (unless noted) to reactions 4 and 5 and is of equal importance.

1. High Mass Meson States

Recent results of π N and $\bar{K}$ N phase shift analysis[1,2] indicate that there are a large number of strongly overlapping resonances with increasing mass without clear peaks in the π N or $\bar{K}$ N effective mass distribution. On the otherhand the CERN Boson Spectrometer (C.B.S.) indicates[3] the existence of a series of narrow well separated boson resonances in the S = 0 T = I states up to 4 GeV. The above differences may be interpreted several ways.

a., Both the absence of the spin of the target pion and certain angular momentum states (Bose statistics) reduces the density of available resonance states at a given centre of mass momentum and isotopic spin.

b., The simple fact that the total interaction potential V_T attains contribution from a short and long range terms

$$V_T (r) = V(r) + \frac{\ell(\ell + 1)}{2\sqrt{s}\; r^2}$$

where $\sqrt{s}$ denotes the mass of the resonance. Thus for the same momentum and interaction radius the lighter system would have a narrower width, which is considerable smaller for the $\pi\pi$ system than for the π N system only for low values of s.

c., The structure of the nucleon is different from that of a pion.

d., It may be that the $\pi\pi$ system is just as complex as the π N system and $\bar{K}$ N system. The narrow peaks observed by C.B.S. may of course lie on other states with larger widths requiring that a phase shift analysis must be carried out. Thus the determination of positions, widths, and G parities in the $\pi^+ \pi^-$ (K + $\bar{K}$) final state will be extremely important to the fundamental under-

standing of the strong interaction. From what is said in section "a" and the fact that the $\pi^+\pi^-$ system is a super-position of $T = 0$, 1, and 2 we can obtain both even and odd orbital angular momentum states. Thus the $I = 0$ states may not have the same narrow characteristics as the C.B.S. results.

We are currently involved in looking for high mass boson resonances in two bubble chamber experiments of 10 events/μb each.[4] Fig. 1 shows the final data from the reaction

$$\pi^+ p \rightarrow p\pi^+ \pi^o \text{ at 13 GeV/c}$$
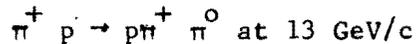
There appears to be indications of structures in the g, S and T regions. The significance is near 4 standard deviations for each which is not as significant as one would like. We have an extension to this experiment approved of 25 events/μb which should definitely settle the status of these enhancements. Table 1 shows the fitted values for the masses and widths obtained assumming the structures seen are resonant. Fig. 2 shows that the dipion system is produced very peripherally and that Cos θ $\pi\pi$ is strongly peaked forward and backward. In addition Fig. 3 shows a compiled mass histogram for the reaction $\pi^+ n \rightarrow p\pi^+\pi^-$ including our data at 13 GeV/c. Strong ρ and f production can be seen with possible hints of other structures at higher masses. The $K^o K^-$ final states have a cross section about a factor of 30 down on the $\pi\pi$ final state and hence no significant structures have been seen. It is an important point, however, that essentially nothing is known about reaction 2 at masses above about 1.7 GeV. It has generally been true that the signal to background ratio is much better for resonances in the $K \bar{K}$ final state than in the $\pi\pi$ final state.

In summary, high statisics for reactions 1, 2, and 3 will yield a spectrum of meson states with $S = 0$ ($S = 1$) together with their G parity and couplings to the meson-meson systems.

Exotic States

In reaction 3 and 5 we will be able to study the T = 2 (3/2) states to look for possible resonances. This is of fundamental importance to all current theoretical models and it is possible that they may be produced in the high mass regions.

Exchange Mechanisms

The presently known and dominant amplitudes for single pion production are the Absorption Modified One Pion Exchange (O.P.E.)[5] and the Diffraction Dissociation (D.D.)[6] amplitudes. Since we study the even G parity ππ system the diffraction dissociation amplitude is confined to the nucleon vertex. Our experience at 13 GeV/c is that while the nucleon dissociation may bias the ππ angular distribution it cannot make a narrow ππ enhancement. It is also an experimental fact that the distribution of the higher mass boson systems is almost completely peaked forward while the biases from the difraction dissociation (N's) affect the backward angular region. Thus we are reasonably sure that if we find narrow structures we will be able to isolate the O.P.E. amplitude from the D.D. amplitude, if O.P.E. is the dominant amplitude. Since our long standing interest is ππ interactions[7,8] and πV→ ππ (V is vector meson) once an enhancement is isolated we can use the absorption modified Regge model[9] to ascertain the O.P.E. or O.V.E. nature of the interaction, or if it is superposition of the two we will be able to unravel the O.P.E. amplitude for further study.

Vector Meson Dominance

The availability of high energy photon beams received theoretical as well as experimental interest in the Vector Meson Dominance Model[10]. (V.M.D.) Time reversal invariance and V. M. D. relates the

$$\gamma + N \rightarrow \pi + N$$

and

$$\pi + N \rightarrow \rho_{tr} + N$$

reaction amplitudes. We have tested V. M. D. by comparing the asymmetry para-

meter, obtained in polarized photopion production, with the ρ^o production dens-
ity matrix elements in the helicity frame[11]. It indicated that if V. M. D. is to
hold, mass extrapolation must be taken into account[12]. The sudden rise in the
photopion production crossection would imply a similar rise in the transverse
ρ^o differential crossection. At low incident beam momentum it is not possible
to measure directly the shape of the differential crossection for momentum trans-
fers less than μ^2 because the minimum momentum transfer is large. Thus up to
now we have relied on the theoretical result that Ball's invariant amplitudes[13]
have no kinematical singularities and the experimental fact that the reduced Ball
amplitudes, after explicit kinemetic factors are seperated out, vary smoothly as
function of Δ^2 in the One Pion Exchange frame. Using the above facts and an extro-
polation method in the O.P.E. frame we have observed a forward peak in the
helicity frame.[14]

At high energies the minimum momentum transfer is extremly small, and thus
the above test can be carried out <u>without</u> relying on extrapolation procedures.

$\pi\pi$Scattering-Pomeron Parameters and Spin-Parity Determinations.

In our study of $\pi^\pm p \rightarrow \pi^\pm \pi^0 N$ reactions we have found that as one goes
to higher $\pi\pi$ masses the $\pi\pi$ angular distribution becomes more and more forward
peaked. The most plausible interpretation is that the $\pi\pi$ scattering becomes
diffractive and presumably dominated by Pomeron exchange[15,16]. Above the
g region the angular distribution peaks extremly forward, even in the resonance
region of the S and T. This suggests that one cannot study the resonance re-
gion without obtaining first the quantitive description of the diffraction
scattering, with which the resonances interfere with. Since up to now no
convincing evidence has been found for a T = 2 resonance, the $\pi^+ \pi^+$ system is the
best system to obtain the parameters of diffraction scattering. Since there
is a relation between the Pomeron contribution in the $\pi^+ \pi^+$ and $\pi^+ \pi^o$ channels[16]

we will be able to investigate the $\pi^+ \pi^o$ final state. The $\pi^+ \pi^o$ final state is

a mixture of $T = 1$ and $T = 2$ amplitudes. The source of the asymmetry in the

$\pi^+ \pi^o$ angular distribution is due to the interference between the known $T = 2$

amplitude and the unknown $T = 1$ amplitude. From our preliminary data it appears

as if even the resonating partial wave has a considerable contribution from the

diffraction scattering. To proceed we assume (as a first approximation[16])

that the information obtained for the $\pi^+ \pi^o$ diffraction scattering can be used

to charaterise the diffraction part of the $\pi^+ \pi^o$ scattering even in the odd

partial waves. Using the K matrix unitarization method[17] we then determine which

partial wave resonates.

The $\pi^+ \pi^-$ system is a superposition of $T = 2, 1, 0$ amplitudes, thus it

contains non exotic, even possible resonating partial waves. Since the $T = 2$,

and $T = 1$ states have been studied the $T = 0$ component of the $\pi^+ \pi^-$ system can

be analysed. From the above outline of how we intend to carry out the analysis,

the following sequence arises

	Reaction	Boson System	Information Obtained
1	$\pi^+ \pi^+ n$	$\pi^+ \pi^+$	$A^2(s,t,\Delta^2), \alpha_p^2(t)$
2	$\pi^+ \pi^o p$	$\pi^+ \pi^o$	$A^2(s,t,\Delta^2), A^1(s,t,\Delta^2)$ $\alpha_p^2(t), \alpha_p^1(t)$
3	$\pi^+ \pi^- n$	$\pi^+ \pi^-$	$A^2, A^1, A^o, \alpha_p^2, \alpha_p^1$

Once the above sequence of final states has been analysed we intend to carry out

our analysis for the $K^+ p$ system in a similar fashion.

We have a long history of work in the field of $\pi\pi$ phase shifts and re-

lated topics such as vector dominance. We have studied the reaction

from 2 - 5 GeV/c up to our present energy of 13 GeV/c. The characteristics

of this interaction are changing markedly and we expect this to continue to

higher energies. We also expect the Physics obtained will give us a fundamental insight into the nature of strong interactions.

Summary

For the last few years we have devoted a great amount of effort to studying the diboson systems in the reactions discussed earlier.

1. We have combined the Goebel-Chew-Low conjecture, the Absorption Model and the Wick expansion to obtain a mathematical framework to study the S-wave $\pi\pi$ system.[18] The first determination of δ_0^0 was obtained by the Purdue $\pi\pi$ group.[7]

2. The first evidence for ω exchange in ρ^- production (with good statistics) was done at Purdue.[8]

3. The observation of the P(1080) in the $\pi^+\pi^-$ angular distribution[19] at two beam momenta.

4. Break down of the vector meson dominance model at two beam momenta[11].

5. Determination of the T = 0 to T = 2 scattering length ratio[20] and the off mass shell dependence of the S-wave $\pi\pi$ amplitude was done by our $\pi\pi$ group using the Argonne-Berkeley-Colorado-Pennsylvania-Purdue-Toronto-Wisconsin (ABCPPT) data. We tested both the current algebra[20] and Veneziano model. predictions.[21]

6. Amalgamating the ideas mentioned in item 1 with that of Ball's description[13] we have worked out the theoretical and experimental procedure to show the existence of a forward peak in ρ^0 production[14]. Again we have used the ABCPPT data.

7. We have shown that the high boson resonances exist in the positive G parity state[4].

8. Finally we initiated the collaboration with Argonne National Laboratory to organize the $\pi\pi$ and Kπ Conference at Argonne. The interest in

the topic can be **gauged by its** international character and the **world-wide** demand for the proceedings.

Judging on the past **successes** of this field, we are convinced even more profitable physics will come from the high energies. Including a further understanding of the fundamental $\pi\pi$ interaction, and the nature of the Pomeron. This experiment represents a natural extension of our work in currently available energy regions.

Experimental Details

Cross Sections and Rates

The cross section for reaction (1) is known up to 25 GeV/c and is behaving in a predictable way. Fig. 4 shows this cross section and also the ρ^0 contribution. In general, the overall cross section for the reaction is following as $p^{-1.3}$ while a specific resonant channel falls as p^{-2}. This is probably because as the momentum goes up, the overall channel phase space is increasing which moderates the normal p^{-2} fall off. We will assume that the other reactions behave in a similar way and for reaction (2) and (3) we will use our cross sections at 13 GeV/c and extrapolate from that point.

Reaction	Cross Section μb		Number of Events/pulse	
	50 GeV/c	100 GeV/c	50 GeV/c	100 GeV/c
$\pi^- p \to n\pi^+\pi^-$	90	45	90	45
$\pi^- p \to nK^+K^-$	1	.50	1	.50
$\pi^+ p \to n\pi^+\pi^+$	4	2	4	2
$K^- p \to nK^-\pi^+$	36	18	36	18
$K^+ p \to nK^+\pi^+$	3	1.5	3	1.5

The number of events is calculated for a 1 foot path length of hydrogen with 10^6 pions/4 secs. This is the flux that the high energy high resolution beam in area 2 will give. The number of K^-, and K^+ events will depend on the available flux.

The actual number of events depends on the memory times etc. of the detection system and the efficiencies of the triggers. It appears, however, that in a 100 hours of running one could accumulate 100,000 events of $n\pi^+\pi^-$.

It is more difficult to estimate exactly the event rate, since it depends on the ability to reject the background, the characteristics of which are unknown. This type of experiment has been done successfully at CERN (Hyams group) and also at other laboratories with approximately 50% bad triggers.

Triggers

The most useful trigger appears to be multiplicity and, in addition, trying to anti-out π^0 events with lead scintillator sandwich counters. Triggering on the neutron, in a way that will not seriously bias either angular or t regions, is probably difficult.

Computing.

The system would need an on-line computer but the main bulk of the analysis would be done away from N.A.L.

Apparatus

The kinematics of the events shows that the events consist of a
high energy secondary with a momentum near to the beam energy, together
with a low energy secondary of $1 \rightarrow 5$ GeV. The apparatus that is necessary,
therefore, consists of:

1) Beam analyzing and defining counters.

2) Liquid hydrogen target.

3) A high momentum arm with magnets and wire planes and Čerenkov counters.

4) A low momentum device around the target. This could be a streamer
   chamber or wire planes, together with π^o anti counters.

In our Monte-Carlo calculation, APPENDIX B, we have used our
13 GeV π^+p and π^+d data and tried to extrapolate to higher momenta.
The amplitudes used in part of APPENDIX B come from Ref. 22. This
analysis indicates that the high momentum arm needs the characteristics
of spectrometers already proposed, e.g. Ref. 23. The low momentum
arm has more solutions and one could probably use a streamer chamber,
or wires planes or possibly even a rapid cycling bubble chamber. We
are currently looking into the characteristics of these systems.

The apparatus described is particularly useful for the higher mass regions
> 2 GeV dominated by diffractive processes and high angular momentum states.
In the low mass region the Q value for the breakup is smaller and lower partial
waves dominate. This leads to a much higher incidence of events where both
pions have momenta greater than 10 GeV/c (for 50 GeV/c incident). This type
of event leads to the requirement of being able to measure two 'high' momentum
secondaries which would be difficult with the apparatus defined above. One
solution would be to provide a much larger visual detector, such as a streamer
chamber with large aperture magnet perhaps 1m x 1m x 5 metres. One could
then still use a high momentum arm for the events with one pion near to the beam
momentum. The resolution may be good enough to measure the symmetric decays

all inside the streamer chamber. This would fit in with our POLLY system
and could be automatically scanned and analyzed in conjunction with a magnetic
tape from the external wire planes.

Contribution from Purdue

In order to implement the physics we wish to do, it seems clear we will have to collaborate with other people interested in building a spectrometer system. The facilities and man power we could bring to such a collaboration are the following:

1) Manpower - I would expect at least 4 people would actively work on it and have up to two physicists at N.A.L. permanently (perhaps on a rotation basis) plus more in the summer or when the experiment is under way.

2) We could build in house some hardware e.g. Cerenkov counters or possibly a streamer chamber or wire planes depending on need.

3) We do not possess computers which we could bring to N.A.L. but we do have our own 360/40, 360/44 system for off-line computing.

4) If part of the spectrometer is a visual device, we will be able to use our POLLY system and our considerable experience with analysis of film. This could be done in an automatic scanning mode.

5) The money situation is not clear but we would certainly intend to put a significant part of our budget into N.A.L. experiments.

APPENDIX A

Study of Two Body Final States

This type of apparatus is well suited to the study of all two body final states, since they are dominated by low t. In principle, just the high momentum arm of the apparatus is sufficient to study this type of reaction such as elastic scattering. Other possible reactions are:-

$$\pi^- p \to K^- \Sigma^+$$
$$\pi^+ p \to K^+ \Sigma^+$$
$$K^- p \to \pi^- \Sigma^+$$
$$\qquad \to K^+ \Xi^-$$
$$K^+ p \to \pi^+ \Sigma^+$$
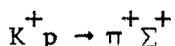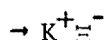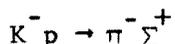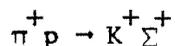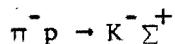
With the additions of a detector around the target, the decay and polarization of, for example, the Ξ^- could be studied.

The cross sections and the shape and structure of the differential cross sections for these reactions will yield sensitive tests of the Regge[24] cut and duality[25] models.

We would propose that the high momentum arm of the spectrometer be implemented first and some runs taken on the above reactions. The addition of the low momentum arm around the target would enable us to study the low momentum hyperon. These reactions, besides giving valuable data, will provide a first step in studying the reactions which form the basis of this proposal.

APPENDIX B

Monte Carlo Studies

The computer program FOWL was run using two different amplitudes. The diagram is shown in Fig. 5. The amplitude A_0 is

$$A_0 = c \exp\left[\frac{5}{2}(t_{\pi\pi} + t_{pn})\right]$$

$$t_{\pi\pi} = (P_{\pi^+} - P_{beam})^2$$

$$t_{pn} = (P_n - P_{target})^2$$

Although the exchange trajectories have not been specified in A_0, this model reproduces the lower energy data reasonably well when the Pomeron is not exchanged. The amplitude A_p applies to the same diagram but includes the effect of Pomeron exchange.

$$A_p = A_0 \, S_{\pi\pi} \, S_{n\pi}{}^{\alpha\pi(t_{pn})}$$

$$S_{\pi\pi} = (P_{\pi^+} + P_{\pi^-})^2$$

$$S_{n\pi} = (P_n + P_{\pi^+})^2$$

$$\alpha_\pi(t) = -0.02 + t$$

The pi-pi effective mass distributions for these two amplitudes are shown in Fig. 6 and 7. Pomeron exchange has the effect of enhancing high pi-pi effective masses.

The general kinematic characteristics of the reaction are shown in Table 1. In the case of amplitude A_0, the reaction is characterized by two pions produced in the beam direction. One has momentum within 10% of the incident beam momentum and the other pion has momentum less than 2 GeV/c. The kinematic characteristics of the reaction assuming A_p, are that one pion is produced along the beam direction with momentum within 10% of the beam momentum, while the other pion is produced with a momentum less than 1 GeV/c over a wide solid angle.

Further studies of a more general character are underway. These include:

1) Aperture, magnetic fields etc. necessary to cover particular regions of four momentum transfer and Cos $\theta_{\pi\pi}$.

2) Resolutions of different types of detectors, particularly the detector

around the target.

3) Overall mass and scattering angle resolutions.

4) Calculation of the number of events and the regions to be covered to differentiate high spins from one another.

5) Study of experiments underway at current energy such as the CERN-Munich experiment. The rho production (E41) experiment at SLAC by Richter's group and the apparatus being set up by Diebold at Argonne. All these experiments intend to study the final states $\pi^-\pi^+n$ and K^-K^+n at energies currently available.

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J. Rosner, Phys. Rev. Letters $\underline{22}$, 689 (1969).

TABLE 1

STATE OBSERVED	MASS IN GeV/c^2	Γ_{obs} IN GeV/c^2	Γ_{phys} IN GeV/c^2	CROSS SECTION IN μb
ρ^+	0.771 ± 0.025	0.188 ± 0.030	0.145 ± 0.038	54.0 ± 10.3
R_1^+	1.652 ± 0.015	0.061 ± 0.020	0.040 ± 0.032	8.7 ± 2.3
S'^+	1.975 ± 0.012	0.045 ± 0.020	≤ 0.052 AT 90 % C.L.	6.5 ± 1.8
T'^+	2.157 ± 0.010	0.078 ± 0.018	0.068 ± 0.022	10.5 ± 3.2

TABLE   2

AMPLITUDE A_o			AMPLITUDE A_p		
Beam Momentum	Kinematic Characteristic	Percentage of Events	Beam Momentum	Kinematic Characteristic	Percentage of Events
50 GeV/c	P+<1.5 GeV/c	28%	50 GeV/c	P+<1.5 GeV/c	87%
	P->45.5 GeV/c	50%		P->45.5 GeV/c	87%
	θ+< 10°	75%		θ+< 60°	56%
	θ-< 10 mrad	66%		θ-< 10 mrad	73%
100 GeV/c	P+< 1.5 GeV/c	23%	100 GeV/c	P+< 1.5 GeV/c	85%
	P-> 91 GeV/c	57%		P-> 91 GeV/c	99%
	θ+< 10°	36%		θ+< 60°	51%
	θ-< 10 mrad	96%		θ-< 10 mrad	99%

Notation: P + = lab momentum of π^+, P- = lab momentum of π^-

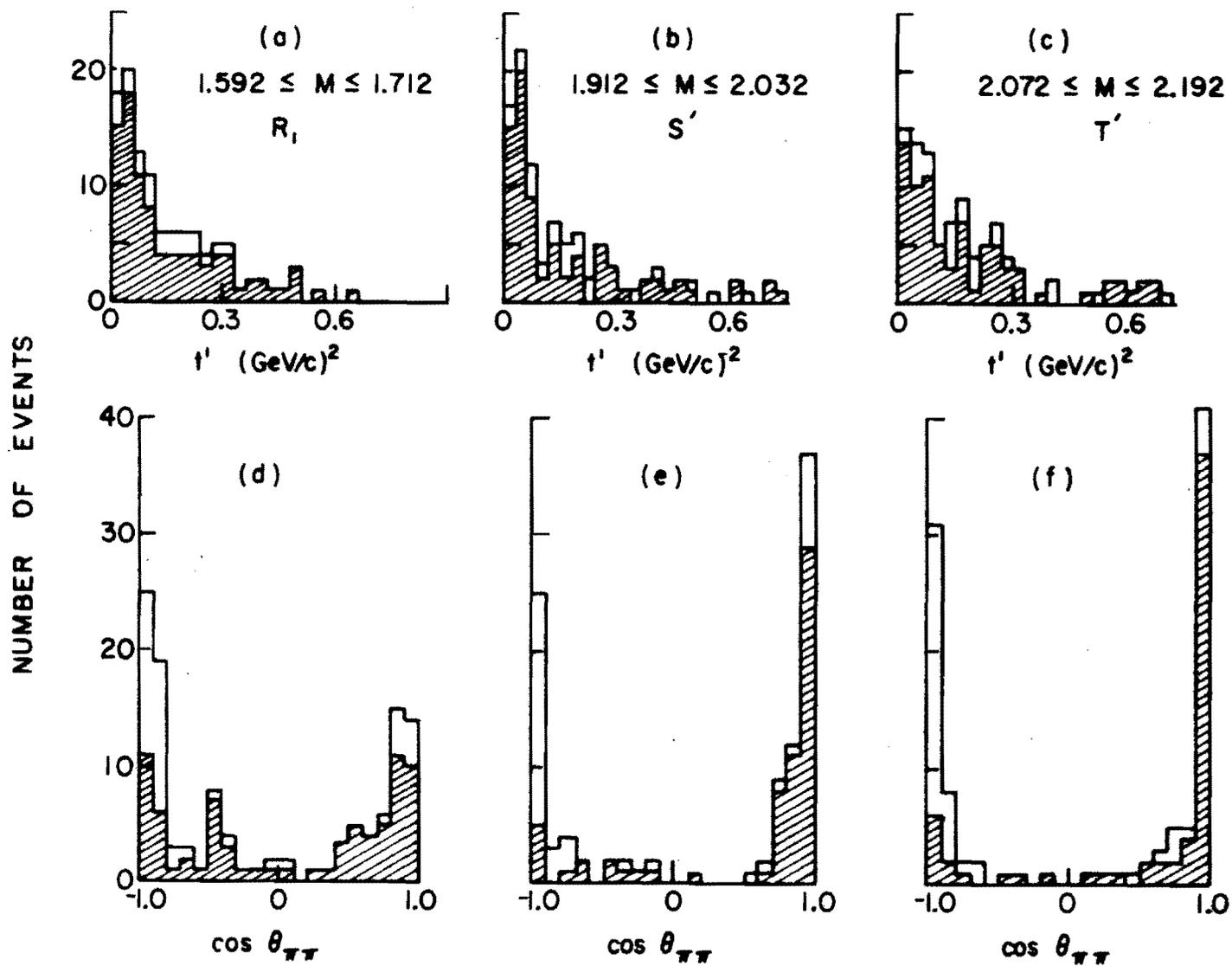θ + = lab scattering angle of π^+, θ - = lab scattering angle of π^-.
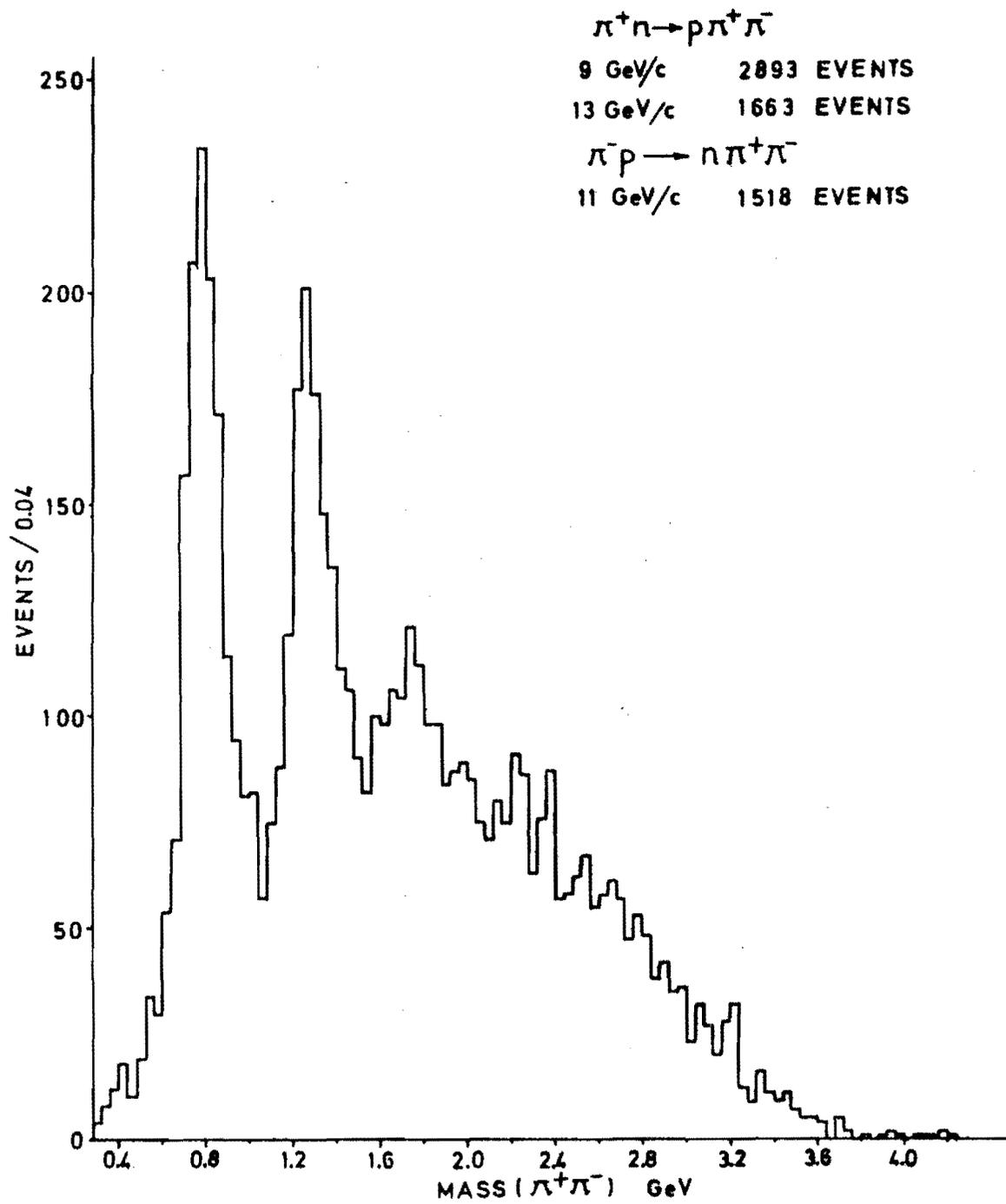
Fig. 1

Fig. 2

(a) $1.592 \leq M \leq 1.712$ R_1

(b) $1.912 \leq M \leq 2.032$ S'

(c) $2.072 \leq M \leq 2.192$ T'

t' $(GeV/c)^2$

t' $(GeV/c)^2$

t' $(GeV/c)^2$

(d)

(e)

(f)

$\cos \theta_{\pi\pi}$

$\cos \theta_{\pi\pi}$

$\cos \theta_{\pi\pi}$

NUMBER OF EVENTS

$\pi^+ n \rightarrow p\pi^+\pi^-$

| 9 GeV/c | 2893 EVENTS |
| 13 GeV/c | 1663 EVENTS |

$\pi^- p \rightarrow n\pi^+\pi^-$

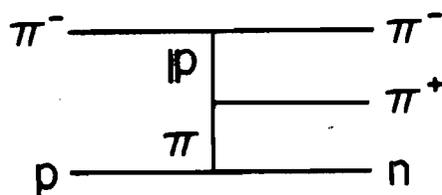| 11 GeV/c | 1518 EVENTS |

Fig. 3

Fig. 4

$$A_o = c\, e^{\frac{5}{2} t_{\pi\pi}} e^{\frac{5}{2} t_{pn}}$$

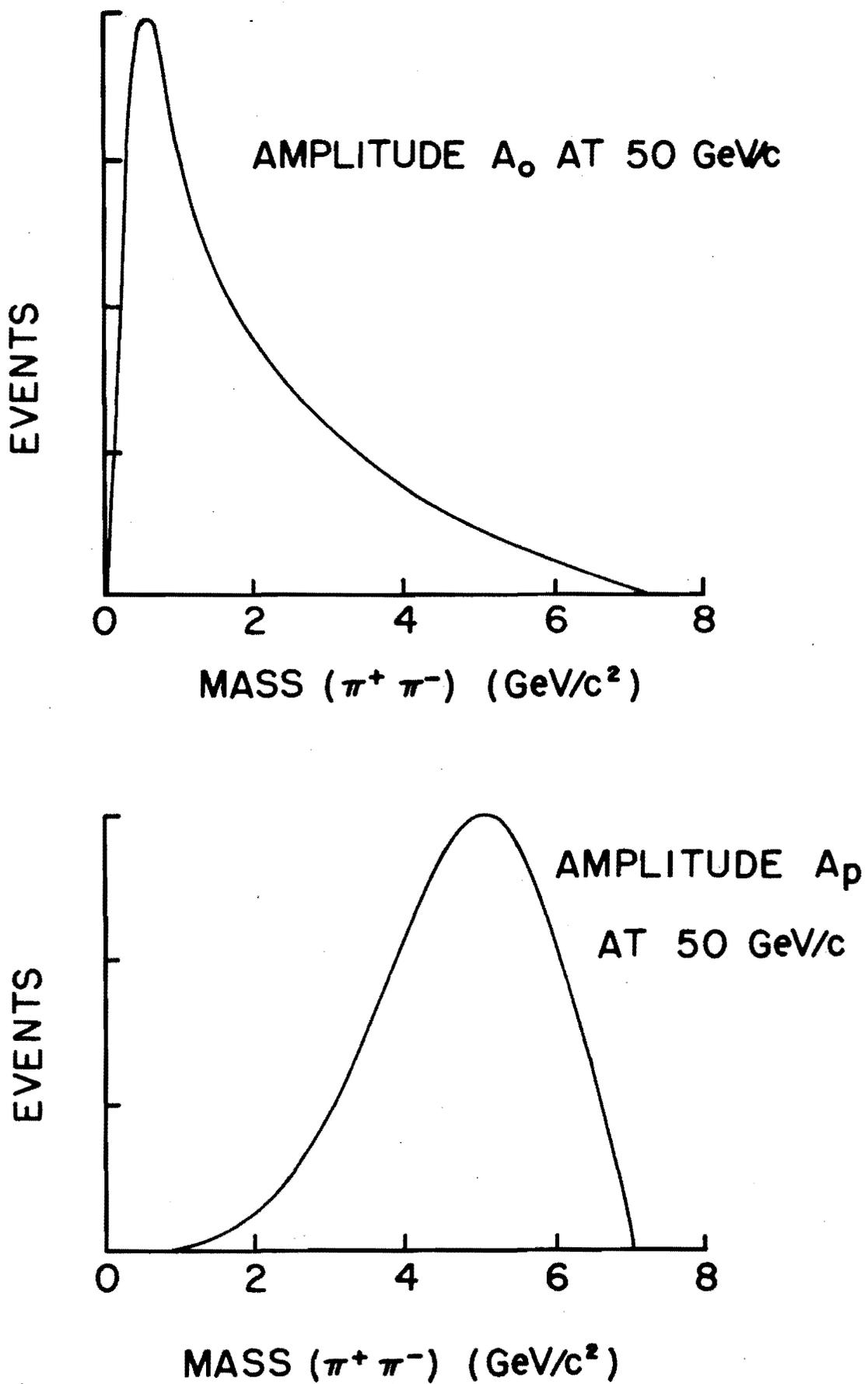$$A_p = A_o\, s_{\pi\pi}\, s_{n\pi}{}^{\alpha_\pi(t_{pn})}$$
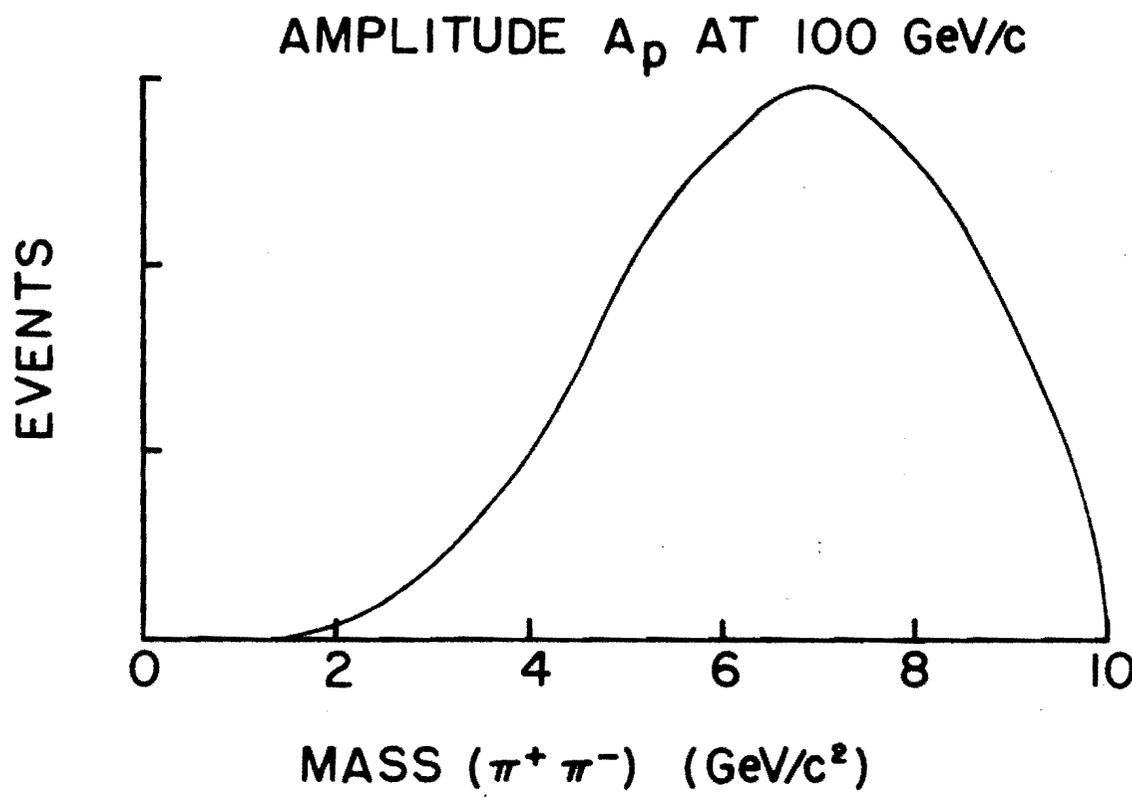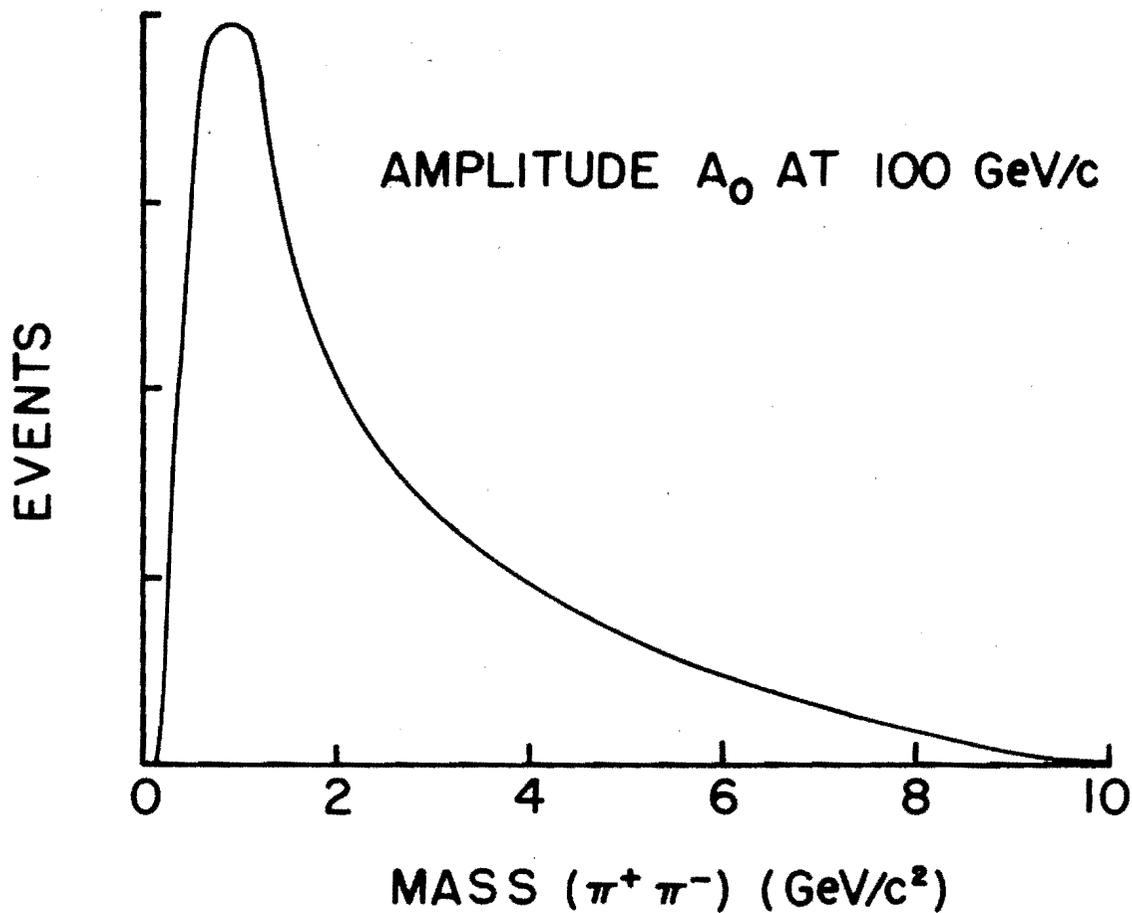
Fig. 5

Fig. 5

AMPLITUDE A_o AT 50 GeV/c

EVENTS

MASS ($\pi^+ \pi^-$) (GeV/c²)

AMPLITUDE A_p

AT 50 GeV/c

EVENTS

MASS ($\pi^+ \pi^-$) (GeV/c²)

Fig. 6

AMPLITUDE A_0 AT 100 GeV/c

EVENTS

MASS ($\pi^+ \pi^-$) (GeV/c^2)

AMPLITUDE A_p AT 100 GeV/c

EVENTS

MASS ($\pi^+ \pi^-$) (GeV/c^2)

Fig. 7