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Abstract The general principles of scaling are discussed, followed by a survey of the important dimensionless parameters of fluid dynamics including radiation and magnetic fields, and of non-LTE spectroscopy. The values of the parameters are reviewed for a variety of astronomical and laboratory environments. It is found that parameters involving transport coefficients — the fluid and magnetic Reynolds numbers – have enormous values for the astronomical problems that are not reached in the lab. The parameters that measure the importance of radiation are also scarcely reached in the lab. This also means that the lab environments are much closer to LTE than the majority of astronomical examples. Some of of the astronomical environments are more magnetically dominated than anything in the lab. The conclusion is that a good astronomical environment for simulation in a given lab experiment can be found, but that the reverse is much more difficult.

Keywords hydrodynamics · radiation · scaling

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1 Introduction

Radiation hydrodynamics is the discipline in which not only the material fluid but also the radiation (photons or neutrinos) must be treated dynamically. Since the speed of light is so large, it is tempting and often successful to neglect the fluid velocity in the dynamical equation for the radiation. Accounting for the effects thereby ignored is the business of radiation hydrodynamics. Chief among these are the advective flux of radiation energy and the subtraction of momentum and energy from the radiation when it exerts a force on the material. From a computational point of view, the proper accounting for the velocity effects is one of two main challenges in radiation hydrodynamics; the other challenge is meeting the requirement of a full transport solution with all the spectral and angular detail that the radiation field possesses. The latter challenge is faced even when the fluid velocity is negligible. The computational complexity of radiation hydrodynamics is the motivation for seeking laboratory analogues of astronomical environments for which, owing to the radiation hydrodynamic effects, numerical simulations are very difficult; the analogue experimental results can provide benchmarks for the simulations.

In this paper I will give a quick review of the principle of scaling for physical systems described by a small set of partial differential equations. The central point is the non-dimensionalization of the equations, which leads to a minimum set of non-dimensional parameters the values of which must all match for two physical systems in order for one system to be the scaled version of the other. Next I provide a list of possibly relevant dimensionless parameters that arise in describing various astronomical environments. After a brief explanation of the parameters I provide a table of the parameter values for several astronomical environments and also several laboratory environments that may be proposed as scaling candidates for the astronomical ones. The discussion of this table is the main point of this paper, and after the discussion I offer a short conclusion.

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2 The principle of scaling

The scaling concept is described as follows: It is assumed that our physical system is described fully and with sufficient accuracy by providing the values of a few fields, such as the density, fluid velocity, perhaps magnetic induction, *etc.*, as functions of a few independent variables, such as coordinates x, y, z and time t. It is also assumed that to sufficient accuracy the fields obey a certain set of partial differential equations over these coordinates. These equations may be put into non-dimensional form by expressing each field or coordinate as the product of a representative value and a dimensionless function. When the transformed equations are simplified, the dimensional representative values can be grouped together into dimensionless combinations, which are the fundamental parameters of the problem. The two systems can be scaled versions of each other if the non-dimensional partial differential equations that describe them are identical, and, in particular, if the dimensionless parameters are identical. This means that the same set of physical processes is an accurate description of both systems, and that the relative magnitudes of the different processes that are included are also identical between the two systems.

The test for scaling is therefore this: identify the relevant dimensionless parameters and test each of them for equality.

Each dimensionless parameter can be expressed as the ratio of two physical quantities that appear in the governing equations and which have the same dimensions. When the dimensionless ratio is either extremely large or extremely small, it means that one of the physical quantities is negligible compared with the other. In this case the equations could be simplified by discarding the negligible term(s). So when we test for equality of the dimensionless parameters for two systems, we can ignore a parameter that is different for the two systems if it happens to be extremely large or extremely small in both; that parameter involves physics that is not actually relevant for these systems.

3 The astronomical environments

Astronomical bodies have characteristic length and time scales that are huge compared with terrestrial laboratories, of course, but there is also a great dynamic range among them. But just to pick one example, consider the interstellar medium. The typical length scale is roughly one parsec,[1] and the typical time scale is very roughly 1000 years. Each of these numbers is 3×10^{19} larger than laser experiment scales of 1 mm and 1 nanosecond. The density may be 1–1000 particles per cubic centimeter, which is at least 10^{19} times less than the typical laser target density $10^{22}\,\mathrm{cm}^{-3}$. It is a stringent test of scaling to span nineteen orders of magnitude!

For the present discussion I have selected eight astronomical environments as candidates for scaling to the lab: (1) warm interstellar medium; (2) a dense interstellar cloud; (3) a stellar photosphere; (4) an interior point in a stellar envelope; (5) the accretion disk around an active galactic nucleus; (6) an x-ray binary accretion disk; (7) in a neutron star accretion column; and (8) a point in the wind of a hot star. Table 1 shows the characteristic properties and dimensions of the environments. The units are cgs, kelvins and Gauss, and the particle density is atoms per cubic centimeter.

[1] 1 parsec (pc) is 3.08568×10^{18} cm.

Table 1 Selected astronomical environments

environ	length	velocity	# density	temp	B
warm ISM	3×10^{18}	10^{7}	1	10^{4}	10^{-5}
dense cld	3×10^{18}	5×10^{5}	10^{3}	10^{2}	10^{-4}
stellar atm	10^{9}	10^{7}	10^{15}	10^{4}	10^{2}
stellar env	10^{10}	10^{7}	10^{18}	10^{6}	10^{2}
AGN disk	2×10^{13}	3×10^{8}	10^{12}	10^{7}	10^{6}
XRB disk	10^{6}	3×10^{7}	3×10^{21}	10^{7}	10^{6}
NS acc col	4×10^{4}	3×10^{9}	10^{23}	10^{8}	10^{12}
stellar wnd	10^{12}	10^{8}	10^{11}	10^{5}	5×10^{1}

Table 2 Selected laboratory environments

environ	length	velocity	# density	temp	B
burn thru	10^{-3}	10^{6}	10^{24}	6×10^{5}	10^{6}
Ω hohlraum	10^{-2}	10^{7}	10^{22}	10^{6}	10^{6}
NIF hohl	3×10^{-2}	2×10^{7}	10^{22}	3×10^{6}	10^{6}
Z expt	10^{-1}	10^{7}	10^{22}	10^{6}	10^{6}
short pulse	10^{-3}	10^{8}	10^{24}	10^{7}	10^{8}

4 Some laboratory environments

I will consider here a few selected laboratory environments that have been employed or proposed for laboratory astrophysics experiments. These are: a burn-through foil that might be inserted in an Omega hohlraum wall; plasma at critical density in a modest-temperature hohlraum; the same thing but sized for a NIF hohlraum; the same thing on the Z pulsed-power machine; conditions produced by a short-pulse laser. Table 2 lists the characteristics chosen to represent the different experiments. As above, the units are cgs-kelvin-Gauss.

5 Relevant dimensionless parameters

The physical processes that dominate the behavior in an astronomical environment that can be scaled to the laboratory are necessarily simple: ideal gas dynamics with radiation flow and perhaps MHD. The gas dynamics by itself introduces one parameter, the Mach number $\mathscr{M} = u/c_s$, in which $c_s = (\gamma p/\rho)^{1/2}$ is the adiabatic sound speed. Viscosity might be significant, in which case the Reynolds number $Re = \rho u L/\mu$ is a relevant parameter.

We suppose that molecular heat conduction is negligible compared with the radiative heat flux, so we need not be concerned with the Prandtl number. The radiative flux is described with the Boltzmann number, which is the ratio of the convective heat flux to the 1-way radiative flux $\sigma_B T^4$: $Bo = \rho u C_p/(\sigma_B T^3)$. In some of the environments the radiation mean free path λ_p is short and in some it is long; the optical depth parameter $\tau = L/\lambda_p$ is the measure of it. The mean free path is $\lambda_p = 1/(\kappa\rho L)$ in terms of the opacity κ, which may be the Rosseland mean or some other fiducial value. When the optical depth is large, the radiation is said to be in the diffusion limit, and the net radiative flux can be computed from a heat-conduction-like formula. The ratio of the convective flux to the diffusive radiative flux is the Péclet number $Pe = (3/4)\tau\rho u C_p/(\sigma_B T^3) = (3/4)\tau Bo$. In the optically thin limit the diffusion expression for the radiative flux is not appropriate, and the relevant parameter is the ratio of the optically-thin radiative cooling time to the flow time; I call this the Newton cooling number. It is $Nc = Bo/(4\tau)$. Whichever is larger of Pe and Nc is the relevant one.

When the magnetic field is significant the equations of MHD replace the Euler equations. An additional parameter appears for ideal MHD, the plasma beta, $\beta = 8\pi p/B^2$. If the electrical conductivity is not effectively infinite then the equations of resistive MHD must be used, and an additional parameter is the magnetic Reynolds number, $Rm = \mu_0 uL/\eta = 4\pi\sigma uL/c^2$, in which η is the electrical resistivity in SI units, and σ is the conductivity in Gaussian units (s^{-1}). These are the relevant parameters in a collisional plasma. In the weak-collision regime there are additional parameters, such as the Larmor radius divided by L and the collision frequency times L/u, as discussed in these proceedings by Ryutov [2].

6 Scaling parameters for astronomical and lab environments

I have evaluated the eight parameters discussed earlier, τ, $\mathscr{M}$, Re, Rm, Bo, Pe, Nc and β, for the various astronomical and laboratory environments. These are shown in Table 3. The way in which we would like to use these tables is to look up the astronomical environment we want to simulate in Table 3, and then find a laboratory environment in the table that has similar values of the scaling parameters. But we see at a glance that the scaling parameters for the astronomical environments have a huge dynamic range while the laboratory parameters do not.

Some of the notable differences between that astronomical and laboratory environments that are seen in Table 3 are these: The transport parameters Re and Rm, the ordinary and magnetic Reynolds numbers, are much larger in all the astronomical environments. In view of the comments earlier about very large parameters, it seems that viscosity and resistivity can quite generally be neglected in the astronomical environments, while this may not be true in the laboratory. The Boltzmann number is generally very small for the astronomical environments; this means that they are radiation-dominated. The derived values of the Péclet and Newton-cooling numbers are also small. In the laboratory, because the density is so much higher, radiation is generally not dominant. This is a major impediment to simulating astronomical radiation hydrodynamics problems in laboratory experiments. Also because of the high density, the lab environments have a great difficulty achieving a low β. Some, but not all, astronomical environments are very highly magnetized, which is not true of the laser and pulsed-power experiments considered here.

Leaving aside the radiation and magnetic field effects, in other words just looking at the Mach number, we see a better overlap between astronomical and laboratory parameter values. So pure gas dynamics looks promising for scaling astronomical problems to the laboratory.

7 Non-LTE and astronomical spectra

Another area in which we would hope to simulate an astronomical problem in the lab is in plasma spectroscopy: Can we create a radiation source in the lab of which the spectrum would be a good match to that of the astronomical object?

Scaling spectroscopy is harder than scaling hydrodynamics. Replacing one element by another does not work very well, since complex spectra are unique; hydrogenic spectra are the exception. So we suppose that the ions of interest are not hydrogen-like, and that the same element will be used in the simulation that occurs in the astronomical problem. Since atomic excitation and ionization depend on the ratio of the ionization potential to kT, we conclude that T will also not be scaled.

The competition between collisional excitation and de-excitation processes and their radiative counterparts is the heart of non-LTE excitation and ionization equilibrium. The scaling parameters that express this competition are the ε values defined by

$$\varepsilon = \frac{N_e C_{u\ell}}{A_{u\ell}} ,$$

in which $u \to \ell$ is an atomic transition forming a spectral line, $C_{u\ell}$ is the rate coefficient for collisional de-excitation and $A_{u\ell}$ is the spontaneous radiative decay rate. In order for the emitted spectrum to match, all the values of ε should match. If Van Regemorter's [4] semi-empirical formula is used to approximate the collisional rate, then the dependence on $A_{u\ell}$ cancels out and the result is $\varepsilon \propto N_e\lambda_{\mathrm{ph}}^3$, where λ_{ph} is the photon wavelength. If this wavelength is centrally located with respect to the Planck distribution, then $\lambda_{\mathrm{ph}} \propto 1/T$, and $\varepsilon \propto N_e/T^3$. We see that $\varepsilon \propto Bo/(T\mathscr{M})$. This is bad news for scaling spectra, since Bo for the astronomical plasmas is quite out of the range of the lab values.

In nebulae in our galaxy, and in emission-line regions of active galactic nuclei and elsewhere, we have conditions very far from LTE in which the plasma is strongly photoionized by a diluted but energetic radiation field. The plasma temperature comes to an equilibrium in which photoionization heating balances radiative cooling, mostly in line emission. The ionization balance and the temperature then depend, for a given shape of the ionizing spectrum, on the ionization parameter Ξ defined by [3,1]

$$\Xi = \frac{U}{\rho C_p T} .$$

Here U is the diluted radiation energy density. We see that Ξ is about the same as $u/(cBo)$, so that, again, the Boltzmann number is the important scaling parameter for non-LTE. The

Table 3 Scaling parameters for astronomical and laboratory environments

environ	τ	$\mathscr{M}$	Re	Rm	Bo	Pe	Nc	β
warm ISM	10^{-6}	10^{1}	10^{7}	10^{19}	3×10^{-17}	2×10^{-23}	7×10^{-12}	3×10^{-1}
dense cld	10^{-3}	6	7×10^{13}	6×10^{14}	10^{-9}	10^{-12}	4×10^{-7}	3×10^{-2}
stellar atm	10^{1}	10^{1}	5×10^{12}	4×10^{9}	3×10^{-2}	3×10^{-1}	5×10^{-4}	3
stellar env	3×10^{3}	1	5×10^{11}	4×10^{13}	3×10^{-5}	7×10^{-2}	2×10^{-9}	3×10^{5}
AGN disk	5	10^{1}	7×10^{7}	6×10^{19}	8×10^{-13}	3×10^{-12}	4×10^{-14}	3×10^{-8}
XRB disk	10^{3}	1	10^{9}	4×10^{11}	3×10^{-4}	2×10^{-1}	6×10^{-8}	8×10^{1}
NS acc col	10^{3}	4×10^{1}	6×10^{8}	5×10^{13}	8×10^{-4}	8×10^{-1}	2×10^{-7}	3×10^{-8}
stellar wnd	3×10^{-2}	4×10^{1}	10^{10}	10^{15}	3×10^{-8}	7×10^{-10}	2×10^{-7}	10^{-2}
burn thru	7×10^{1}	2×10^{-1}	2×10^{4}	2×10^{-1}	10^{1}	8×10^{2}	5×10^{-2}	2×10^{3}
Omega hohl	9×10^{-3}	1	3×10^{3}	5×10^{1}	2×10^{-1}	10^{-3}	5	3×10^{1}
NIF hohl	10^{-3}	1	10^{3}	2×10^{3}	10^{-2}	10^{-5}	3	9×10^{1}
Z expt	9×10^{-2}	1	3×10^{4}	5×10^{2}	2×10^{-1}	10^{-2}	5×10^{-1}	3×10^{1}
short pls	10^{-2}	4	10^{3}	10^{3}	3×10^{-1}	3×10^{-3}	5	3

typical values of Ξ in nebulae and active galactic nuclei are 10^2–10^3, while the values in the lab environments are closer to 10^{-1}.

8 Prospects

We have seen that the dynamic range of the scaling parameters for the astronomical environments is very large indeed, much larger than the range among the available laboratory experiments. A large part of the range covered by the astronomical environments is inaccessible in the laboratory. This means that the the odds that a given astronomical environment can be simulated in the laboratory are not good. However, the odds that a given laboratory environment has an analogue in astronomy are much better.

Some processes do not scale very well — the viscosity and resistivity effects are generally much smaller in the astronomical environments, and both radiation and magnetic fields tend to be stronger (small Bo and β) in the astronomical cases. Scaling appears to be most successful for pure gas dynamics. Within these limitations the prospects for scaling are good.

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