Friction Factor Measurements in an Equally Spaced Triangular Tube Array

P Vassallo, P Symolon

Friction Factor Measurements
in an Equally Spaced Triangular Tube Array

Peter Vassallo[*] and Paul Symolon

Lockheed Martin Corporation
Schenectady, New York, 12301, USA

Abstract

Friction factor data for adiabatic cross-flow of water in a staggered tube array was obtained over a Reynolds number range (based on hydraulic diameter and gap velocity) of about 10,000 to 250,000. The tubes were 12.7mm (0.5 inch) outer diameter, in a uniformly spaced triangular arrangement with a pitch-to-diameter ratio of 1.5. The friction factor was compared to several literature correlations, and was found to be best matched by the Idelchik correlation. Other correlations were found to vary significantly from the test data. Based on the test data, a new correlation is proposed for this tube bundle geometry which covers the entire Reynolds number range tested.

Keywords: friction factor, tube bundle, pressure drop, staggered array

* Corresponding author. Tel.: 518-395-6799; fax: 518-395-5353
  E-mail address: vassall@kapl.gov

Introduction

Pressure drop across a tube bank is one of the primary design considerations for shell and tube heat exchangers. The pressure drop is a function, among other things, of tube arrangement and packing as well as total system flow. Friction factor correlations have been used for many years to determine the pressure loss based on relevant design parameters. These correlations offer a means of assessing the pressure losses quickly without the need for expensive and time consuming computational methods.

A summary of early work is provided by Chilton and Generaux[1]. In this paper, a general equation for friction factor, based on tube gap spacing and gap velocity, was developed by fitting a curve to eight data sets. Later, Gunter and Shaw [2] considered a broader set of data, and concluded that the data best collapsed using an equivalent hydraulic diameter as well as transverse and longitudinal pitch to diameter ratios. Friction factors for bare tubes of diameters between 0.5 and 127mm were included, with transverse and longitudinal pitches ranging from 1.25 to 5 diameters.

Perhaps the most widely referenced correlation is that provided by Zukauskas [3]. This correlation, in graphical form, has been reprinted in numerous heat transfer and heat exchanger design manuals [4,5]. The correlation is plotted as a function of pitch and spacing as well as Reynolds number based on gap velocity and tube diameter. It covers a wide range of Reynolds number, from 10 to 1,000,000. Another useful, well regarded resource for pressure losses in a variety of geometries is provided by Idelchik [6]. Here, a method for obtaining friction factor in staggered tube arrays is provided based on gap

velocity, tube diameter and numerous geometric factors.

The tube configuration currently being considered is made up of an equally spaced, staggered triangular array with tube OD = 12.7mm. Several methods, including those described above, were used to determine friction factor for this geometry, and a large spread was observed for the Reynolds number range of interest (i.e., between 10,000 and 250,000). The data of Kays and London [7] which most closely matched our test configuration, although limited to Reynolds numbers less than 23,000, indicated a relatively low friction factor, nearly half that of Chilton-Generaux. The predictions of Gunter-Shaw, Zukauskas, and Idelchick lay in between, each successively giving decreasing values of friction factor. As a result of the spread of these predictions and the lack of high Reynolds number data, a test was performed to obtain more specific data for the desired tube geometry.

Calculation of Tube Bundle Geometric Parameters

Figure 1 illustrates the nomenclature associated with modeling pressure drop in a triangular tube array.
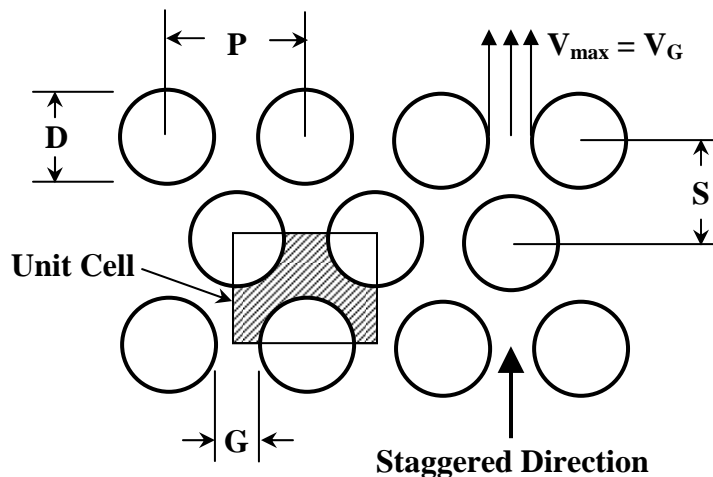
Figure 1. Schematic of Triangular Tube Array

A single unit cell of the rod array (the cross hatched region in Figure 1) can be analyzed to derive the volume porosity and hydraulic diameter. For uniformly spaced triangular rod arrays ($S = \frac{\sqrt{3}}{2}P$):

$$\gamma_v = \frac{fluid\ volume}{total\ volume} = 1 - \frac{\pi}{2\sqrt{3}}\left(\frac{D}{P}\right)^2 \qquad (1)$$

$$D_v = \frac{4\ x\ fluid\ volume}{wetted\ area} = \left[\frac{2\sqrt{3}}{\pi}\left(\frac{P}{D}\right)^2 - 1\right]D \qquad (2)$$

For a triangular array with D=12.7mm and P/D=1.5, Eq. (1) gives γ_v=0.597 and Eq. (2) gives D_v=18.8mm. These values are derived for a large tube array free of wall effects. If the walls of the test section are included in the wetted area calculation, the volume porosity is unchanged and the volumetric hydraulic diameter decreases 9% to 17.3mm. However, while the wetted area contributes to the hydraulic diameter, it is believed to have an insignificant effect on the pressure drop, because crossflow resistance is dominated by the form drag of the tubes. Therefore, the infinite bundle hydraulic diameter of 18.8mm has been used in the data reduction calculations.

Experimental Description

Test Section. A sketch of the test section is shown in Figure 2. The test section consisted of a stainless steel rectangular duct, 30.48cm (12 inches) by 11.43cm (4.5 inches) in cross section and 182.88cm (72 inches) long. The test section was oriented horizontally, with the tubes parallel to the floor. The test section included a flow

straightener at the inlet, consisting of a perforated plate followed by a bundle of 9.5mm

(3/8 inch) OD thin walled tubes. The test section contained 60 rows of tubes with six

tubes per row, arranged in a triangular pitch. The surface of the tubes was nominally

smooth. The rows alternate between a row with five full tubes with two half tubes welded

to the wall, and a row with six full tubes. Pressure tap locations are shown in Figure 2;

the taps were flush to the sidewall of the test assembly. An acrylic window was included

in the test section for future visualization or laser measurements.
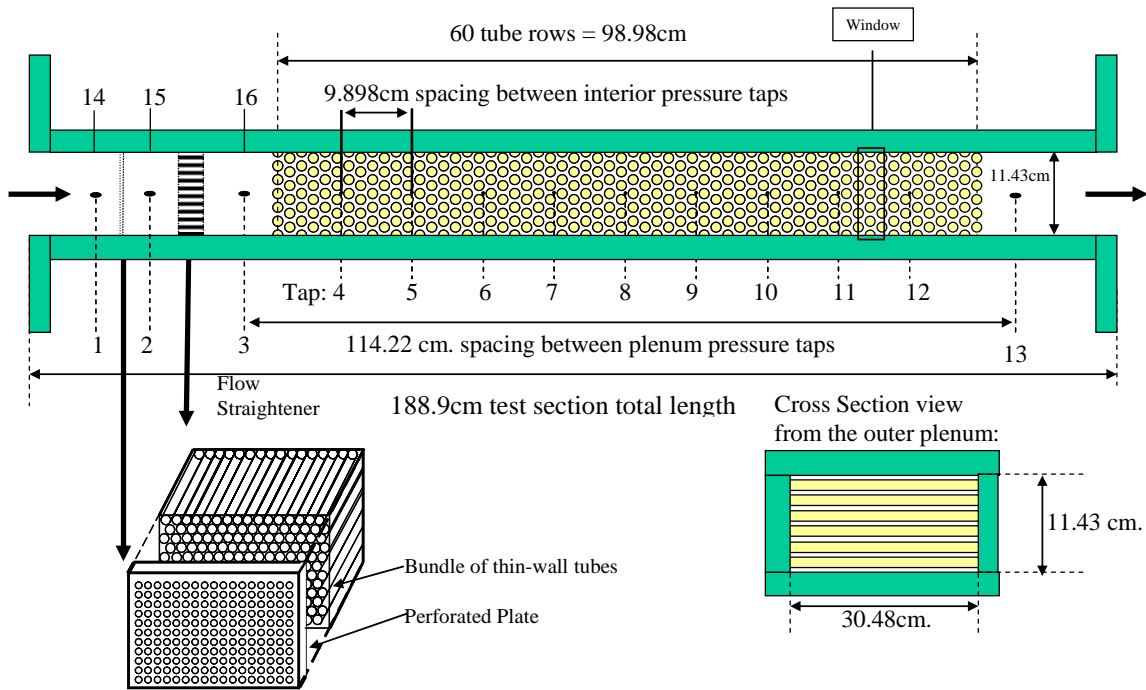

Figure 2: Test Section Schematic (not to scale)

Instrumentation. Calibrated differential Rosemount transducers were used to

measure the pressure drop with an uncertainty of +/-1% of reading or +/-0.01 psi,

5

whichever was greater. Care was taken to thoroughly bleed the transducers so that no air bubbles were present. Measurements taken at zero flow confirmed that no measurement bias due to trapped air was present (i.e., the data indicated <0.01 psi at zero flow).

A Venturi flow device calibrated for tube bundle Re>10,000 was used to measure the flow rate with an uncertainty of +/-1%. The primary Venturi used to take the flow rate data was checked with a second in-line Venturi and the two measurements agreed to within 1% for the applicable range of the second Venturi (i.e., Re>100,000). Standard Type-K thermocouples were used to measure the water temperature (within +/-1.1°C).

As a check of the pressure data accuracy, the 10 bundle incremental pressure drops were added together, and compared to the total plenum to plenum $\Delta P_{3\text{-}13}$. The difference was at most a 2% error for the lowest flow tested. For higher flows where pressure drops are greater, the difference was generally within two tenths of a percent.

Experimental Method for Obtaining Friction Factor. For convenience in making comparisons, the incremental pressure drops were converted to loss factors by dividing the pressure drop by the dynamic head:

$$K = \frac{\Delta P}{\frac{1}{2}\rho V^2} \tag{3}$$

For the dynamic head, we used either the average velocity in the tube gaps (V_G) if the ΔP was in the bundle, or the average velocity in the un-rodded part of the duct ($3V_G$) if the

6

ΔP was across the perforated plate or flow straightener.

For comparison purposes, the loss factor is plotted versus position in the test section in Figure 3. All the data taken (i.e., at different flows and temperatures) is plotted to show the overall trends, as well as the mean results. For tap 3 to 4, the pressure drop is higher because the flow must be accelerated (by a factor of 3) from the open duct velocity to the gap velocity. Conversely, for tap 12 to 13, the flow decelerates by the same amount, causing a pressure recovery, and a smaller loss factor.

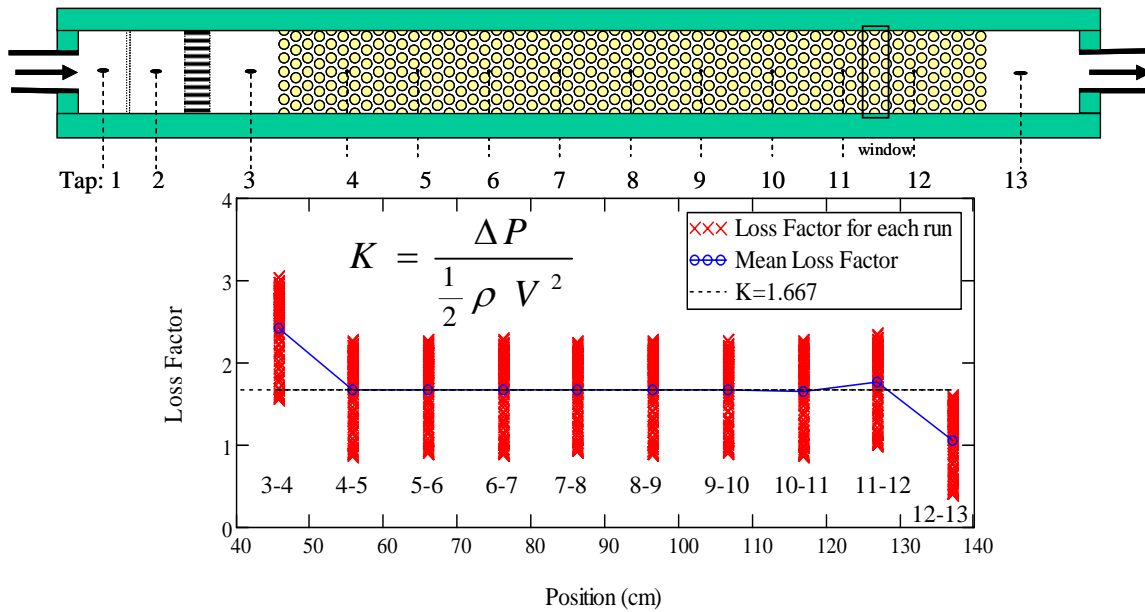**Figure 3 Loss Factor versus Position in the test section**

It is also observed in Figure 3 that, for the incremental pressure drops within the bundle, the loss factor from tap 11 to tap 12 is greater than the other seven bundle incremental loss factors by 5% (on the average). This may have been due to a flow perturbation at the window, or possibly a defect in tap 11 or 12, such as a burr. For this reason, the friction

factor was based on the sum of the pressure drops from tap 4 to tap 11 rather than tap 4 to tap 12. The Darcy friction factor is therefore defined as:

$$f = \frac{2\Delta P_{4-11}}{\rho(Q/A_G)^2} \frac{D_v}{L_{4-11}} \tag{4}$$

Experimental Uncertainty in the Friction Factor. By propagating errors through Eq. (4), the error in friction factor can be expressed in terms of the error in the measured parameters. Neglecting density uncertainty and uncertainty in the length between pressure taps:

$$\frac{\varepsilon_f}{f} = \sqrt{\left[\frac{\varepsilon_{\Delta P}}{\Delta P}\right]^2 + 4\left\{\left[\frac{\varepsilon_Q}{Q}\right]^2 + \left[\frac{\varepsilon_A}{A_{gap}}\right]^2\right\} + \left[\frac{\varepsilon_{D_v}}{D_v}\right]^2} \tag{5}$$

The uncertainty for the pressure drop and flow rate measurements within the calibrated range of the primary Venturi (i.e., $Re>11,000$) was +/-1%. The effect on flow area of a rod being at an off nominal position was studied and showed that a 0.127mm error in rod position resulted in less than a tenth of a percent change in gap flow area. From test section design tolerances, the uncertainties in flow area and hydraulic diameter were estimated to be +/-1%. Using these values in Eq. (5), the uncertainty in friction factor is +/-3.2%. A similar propagation of errors on Reynolds number yields:

$$\frac{\varepsilon_{\text{Re}}}{\text{Re}} = \sqrt{\left[\frac{\varepsilon_Q}{Q}\right]^2 + \left[\frac{\varepsilon_A}{A_{gap}}\right]^2 + \left[\frac{\varepsilon_{D_v}}{D_v}\right]^2 + \left[\frac{\varepsilon_v}{\upsilon}\right]^2} \qquad (6)$$

Based on the uncertainty in loop temperature (+/-1.1°C), the uncertainty in viscosity is +/-3% for the range of loop temperatures studied. The overall uncertainty in Reynolds number is therefore +/-3.5%.

Results and Discussion

The friction factor data (119 points total) is plotted versus Reynolds number in Figure 4 with the error bars determined from Eqs. (5) and (6). Temperatures were varied from 10°C to 45°C to extend the range of Reynolds number and to confirm no temperature sensitivities existed. The scatter in the data is seen to be within the specified measurement uncertainty.
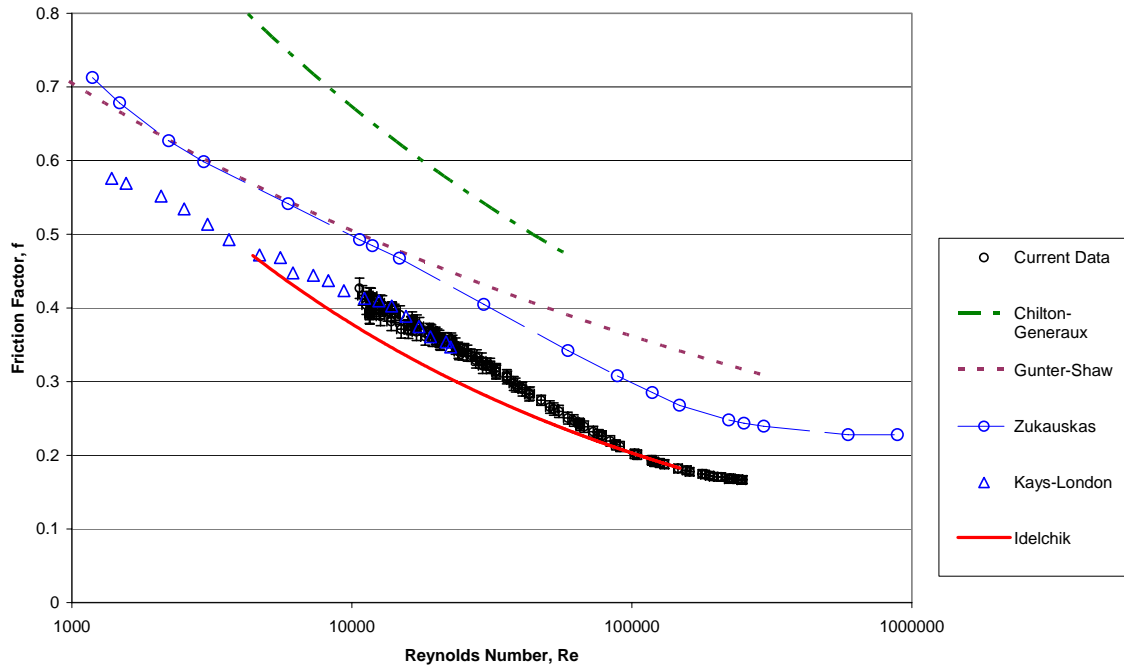
Figure 4: Cross-flow Friction Factor Data vs. Other Sources

Also included in Figure 4 are predictions from other sources to provide context for the present data. The points for Zukauskas [3] and Kays and London [4] were taken directly from the graphs provided in those references. The Zukauskas method matched the current tube geometry while the Kays and London data closely matched it (P/D=1.5 and S/D=1.25). The correlations for Idelchik, Chilton-Generaux, and Gunter-Shaw are summarized as follows:

Idelchik Method. For $3x10^3 < \text{Re}_D < 10^5$:

$$f_I = K \, \text{Re}_D^{-0.27} \, N \tag{7}$$

10

$$\bar{S} = \frac{(P-D)}{\left(\left(\sqrt{0.25P^2 + S^2}\right) - D\right)} = 1 \tag{8}$$

For $P/D \geq 1.44$ and $0.1 < \bar{S} < 1.7$:

$$K = 3.2 + 0.66\left(1.7 - \bar{S}\right)^{1.5} = 3.59 \tag{9}$$

$$f = f_I \frac{D_v}{L} \tag{10}$$

$$\text{Re} = \text{Re}_D \frac{D_v}{D} \tag{11}$$

Chilton and Generaux Method. For $50 < \text{Re}_G < 2x10^4$:

$$f_{C-G} = 3\,\text{Re}_G^{-0.2}\,N \tag{12}$$

$$f = f_{C-G} \frac{D_v}{L} \tag{13}$$

$$\text{Re} = \text{Re}_G \frac{D_v}{G} \tag{14}$$

Note: the correlation was based on data for 1.25<P/D<5.

Gunter-Shaw Method. For isothermal flow in an equally spaced triangular tube array and $5x10^2 < \text{Re}_v < 3x10^5$:

$$f = 1.92\,\text{Re}_v^{-0.145}\left(\frac{D_v}{P}\right)^{0.4} \tag{15}$$

As seen in Figure 4, the data is best matched overall by Idelchik, especially for Re >

80,000. For Re < 80,000, the data departs slightly from Idelchik, and exhibits a curvature

similar to Zukauskas. The Kays and London data is in excellent agreement within the

range 10,000 < Re < 23,000 and Chilton-Generaux is noticeably higher. Except for

Chilton-Generaux, which has no explicit sensitivity to pitch or spacing ratios, it is unclear

why such variation exists. As far as the data is concerned, differences in working fluid,

errors in density or viscosity, flow-pressure measurement errors, unknown wall effects,

variations in tube pitch and/or spacing, limited number of tube rows, and uncertainties in

surface roughness are possible culprits, all of which make it difficult to obtain universal

correlations. It does appear, that for most accurate results, testing in the particular

geometry of interest is recommended. Based on this, a new correlation was developed

for our data as follows:

$$f = \begin{cases} \dfrac{3.6862}{\mathrm{Re}_v^{0.2337}} & if \quad \mathrm{Re}_v < 27{,}582 \\[4mm] 0.1527 + 0.818\left[1 - \left(\dfrac{\mathrm{Re}_v}{10^6}\right)^{0.3532}\right]^{4.4974} & if \quad 27{,}582 \le \mathrm{Re}_v \le 10^6 \\[4mm] 0.1527 & if \quad \mathrm{Re}_v > 10^6 \end{cases} \qquad (16)$$

Note that the traditional power law relationship between *f* and *Re* accurately matches the

data up to *Re* of about 30,000, (where fitting yields *f=3.6862/Re$^{.2337}$*). At higher *Re* the

alternative equation shown above was required to obtain a good fit.

This fit matches the data to within +/-5% (effectively the uncertainty of the data), and the

RMS error in *f* is 0.00393. In practice, Eq. (16) is valid for Reynolds numbers ranging

from 10,000 to 250,000, where data was taken. Use of the correlation outside the data

range should take into consideration additional uncertainty. However, based on the trends

observed in the Zukauskus correlation (which is based on air data up to a million *Re*), the

correlation may be extrapolated up to a million Reynolds number. For *Re* greater than a

million a constant value of *f=0.1527* is assumed, based on similar high Re trends in the

Zukauskus correlation and the present data.

As a sensitivity study, the pressure drop across the tube bundle was calculated using the

friction factors in Figure 4 and compared to the pressure drop for the hypothetical case

with the tubes removed and flow held constant. The results are presented in Table 1 for a

range of Reynolds number, indicating a ΔP ratio between 1000 and 1500. Thus, even

though the tube bundle friction factor is only an order of magnitude higher than the

standard duct friction factor, the velocity and L/D effects contribute to an overall pressure

drop that is three orders of magnitude higher.

Q (m³/s)	With Tubes				Without Tubes				ΔP_{ratio}
	Re	f	V (m/s)	ΔP (kPa)	Re	f	V (m/s)	ΔP (Pa)	
8.6E-3	10,645	0.427	0.74	0.6	32,280	0.0231	0.25	0.4	1475
3.3E-2	47,440	0.275	2.89	5.9	316,930	0.0162	0.94	4.3	1357
4.4E-2	104,505	0.201	3.79	7.6	143,870	0.0162	1.26	7	1087
9.4E-2	246,386	0.166	8.1	28.7	316,930	0.0146	2.69	26.9	1068

Table 1. Calculated pressure drop in test section described in Figure 2 for two cases: with and without tubes. The density was taken to be 1000 kg/m3; With tubes: L/D$_v$=5.27, A=0.0116m^2; Without tubes: L/D$_H$=0.6, A=0.035m^2.

Conclusions

Experiments have been carried out to determine the friction factor for cross flow in an equally spaced triangular tube array with P/D=1.5. This data extends the Reynolds number range of previous data taken by Kays and London in a similar tube geometry by a factor of ten (up to Re = 250,000). In the overlap region between data sets, (10,000 < Re < 22,500) excellent agreement was noted. However, there is a wide variation in prediction of pressure drop in this geometry when employing available correlations, that is, nearly a factor of two in friction factor. The Idelchik correlation best matched the data for Re > 80,000 while the Zukauskas method best matched the shape of the data between 10,000 < Re < 250,000. A new correlation was presented which matches the data to within +/-5% over the full Reynolds number range. Since there are numerous factors which impact the overall friction factor, particularly tube spacing and surface roughness, future testing in a variety of tube configurations may allow for construction of a more accurate, general correlation which covers a wider range of tube geometries.

Nomenclature

A	Cross flow area, m^2
A_G	Flow area in gap between tubes, m^2
D	Tube outside diameter (mm)
D_v	Tube bundle hydraulic diameter, defined volumetrically (mm)
f	Darcy friction factor, based on D_v
G	Gap spacing between tubes (mm)
N	Number of tube rows between ΔP taps
P	Distance between tubes centers (pitch, mm)
Q	Volumetric flow rate (m^3/s)
Re	Reynolds number, based on gap velocity and hydraulic diameter
Re_D	Reynolds number, based on gap velocity and tube diameter
Re_G	Reynolds number, based on gap velocity and gap spacing
S	Row to row spacing between tubes (inches)

V_G Average velocity in the gaps between tubes (m/sec)

ρ Density (kg/m^3)

μ Dynamic viscosity (kg/s-m)

υ Kinematic viscosity (m^2/s)

$\varDelta P$ Pressure drop (Pa)

γ_v Volume porosity

Acknowledgements:

References

1)   Chilton, T.H., and Generaux, R.P., 1933, "Pressure Drops Across Banks of Tubes," AIChE Transactions, **29**, pp. 161-173.

2)   Gunter, A.Y., and Shaw, W.A., 1945, "A General Correlation of Friction Factors of Various Types of Surfaces in Cross Flow," ASME Transactions, **67**, pp. 643-660.

3)   Zukauskas, A., 1987, "Heat Transfer from Tubes in Cross Flow," Adv. Heat Transfer, pp. 87-159.

4)   A. Zukauskus and R. Ulinskas, 1983, "Banks of plain and finned Tubes," Section 2.2.4 in *Heat Exchanger Design Handbook*, Published by Hemisphere Publishing Corporation.

5)   Incropera and DeWitt, 1981, *Fundamentals of Heat Transfer*, John Wiley and Sons.

6)   Idelchik, I.E., 1986,   *Handbook of Hydraulic Resistance*, Second Edition, Hemisphere Publishing Corp.

7)   Kays and London, 1964, *Compact Heat Exchangers,* Second Edition, McGraw

        Hill Inc.