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Parallel H1-based auxiliary space AMG solver for H(curl) problems

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PARALLEL $\boldsymbol{H}^1$-BASED AUXILIARY SPACE AMG SOLVER FOR $\boldsymbol{H}(\mathrm{curl})$ PROBLEMS

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Abstract. This report describes a parallel implementation of the auxiliary space methods for definite Maxwell problems proposed in [4]. The solver, named AMS, extends our previous study [7]. AMS uses ParCSR sparse matrix storage and the parallel AMG (algebraic multigrid) solver BoomerAMG [1] from the *hypre* library. It is designed for general unstructured finite element discretizations of (semi)definite $\boldsymbol{H}(\mathrm{curl})$) problems discretized by Nédélec elements. We document the usage of AMS and illustrate its parallel scalability and overall performance.

1. Introduction

Based on a proper stable decomposition of the lowest order Nédélec (edge) finite element space, Hiptmair and Xu introduced in [4] an auxiliary space preconditioner for $\boldsymbol{H}(\mathrm{curl})$ problems, which in addition to standard smoothing utilizes efficient auxiliary space solvers for related Poisson problems. Their method borrows the main tool from the auxiliary mesh preconditioners in [6] and [3], namely, the interpolation operator $\boldsymbol{\Pi}$, which maps vector fields into the the Nédélec finite element space $\boldsymbol{V}_h$. Motivated by these developments, we considered several options for constructing unstructured mesh AMG preconditioners for $\boldsymbol{H}(\mathrm{curl})$-problems and reported the computational results in [7]. The present report describes a parallel implementation of these methods in the *hypre* library.

We are interested in solving the following variational problem stemming from Maxwell's equations:

$$\text{Find } \mathbf{u} \in \boldsymbol{V}_h \ : \qquad (\alpha \, \mathrm{curl}\, \mathbf{u}, \mathrm{curl}\, \mathbf{v}) + (\beta\, \mathbf{u}, \mathbf{v}) = (\mathbf{f}, \mathbf{v})\,, \qquad \text{for all } \mathbf{v} \in \boldsymbol{V}_h\,. \tag{1.1}$$

Here $\alpha > 0$ and $\beta \geq 0$ are scalar coefficients. We allow β to be zero in part or the whole domain. In either case the resulting matrix is only semidefinite, and for solvability the right-hand side should be chosen to satisfy compatibility conditions.

Let $\boldsymbol{A}$ and $\mathbf{b}$ be the stiffness matrix and the load vector corresponding to (1.1). Then the resulting linear system of interest reads,

$$\boldsymbol{A}\,\mathbf{x} = \mathbf{b}\,. \tag{1.2}$$

Note that the coefficients α and β are naturally associated with the "stiffness" and "mass" terms of $\boldsymbol{A}$.

Besides $\boldsymbol{A}$ and $\mathbf{b}$, our solution method for (1.2) requires the following additional information:

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(1) The discrete gradient matrix G representing the edges of the mesh in terms of its vertices. G has as many rows as the number of edges in the mesh, with each row having two nonzero entries: $+1$ and -1 in the columns corresponding to the vertices composing the edge. The sign is determined based on the orientation of the edge. We require that G includes all (interior and boundary) edges and vertices.
(2) The representations of the constant vector fields $(1,0,0)$,$(0,1,0)$ and $(0,0,1)$ in the $\boldsymbol{V}_h$ basis, given as three vectors: G_x, G_y, and G_z. Note that since no boundary conditions are imposed on G, we can compute the above vectors ad $G_x = G_h x$, $G_y = G_h y$ and $G_z = G_h z$, where x, y, and z are vectors representing the coordinates of the vertices of the mesh.

In addition to the above quantities, our solver can utilize the following (optional) information:

(3) The Poisson matrices A_α and A_β, corresponding to assembling of the forms $(\alpha\nabla u, \nabla v) + (\beta u, v)$ and $(\beta\nabla u, \nabla v)$ using standard linear finite elements on the same mesh.

As discussed in [7], based on the above quantities we proceed with the construction of the following additional objects:

- A_G – either the matrix $G^T\boldsymbol{A}G$ or the Poisson matrix A_β (if given).
- B_G – an efficient solver for A_G.
- $\boldsymbol{\Pi}$ – the matrix representation of the interpolation operator from vector linear to edge finite elements.
- $\boldsymbol{A_\Pi}$ – either the matrix $\boldsymbol{\Pi}^T\boldsymbol{A}\boldsymbol{\Pi}$ or the block-diagonal matrix $\mathrm{diag}(A_\alpha)$ (if given).
- $\boldsymbol{B_\Pi}$ – an efficient solver for $\boldsymbol{A_\Pi}$.

The solution procedure then is a three-level method using smoothing in the original edge space and subspace corrections based on B_G and $\boldsymbol{B_\Pi}$. We can employ a number of options here utilizing various combinations of the smoother and solvers in additive or multiplicative fashion. If β is identically zero one can skip the subspace correction associated with G, in which case the solver is a two-level method.

The remainder of the report is organized as follows. The parallel version of the solver, as implemented in the *hypre* library is described in Section 2. Section 3 contains a set of numerical experiments demonstrating the parallel scalability and overall performance of the method. Finally, we summarize the results in the concluding Section 4.

2. The AMS solver in *hypre*

In this section we discuss the parallel implementation of the $\boldsymbol{H}^1$-based auxiliary space method in the *hypre* library under the name AMS (Auxiliary space Maxwell Solver). AMS can be used either as a solver or as a preconditioner. We concentrate here on the sequence of *hypre* calls needed to create and use it as a solver. We start with the allocation of the `HYPRE_Solver` object:

```
HYPRE_Solver solver;
HYPRE_AMSCreate(&solver);
```

Next, we set a number of solver parameters. Some of them are optional, while others are necessary in order to perform the solver setup.

AMS offers the option to set the space dimension. By default we consider the dimension to be 3. The only other option is 2, and it can be set with the function given below. We note that a 3D solver will still work for a 2D problem, but it will be slower and will require more memory than necessary.

```
HYPRE_AMSSetDimension(solver, dim);
```

The user is required to provide the discrete gradient matrix G. AMS expects a matrix defined on the whole mesh with no boundary edges/nodes excluded. It is essential to **not** impose any boundary conditions on G. Regardless of which *hypre* conceptual interface was used to construct G, one can always obtain a ParCSR version of it. This is the expected format in the following function.

```
HYPRE_AMSSetDiscreteGradient(solver, G);
```

In addition to G, we need one additional piece of information in order to construct the solver. The user has the option to choose either the coordinates of the vertices in the mesh or the representations of the constant vector fields in the edge element basis. In both cases three *hypre* parallel vectors should be provided. For 2D problems, the user can set the third vector to NULL. The corresponding function calls read:

```
HYPRE_AMSSetCoordinateVectors(solver,x,y,z);
```

or

```
HYPRE_AMSSetEdgeConstantVectors(solver,
                                one_zero_zero,
                                zero_one_zero,
                                zero_zero_one);
```

The vectors `one_zero_zero`, `zero_one_zero` and `zero_zero_one` above correspond to the constant vector fields $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$.

The remaining solver parameters are optional. For example, the user can choose a different cycle type by calling

```
HYPRE_AMSSetCycleType(solver, cycle_type); /* default value: 1 */
```

The available cycle types in AMS are:

- `cycle_type=1`: multiplicative solver (01210)
- `cycle_type=2`: additive solver $(0+1+2)$
- `cycle_type=3`: multiplicative solver (02120)
- `cycle_type=4`: additive solver $(010+2)$
- `cycle_type=5`: multiplicative solver (0102010)
- `cycle_type=6`: additive solver $(1+020)$
- `cycle_type=7`: multiplicative solver (0201020)
- `cycle_type=8`: additive solver $(010+020)$

Here we use the following convention for the three subspace correction methods: 0 refers to smoothing, 1 stands for BoomerAMG based on B_G, and 2 refers to a call to BoomerAMG for $\boldsymbol{B_\Pi}$. The abbreviation $xyyz$ for $x,y,z \in \{0,1,2\}$ refers to a multiplicative subspace correction based on solvers x, y, y, and z (in that order). The abbreviation $x+y+z$ stands for an additive subspace correction method based on x, y and z solvers. The additive cycles are meant to be used only when AMS is called as a preconditioner. In our experience the choices `cycle_type=1` and `cycle_type=5` often produced fastest solution times, while `cycle_type=7` resulted in smallest number of iterations.

Additional solver parameters, as the maximum number of iterations, the convergence tolerance and the output level, can be set with

```
HYPRE_AMSSetMaxIter(solver, maxit);     /* default value: 20 */
HYPRE_AMSSetTol(solver, tol);           /* default value: 1e-6 */
HYPRE_AMSSetPrintLevel(solver, print);  /* default value: 1 */
```

More advanced parameters, affecting the smoothing and the internal AMG solvers, can be set with the following three functions:

```
HYPRE_AMSSetSmoothingOptions(solver, 2, 1, 1.0, 1.0);
HYPRE_AMSSetAlphaAMGOptions(solver, 10, 1, 3, 0.25);
HYPRE_AMSSetBetaAMGOptions(solver, 10, 1, 3, 0.25);
```

For (singular) problems where $\beta = 0$ in the whole domain, different (in fact simpler) version of the AMS solver is offered. To allow for this simplification, use the following *hypre* call

```
HYPRE_AMSSetBetaPoissonMatrix(solver, NULL);
```

If β is zero only in parts of the domain, the problem is still singular, but the AMS solver will try to detect this and construct a non-singular preconditioner.

Two additional matrices are constructed in the setup of the AMS method—one corresponding to the coefficient α and another corresponding to β. This may lead to prohibitively high memory requirements, and the next two function calls may help to save some memory. For example, if the Poisson matrix with coefficient β (denoted by `Abeta`) is available then one can avoid one matrix construction by calling

```
HYPRE_AMSSetBetaPoissonMatrix(solver, Abeta);
```

Similarly, if the Poisson matrix with coefficient α is available (denoted by `Aalpha`) the second matrix construction can also be avoided by calling

```
HYPRE_AMSSetAlphaPoissonMatrix(solver, Aalpha);
```

Note the following regarding these functions:

- Both of them change their input. More specifically, the diagonal entries of the input matrix corresponding to eliminated degrees of freedom (due to essential boundary conditions) are penalized.
- `HYPRE_AMSSetAlphaPoissonMatrix` forces the AMS method to use a simpler, but weaker (in terms of convergence) method. With this option, the multiplicative AMS cycle is not guaranteed not converge with the default parameters. The reason for this is the fact the solver is not variationally obtained from the original matrix (it utilizes the auxiliary Poisson–like matrices `Abeta` and `Aalpha`). Therefore, it is recommended in this case to use AMS as preconditioner only.

After the above calls, the solver is ready to be constructed. The user has to provide the stiffness matrix $\boldsymbol{A}$ (in ParCSR format) and the *hypre* parallel vectors $\mathbf{b}$ and $\mathbf{x}$. (The vectors are actually not used in the current AMS setup.) The setup call reads,

```
HYPRE_AMSSetup(solver, A, b, x);
```

It is important to note the order of the calling sequence. For example, do **not** call `HYPRE_AMSSetup` before calling `HYPRE_AMSSetDiscreteGradient` and one of the functions `HYPRE_AMSSetCoordinateVectors` or `HYPRE_AMSSetEdgeConstantVectors`.
Once the setup has completed, we can solve the linear system by calling

```
HYPRE_AMSSolve(solver, A, b, x);
```

Finally, the solver can be destroyed with

```
HYPRE_AMSDestroy(&solver);
```

More details can be found in the files `ams.h` and `ams.c` located in the `parcsr_ls` directory of *hypre* .

3. Numerical experiments

In this section we present results from numerical experiments with AMS used as a preconditioner in PCG. We consider two setups: one using the default solver parameters, and one with an alternative set of parameters. The default solver parameters were already described in the preceding section, but for completeness we give here their corresponding sequence of *hypre* calls (it is assumed that the user has installed the *hypre* library [5]):

```
#include "parcsr_ls.h"

HYPRE_ParCSRMatrix A;
HYPRE_ParVector b, x;
HYPRE_ParCSRMatrix G;
HYPRE_ParVector one_zero_zero, zero_one_zero, zero_zero_one;

/* Declare solver and preconditioner objects */
HYPRE_Solver solver, precond;

/* Create PCG solver */
HYPRE_ParCSRPCGCreate(MPI_COMM_WORLD, &solver);
HYPRE_PCGSetTol(solver, 1e-6); /* convergence tolerance */

/* Create AMS preconditioner */
HYPRE_AMSCreate(&precond);

HYPRE_AMSSetMaxIter(precond, 1); /* do exactly one iteration */
HYPRE_AMSSetTol(precond, 0.0);
HYPRE_AMSSetPrintLevel(precond, 0);

HYPRE_AMSSetDiscreteGradient(precond, G);
HYPRE_AMSSetEdgeConstantVectors(precond,
                                one_zero_zero,
                                zero_one_zero,
                                zero_zero_one);
if (beta_is_identically_zero)
   HYPRE_AMSSetBetaPoissonMatrix(precond, NULL);

HYPRE_AMSSetCycleType(precond, 1);
HYPRE_AMSSetSmoothingOptions(precond, 2, 1, 1.0, 1.0);
HYPRE_AMSSetAlphaAMGOptions(precond, 10, 1, 3, 0.25);
HYPRE_AMSSetBetaAMGOptions(precond, 10, 1, 3, 0.25);

/* Set preconditioner */
```

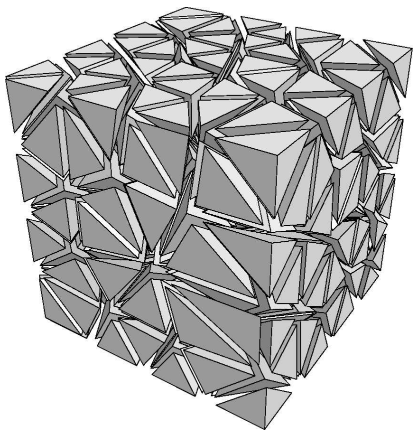


FIGURE 1. Initial unstructured tetrahedral mesh on the unit cube.

```
HYPRE_PCGSetPrecond(solver,
                    (HYPRE_PtrToSolverFcn) HYPRE_AMSSolve,
                    (HYPRE_PtrToSolverFcn) HYPRE_AMSSetup,
                    precond);

/* Setup */
HYPRE_ParCSRPCGSetup(solver, A, b, x);

/* Solve */
HYPRE_ParCSRPCGSolve(solver, A, b, x);
```

The alternative set of parameters differs only in the following calls:

```
HYPRE_AMSSetCycleType(precond, 7);
HYPRE_AMSSetAlphaAMGOptions(precond, 6, 0, 6, 0.25);
HYPRE_AMSSetBetaAMGOptions(precond, 6, 0, 6, 0.25);
```

As seen from above, in both cases the relative tolerance in PCG was 10^{-6}. The input matrices and vectors were constructed in parallel using the unstructured finite element package aFEM. In our experiments, we tried to keep the problem size per processor approximately the same (while increasing the number of processors), although the resulting load balance varied somewhat with the number of processors.

We tested both versions of AMS on problems with constant and variable coefficients and recorded the results using the following notation:

- np denotes the number of processors in the run,
- N is the total size of the problem,
- n_{it} is the number of PCG iterations,
- t_{setup}, t_{solve} and t denote the average times (in seconds) needed for setup, solve and time to solution (setup and solve), respectively, on a machine with 2.4GHz Xeon processors.

3.1. **Constant coefficients.** First we consider a simple constant coefficients problem with $\alpha = \beta = 1$. The domain is the unit cube meshed with an unstructured tetrahedral mesh. The initial coarse mesh, before serial or parallel refinement is shown in Figure 1.

	AMS				AMG			
np	N	n_{it}	t_{setup}	t_{solve}	N	n_{it}	t_{setup}	t_{solve}
`default solver parameters`								
1	105,877	11	2.5s	4.9s	17,478	13	0.1s	0.3s
2	184,820	12	3.5s	8.4s	29,059	14	0.2s	0.5s
4	293,224	13	2.9s	6.9s	43,881	15	0.1s	0.4s
8	697,618	14	4.2s	9.7s	110,745	18	0.2s	0.7s
16	1,414,371	16	4.6s	11.0s	225,102	18	0.3s	0.8s
32	2,305,232	16	3.9s	9.7s	337,105	20	0.4s	0.9s
64	5,040,829	18	5.2s	12.8s	779,539	22	0.5s	1.4s
128	10,383,148	19	6.5s	15.9s	1,682,661	23	0.8s	1.8s
256	18,280,864	21	7.3s	17.0s	2,642,337	25	1.1s	2.1s
512	38,367,625	23	9.0s	22.0s	5,845,443	28	1.7s	2.8s
1024	78,909,936	25	17.9s	33.2s	12,923,121	30	4.0s	4.8s
`alternative solver parameters`								
1	105,877	4	5.5s	8.8s	17,478	6	0.4s	0.4s
2	184,820	4	9.8s	13.5s	29,059	7	0.7s	0.7s
4	293,224	4	10.5s	10.8s	43,881	7	1.2s	0.8s
8	697,618	5	21.2s	18.3s	110,745	7	2.1s	1.1s
16	1,414,371	5	38.0s	20.5s	225,102	7	3.9s	1.3s
32	2,305,232	5	53.8s	19.4s	337,105	8	7.3s	1.9s
64	5,040,829	6	79.3s	25.3s	779,539	8	10.7s	2.5s

TABLE 1. Numerical results for the problem with constant coefficients ($\alpha = \beta = 1$) on a cube.

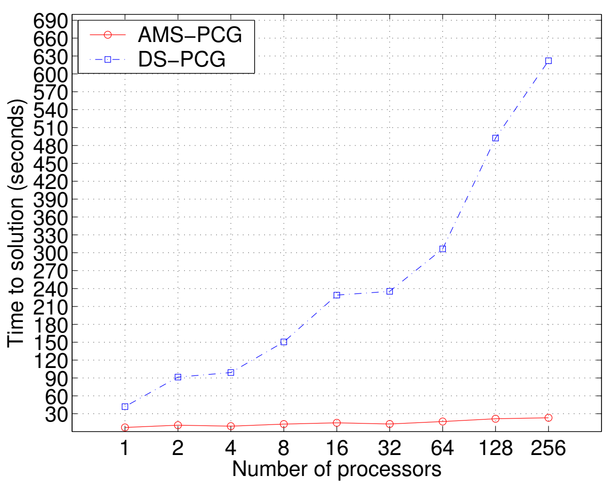
To better assess the quality of the method, we compare the AMS preconditioner with the BoomerAMG preconditioner in *hypre* applied to a Laplace problem discretized with linear finite elements on the same mesh. The results listed in Table 1 show that the behavior of AMS is qualitatively similar to that of BoomerAMG. This trend, observed in all our experiments, is to be expected since BoomerAMG is used in the auxiliary space solves of the AMS solver.

With the default parameters, the number of AMS iterations increases slightly, but the total run time grows slowly and remains less than a minute. The alternative set of solver parameters gives constant number of iterations, but the setup time increases (almost linearly) with the number of processors. The differences between the two sets of parameters are further investigated in Table 2, where we see that the default parameters result in bounded operator complexities for the two internal AMG methods, whereas the alternative parameters lead to high operator complexities. For the definition of grid and operator complexities, and their influence on AMG performance, we refer to [1].

The scalability of the AMS preconditioner is clearly demonstrated in Figure 2, where we compare it with a diagonally scaled PCG solver. We only include results for up to 256 processors, since on 512 processors (around 38M unknowns) the diagonally scaled PCG did not converge in $100,000$ iterations.

np	N	n_{it}	t_{setup}	t_{solve}	$grid_G$	op_G	$grid_{\boldsymbol{\Pi}}$	$op_{\boldsymbol{\Pi}}$
`default solver parameters`								
1	105,877	11	2.5s	4.9s	1.04	1.08	1.05	1.10
2	184,820	12	3.5s	8.4s	1.04	1.08	1.05	1.10
4	293,224	13	2.9s	6.9s	1.04	1.08	1.05	1.10
8	697,618	14	4.2s	9.7s	1.04	1.09	1.05	1.10
16	1,414,371	16	4.6s	11.0s	1.04	1.10	1.06	1.12
32	2,305,232	16	3.9s	9.7s	1.05	1.11	1.06	1.13
64	5,040,829	18	5.2s	12.8s	1.04	1.08	1.04	1.10
128	10,383,148	19	6.5s	15.9s	1.05	1.12	1.06	1.15
256	18,280,864	21	7.3s	17.0s	1.06	1.13	1.07	1.16
512	38,367,625	23	8.7s	20.6s	1.04	1.09	1.04	1.10
1024	78,909,936	25	17.9s	33.2s	1.05	1.12	1.06	1.17
`alternative solver parameters`								
1	105,877	4	5.5s	8.8s	1.79	4.70	1.79	4.45
2	184,820	4	9.8s	13.5s	1.80	4.86	1.83	4.96
4	293,224	4	10.5s	10.8s	1.83	5.77	1.77	4.83
8	697,618	5	21.2s	18.3s	1.89	6.67	1.82	5.94
16	1,414,371	5	38.0s	20.5s	1.92	7.31	1.91	6.82
32	2,305,232	5	53.8s	19.4s	1.97	8.53	1.89	6.80
64	5,040,829	6	79.3s	25.3s	1.80	7.35	1.75	6.44

TABLE 2. Grid and operator complexities for the subspace solvers in the problem from Table 1.



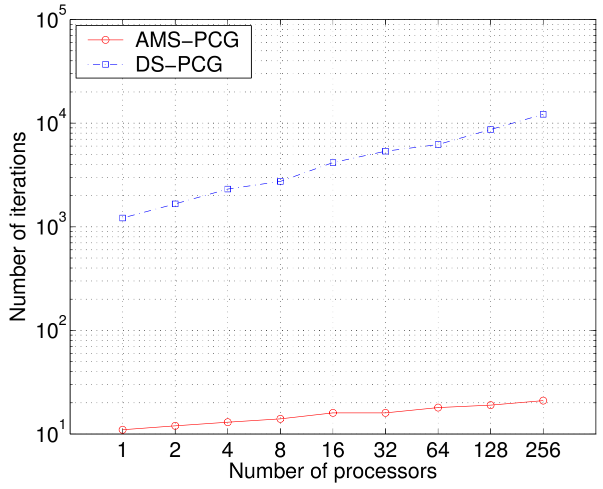
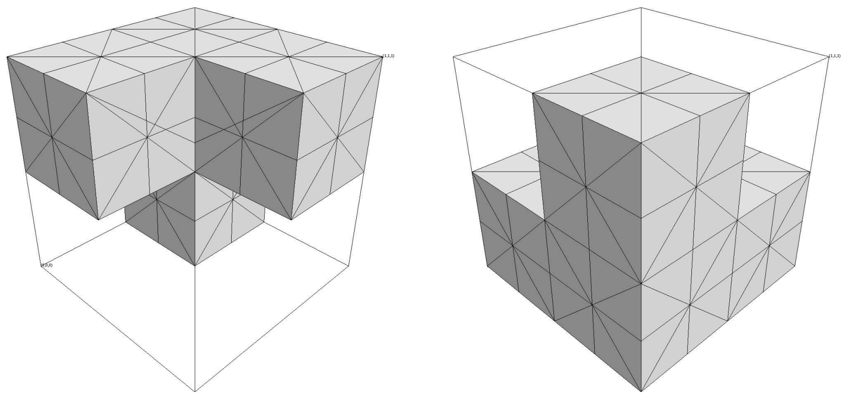


FIGURE 2. Time to solution (left) and number of iterations (right) for the constant coefficients problem on a cube: AMS versus Jacobi preconditioners.

3.2. **Variable coefficients.** Next we consider a problem where α and β have different values in two regions of the domain. The geometry and the results with the two sets of parameters are presented in Tables 3–4. Note that this particular test problem was reported to be problematic for geometric multigrid in [2].



np	N	p									t
		-8	-4	-2	-1	0	1	2	4	8	
`default solver parameters`											
$\alpha = 1, \beta \in \{1, 10^p\}$											
1	83,278	9	9	9	9	9	9	10	11	11	5s
2	161,056	10	10	10	10	10	10	10	11	11	9s
4	296,032	11	12	12	12	11	11	12	13	13	9s
8	622,030	13	13	13	12	12	12	13	15	14	12s
16	1,249,272	13	13	13	13	13	13	13	15	14	13s
32	2,330,816	15	15	15	15	15	15	15	16	15	14s
64	4,810,140	16	16	16	16	16	15	16	18	17	17s
128	9,710,856	16	16	16	16	16	16	16	17	17	23s
256	18,497,920	19	19	19	19	19	19	19	21	20	27s
512	37,864,880	21	20	20	20	20	20	20	23	22	32s
1024	76,343,920	20	20	20	20	20	20	20	21	21	56s
$\beta = 1, \alpha \in \{1, 10^p\}$											
1	83,278	10	10	11	10	9	11	12	13	13	6s
2	161,056	10	10	11	10	10	11	12	12	12	10s
4	296,032	11	11	13	12	11	13	14	15	15	11s
8	622,030	13	13	15	14	12	14	16	16	16	14s
16	1,249,272	13	13	14	14	13	14	15	16	16	15s
32	2,330,816	14	15	16	16	15	17	17	18	18	16s
64	4,810,140	16	17	18	17	16	18	19	19	20	20s
128	9,710,856	14	17	17	17	16	18	18	18	19	26s
256	18,497,920	17	19	20	20	19	21	21	22	22	29s
512	37,864,880	19	20	22	22	20	23	24	24	25	36s
1024	76,343,920	17	20	21	21	20	22	23	22	23	76s

TABLE 3. Number of iterations for the problem on a cube with α and β having different values in the shown regions (cf. [2]).

We observe that the number of iterations is not very sensitive to the magnitude of the jumps in the coefficients. Again, the default parameters lead to overall fastest solution times. For example, we solved a problem with more than 76 million unknowns and 8 orders of magnitude jumps in the coefficients in less than a minute. In contrast, the alternative set of parameters gives almost constant number of iterations, independent of α, β and the problem size, but its overall run time increases similarly to the previous example.

np	N	p									t
		-8	-4	-2	-1	0	1	2	4	8	
`alternative solver parameters`											
$\alpha = 1, \beta \in \{1, 10^p\}$											
1	83,278	5	4	4	4	4	4	4	5	5	12s
2	161,056	5	4	4	4	4	4	4	5	5	22s
4	296,032	5	5	5	5	5	4	4	5	5	18s
8	622,030	6	5	5	5	5	5	5	5	5	41s
16	1,249,272	6	5	5	5	5	5	5	5	6	59s
32	2,330,816	6	6	6	6	6	6	6	6	6	44s
64	4,810,140	7	5	5	5	5	5	5	6	6	99s
128	9,710,856	6	6	5	6	6	5	6	6	6	184s
$\beta = 1, \alpha \in \{1, 10^p\}$											
1	83,278	8	7	5	4	4	4	4	5	5	10s
2	161,056	8	8	6	4	4	4	5	5	5	20s
4	296,032	8	8	5	5	5	5	5	5	5	18s
8	622,030	9	8	6	5	5	5	5	5	5	36s
16	1,249,272	9	8	6	5	5	5	6	6	6	52s
32	2,330,816	9	8	6	6	6	6	6	6	6	45s
64	4,810,140	9	9	6	5	5	6	6	6	6	80s
128	9,710,856	9	9	7	6	6	6	6	6	6	164s

Table 4. Results with the alternative solver parameters for the problem from Table 3.

3.3. **Singular problems.** Finally, we report results on problems where β is identically zero in the domain (Table 5) or in part of it (Table 6). In both cases $\alpha = 1$.

The iteration counts in Table 5 are very similar to those in Table 1. As mentioned earlier, when $\beta = 0$ the solver reduces to a two-level method, which results in the smaller setup and solution times compared to Table 1.

When β is zero only in part of the domain, we can not use the simpler two-level method, but the results shown in Table 6 are still very good and comparable with the previous experiments.

4. Conclusion

The AMS preconditioner in *hypre* is a scalable solver for definite (and semidefinite) Maxwell problems discretized with the lowest order edge finite elements. It requires some

	AMS				AMG			
np	N	n_{it}	t_{setup}	t_{solve}	N	n_{it}	t_{setup}	t_{solve}
`default solver parameters`								
1	105,877	11	2.0s	3.7s	17,478	13	0.1s	0.3s
2	184,820	12	2.8s	6.1s	29,059	14	0.2s	0.5s
4	293,224	13	2.3s	5.1s	43,881	15	0.1s	0.4s
8	697,618	15	3.3s	7.3s	110,745	18	0.2s	0.7s
16	1,414,371	16	3.8s	7.9s	225,102	18	0.3s	0.8s
32	2,305,232	17	3.3s	7.0s	337,105	20	0.4s	0.9s
64	5,040,829	19	4.5s	9.2s	779,539	22	0.5s	1.4s
128	10,383,148	20	5.4s	11.1s	1,682,661	23	0.8s	1.8s
256	18,280,864	23	5.9s	13.1s	2,642,337	25	1.1s	2.1s
512	38,367,625	24	10.4s	14.9s	5,845,443	28	1.7s	2.8s
1024	78,909,936	26	10.8s	21.6s	12,923,121	30	4.0s	4.8s
`alternative solver parameters`								
1	105,877	5	4.8s	4.8s	17,478	6	0.4s	0.4s
2	184,820	6	8.4s	8.7s	29,059	7	0.7s	0.7s
4	293,224	7	9.1s	7.7s	43,881	7	1.2s	0.8s
8	697,618	6	18.4s	9.7s	110,745	7	2.1s	1.1s
16	1,414,371	7	33.4s	12.4s	225,102	7	3.9s	1.3s
32	2,305,232	8	46.1s	12.5s	337,105	8	7.3s	1.9s
64	5,040,829	7	70.3s	13.1s	779,539	8	10.7s	2.5s

Table 5. Numerical results for the singular problem on a cube with $\alpha = 1$ and $\beta = 0$. The geometry is the same as in Figure 1.

additional user input: the discrete gradient matrix and the coordinates of the vertices, but it can handle both variable coefficients and singular problems with zero conductivity. In fact, our experiments demonstrate that the performance of AMS on such problems is very similar to that on problems with $\alpha = \beta = 1$.

The general conclusion from the numerical results is that with its default parameters, AMS can be quite scalable on hundreds of processors. The alternative set of parameters usually needs significantly less (and constant) number of iterations but its total running time is not as scalable.

The behavior of AMS on $\boldsymbol{H}(\mathrm{curl})$) problems is qualitatively similar to that of BoomerAMG on Laplace problems discretized on the same mesh. Thus, any further improvements in BoomerAMG's parameters, or in AMG in general, will lead to additional improvements in AMS.

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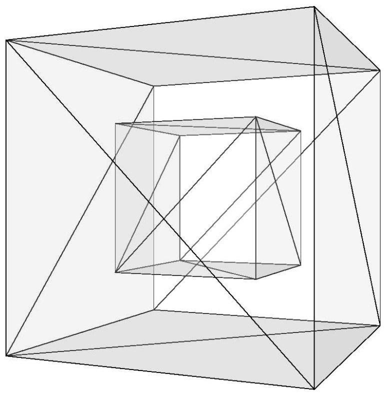

np	N	n_{it}	t_{setup}	t_{solve}
`default solver parameters`				
1	90,496	11	2.0s	4.7s
2	146,055	12	2.7s	7.3s
4	226,984	13	2.3s	6.3s
8	569,578	15	3.7s	9.3s
16	1,100,033	16	3.9s	10.8s
32	1,806,160	17	3.4s	9.1s
64	4,049,344	19	5.1s	12.8s
128	8,123,586	20	6.7s	15.8s
256	14,411,424	22	6.5s	17.2s
512	30,685,829	25	9.5s	22.6s
1024	61,933,284	24	19.5s	44.8s
`alternative solver parameters`				
2	146,055	6	7.9s	14.6s
4	226,984	6	8.4s	11.5s
8	569,578	6	28.4s	21.7s
16	1,100,033	7	35.8s	24.0s
32	1,806,160	7	43.5s	18.5s
64	4,049,344	7	91.7s	31.3s

TABLE 6. Initial mesh and numerical results for the problem on a cube with $\alpha = 1$ and β equal to 1 inside and 0 outside the interior cube.

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