Azimuthal and single spin asymmetry in deep-inelastic lepton-nucleon scattering

Zuo-tang Liang[1,2] and Xin-Nian Wang[2,1]

[1] Department of Physics, Shandong University, Jinan, Shandong 250100, China
[2] Nuclear Science Division, MS 70R0319, Lawrence Berkeley National Laboratory, Berkeley, California 94720

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We derive a general framework for describing semi-inclusive deep-inelastic lepton-nucleon scattering in terms of the unintegrated parton distributions and other higher twist parton correlations. Such a framework provides a consistent approach to the calculation of inclusive and semi-inclusive cross sections including higher twist effects. As an example, we calculate the azimuthal asymmetries to the order of $1/Q$ in semi-inclusive process with transversely polarized target. A non-vanishing single-spin asymmetry in the "triggered inclusive process" is predicted to be $1/Q$ suppressed with a part of the coefficient related to a moment of the Sivers function.

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I. INTRODUCTION

Many interesting phenomena have been observed [1–8] in semi-inclusive deep-inelastic lepton-nucleon scattering (SIDIS), in particular the azimuthal asymmetries in the momentum distribution of the final hadrons and their spin dependence [1–23]. Since most of the studies involve hadrons with $p_\perp \sim 1$ GeV/c, the intrinsic parton transverse momenta (denoted by $k_\perp$) and multiple parton scattering become critical in the perturbative QCD (pQCD) approach.

The effects of intrinsic parton transverse momenta and multiple parton scattering are normally higher-twist. The higher twist effects have been studied extensively in inclusive DIS or lepton-pair production in in hadron-hadron or hadron-nucleus collisions in the past. An elegant and practical framework in terms of collinear expansion has been developed and applied to these processes [24–26]. A factorized form for the cross section is obtained as a convolution of the calculable hard parts with the universal ($k_\perp$-integrated) parton distributions and correlation functions (hereafter referred generally as parton correlation functions) that can be measured in different reactions. The framework is important not only because it provides a practical way of studying higher twist effects but also because it leads to the definitions of the parton correlations in a gauge invariant form.

The gauge invariant parton correlations contain contributions from the initial and final state interactions with soft gluons, which, when extended to include transverse momenta, lead in particular to the single-spin asymmetry in the polarized case [19, 21]. The asymmetry can manifest itself in SIDIS, hence the framework has been generalized to study the azimuthal asymmetries in SIDIS without explicit derivations [16, 23]. It is not clear whether/how the collinear expansion can be made in SIDIS. Many of the generalized formulas used in the literature are not proved and some of them are in fact even erroneous, in particular when higher twist effects are involved.

In this paper, we will clarify the situation by making an explicit derivation that generalizes the factorization to the transverse-momentum-dependent SIDIS. We show that the collinear expansion can also be made in SIDIS and such a derivation leads to a general framework for describing SIDIS including higher twist effects. We present the results and show that they have a number of remarkable properties that are often neglected in the literature. The framework in this study provides a consistent and systematic approach to the pQCD study of SIDIS beyond the leading twist. As an example, we present the calculation of the differential cross section for SIDIS with transversely polarized target to the order $1/Q$. We show in particular that the results imply the existence of a new single-spin asymmetry in the "triggered inclusive process" which can easily be tested experimentally.

The paper is organized as follows. After this introduction, we will make a short review of the key gradients of the collinear expansion technique and its application to inclusive DIS, then show how it can be applied to SIDIS in Sec. II. In Sec. III, as an example of the application of the formulas obtained in Sec. II, we will present the differential cross section and azimuthal asymmetries in SIDIS using unpolarized electron and transversely polarized polarized nucleon. Finally, we make a short summary in Sec.IV.

II. COLLINEAR EXPANSION IN SIDIS

A. Collinear expansion in inclusive DIS

A general framework for studying inclusive DIS including higher twist contributions has been developed in Refs.[24, 25]. We review the key gradients here. We consider the inclusive DIS $e^- p \to e^- X$, and the differential cross section is given by,

$$d\sigma = \frac{e^4}{sQ^4} L^{\mu\nu}(l,l') W_{\mu\nu}(q,p,S) \frac{d^3 l'}{(2\pi)^3 2E_{l'}}, \qquad (1)$$

where l and l' are respectively the four momenta of the incoming and outgoing leptons, p and S are the four momentum and the spin of the incoming proton, q is the four momentum transfer. We neglect the masses and use the light-cone coordinates. The unit vectors are taken as, $\bar{n} = (1,0,0,0)$, $n = (0,1,0,0)$, $n_{\perp 1} = (0,0,1,0)$, $n_{\perp 2} = (0,0,0,1)$. We work in the center of mass frame of the $\gamma^* p$-system, and chose the coordinate system in the way so that, $p = p^+ \bar{n}$, $q = -x_B p + nQ^2/(2x_B p^+)$, and $l_\perp = |\vec{l}_\perp| n_{\perp 1}$, where $x_B = Q^2/2p\cdot q$ is the Bjorken-x and $y = p\cdot q/p\cdot l$. The leptonic tensor $L^{\mu\nu}$ is defined as usual and is given by,

$$L^{\mu\nu}(l,l') = 4[l^\mu l'^\nu + l^\nu l'^\mu - (l\cdot l')g^{\mu\nu}], \tag{2}$$

The hadronic tensor $W_{\mu\nu}$ is defined as,

$$W_{\mu\nu}(q,p,S) = \frac{1}{2\pi}\sum_X \langle p,S|J_\mu(0)|X\rangle\langle X|J_\nu(0)|p,S\rangle(2\pi)^4\delta^4(p+q-p_X), \tag{3}$$

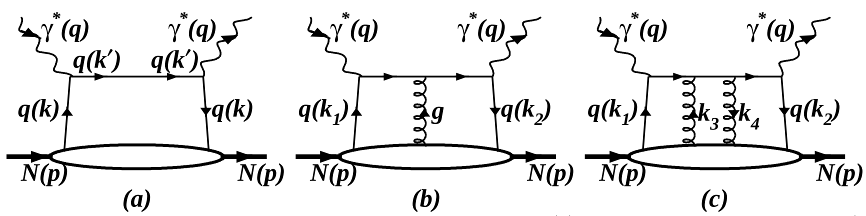

FIG. 1: Feynman diagrams for the cases with exchange of $j = 0,1,2$ gluon(s). The gluon momenta in (c) are: $k_3 = k - k_1$ and $k_4 = k - k_2$.

We consider final state interaction in pQCD so that we have the contributions from the type of diagrams shown in Fig.1. The hadronic tensor $W_{\mu\nu}$ should be written as a sum of the contributions from all the diagrams, i.e., $W_{\mu\nu} = \sum_j W^{(j)}_{\mu\nu}$, where j denotes the number of soft gluons. At the lowest order in pQCD, we have,

$$W^{(0)}_{\mu\nu}(q,p,S) = \frac{1}{2\pi}\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\hat{H}^{(0)}_{\mu\nu}(k,q)\hat{\phi}^{(0)}(k,p,S)], \tag{4}$$

$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \gamma_\mu(\not{k}+\not{q})\gamma_\nu(2\pi)\delta_+((k+q)^2), \tag{5}$$

where δ_+ means that only the positive solution is taken. Similarly, corresponding to Figs. 1(b) and (c), we have,

$$W^{(1)}_{\mu\nu}(q,p,S) = \frac{1}{2\pi}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}[\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q)\hat{\phi}^{(1)}_\rho(k_1,k_2,p,S)], \tag{6}$$

$$W^{(2)}_{\mu\nu}(q,p,S) = \frac{1}{2\pi}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\hat{H}^{(2)\rho\sigma}_{\mu\nu}(k_1,k_2,k,q)\hat{\phi}^{(2)}_{\rho\sigma}(k_1,k_2,k,p,S)], \tag{7}$$

where $\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q) = \sum_{c=L,R}\hat{H}^{(1,c)\rho}_{\mu\nu}(k_1,k_2,q)$, $\hat{H}^{(2)\rho}_{\mu\nu}(k_1,k_2,k,q) = \sum_{c=L,M,R}\hat{H}^{(2,c)\rho}_{\mu\nu}(k_1,k_2,k,q)$, c denotes the different cuts in the diagrams. These hard parts can be read from the diagram and are given by,

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(k_1,k_2,q) = \gamma_\mu(\not{k}_1+\not{q})\gamma^\rho\frac{\not{k}_2+\not{q}}{(k_2+q)^2-i\epsilon}\gamma_\nu(2\pi)\delta_+((k_1+q)^2), \tag{8}$$

$$\hat{H}^{(2,L)\rho\sigma}_{\mu\nu}(k_1,k_2,k,q) = \gamma_\mu(\not{k}_1+\not{q})\gamma^\rho\frac{\not{k}+\not{q}}{(k+q)^2-i\epsilon}\gamma^\sigma\frac{\not{k}_2+\not{q}}{(k_2+q)^2-i\epsilon}\gamma_\nu(2\pi)\delta_+((k_1+q)^2), \tag{9}$$

and so on. The structure of proton is contained only in the matrix elements $\hat{\phi}$'s that are defined as,

$$\hat{\phi}^{(0)}(k,p,S) \equiv \int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\psi(z)|p,S\rangle, \tag{10}$$

$$\hat{\phi}^{(1)}_\rho(k_1,k_2,p,S) \equiv \int d^4y d^4z e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)gA_\rho(y)\psi(z)|p,S\rangle, \tag{11}$$

$$\hat{\phi}^{(2)}_{\rho\sigma}(k_1,k_2,k,p,S) \equiv \int d^4y d^4y' d^4z e^{ik_1y+ik(y'-y)+ik_2(z-y')}\langle p,S|\bar{\psi}(0)gA_\rho(y)gA_\sigma(y')\psi(z)|p,S\rangle. \tag{12}$$

We note that neither of the $\hat{\phi}$'s defined in this way is gauge invariant. To organize the above results in terms of gauge invariant parton correlations, we need to invoke the collinear expansion procedure. This procedure has been developed in Refs.[24, 25], and is carried out in the following steps.

(1) we make a Taylor expansion of the hard parts around $k = xp$, e.g.,

$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \hat{H}^{(0)}_{\mu\nu}(x) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k_\rho}\omega_\rho^{\ \rho'} k_{\rho'} + \frac{1}{2}\frac{\partial^2 \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k_\rho \partial k_\sigma}\omega_\rho^{\ \rho'}\omega_\sigma^{\ \sigma'} k_{\rho'} k_{\omega'} + ... \tag{13}$$

where $x = k^+/p^+$, $\hat{H}^{(0)}_{\mu\nu}(x) \equiv \hat{H}^{(0)}_{\mu\nu}(k,q)|_{k=xp}$, $\partial\hat{H}^{(0)}_{\mu\nu}(x)/\partial k_\rho \equiv \partial\hat{H}^{(0)}_{\mu\nu}(k,q)/\partial k_\rho|_{k=xp}$ and so on; $\omega_\rho^{\ \rho'} \equiv g_\rho^{\ \rho'} - \bar{n}_\rho n^{\rho'}$ is a projection operator so that $\omega_\rho^{\ \rho'} k_{\rho'} = (k-xp)_\rho$.

(2) we decompose the gluon field A_ρ into the longitudinal and transverse components, i.e.,

$$A_\rho(y) = \omega_\rho^{\ \rho'} A_{\rho'}(y) + p_\rho n\cdot A(y)/n\cdot p. \tag{14}$$

(3) we use the generalized Ward identities to relate the derivative of a hard part to that of a higher order, and the product of p with a hard part to that of a lower order, *i.e.*,

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k_\rho} = -\hat{H}^{(1)\rho}_{\mu\nu}(x,x), \qquad \frac{1}{2}\frac{\partial^2 \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k_\rho \partial k_\sigma} = \hat{H}^{(2)\rho\sigma}_{\mu\nu}(x,x,x), \tag{15}$$

$$\frac{\partial \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)}{\partial k_{2\sigma}} = -\hat{H}^{(2)\rho\sigma}_{\mu\nu}(x_1,x_2,x_2), \qquad \frac{\partial \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)}{\partial k_{1\sigma}} = -\hat{H}^{(2)\sigma\rho}_{\mu\nu}(x_1,x_1,x_2), \tag{16}$$

$$p_\rho \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_B - i\epsilon} + \frac{\hat{H}^{(0)}_{\mu\nu}(x_2)}{x_1 - x_B + i\epsilon}, \tag{17}$$

$$p_\rho \hat{H}^{(2)\rho\sigma}_{\mu\nu}(x_1,x_2,x) = \frac{\hat{H}^{(1)\sigma}_{\mu\nu}(x_1,x_2)}{x - x_1 - i\epsilon} + \frac{\hat{H}^{(1)\sigma}_{\mu\nu}(x,x_2)}{x_1 - x + i\epsilon}, \tag{18}$$

$$p_\sigma \hat{H}^{(2)\rho\sigma}_{\mu\nu}(x_1,x_2,x) = \frac{\hat{H}^{(1)\rho}_{\mu\nu}(x_1,x)}{x_2 - x - i\epsilon} + \frac{\hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)}{x - x_2 + i\epsilon}, \tag{19}$$

(4) we rearrange all the terms by adding the contributions with the same hard part together. In this way, we obtain that $W_{\mu\nu}(q,p,S) = \sum_{j=0,1,2}\tilde{W}^{(j)}_{\mu\nu}(q,p,S)$, and,

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \frac{1}{2\pi}\int dx \mathrm{Tr}[\hat{H}^{(0)}_{\mu\nu}(x)\ \hat{\Phi}^{(0)}(x,p,S)]; \tag{20}$$

$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S) = \frac{1}{2\pi}\int dx_1 dx_2 \mathrm{Tr}[\hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'}\hat{\Phi}^{(1)}_{\rho'}(x_1,x_2,p,S)]; \tag{21}$$

$$\tilde{W}^{(2)}_{\mu\nu}(q,p,S) = \frac{1}{2\pi}\int dx_1 dx_2 dx \mathrm{Tr}[\hat{H}^{(2)\rho\sigma}_{\mu\nu}(x_1,x_2,x)\omega_\rho^{\ \rho'}\omega_\sigma^{\ \sigma'}\hat{\Phi}^{(2)}_{\rho'\sigma'}(x_1,x_2,x,p,S)], \tag{22}$$

where the matrix elements $\hat{\Phi}$'s have contributions from all the three diagrams in Fig. 1, and are given by,

$$\hat{\Phi}^{(0)}(x,p,S) = \int dz^- e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\psi(z^-)|p,S\rangle, \tag{23}$$

$$\hat{\Phi}^{(1)}_\rho(x_1,x_2,p,S) = \int dy^- dz^- e^{ix_1p^+y^- + ix_2p^+(z^- - y^-)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y^-)D_\rho(y^-)\mathcal{L}(y^-,z^-)\psi(z^-)|p,S\rangle, \tag{24}$$

$$\begin{aligned}\hat{\Phi}^{(2)}_{\rho\sigma}(x_1,x_2,x,p,S) = &\int dy^- dy'^- dz^- e^{ix_1p^+y^- + ixp^+(y'^- - y^-) + ix_2p^+(z^- - y'^-)} \\ &\times \langle p,S|\bar{\psi}(0)\mathcal{L}(0,y^-)D_\rho(y^-)\mathcal{L}(y^-,y'^-)D_\sigma(y'^-)\mathcal{L}(y'^-,z^-)\psi(z^-)|p,S\rangle,\end{aligned} \tag{25}$$

where $D_\rho(y) = -i\partial_\rho + gA_\rho(y)$ is the covariant derivative. $\mathcal{L}(y,z)$ is the so-called gauge link, which is obtained in the derivation and is given by,

$$\mathcal{L}(y^-,z^-) = 1 + ig\int_{y^-}^{z^-} d\xi^- A^+(0,\xi^-,\vec{0}_\perp) + (ig)^2\int_{y^-}^{z^-} d\xi A^+(0,\xi^-,\vec{0}_\perp)\int_{z^-}^{y^-} dy'^- A^+(0,y'^-,\vec{0}_\perp), \tag{26}$$

Including higher order contributions in g, the gauge link $\mathcal{L}$ is given by,

$$\mathcal{L}(-\infty, z) = Pe^{ig\int_{-\infty}^{z^-} dy^- n\cdot A(z^+,y^-,\vec{z}_\perp)}, \tag{27}$$

which is the well-known path integral representation of the gauge link without transverse displacement. These $\hat{\Phi}$'s are now gauge invariant. They lead to the well-known gauge invariant definitions of the parton correlation functions used in literature.[24–26]

B. Generalization to SIDIS

Now we apply the same procedure to SIDIS. We show why and how the collinear expansion can also be made for SIDIS. To show the main idea, we star with a simple case $e^-p^\uparrow \to e^-qX$, i.e., we do not consider the fragmentation. This is equivalent to consider jet production. The differential cross section for $e^-p^\uparrow \to e^-qX$ can be written as,

$$d\sigma = \frac{\alpha_{em}^2 e_q^2}{sQ^4} L^{\mu\nu}(l,l') W_{\mu\nu}^{(si)}(q,p,S,k') \frac{d^3l' d^3k'}{(2\pi)^4 E_{l'} E_{k'}}, \tag{28}$$

where k' is the 4-momentum of the outgoing quark; and the hadronic tensor $W_{\mu\nu}^{(si)}$ for SIDIS is defined as,

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \frac{1}{2\pi}\sum_X \langle p,S|J_\mu(0)|k'X\rangle\langle k'X|J_\nu(0)|p,S\rangle(2\pi)^4\delta^4(p+q-k'-p_X), \tag{29}$$

where the superscript (si) denotes that it is for SIDIS.

We consider the same final state interaction in pQCD as one did for inclusive DIS so that we have the contributions from the diagrams shown in Fig.1. The hadronic tensor $W_{\mu\nu}^{(si)}$ is written as a sum of the contributions from all the diagrams, i.e., $W_{\mu\nu}^{(si)} = \sum_j W_{\mu\nu}^{(j,si)}$.

$$W_{\mu\nu}^{(0,si)}(q,p,S,k') = \frac{1}{2\pi}\int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\hat{H}_{\mu\nu}^{(0,si)}(k,k',q)\hat{\phi}^{(0)}(k,p,S)], \tag{30}$$

$$W_{\mu\nu}^{(1,si)}(q,p,S,k') = \frac{1}{2\pi}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \sum_{c=L,R} \mathrm{Tr}[\hat{H}_{\mu\nu}^{(1,c,si)\rho}(k_1,k_2)\hat{\phi}_\rho^{(1)}(k_1,k_2,p,S)]; \tag{31}$$

$$W_{\mu\nu}^{(2,si)}(q,p,S,k') = \frac{1}{2\pi}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} \sum_{c=L,M,R} \mathrm{Tr}[\hat{H}_{\mu\nu}^{(2,c,si)\rho\sigma}(k_1,k_2,k)\hat{\phi}_{\rho\sigma}^{(2)}(k_1,k_2,k,p,S)]. \tag{32}$$

where $\hat{H}_{\mu\nu}^{(j,c,si)}$'s denote the hard parts for the semi-inclusive process and are given by,

$$\hat{H}_{\mu\nu}^{(0,si)}(k,k',q) = \gamma_\mu(\not{k}+\not{q})\gamma_\nu(2\pi)^4\delta^4(k'-k-q), \tag{33}$$

$$\hat{H}_{\mu\nu}^{(1,L,si)\rho}(k_1,k_2,q) = \gamma_\mu(\not{k}_1+\not{q})\gamma^\rho\frac{\not{k}_2+\not{q}}{(k_2+q)^2-i\epsilon}\gamma_\nu(2\pi)^4\delta^4(k'-k_1-q), \tag{34}$$

$$\hat{H}_{\mu\nu}^{(2,L,si)\rho\sigma}(k_1,k_2,k,q) = \gamma_\mu(\not{k}_1+\not{q})\gamma^\rho\frac{\not{k}+\not{q}}{(k+q)^2-i\epsilon}\gamma^\sigma\frac{\not{k}_2+\not{q}}{(k_2+q)^2-i\epsilon}\gamma_\nu(2\pi)^4\delta^4(k'-k_1-q), \tag{35}$$

We compare the $W_{\mu\nu}^{(j,si)}$'s with their counterparts $W_{\mu\nu}^{(j)}$ for the inclusive reaction, where an integration over d^3k' is carried out, and we see that they differ from each other only in the hard parts. If we would now apply the same procedure as we did for inclusive DIS to make the collinear expansion, i.e., we make a Taylor expansion for the hard parts $H_{\mu\nu}^{(j,si)}$ directly, we would find out that there are no such relations as given in Eqs.(15-19) for in the semi-inclusive hard parts. We would not be able to reach similar results as given in Eqs.(20-22).

This difficulty can be avoided using the identity,

$$\delta^4(k'-k-q) = 2E_{k'}\delta_+((q+k)^2)\delta^3(\vec{k}'-\vec{k}-\vec{q}), \tag{36}$$

to rewrite the hard parts for the semi-inclusive processes. Using this identity, we have,

$$\hat{H}_{\mu\nu}^{(0,si)}(k,k',q) = \hat{H}_{\mu\nu}^{(0)}(k,q)K(k,k',q), \tag{37}$$

$$K(k',k,q) \equiv 2E_{k'}(2\pi)^3\delta^3(\vec{k}'-\vec{k}-\vec{q}). \tag{38}$$

We see that the difference between $\hat{H}^{(0,si)}_{\mu\nu}(k,k',q)$ and its counterpart $\hat{H}^{(0)}_{\mu\nu}(k,q)$ is only a multiplicative factor $K(k',k,q)$. Correspondingly, for the hadronic tensor, we have,

$$W^{(0,si)}_{\mu\nu}(q,p,S,k') = \frac{1}{2\pi}\int\frac{d^4k}{(2\pi)^4}K(k',k,q)\mathrm{Tr}[\hat{H}^{(0)}_{\mu\nu}(k,q)\hat{\phi}^{(0)}_{\rho}(k,p,S)]. \tag{39}$$

This is also true for higher orders, i.e, for higher j, but the kinematic factor K is different for different cuts. E.g., for $j=1$ and 2, corresponding to Figs. 1(b) and 1(c),

$$\hat{H}^{(1,c,si)\rho}_{\mu\nu}(k_1,k_2,k',q) = \hat{H}^{(1,c)\rho}_{\mu\nu}(k_1,k_2,q)K(k_c,k',q), \tag{40}$$

$$\hat{H}^{(2,c,si)\rho\sigma}_{\mu\nu}(k_1,k_2,k,k',q) = \hat{H}^{(2,c)\rho\sigma}_{\mu\nu}(k_1,k_2,k,q)K(k_c,k',q). \tag{41}$$

where $c=L,R$ or M and $k_L=k_1$, $k_R=k_2$, $k_M=k$. Correspondingly, we have,

$$W^{(1,si)}_{\mu\nu}(q,p,S,k') = \frac{1}{2\pi}\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\sum_{c=L,R}K(k',k_c,q)\mathrm{Tr}[\hat{H}^{(1,c)\rho}_{\mu\nu}(k_1,k_2,q)\hat{\phi}^{(1)}_{\rho}(k_1,k_2,p)]; \tag{42}$$

$$W^{(2,si)}_{\mu\nu}(q,p,S,k') = \frac{1}{2\pi}\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\sum_{c=L,R,M}K(k',k_c,q)\mathrm{Tr}[\hat{H}^{(2,c)\rho\sigma}_{\mu\nu}(k_1,k_2,k,q)\hat{\phi}^{(2)}_{\rho\sigma}(k_1,k_2,k,p,S), \tag{43}$$

This feature is very important since a multiplicative factor does not influence the collinear expansion that are needed to make the definition of the parton correlations gauge invariant. It implies that we can apply the same collinear expansion as that for inclusive processes. More precisely, we do in exactly the same way as we did for inclusive DIS. We go through the steps (1) to (4) and in step (1) we make a Taylor expansion of the hard parts $\hat{H}$'s (for inclusive DIS) but *not* $\hat{H}^{(si)}$'s (for semi-inclusive) around $k=xp$. In this way, we have the same relations between the derivatives of the hard parts to the hard parts of a higher order. However, since the kinetic factor K depends on the cut, so we have to use the relation for different cuts separately, e.g.,

$$\frac{\partial\hat{H}^{(0)}_{\mu\nu}(x)}{\partial k_\rho} = -\hat{H}^{(1,L)\rho}_{\mu\nu}(x,x) - \hat{H}^{(1,R)\rho}_{\mu\nu}(x,x), \tag{44}$$

$$\frac{\partial\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2)}{\partial k_{2\sigma}} = -\hat{H}^{(2,L)\rho\sigma}_{\mu\nu}(x_1,x_2,x_2), \tag{45}$$

$$\frac{\partial\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2)}{\partial k_{1\sigma}} = -\hat{H}^{(2,L)\rho\sigma}_{\mu\nu}(x_1,x_2,x_1) - \hat{H}^{(2,M)\rho\sigma}_{\mu\nu}(x_1,x_2,x_1), \tag{46}$$

$$p_\rho\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2-x_1-i\epsilon}, \tag{47}$$

$$p_\rho p_\sigma\hat{H}^{(2,L)\rho\sigma}_{\mu\nu}(x_1,x_2,x) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{(x_2-x_1-i\epsilon)(x-x_1-i\epsilon)}, \tag{48}$$

$$p_\rho\hat{H}^{(2,R)\rho\sigma}_{\mu\nu}(x_1,x_2,x) = \frac{\hat{H}^{(1,M)\sigma}_{\mu\nu}(x_1,x_2) - \hat{H}^{(1,M)\sigma}_{\mu\nu}(x,x_2)}{x-x_1-i\epsilon}, \tag{49}$$

Finally, we obtain similar results as, $W^{(si)}_{\mu\nu}(q,p,S,k') = \sum_{j=0,1,2}\tilde{W}^{(j,si)}_{\mu\nu}(q,p,S,k')$, and,

$$\tilde{W}^{(0,si)}_{\mu\nu} = \frac{1}{2\pi}\int\frac{d^4k}{(2\pi)^4}K(k',k,q)\mathrm{Tr}[\hat{H}^{(0)}_{\mu\nu}(x)\ \hat{\Phi}^{(0)}(k,p,S)]; \tag{50}$$

$$\tilde{W}^{(1,si)}_{\mu\nu} = \frac{1}{2\pi}\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\sum_{c=L,R}K(k',k_c,q)\mathrm{Tr}[\hat{H}^{(1,c)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'}\hat{\Phi}^{(1)}_{\rho'}(k_1,k_2,p,S)]; \tag{51}$$

$$\tilde{W}^{(2,si)}_{\mu\nu} = \frac{1}{2\pi}\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\sum_{c=L,R,M}K(k',k_c,q)\mathrm{Tr}[\hat{H}^{(2,c)\rho\sigma}_{\mu\nu}(x_1,x_2,x)\omega_\rho^{\ \rho'}\omega_\sigma^{\ \sigma'}\hat{\Phi}^{(2)}_{\rho'\sigma'}(k_1,k_2,k,p,S)]. \tag{52}$$

Because of the existence of the factor K, we can not carry out the integration over k^- and $\vec{k}_\perp$ as we did for inclusive cross section. We have to keep the un-integrated gauge invariant matrix elements $\hat{\Phi}$'s. They have received the contributions from all the three diagrams in Fig. 1, and are given by,

$$\hat{\Phi}^{(0)}(k,p,S) = \int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle, \tag{53}$$

$$\hat{\Phi}^{(1)}_{\rho}(k_1,k_2,p,S) = \int d^4y d^4z e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle, \tag{54}$$

$$\hat{\Phi}^{(2)}_{\rho\sigma}(k_1,k_2,k,p,S) = \int d^4y d^4y' d^4z e^{ik_1y+ik(y'-y)+ik_2(z-y')}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,y')D_\sigma(y')\mathcal{L}(y',z)\psi(z)|p,S\rangle,$$

where $\mathcal{L}(0,z)$ is the gauge link with transverse separation. $\mathcal{L}(0,z)$ is obtained explicitly from the derivation and it is a product of two parts, *i.e.*,

$$\mathcal{L}(0,z) = \mathcal{L}^\dagger(-\infty,0)\mathcal{L}(-\infty,z), \tag{55}$$

$$\mathcal{L}(-\infty,z) = 1 + ig\int_{-\infty}^{z^-} dy^- A^+(z^+,y^-,\vec{z}_\perp) + (ig)^2\int_{-\infty}^{z^-} dy^- A^+(z^+,y^-,\vec{z}_\perp)\int_{-\infty}^{y^-} dy'^- A^+(z^+,y'^-,\vec{z}_\perp), \tag{56}$$

Including higher order contributions in g, we have,

$$\mathcal{L}(-\infty,z) = Pe^{ig\int_{-\infty}^{z^-} dy^- n\cdot A(z^+,y^-,\vec{z}_\perp)}. \tag{57}$$

which is the path integral expression in the case that there is a transverse displacement $\vec{z}_\perp$ [29].

Eqs.(50-52) can be considered as a generalized factorization of transverse momentum dependent SIDIS as a result of the collinear expansion. Consequently, there are two distinct properties in the factorized form:

(A) All the hard parts in the Eqs. (50-52) are only functions of the longitudinal parton momenta. All the information of transverse momenta is contained in the matrix elements $\hat{\Phi}$'s.

(B) The operator $\omega_\rho^{\ \rho'}$ in $\tilde{W}^{(1,si)}_{\mu\nu}$ projects away the longitudinal components of the gauge fields that go into the gauge links.

We want to point out that the final result for the cross section is *not* simply a convolution of the *transverse momentum dependent* lepton-quark scattering cross section with the unintegrated (or $k_\perp$-dependent) quark distributions as in some of the existing literature. Such a convolution can result in double-counting of the effects of transverse momenta.

III. DIFFERENTIAL CROSS SECTION AND AZIMUTHAL ASYMMETRIES FOR SIDIS

The properties (A) and (B) mentioned above lead to a great simplification of $\tilde{W}^{(1,si)}_{\mu\nu}$. It can be shown that

$$H^{(1,L)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'} = \frac{\pi}{2q\cdot p}\delta(x_1-x_B)\omega_\rho^{\ \rho'}\gamma_\mu \not{n}\gamma^\rho \not{\bar{n}}\gamma_\nu \equiv \hat{H}^{(1)\rho'}_{\mu\nu}(x_1), \tag{58}$$

$$H^{(1,R)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'} = \frac{\pi}{2q\cdot p}\delta(x_2-x_B)\omega_\rho^{\ \rho'}\gamma_\mu \not{\bar{n}}\gamma^\rho \not{n}\gamma_\nu = \gamma_0\hat{H}^{(1)\rho'\dagger}_{\nu\mu}(x_2)\gamma_0, \tag{59}$$

is either independent of x_2 or independent of x_1. Hence, we can carry out the integration over d^4k_2 or d^4k_1 in $\hat{\Phi}^{(1)}_\rho(k_1,k_2,p,S)$ and obtain, for the $\mu\leftrightarrow\nu$ symmetric part,

$$\tilde{W}^{(1,si)}_{\mu\nu}(q,p,S,k') = \frac{1}{\pi}\mathrm{Re}\int\frac{d^4k}{(2\pi)^4}K(k',k)\mathrm{Tr}[\hat{H}^{(1)\rho}_{\mu\nu}(x)\hat{\varphi}^{(1)}_\rho(k,p,S)], \tag{60}$$

$$\hat{\varphi}^{(1)}_\rho(k,p,S) = \int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)D_\rho(z)\psi(z)|p,S\rangle, \tag{61}$$

The matrix element $\hat{\varphi}^{(1)}_\rho(k,p,S)$ depends only on one external parton momentum thus is much simpler than the unintegrated counterpart $\hat{\Phi}^{(1)}_\rho(k_1,k_2,p,S)$. This means that, in SIDIS, only $\hat{\varphi}^{(1)}_\rho(k,p,S)$ but not $\hat{\Phi}^{(1)}_\rho(k_1,k_2,p,S)$ is relevant.

To demonstrate the usefulness of the formalism, we now calculate the azimuthal asymmetries to the order of $1/Q$ in SIDIS with unpolarized electron and transversely polarized proton. In this case, we need to consider only the symmetric parts of $W_{\mu\nu}$ and should include the contributions from $\hat{W}^{(1,si)}_{\mu\nu}$. The calculations are in principle straightforward, but a bit involved. We need to first decompose the matrix elements in terms of the Γ-matrices. Since the hard parts contain odd-number of γ's, only the γ_α and $\gamma_5\gamma_\alpha$ terms contribute. We carry out the traces of γ_α and $\gamma_5\gamma_\alpha$ with the hard parts, make the Lorentz contraction with the leptonic tensor $L_{\mu\nu}$ and obtain the differential cross section as,

$$d\sigma = \frac{2\alpha_{em}^2 e_q^2}{Q^4}\frac{dx_B dQ^2 d^3k'}{(2\pi)^3 2E_{k'}}\int\frac{d^4k}{(2\pi)^4}K(k,k')\delta(x-x_B)\big\{\sigma^{(0)\alpha}(x)\Phi_\alpha^{(0)}(k,p,S) + 2\mathrm{Re}\big[\sigma^{(1)\alpha\rho}(x)\varphi^{(1)}_{\alpha\rho}(k,p,S)+\tilde\sigma^{(1)\alpha\rho}(x)\tilde\varphi^{(1)}_{\alpha\rho}(k,p,S)\big]\big\}; \tag{62}$$

where $\Phi_\alpha^{(0)}(k,p,S)=\mathrm{Tr}[\gamma_\alpha\hat\Phi^{(0)}(k,p,S)]/2$, $\varphi^{(1)}_{\alpha\rho}(k,p,S)=\mathrm{Tr}[\gamma_\alpha\hat\varphi^{(1)}_\rho(k,p,S)]/2$, $\tilde\varphi^{(1)}_{\alpha\rho}(k,p,S)=\mathrm{Tr}[\gamma_5\gamma_\alpha\hat\varphi^{(1)}_\rho(k,p,S)]/2$; and $\sigma^{(0)}(x)$, $\sigma^{(1)\alpha\rho}(x)$ and $\tilde\sigma^{(1)\alpha\rho}(x)$ stand for,

$$\begin{aligned}&\sigma^{(0)\alpha}(x) = A(y)q^-n^\alpha + 2(1-y)x_Bp^\alpha + (2-y)yl_\perp^\alpha,\\ &\sigma^{(1)\alpha\rho}(x) = -y^2(l+l')^\alpha l_\perp^\rho/Q^2, \qquad \tilde\sigma^{(1)\alpha\rho}(x) = -iy^2(l+l')^\alpha|\vec l_\perp|n^\rho_{\perp 2}/Q^2,\end{aligned} \tag{63}$$

where $A(y)=1+(1-y)^2$.

The Lorentz structure of $\Phi_\alpha^{(0)}(k,p,S)$ is given by [16, 30],

$$\Phi_\alpha^{(0)}(k,p,S) = p_\alpha f_1 + \omega_\alpha^{\ \alpha'}k_{\alpha'}f_\perp + \varepsilon_{\alpha\beta\gamma\delta}p^\beta k^\gamma S^\delta f_{1T}^\perp/M, \tag{64}$$

where M is the nucleon mass, the f's on the r.h.s. of the equations are Lorentz scalars and are functions of $k\cdot p$ and k^2. Note that by integrating $\Phi_\alpha^{(0)}$ over k^-, we obtain the $k_\perp$-dependent quark distributions $\Phi_\alpha^{(0)}(x,\vec k_\perp)$ and that $\int p^+dk^- f_{1T}^\perp$ is just the Sivers function[27] discussed in connection with single-spin asymmetries.

There are much more terms for the Lorentz structure of $\varphi^{(1)}_{\alpha\rho}$ and $\tilde\varphi^{(1)}_{\alpha\rho}$. We made a complete examination of them and found out that, to the order of $1/Q$, the contributing terms are,

$$\begin{aligned}\varphi^{(1)}_{\rho\alpha}(k,p,S) &= k_\rho p_\alpha\varphi_\perp^{(1)} + Mp_\alpha\varepsilon_{\perp\rho\delta}S^\delta\varphi^{(1)}_{\perp s1} + p_\alpha\varepsilon_{\perp\gamma\delta}(k_\rho k^\gamma - k_\perp^2 g_\rho^{\ \gamma})S^\delta\varphi^{(1)}_{\perp s2}/M + ..., \quad (65)\\ \tilde\varphi^{(1)}_{\rho\alpha}(k,p,S) &= ip_\alpha\varepsilon_{\perp\rho\gamma}k^\gamma\tilde\varphi^{(1)}_\perp + iMp_\alpha S_\rho\tilde\varphi^{(1)}_{\perp s1} + ip_\alpha\varepsilon_{\perp\rho\beta}\varepsilon_{\perp\gamma\delta}(k^\beta k^\gamma - k_\perp^2 g^{\beta\gamma})S^\delta\tilde\varphi^{(1)}_{\perp s2}/M + ...,\end{aligned}$$

where $\varepsilon_{\perp\rho\alpha}=\varepsilon_{\rho\alpha\gamma\delta}\bar n^\gamma n^\delta$, and all the φ's and $\tilde\varphi$'s on the r.h.s. of the equations are Lorentz scalars and functions of $k\cdot p$ and k^2.

Equation of motion relates the φ's and $\tilde\varphi$'s to f's as,

$$xf_\perp = -\varphi^{(1)}_\perp + \tilde\varphi^{(1)}_\perp, \qquad xp\cdot kf_{1T}^\perp = -M^2(\varphi^{(1)}_{\perp s1}+\tilde\varphi^{(1)}_{\perp s1}). \tag{66}$$

Using the above expansion of the parton matrix elements, one obtains that, to the order $1/Q$,

$$\begin{aligned}d\sigma \ =\ & \frac{2\alpha_{em}^2e_q^2}{Q^4}\frac{dx_BdQ^2p^+dk^-d^2k_\perp}{(2\pi)^4}\Big\{A(y)\big[f_1+\frac{|\vec k_\perp|}{M}f_{1T}^\perp\sin(\phi-\phi_s)\big] \quad (67)\\ &-\frac{MB(y)}{Q}\big[\frac{2|\vec k_\perp|}{M}(x_Bf_\perp - \tilde\varphi^{(1)}_\perp)\cos\phi + \frac{\vec k_\perp^2}{2M^2}(\varphi^{(1)}_{\perp s2}-3\tilde\varphi^{(1)}_{\perp s2})\sin\phi_s + \frac{\vec k_\perp^2}{2M^2}(\varphi^{(1)}_{\perp s2}+\tilde\varphi^{(1)}_{\perp s2})\sin(2\phi-\phi_s)\big]\Big\},\end{aligned}$$

where $B(y)=2(2-y)\sqrt{1-y}$, ϕ and ϕ_s are respectively the azimuthal angle of $\vec k_\perp$ and $\vec S_\perp$ relative to the lepton plane; $\vec k_\perp=\vec k'_\perp$, $k^-=k'^- - q^-$ and $k^+=x_Bp^+$.

From Eq.(67), one notes that the leading contribution to the azimuthal angle dependence comes only from the leading twist quark distribution $\Phi_\alpha^{(0)}$ while the higher order ($1/Q$) receives contribution from both the leading and the next leading twist parton matrix elements. In view that the energies of the current polarized experiments, in particular at HERMES [6], are not very high, these $1/Q$-terms can be very important.

The $\sin\phi_s$-terms in Eq.(67) warrants a special examination since they correspond to single-spin asymmetry. It can be seen more clearly from the cross section integrated over ϕ, i.e,

$$d\sigma = \frac{\alpha_{em}^2e_q^2}{Q^4}\frac{dx_BdQ^2p^+dk^-d\vec k_\perp^2}{(2\pi)^3}\Big\{A(y)f_1 - \frac{B(y)\vec k_\perp^2}{2MQ}(\varphi^{(1)}_{\perp s2}-3\tilde\varphi^{(1)}_{\perp s2})\sin\phi_s\Big\}. \tag{68}$$

The single-spin asymmetry A_N is the difference between the cross section at $\phi_s=\pi/2$ and that at $\phi_s=3\pi/2$ divided by their sum. We see that there exists a finite

A_N for SIDIS at a given k^- at the order M/Q. This corresponds to one of the asymmetries discussed in [17]. Such asymmetry can provide important information on the unintegrated parton correlations at given k^- and $k_\perp$. The value of k^- could be determined from $k^- = k'^- - q^-$ by measuring the momenta of the scattered lepton and single jet. This could be done at very high energies where one can reconstruct the single jet event. However it might be very difficult at lower energies.

If one integrates over k^- and $\vec{k}_\perp$ to obtain the inclusive cross section for $e^- p \to e^- X$, these $\sin\phi_s$-terms should vanish as demanded by the parity and time reversal invariance of the inclusive hadronic tensor. This implies that the integration of $\varphi^{(1)}_{\perp s}$ and $\tilde{\varphi}^{(1)}_{\perp s}$ over k^- and $k_\perp$ should vanish for unrestricted ranges of k^- and $k_\perp$, leading to zero single-spin asymmetry for inclusive DIS. However, if we integrate over k^- only for a restricted range in which the outgoing partons are time-like, *i.e.* for $0 < k'^- < \infty$, the result might be non-zero. In practice, this is equivalent to identifying a large momentum final hadron in the photon direction to guarantee that the outgoing parton is time-like. We call such events as "triggered inclusive process" and denotes it by $e^- p \to e^- + h_{trig} + X$. The averaged single-spin asymmetry could be finite for such triggered inclusive DIS.

At the hadron level, such a single-spin asymmetry corresponds to a spin-dependent term in the hadronic tensor $W^{(trig)}_{\mu\nu}(q, p, S)$, i.e.,

$$\frac{1}{2M} W^{(S,trig)}_{\mu\nu}(q,p,S) = \frac{1}{2p\cdot q}[\varepsilon_{\perp\mu\gamma}(q_\nu + 2x_B p_\nu) + \varepsilon_{\perp\nu\gamma}(q_\mu + 2x_B p_\mu)] S^\gamma G_s(x_B, Q^2). \quad (69)$$

and the single-spin spin asymmetry $A^{(trig)}_N$ is given by,

$$A^{(trig)}_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = B(y)\frac{x_B M}{Q}\frac{G_s}{F_1}, \quad (70)$$

where F_1 is the normal spin averaged structure function. In terms of the parton correlations discussed above, G_s is given by,

$$x_B G_s = -\int \frac{d^4k}{(2\pi)^4}\delta(x - x_B)\frac{\vec{k}^2_\perp}{2M^2}(\varphi^{(1)}_{\perp s2} - 3\tilde{\varphi}^{(1)}_{\perp s2}). \quad (71)$$

The integration limit for k^- is $-q^- < k^- < \infty$. Experimental measurements of the above averaged single-spin asymmetry in the triggered DIS would lead to useful information on the spin structure of nucleon.

If we neglect the final-state soft gluon interactions, *i.e.* set $g = 0$, the covariant derivatives in Eq. (61) become normal derivatives. We then have $\hat{\varphi}^{(1)}_\rho = -k_\rho \hat{\phi}^{(0)}$, and $-\varphi^{(1)}_\perp = f_1 = x f_\perp$, $\varphi^{(1)}_{\perp s1} = \varphi^{(1)}_{\perp s2} = 0$, $\tilde{\varphi}^{(1)}_\perp = \tilde{\varphi}^{(1)}_{\perp s1} = \tilde{\varphi}^{(1)}_{\perp s2} = 0$. Hence, only $A(y) f_1 + 2B(y)(|\vec{k}_\perp|/Q) f_1 \cos\phi$ are left in Eq.(67) which is the result in Ref. [10].[31] The difference between this and the full result given by Eq.(67) comes from the final-state soft gluon interactions. Data on $\langle \cos\phi \rangle$ obtained in unpolarized experiments[1, 3, 4] suggest the existence of such QCD contributions. It is also obvious that fragmentation can contribute to the azimuthal asymmetries discussed above. A complete formalism for SIDIS should include fragmentation. Such an extension is underway.

IV. SUMMARY

In summary, we derived the factorized form of the cross section for semi-inclusive deep-inelastic lepton-nucleon scattering as a convolution of the hard parts with the gauge invariant unintegrated parton distributions and higher twist parton correlations. As a consequence of the collinear expansions, the hard parts depend only on the longitudinal components of the parton momenta. The gauge invariant parton distributions receive contributions from initial and final state interactions and provide the only dependence on the initial parton transverse momentum. Results for azimuthal angle dependence to the order of $1/Q$ in reactions with transversely polarized targets are given. A novel single-spin asymmetry for the "triggered inclusive process" $e^- p^\uparrow \to e^- + h_{trig} + X$ is predicted to the order of M/Q.

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