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Application of Morse Theory to Analysis of Rayleigh-Taylor Topology

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Abstract: We present a novel Morse Theory approach for the analysis of the complex topology of the Rayleigh-Taylor mixing layer. We automatically extract bubble structures at multiple scales and identify the resolution of interest. Quantitative analysis of bubble counts over time highlights distinct mixing trends for a high-resolution Direct Numerical Simulation (DNS) [1].

1 THE MORSE COMPLEX

We partition the mixing layer from a DNS of Rayleigh-Taylor instability [1] using a fundamental topological construct called a Morse complex. Given a function F(x) on a surface S (see Fig. 1.1), the Morse complex partitions S into regions in which all steepest ascending lines end at a single maximum. The domain S for this application is the levelset *density = 2.98.* The density in the problem ranges from 1.0 to 3.0. The function F(x) is the axis opposite to gravity.

The resulting Morse complex is a patchwork depiction of the isosurface in which each patch is assigned a color (Fig. 1.1, center), and the feature in the middle of the patch may be considered a bubble. For more complicated surface structure, as occurs later in the development of the flow, the structures are less distinct. The surface exhibits bumps upon bumps, and the concept of a simple bubble is not applicable.

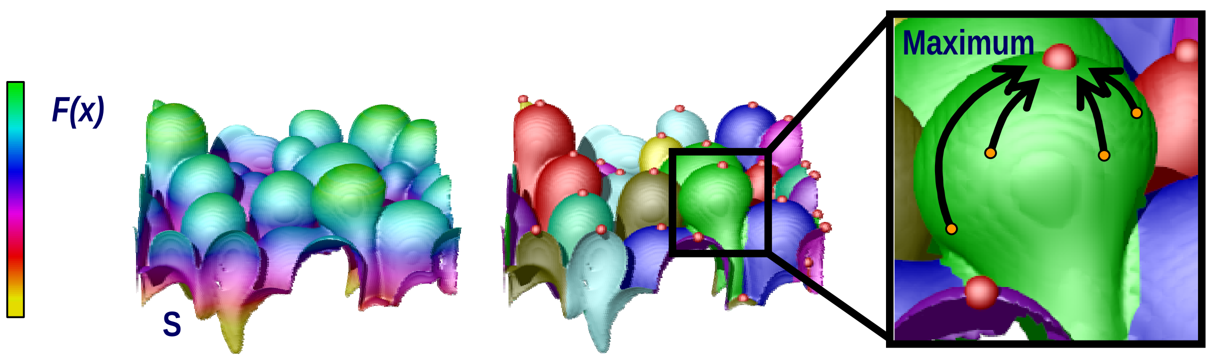

Fig. 1.1. F(x), indicated by the color bar, shown on a surface S (left); Morse complex cells indicated by distinct colors (center); in each Morse complex cell, all steepest ascending lines converge to one maximum (right).

2 TOPOLOGICAL PERSISTENCE

To handle the additional late-time (multi-scale) structure within a bubble framework, we incorporate the concept of *topological persistence*. The topological features are ranked by persistence, the difference in F(x) between a maximum and an adjacent saddle. Persistences *p1* and *p2* of two features are shown below (Fig. 2.1). We remove lower persistence features by contracting pairs of adjacent maxima into one. The corresponding Morse cells are merged to form a coarser partitioning of the domain.

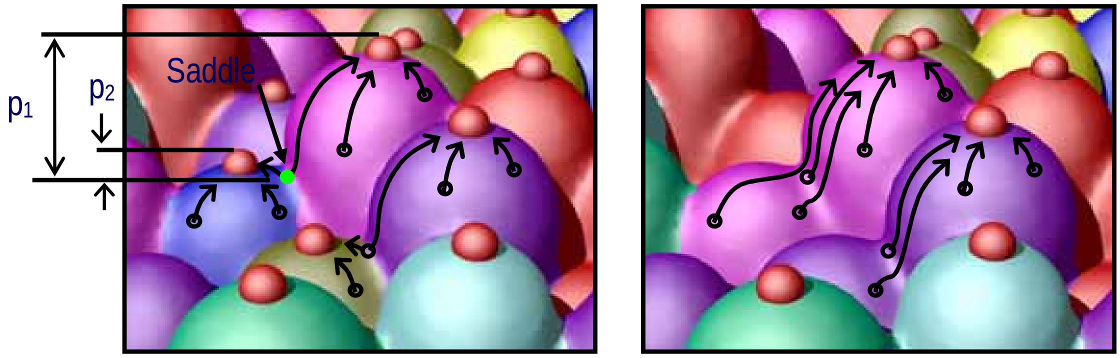

Fig. 2.1. Contracting low-persistence critical points reduces the Morse complex into a tiling that segments *S* into bubbles.

3 TEMPORAL REALIZATIONS

The Morse-complex approach was applied to the DNS data over a range of simulation times. Examples of the color-coded surface at a variety of times are shown in Figs. 3.1 and 3.2. A reduced version of the entire surface is at the upper left in Fig. 3.1, along with close-ups and a three-dimensional rendition. These visualizations were used for validation of the bubble-identification algorithm, which also produced bubble counts as a function of time.

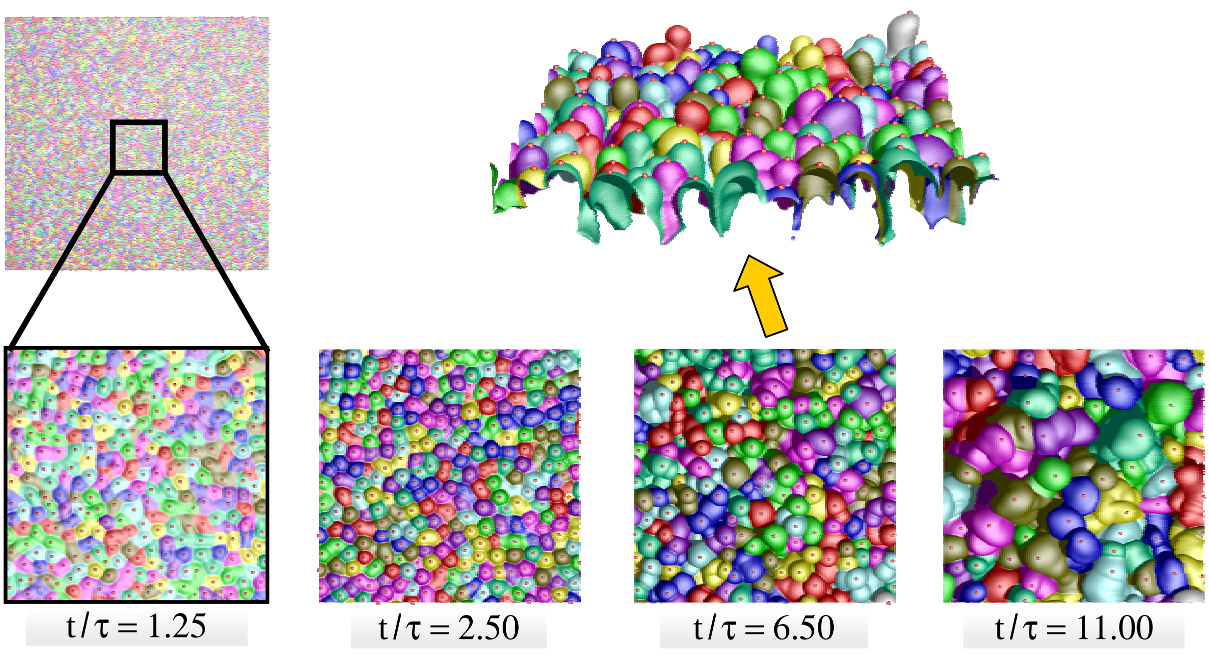

Fig. 3.1. Images of the bubble surface at four select times, enlarged to show just 4% of the total area (top left). The third time realization is also displayed in a three-dimensional rendition.

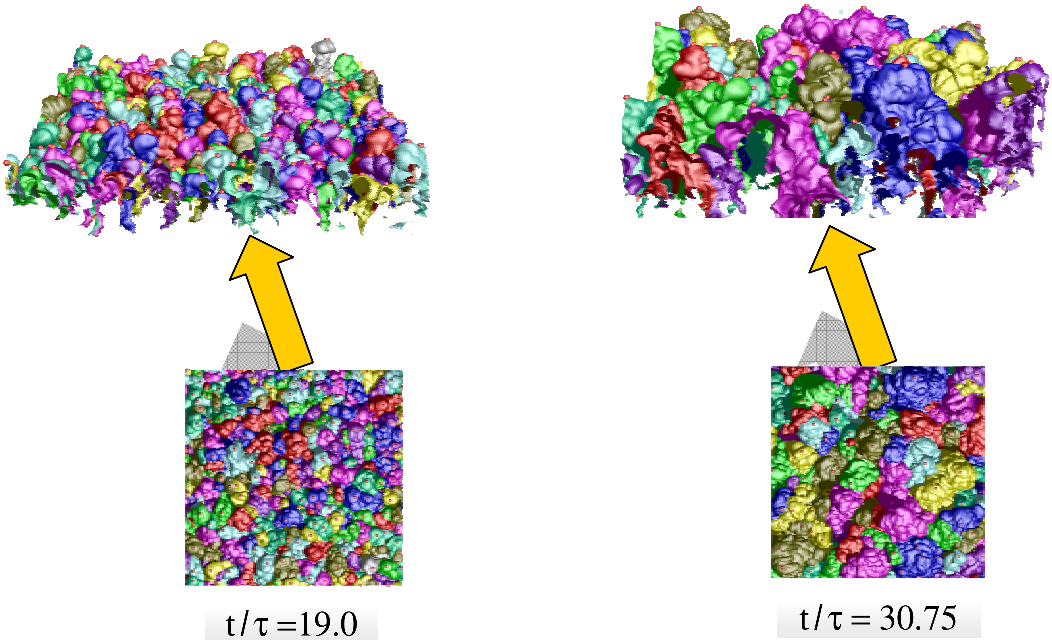

Fig. 3.2. Images of the bubble surface at two additional select times, showing 60% of the total area.

4 QUANTITATIVE TIME ANALYSIS

We analyze trends in a DNS Rayleigh-Taylor simulation (Cabot and Cook, 2006) by counting bubble structures. Shown are the number of Morse maxima vs. nondimensional time (green plot), and the local slope of that curve (red plot). The slope curve highlights several stages in the development of the interface topology, suggestive of earlier findings [2].

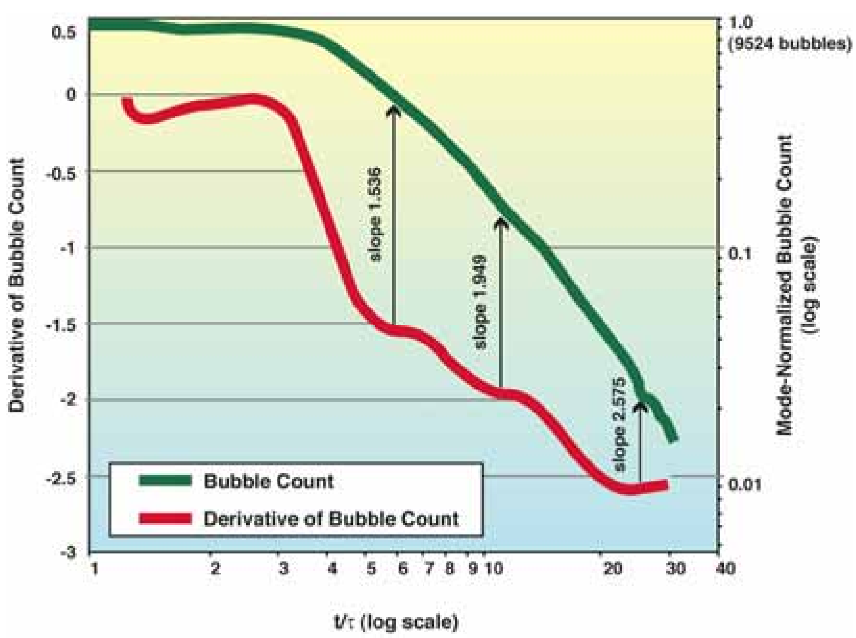

Fig. 4.1. Bubble count vs. time and the corresponding derivative curve.

5 SUMMARY

We have applied the Morse-complex approach to the analysis of simulation data of a Rayleigh-Taylor flow. Bubble counts derived from the analysis display a series of stages, or regimes, for the flow, much as regimes of the flow have been identified previously [2]. At the earliest times, the bubbles grow in an essentially independent manner, followed by a period of interaction during which they begin to feel the presence of their neighbors, develop roll-ups, and become much more convoluted. That is followed by a stage of weak turbulence that transitions to fully turbulent, late-time behavior. It is interesting to observe the regimes manifest themselves in the bubble-count statistics, which was unexpected.

One of our primary interests in pursuing this work was to explore the scaling of the number of bubbles with time. We find that the power-law behavior of the bubble count is much less steep power laws than may have been anticipated. The power-law scaling of these bubble counts has implications for bubble-based models of Rayleigh-Taylor instability. In particular, if the bubbles "tile" the area, then the number of bubbles should scale like the inverse of the square of the bubble radius. If it is further assumed that the characteristic bubble radius scales proportional to the layer width, and the mixing layer width scales like t^2, then the number of bubbles should be proportional to t^{-4}, i.e., a slope of -4 on the log-log plots of bubble count vs. time. Likewise, alternative scalings of the bubble radius will produce other power laws.

Our results do not exhibit slopes nearly as negative as -4, so we conclude that the destruction of bubbles (by whatever means, such as merging or competition) is slower than suggested by a bubble radius proportional to the layer width. We have employed another analysis approach to the bubble-counting problem, with similar findings, that appears elsewhere in this Proceedings [3].

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REFERENCES

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[2] Cook, et al., *The mixing transition in Rayleigh-Taylor instability*, JFM **511** (2004), 333 - 362.

[3] Miller, et al., *Bubble Counts for Rayleigh-Taylor Instability Using Image Analysis*, elsewhere in this Proceedings.