Low Noise Borehole Triaxial Seismometer
Phase II

U.S. DEPARTMENT OF ENERGY
SBIR GRANT
DE-FG02-03ER83667

Summary

This report describes the preliminary design and the effort to date of Phase II of a Low Noise Borehole Triaxial Seismometer for use in networks of seismic stations for monitoring underground nuclear explosions. The design uses the latest technology of broadband seismic instrumentation. Each parameter of the seismometer is defined in terms of the known physical limits of the parameter. These limits are defined by the commercially available components, and the physical size constraints. A theoretical design is proposed, and a preliminary prototype model of the proposed instrument has been built. This prototype used the sensor module of the KS2000. The installation equipment (hole locks, etc.) has been designed and one unit has been installed in a borehole. The final design of the sensors and electronics and leveling mechanism is in process. Noise testing is scheduled for the last quarter of 2006.

LOW NOISE BOREHOLE TRIAXIAL SEISMOMETER

U.S. DEPARTMENT OF ENERGY
SBIR GRANT
DE-FG02-0383667

GEOTECH INSTRUMENTS, LLC

10755 SANDEN DR.

DALLAS, TEXAS 75235

Table of Contents

1 RESPONSE OF MASS SPRING SYSTEM

For the discussion of the seismometer requirements, it will be assumed throughout this discussion that the suspension is a pendulous suspension. It is the more complex case, and the straight line or linear suspension where the mass moves perpendicular to the base of the instrument can easily be derived from the pendulous case.

The summation of torques about the pivot point can be written as (see Figure 1-1)

Equation 1-1 $$m\bar{r}\ddot{y} = m\bar{r}q\ddot{\theta} + d\dot{\theta} + T\theta$$

where
$\bar{r}$ = distance from pivot point to center of mass - m
q = distance from pivot point to center of percussion of mass - m
m = mass of suspension - kg
T = torsion spring rate of suspension - Nm/rad
θ= angular position of suspension relative to seismometer frame - rad
y = motion of ground or base of seismometer - m
d = damping torque constant - Nm/(rad/sec)

The "dot" and "double dot" is used indicate the first and second derivatives with respect to time. Letting s = jω, the steady state response equation can be written as,

Equation 1-2 $$\frac{\theta}{\ddot{y}} = \frac{1}{q}\frac{1}{(s^2 + 2\lambda_m\omega_m s + \omega_m^2)},$$

where $$2\lambda_m\omega_m = \frac{d}{m\bar{r}q}$$

$$\omega_m^2 = \frac{T}{m\bar{r}q}.$$

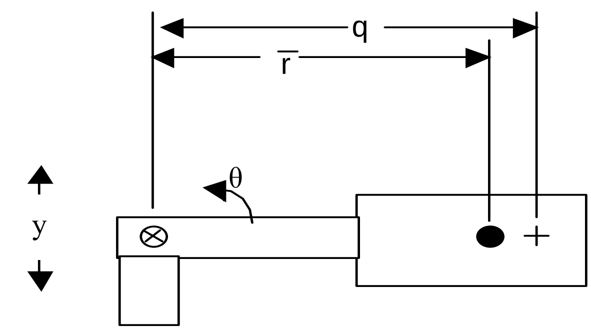


Figure 1-1 Basic dimensions of a pendulous suspension

2 MINIMUM MASS REQUIREMENTS

This section will define the minimum mass required for the thermal noise of the mass/suspension of the seismometer to be less than a given ground acceleration.

An expression for the thermal noise of the instrument can be obtained if we make the following assumptions;

a. The instrument is in thermal equilibrium. This condition includes all components of the system, however remotely located they are from each other.

b. The energy can be defined by continuous quadratic functions of the suspensions position and velocity components.

If these assumptions are true, then the equipartition theorem of thermodynamics states that the energy associated with each defining coordinate (i.e. the position and velocity) is $1/2kT_o$ where T_o is the absolute temperature and k is Boltsmann's constant.

It will also be assumed that the spring and mass do not dissipate any energy. That is the spring and mass exchange potential and elastic energy without losses. The model will assume that regardless of the source of all energy dissipation forces, they are proportional to velocity, and therefore, only the velocity damping mechanism can absorb as well as deliver energy. In other words, air damping, electrical circuit damping, spring and flexure pivot damping are all proportional to velocity.

Assume the instrument is in thermal equilibrium and the mass is oscillating with random frequencies and velocities, and there is no seismic input to the frame.

The energy of the system can be described by defining the amplitudes and velocities of all frequencies of motion, and for each of these frequencies the energy is kT_o. ($1/2kT_o$ each for potential and kinetic energy.)

The power at frequency f is

Equation 2-1 $$P_f = kT_o.$$

If Z_s is the mechanical impedance of the instrument, then the mean squared torque driving the instrument at this frequency is

Equation 2-2 $$\overline{M}^2 = Z_s^2 \overline{\dot{\theta}}_f^2$$

where $\overline{\dot{\theta}}_f^2$ is the mean squared velocity at frequency f.

The power received or delivered by the damper at this frequency is

Equation 2-3 $$P_f = \frac{\overline{M}^2}{Z_d} = \frac{Z_s^2 \overline{\dot{\theta}}_f^2}{Z_d}$$

where $Z_d = 2\lambda_m \omega_m m\bar{r}q$ is the impedance of the damper.

The kinetic energy within an incremental bandwidth df is then proportional to this power, or,

Equation 2-4
$$(1/2)m\bar{r}q\bar{\dot{\theta}}_f^2 df = \beta m\bar{r}q(1/2)P_f \frac{Z_d}{Z_s^2} df$$
$$= \beta m\bar{r}q(1/2)kT_o \frac{Z_d}{Z_s^2} df$$

where Equation 2-1 and Equation 2-3 are substituted for $\bar{\dot{\theta}}_f^2$ and P_f.

If Equation 2-4 is integrated over all frequencies, the right side will integrate to $(1/2)kT_o$ as it should to satisfy the equipartition theorem if $\beta = 4$.

With $\beta = 4$, Equation 2-4 can be solved for $\bar{\dot{\theta}}_f^2$ and substituted into Equation 2-2 to give

Equation 2-5 $$\overline{M}_{nf}^2 = 4(2\lambda_m \omega_m m\bar{r}q)kT_o ,$$

where $\overline{M}_{nf}^2$ is the mean square noise torque acting on the suspension.

Now if the mean squared torque acting on the suspension from ground acceleration is $\bar{\ddot{y}}_f^2 (m\bar{r})^2$, then

Equation 2-6 $$\bar{\ddot{y}}_f^2 (m\bar{r})^2 \geq 4(2\lambda_m \omega_m m\bar{r}q)kT_o .$$

Let $2\lambda_m = 1/Q$ and $\omega_m = 2\pi/p_m$ where p_m is the natural period of the suspension. Then

Equation 2-7 $$\bar{\ddot{y}}_f^2 \geq \frac{8\pi kT_o q}{mp_m Q\bar{r}} .$$

Rewriting Equation 2-7 as

Equation 2-8 $$\frac{m\bar{r}}{q} \geq \frac{8\pi kT_o}{\bar{\ddot{y}}_f^2 p_m Q} .$$

An interesting characteristic of the pendulous suspension is that a larger mass is required than for a linear suspension because as will be shown later q is always greater than $\bar{r}$.

3 MAXIMIZING SENSITIVITY

From the parallel axis theorem for moving the reference point for the moment of inertia of a mass about its center of mass to a point located $\bar{r}$ from the center of mass,

Equation 3-1 $I_o = I_{cm} + m\bar{r}^2$,

where I_o = moment of inertia about pivot point located $\bar{r}$ from the center of mass

I_{cm} = moment of inertia of mass m about its center of mass.

Equation 3-1 can be written in terms $\bar{r}$, q, and κ_{cm} the radius of gyration of the mass about it center of mass as,

Equation 3-2 $\bar{r}q = \kappa_{cm}^2 + \bar{r}^2$ or,

$$q = \frac{\kappa_{cm}^2}{\bar{r}} + \bar{r}.$$

Differentiating Equation 3-2, setting the result to zero shows that the minimum q occurs at $\bar{r} = \kappa_{cm}$. Substituting this value into Equation 3-2 shows the minimum q is given by,

Equation 3-3 $q = 2\bar{r} = 2\kappa_{cm}$.

Note also from Equation 3-2 that q can never be less than $\bar{r}$.

A plot of Equation 3-2 in normalized form is given in Figure 3-1. In addition to verifying the minimum value of q, the plot shows that q approaches infinity as $\bar{r}$ approaches zero, and as $\bar{r}$ approaches infinity, q becomes equal to $\bar{r}$. This latter fact agrees with what is expected for a simple pendulum.

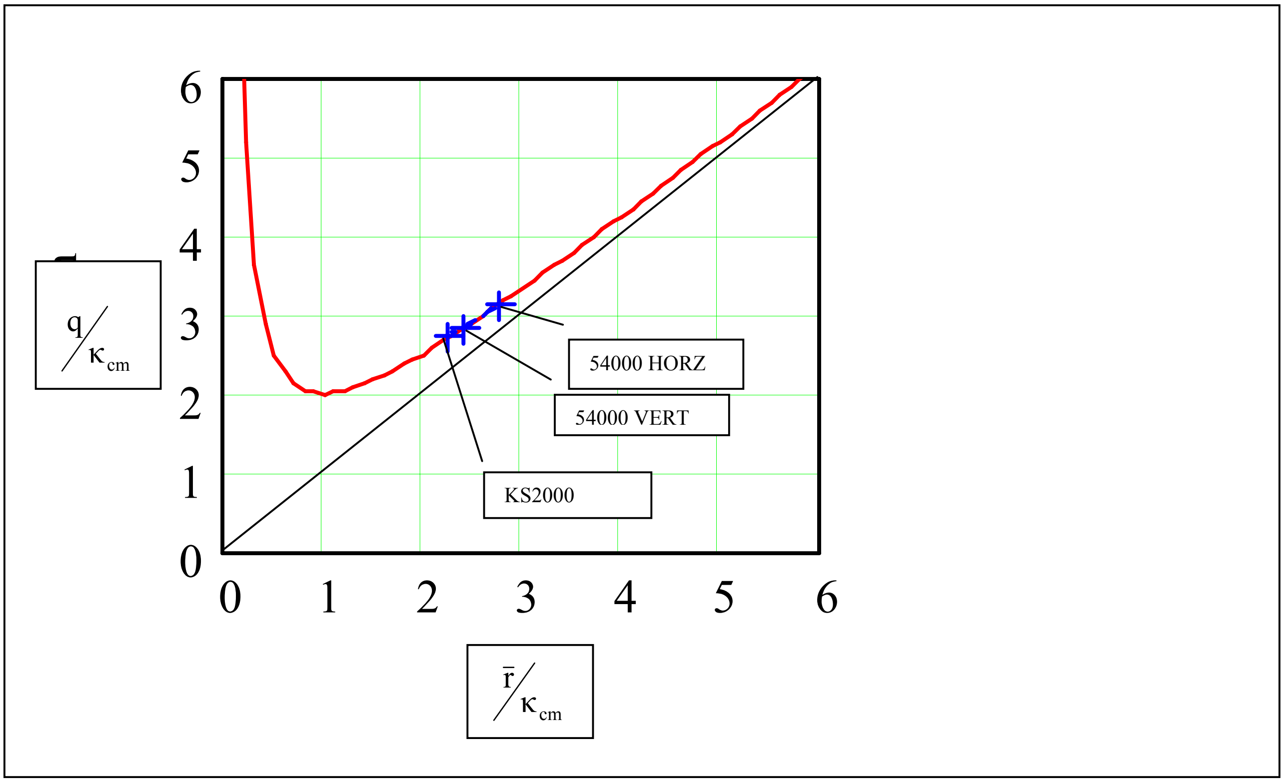


Figure 3-1 Relationship of q, $\bar{r}$, and κ_{cm} for maximizing angular sensitivity of the suspension.

The power dissipated by the damping mechanism is given by,

Equation 3-4 $P_d = 2\lambda\omega_m m\bar{r}q\bar{\dot{\theta}}^2$,

Or the energy per unit ground velocity squared is given by,

Equation 3-5 $E = \frac{2\lambda\omega_m m}{(s^2 + 2\lambda\omega_m s + \omega_m^2)^2}\frac{\bar{r}}{q}$.

Therefore, to maximize the power delivered by an instrument with a given λ, ω_m , and m,
The ratio $\bar{r}$ /q should be maximized.

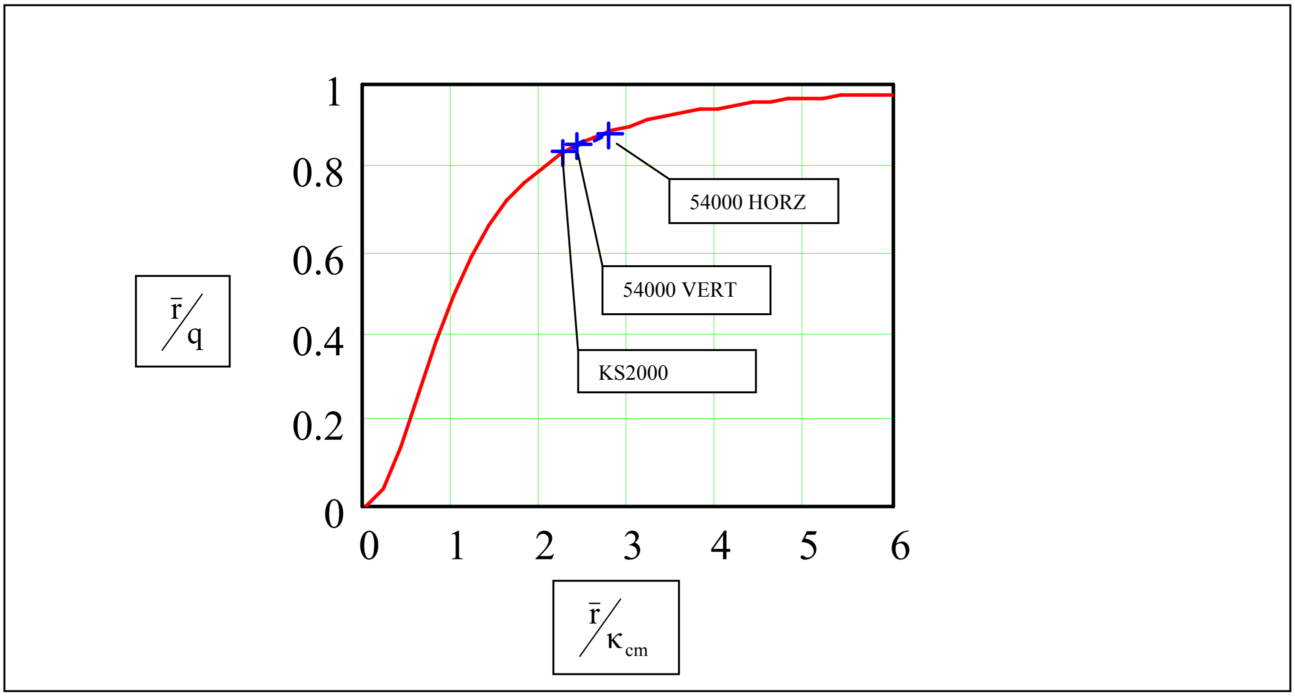
Define,

Equation 3-6 $U = \frac{\bar{r}}{q} = \frac{\bar{r}^2}{\kappa_{cm}^2 + \bar{r}^2}$.

Differentiating Equation 3-6 and setting it to zero will result in, Equation 3-7 $\frac{\kappa_{cm}}{\bar{r}} = 0.$

Since κ_{cm} cannot be zero except for a point mass, $\bar{r}$ must be infinite. This requirement simply means that the center of mass should be as far as possible from the pivot point to maximize power from the suspension. A normalized plot of Equation 3-6 is shown in Figure 3-2. Little is to be gained by making $\bar{r}$ greater than two or three times κ_{cm}.

Figure 3-2 Relationship of q, $\bar{r}$, and κ_{cm} for maximizing power sensitivity of the suspension.



Should the instrument be designed to maximize power, or maximize angular sensitivity?

If we maximize angular sensitivity (i.e. $q = 2\bar{r}$) the minimum mass requirement from Equation 2-8 becomes

Equation 3-8 $m \geq (2)\frac{8\pi k T_o}{\bar{\ddot{y}}_f^2 p_m Q},$

where for maximum power (i.e. $q = \bar{r}$)

Equation 3-9 $m \geq \frac{8\pi k T_o}{\bar{\ddot{y}}_f^2 p_m Q}.$

Thus, what is the compromise? In a borehole instrument, maximizing $\bar{r}$ to satisfy Equation 3-9 becomes difficult. On the other hand, doubling the mass to satisfy Equation 3-8 is also not particularly desirable. Once p_m and Q have been given reasonable values, and $\bar{\ddot{y}}_f^2$ has been defined, then the minimum mass required will be between the two values given by equations Equation 3-8 and Equation 3-9.

It is constructive to consider what has been done in past designs of pendulous seismometers. The KS54000 Borehole Seismometer has $q/\bar{r}$ values of 1.1756 and 1.1327 for the vertical and horizontal suspensions respectively. The $\bar{r}/\kappa_{cm}$ values for the KS54000 vertical and horizontal suspensions are 2.3860 and 2.7455 respectively. The $q/\bar{r}$ for the KS2000 Broadband Seismometer is 1.1957 and the $\bar{r}/\kappa_{cm}$ value is 2.260. These values are plotted in Figure 3-1 and Figure 3-2. Thus, within the limits of present design volumes and dimensions it has not been difficult to achieve a $q/\bar{r}$ ratio approximately one.

Equation 3-8 and Equation 3-9 clearly show that to detect a given noise spectra $\ddot{y}_f^2$, the values of m, p, Q as well as the ratio $q/\bar{r}$ can be manipulated within reason to achieve the desired goal. That is

Equation 3-10 $$mp_m Q \frac{\bar{r}}{q} \geq \frac{8\pi k T_o}{\ddot{y}_f^2}.$$

The product of $p_m Q$ ranges from 20 sec. for the KS2000 design to 130 sec for the KS54000 suspension. With $\bar{r}/q$ in the range of .83 (see Figure 3-2), T_o = 300 ºK, and $p_m Q$ = 20 sec.,

Equation 3-11 $$m \geq \frac{6.26x10^{-21}}{\ddot{y}_f^2}.$$

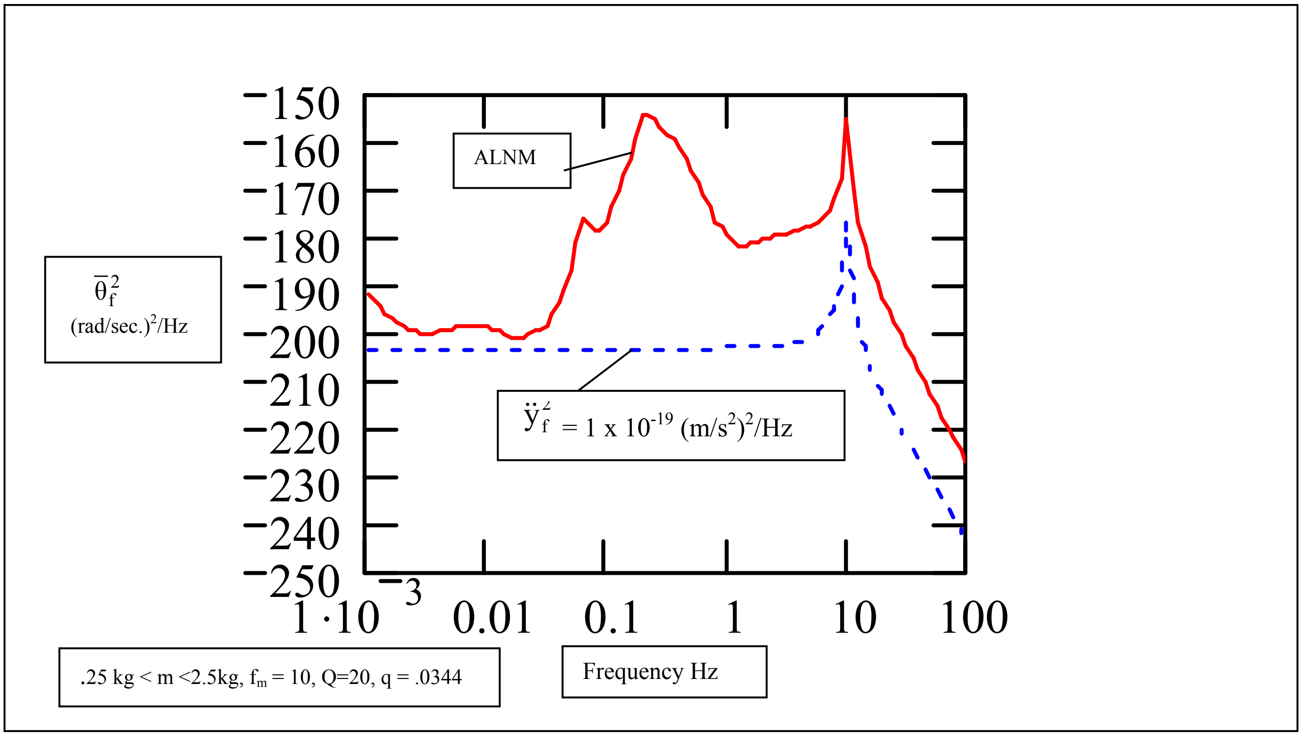
For a white noise threshold of 1 x 10^{-19} (m/sec^2)/Hz, the mass must be greater than .063 kilogram, and for a noise threshold of 1 x 10^{-20} (m/sec^2)/Hz, the mass must be greater than .626 kilogram.

For $p_m Q$ = 130 sec, the mass must greater than .0096 kg for 1 x 10^{-19} (m/sec^2)/Hz, or .096 kg for 1 x 10^{-20} (m/sec^2)/Hz.

For the vertical KS54000 with a mass of 0.367 kg, $p_m Q$ = 130 sec and $q/\bar{r}$ = 1.1756, the thermal noise threshold is 2.56 x 10^{-21} (m/sec^2)^2/Hz. For the KS2000 with mass = .184 kg, $p_m Q$ = 20, and $q/\bar{r}$ = 1.1957 the thermal noise threshold is 3.38 x 10^{-20}.

The output of the suspension to a white noise threshold of 1 x 10^{-19} (m/sec^2)/Hz and to the Albuquerque Low Noise Model (ALNM) is shown in Figure 3-3 for a suspension with a T_o of .1 sec., and Q of 20, and a q of .0344 m. The noise threshold of the suspension is below the low noise model over the entire bandwidth as expected. This will not be true for the noise of the mass position detector as will be shown in the next section. The rms value of the angular random motion of the mass in Figure 3-3 is 6.24 x 10^{-12} radian. If a displacement sensor is located .03 m from the pivot of the suspension the random displacement between frame and mass is 1.87 x 10^{-13} m rms (approximately 1 x 10^{-12} m p-p). These last calculations give an order of magnitude for the resolution required of the mass position sensor.

Figure 3-3 Thermal noise referred to angular output of suspension.



4 DETECTING THE SUSPENSION MOTION

Once the suspension has been designed to have a threshold signal greater than the thermal noise, the next step is to convert the angular output of the suspension into a useable electrical or digital signal.

For an analog design assume the amplifier at the output of the detector has a noise spectra referred to its input of E_{pa} volts^2/Hz, and that the noise is white noise without 1/f noise.

The sensitivity required of the detector will then be,

Equation 4-1 $$K_B \geq \sqrt{\frac{E_{pa}}{\overline{\theta}_f^2}} \text{ Volts/radian.}$$

Table 1 lists a range of minimum values for K_B for different preamplifier noise levels, and different signal threshold values for the suspension described in Figure 3-3.

Table 1 Range of minimum values for detector sensitivity

K_B Volts/radian	$\overline{E}_{pa}^2$ Volts^2/Hz	$\overline{\theta}_f^2$ $(f < f_m)$rad^2/Hz	$\overline{\ddot{y}}_f^2$ (m/s^2)^2/Hz
429	1×10^{-18}	5.42×10^{-24}	1×10^{-19}
1358	1×10^{-18}	5.42×10^{-25}	1×10^{-20}
1358	1×10^{-17}	5.42×10^{-24}	1×10^{-19}
4295	1×10^{-17}	5.42×10^{-25}	1×10^{-20}

Using Equation 1-2 and Equation 4-1 the E_{pa} noise can be referred to the input of the suspension,

Equation 4-2 $$\overline{\ddot{y}}_f^2 = q^2 (s^2 + 2\lambda_m \omega_m s + \omega_m^2)^2 \frac{\overline{E}_{pa}^2}{K_B^2}.$$

Figure 4-1shows the affect on the $\overline{E}_{pa}^2$ noise threshold of varying the natural frequency, ω_m. Decreasing the natural frequency of the suspension has a remarkable affect on the low frequency threshold, but it has little effect on where the threshold crosses the ALNM at the high frequencies.

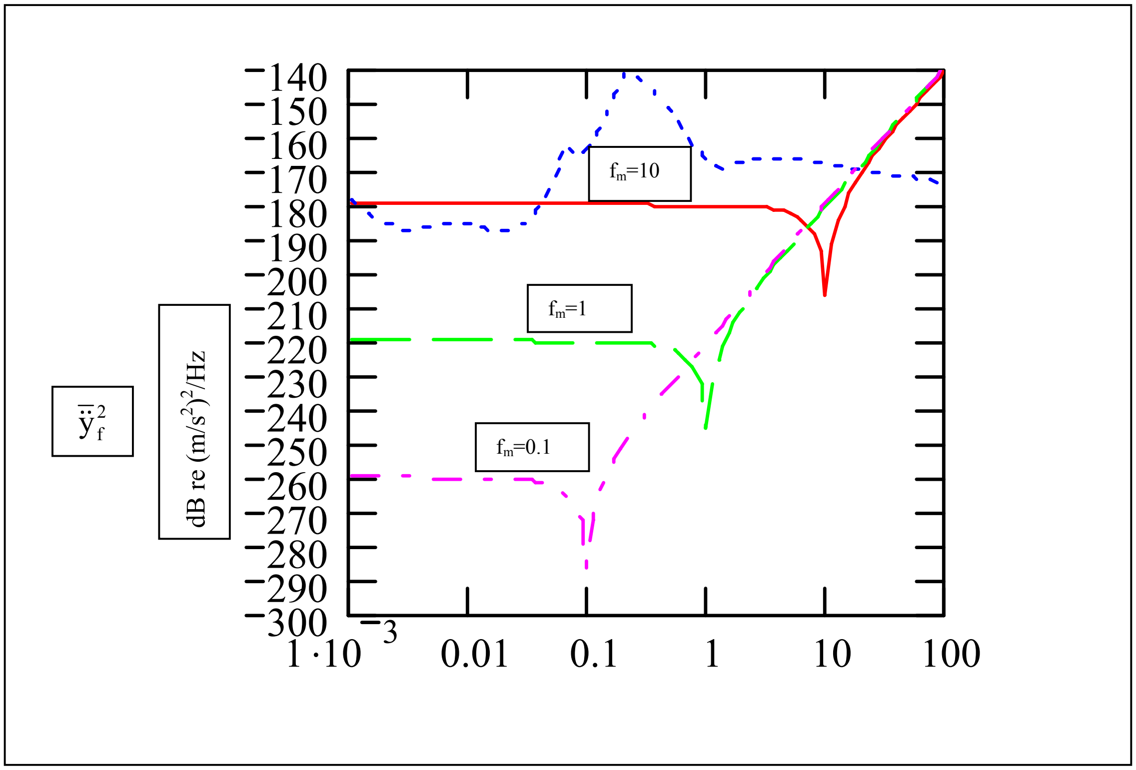


Figure 4-1 E_{pa} noise threshold versus f_m.

In , Figure 4-2 the natural frequency of the suspension is held constant and K_B is the variable. From an examination of the last two figures, there might be a combination of K_B and f_m that could provide a threshold that is below the ALNM over the frequency band of interest.

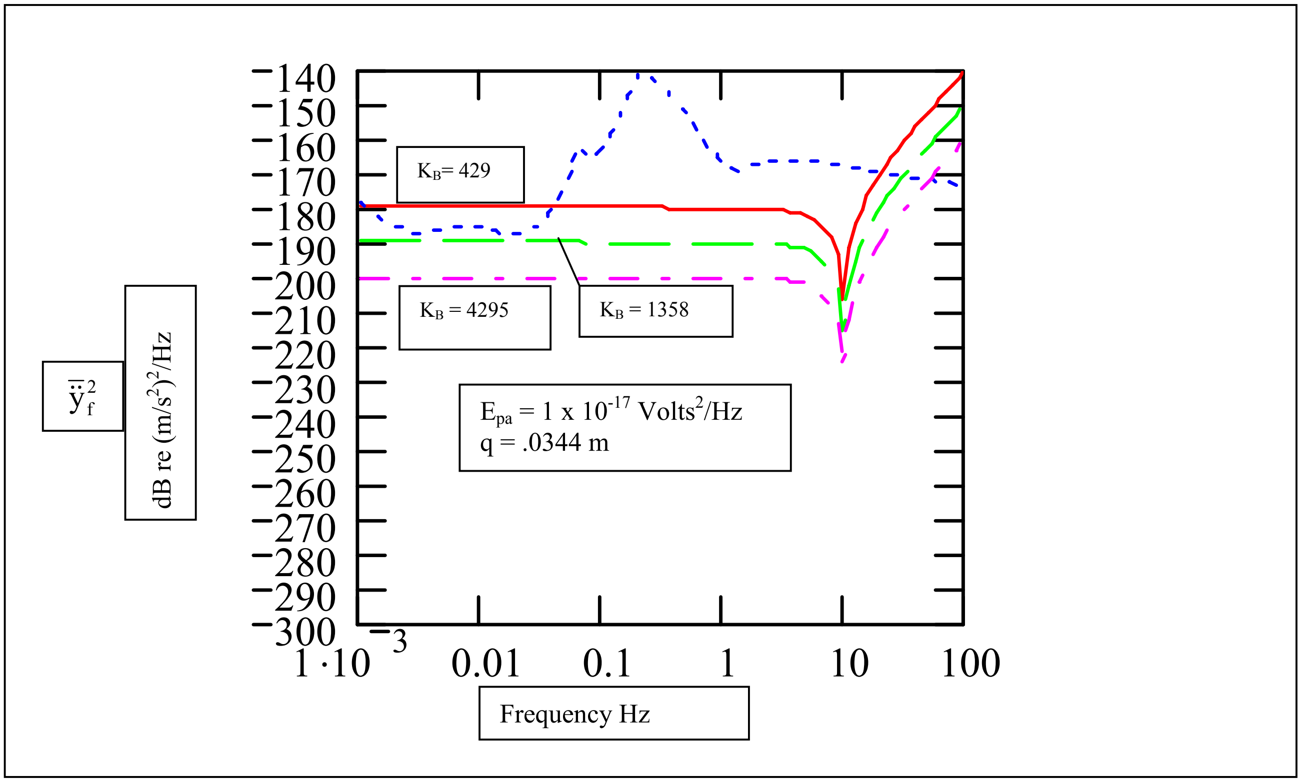


Figure 4-2 Epa noise threshold vs. K_B

It should be restated that Figure 4-1 and Figure 4-2 assume the ideal theoretical situation of no 1/f noise and possibly K_B values that may not be realistic.

Figure 4-3 shows the theoretical E_{pa} noise thresholds of the KS54000 and the KS2000 suspensions. Other noise sources in the system will generally keep the design from seeing these thresholds. In particular, the thermal noise threshold of the KS54000 is -200 dB worst case, and typically it is -204 dB. The typical thermal noise threshold of the KS2000 is -195 dB. It is obvious that the KS54000 suspension has much more sensitivity at the low frequencies than is required, and that the KS2000 suspension has far better potential at the higher frequencies.

Also shown in Figure 4-3 is a theoretical model of a suspension plus detector that could have a noise level 15 dB below the ALNM over the frequency from dc to 16 Hz. Again the assumption of no additional noise added by other elements of the system applies.

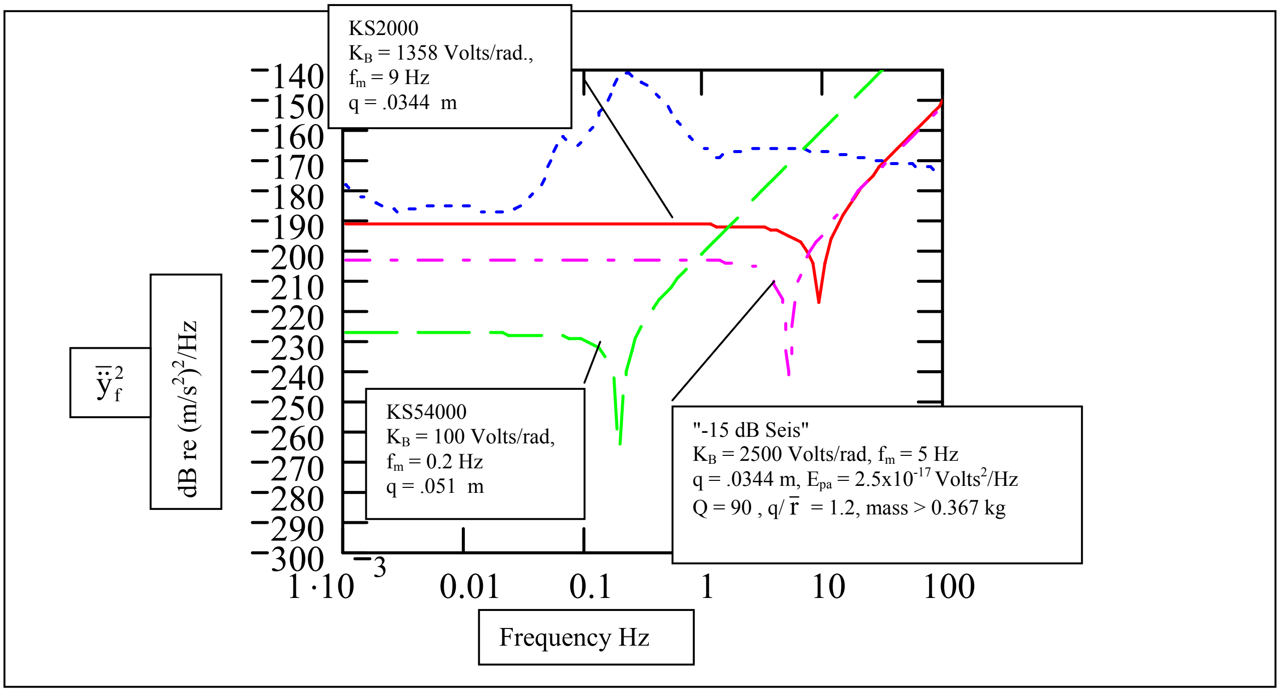


Figure 4-3 Theoretical E_{pa} noise threshold of KS54000 and KS2000 suspensions

5 MASS MOTION TRANSDUCER

This section will begin with the classical capacitor bridge transducer. This approach will set the requirements for other possible sensors.

Figure 5-1 is a schematic diagram of a variable displacement capacitor bridge transducer.

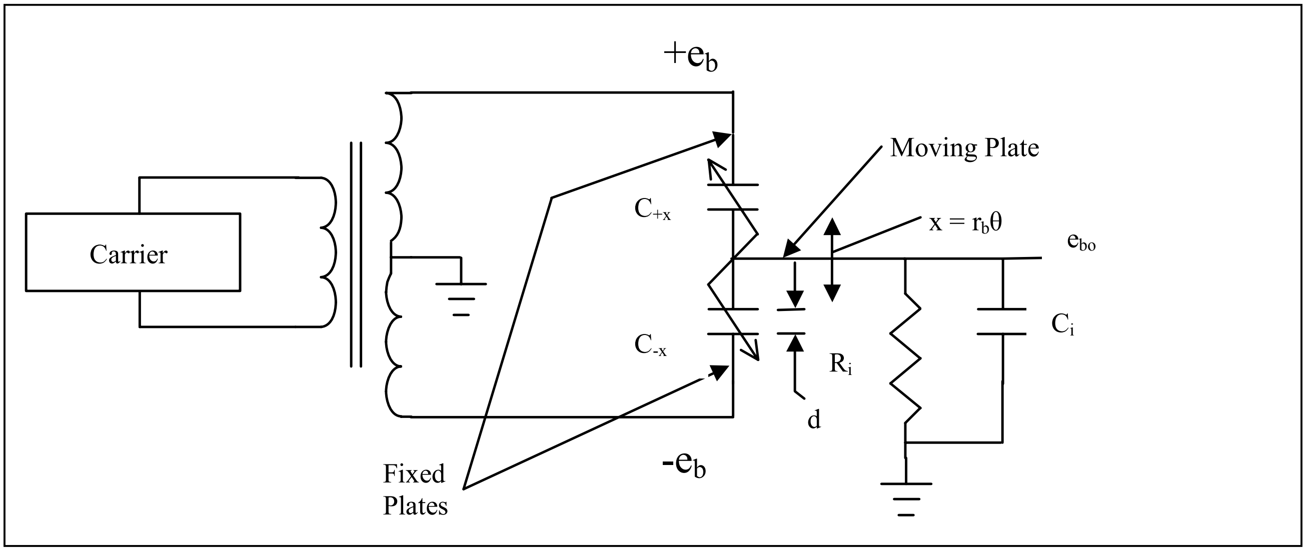


Figure 5-1 Variable displacement capacitor bridge transducer.

The moving plate of the differential capacitor is attached to the mass of the suspension, and moves in proportion to the angular displacement of the mass relative to the frame. The fixed plates of the capacitor are attached to the frame and are driven by the sinusoidal carrier signal. The fixed plate drive signals, $+e_b$ and $-e_b$, are 180 degrees out of phase. It is assumed that the impedance of the carrier source is small compared to the impedance of C_{+x} and C_{-x}, and that R_i is large compare to these impedances. The input capacitance of the preamp, C_i, should be much smaller than C_{+x} and C_{-x}. If it is not, the sensitivity of the transducer will be degraded. The output of the bridge circuit is a carrier signal that is modulated by the motion of the mass. Ideal demodulation of this signal will recover the mass motion signal with correct amplitude and phase without introducing 1/f noise.

Let A equal the area between the plates and ε equal the permittivity between the plates, then

Equation 5-1 $$C_{+x} = \frac{\varepsilon A}{d - x}\text{, and}$$

$$C_{-x} = \frac{\varepsilon A}{d + x}.$$

Then considering each half of the bridge separately and including the capacitance C_i, the bridge sensor sensitivity is

Equation 5-2 $$K_B = \frac{r_b e_b}{d} \frac{1}{\left(1 - \frac{x^2}{d^2}\right)\left(1 + \frac{C_i}{2C_0}\right)},$$

Where C_0 is the capacitance of C_{+x} and C_{-x} at x = 0, and r_b is distance from the pivot point of the suspension to the center of the transducer.

If x <<< d and C_i <<< C_0, Equation 5-2 becomes in the ideal case

Equation 5-3 $$K_B = \frac{r_b e_b}{d}$$ Volts rms/radian.

Figure 4-2 shows that for a given f_0, increasing K_B will decrease the noise level across the bandwidth and is particularly helpful at the higher frequencies. For a borehole instrument, r_b is limited by the dimensions to 2 to 4 cm. The plate drive voltage, e_b, is limited by the arc over voltage across the gap dimension, d. The power supply voltages available also limits d . The gap dimension is limited by the precision of the parts and the allowable drift in mass position resulting from external sources (e.g. temperature, tilt, etc). The gap is also limited by the second order term in Equation 5-2 and the linearity requirements of the sensor. Decreasing d to a minimum that is acceptable not only increases K_B, but it also increases C_0 to further attenuate the effect of C_i. C_0 is also increased by increasing the area of the capacitor plates (a multiple plate capacitor is a possibility).

For r_b = .02 m, e_b = 40 Volts rms, and d = 2.54 x 10^{-4} m, the bridge sensitivity K_b is on the order of 3000 Volts rms/radian. The maximum nonlinearity will occur at the peak value of x, which is a function of the feedback circuit.

Figure 5-2 is a schematic of a variable area capacitance bridge sensor. Let the nominal area between a fixed plate and the moving plate (at x = 0) be a dimension c times the dimension d. Let the differential area be x times c. Then

Equation 5-4 $$C_{+x} = \varepsilon \frac{c}{b}(d + x),$$ and

$$C_{-x} = \varepsilon \frac{c}{b}(d - x)$$

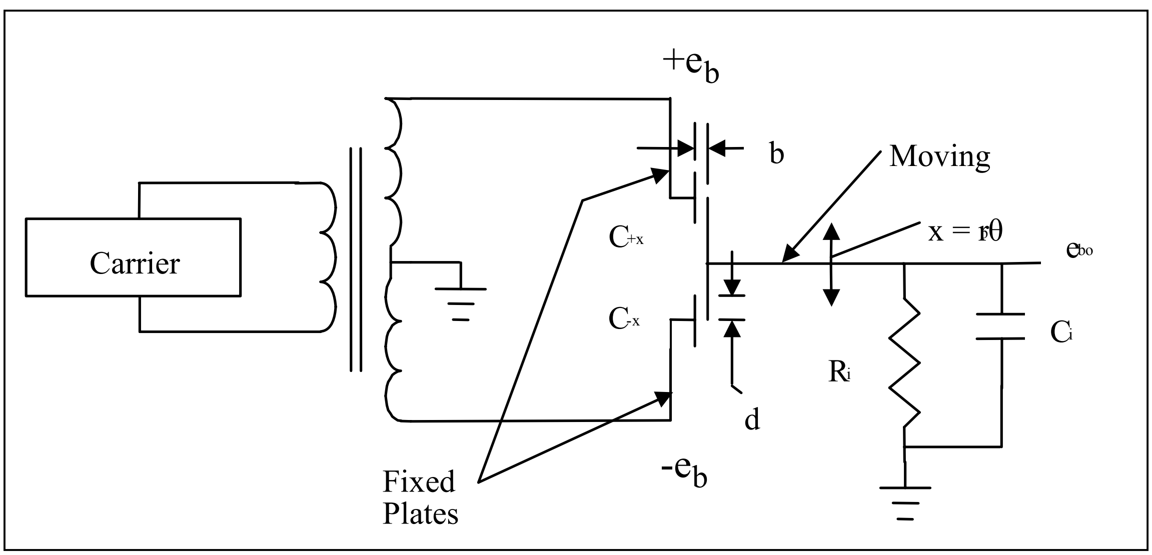


Figure 5-2 Variable area capacitor bridge transducer

Then considering each half of the bridge separately and including the capacitance C_i, the variable area bridge sensor sensitivity is

Equation 5-5 $$K_B = \frac{r_b e_b}{d} \frac{1}{\left(1 + \frac{C_i}{2C_0}\right)}$$

The one advantage the variable area transducer has over the variable displacement sensors is the lack of the nonlinearity term involving x^2. A significant disadvantage is that the dimension d can never be as small as in the variable displacement because the fringing effect at the plate edges will introduce significant nonlinearity. A multiple plate capacitor is usually required to get C_0 much greater than C_i. For $r_b = .02$ m, $e_b = 40$ Volts rms, and d = .003 m, and $C_o >> C_i$, the sensitivity will be on the order of 270 Volts rms/radian.

The variable displacement capacitor sensor is the only one of the two that can meet the criteria of 2500 Volts rms required by the "-15 dB seis" defined in Figure 4-3.

This analysis of the capacitor sensors can also define the criteria required by any other sensor that might be proposed. If the sensor with $K_B = 2500$ Volts rms/radian is combined with a preamp with a noise level of 2.5×10^{-17} Volts^2/Hz, the detection threshold is 4×10^{-24} radians^2/Hz, or if the sensor is located .02 m from the pivot point, the displacement threshold for x is 1.6×10^{-27} m^2/Hz. For a 50 Hz bandwidth, this threshold is 2.8×10^{-13} m rms, or approximately 1.4×10^{-12} m p-p.

At this point, basic physics suggest that a suspension and sensor that could be used in a borehole seismometer with a "-15 dB" threshold relative to the ALNM over the bandwidth required is feasible. Experience with seismometers that are available in today's market provide some confidence that such a suspension and sensor can be built with the proper selection of materials and design expertise.

The next step is to design a workable circuit that will stabilize the suspension and provide the bandwidth required.

The conventional analog force feedback circuits will be analyzed and the criteria set for use in this application.

6 CONTROL LOOP CIRCUITS

The block diagram in Figure 6-1 is a typical analog feedback control loop used to provide a signal proportional to ground acceleration.

The equation describing the response of this circuit is

Equation 6-1 $$S_a = \frac{E_o}{\ddot{y}} = \frac{K_s K_B K_{pa} K_{LD}}{1 + K_s K_B K_{pa} K_{LD} \frac{K_m}{m\bar{r}} \left(\frac{1}{R_K} + C_K s \right)},$$

where $$K_s = \frac{1}{q\left(s^2 + 2\lambda_m \omega_m s + \omega_m^2\right)}.$$

Multiplying thru by $\left(s^2 + 2\lambda_m \omega_m s + \omega_m^2\right)$ and letting $K_B K_{pa} K_{LD}$ be the forward loop gain K_{fl} Equation 6-1 becomes

Equation 6-2 $$S_a = \frac{K_{fl}/q}{s^2 + \left(2\lambda_m \omega_m + \frac{K_{fl} K_m C_K}{m\bar{r}q} \right) s + \left(\omega_m^2 + \frac{K_{fl} K_m}{m\bar{r}q R_K} \right)}.$$

Let

Equation 6-3 $$2\lambda_c \omega_c = \left(2\lambda_m \omega_m + \frac{K_{fl} K_m C_K}{m\bar{r}q} \right), \text{ and}$$

$$\omega_c^2 = \left(\omega_m^2 + \frac{K_{fl} K_m}{m\bar{r}q R_K} \right).$$

The closed loop natural frequency is ω_c, and λ_c is the closed loop damping. If $\omega_c >> \omega_m$, and $\lambda_m << 1$, then Equation 6-2 becomes,

Equation 6-4 $$S_a = \frac{K_{fl}/q}{s^2 + \left(\frac{K_{fl} K_m C_K}{m\bar{r}q} \right) s + \left(\frac{K_{fl} K_m}{m\bar{r}q R_K} \right)}$$

$$= \frac{K_{fl}/q}{s^2 + 2\lambda_c\omega_c s + \omega_c^2}$$

The most efficient torque transducer is a coil and magnet type located at some distance r_m from the pivot point. Let the force constant of the coil-magnet combination be G N/A, then Equation 6-4 becomes

Equation 6-5 $$S_a = \frac{K_{fl}/q}{s^2 + \left(\frac{K_{fl}Gr_mC_K}{m\bar{r}q}\right)s + \left(\frac{K_{fl}Gr_m}{m\bar{r}qR_K}\right)}.$$

It is assumed that the resistance of the force transducer coil is much less than R_K.

An examination of Equation 6-5 shows that all of the constants are set by electronic components or dimensional components that are reasonably stable with temperature. For example the sensitivity for frequencies less than ω_c is,

Equation 6-6 $$S_a = \frac{m\bar{r}R_K}{r_mG} \text{ Volts/(m/sec}^2\text{)}.$$

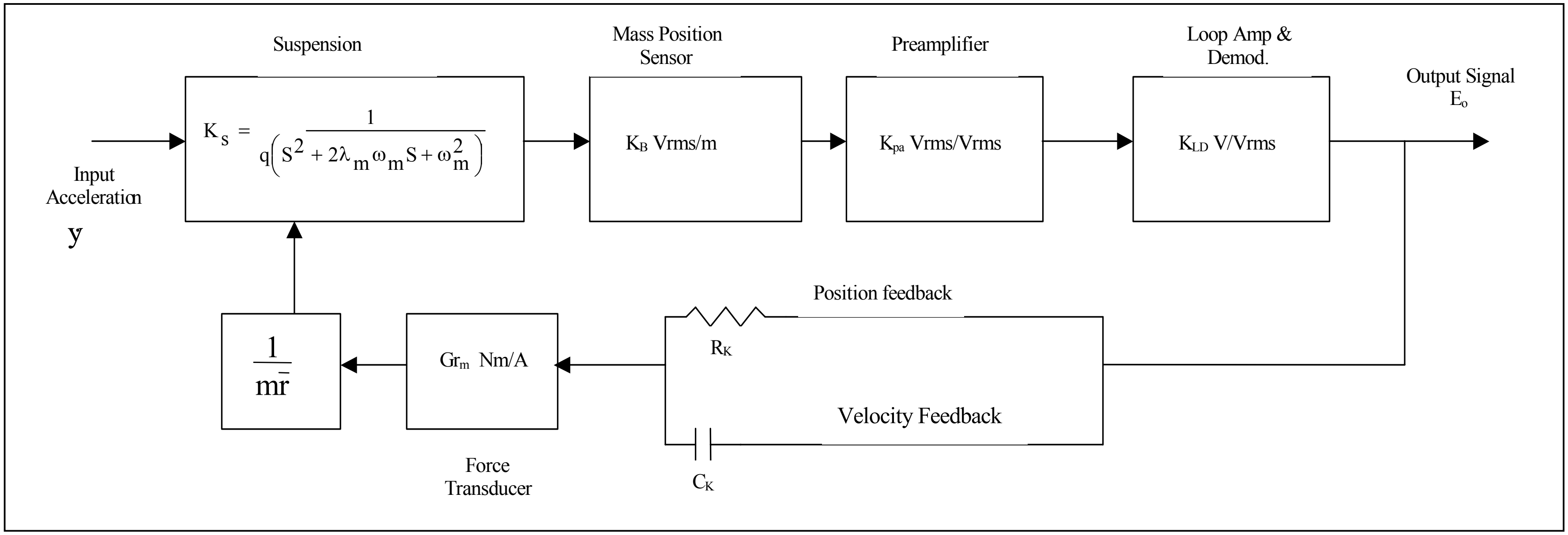


Figure 6-1 Basic loop block diagram.

The sensitivity is no longer a function of the mechanical suspension. The mass, m, and the dimensions r_m and $\bar{r}$ (they could be equal) are very stable, R_K is a resistor, and G is a function of the length of the wire in the magnet gap and the flux density in the gap. This stability of the sensitivity and the ability to set the high frequency corner, ω_c, and the damping, λ_c, is the main advantage of using a control loop around the suspension.

The sensitivity S_a is set by the required detection threshold and the input noise of the recording instrument, digitizer, etc. If the recording instrument noise is E_{dgn} Volts^2/Hz, then

Equation 6-7 $$S_a = \sqrt{\frac{E_{dgn}}{\ddot{y}^2}}.$$

For example, if the threshold is to be -202 dB (see "-15 dB" seis" Figure 4-3), and if the noise E_{dgn} is -144 dB re 1 Volt^2/Hz, then the minimum S_a is 794 Volts/(m/sec^2). This is the minimum if the digitizer is connected at E_o.

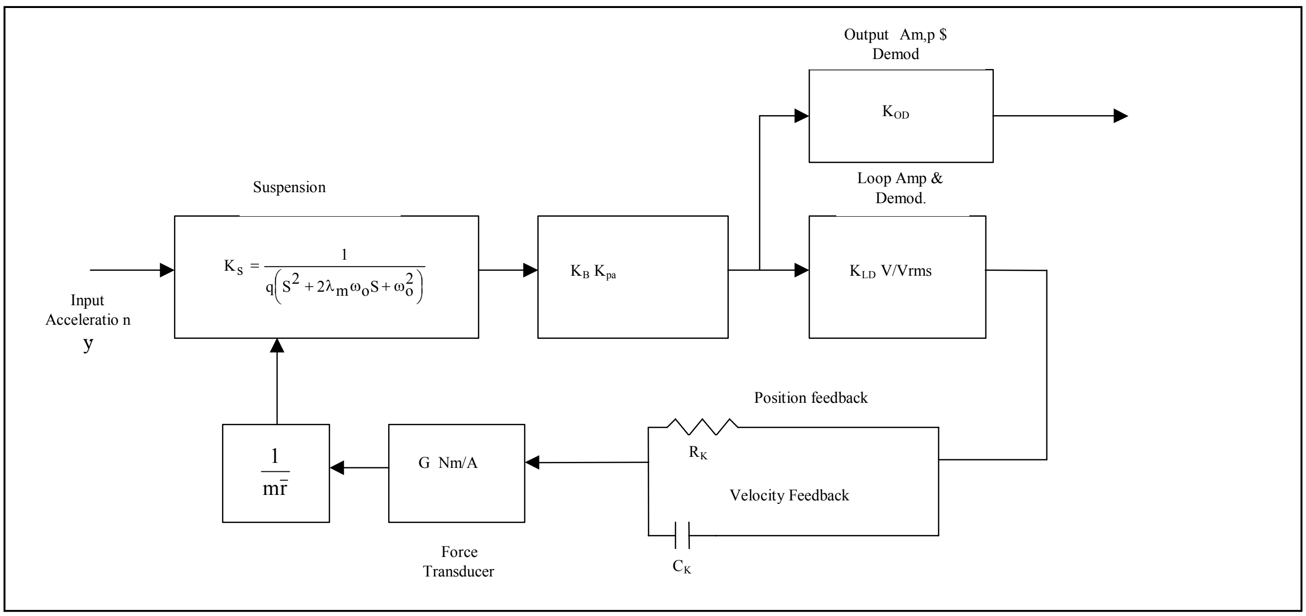


Figure 6-2 Loop diagram with alternate output.

It is possible and may be desirable to extract the signal at a different point in the loop as shown in Figure 6-2. This configuration allows the output amplifier/demodulator, K_{OD}, to have a different gain and frequency response than the loop amplifier/demodulator, K_{LD}. For example the output amplifier is usually ac coupled, and in some cases includes an integrator for a signal that is proportional to velocity. Many times the requirements of the loop amplifier for bandwidth, stability, etc. are contrary to the gain required to match the seismometer output to the digitizer or other recording instrument.

Another possibility for a control loop is to incorporate the digitizer into the control loop as shown in Figure 6-3.

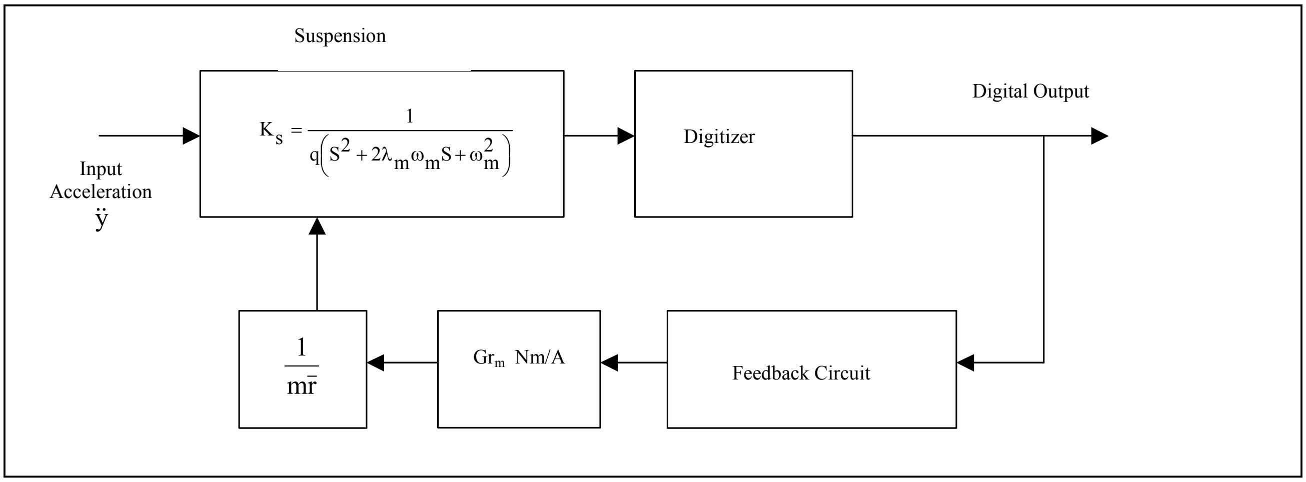


Figure 6-3 Digital loop block diagram

This subject was not considered in Phase II. One advantage of this design is that the signal is digitized at the lowest noise point in the loop. It is recommended that type loop be considered in a separate study.

Regardless of the control loop used, the noise of the various circuits and their influence on the detection threshold of the instrument must be analyzed.

7 MAGNET/COIL FEEDBACK TORQUE TRANSDUCER

The most compact force feedback transducer for this application is a coil and magnet transducer. In section 6 the force constant G and the feed back resistor R_K was introduced. In this section some relationships of these values to the other parameters will be explained and explored.

The minimum sensitivity S_{Amin} can be determined from Equation 6-7 and then from Equation 6-6 the following can be written

Equation 7-1 $$\frac{G}{R_K} = \frac{m\bar{r}}{S_{A\min} r_m}.$$

The values of m, $\bar{r}$ and indirectly r_m are determined from the thermal noise requirements of section 2. (It is assumed that the back emf or damping of the coil and magnet is included in the Q criteria.) Therefore Equation 7-1 defines the ratio G/R_K.

It can be shown that the value of the damping or Q associated with the coil and feedback resistor can be written as

Equation 7-2 $$Q_{fbc} = \frac{m\bar{r}q\omega_m}{r_m^2}\frac{R_K}{G^2}$$

Substituting this equation into Equation 2-7 will give the thermal noise spectra associated with the back emf damping as

Equation 7-3 $$\bar{\ddot{y}}_{fbc}^{\,2} = \frac{4kT_o r_m^2}{m\bar{r}^2}\frac{G^2}{R_K}.$$

Substituting Equation 7-1 into Equation 7-3 gives

Equation 7-4 $$\bar{\ddot{y}}_{fbc}^{2} = \frac{4kT_o}{S_{A\min}^{\;2}} R_K.$$

If the suspension is in a vacuum so that there is no air damping, this noise source could be significant. It does place a maximum value on the feedback resistance R_K. That is,

Equation 7-5 $$R_K \le \frac{S_{A\min}^2 \bar{\ddot{y}}_{fbc}^2}{4kT_o}.$$

The maximum peak acceleration that the loop can handle is determined as follows. Let the maximum positive or negative peak voltage at E_o be E_m. Then the maximum current through the coil is

Equation 7-6 $$I_m = \frac{E_m}{R_K}.$$

It is assumed that the amplifier driving R_K has this capability to deliver this current. The maximum torque generated by this current is

Equation 7-7 $$T_m = \frac{Gr_m}{R_K} E_m.$$

Equating this torque to the maximum acceleration torque gives

Equation 7-8 $$\ddot{y}_m = \frac{Gr_m}{m\bar{r}R_K} E_m.$$

Solving for G

Equation 7-9 $$G \geq \frac{m\bar{r}\ddot{y}}{E_m r_m} R_K$$

Substituting Equation 7-1 into Equation 7-8gives

Equation 7-10 $$\ddot{y}_m = \frac{E_m}{S_{A\,min}}.$$

Equation 7-5 suggests that R_K should be made as small as possible for minimum thermal noise. In conjunction with Equation 7-1 this criteria would make G small, but Equation 7-9 places a lower limit on G as a function of R_K. There are however other criteria on R_K.

First R_K should be much greater than the coil resistance that is in series with R_K. The coil is wound with copper wire and is subject to greater change with temperature than R_K. The series combination has a much smaller variation with temperature. Another requirement comes from Equation 6-3.
If we assume that ω_m is much less than ω_c and that λ_m is very small compared to the total closed loop damping and define $K_m = Gr_m$, then

Equation 7-11 $$\omega_c^2 = \frac{K_{fl} r_m}{m\bar{r}q} \frac{G}{R_K}$$ and

Equation 7-12 $$\lambda_c = \frac{K_{fl} r_m}{m\bar{r}q} \frac{GC_K}{2\omega_c}.$$

Substituting Equation 7-11 into Equation 7-12 gives

Equation 7-13 $$R_K C_K = \frac{2\lambda_c}{\omega_c}.$$

The resistor R_K should be large enough that C_K is not unreasonably large.

Equation 7-11 can be written in terms of S_{Amin} by substituting Equation 7-1 into Equation 7-11.

Equation 7-14 $$\omega_c^2 = \frac{K_{fl}}{qS_{A\,min}}$$

It is interesting to repeat Equation 7-1, Equation 7-10 and Equation 7-14 and note the dependence on S_{Amin}.

$$\frac{G}{R_K} = \frac{m\bar{r}}{S_{A\,min} r_m}$$

$$\ddot{y}_m = \frac{E_m}{S_{A\,min}}$$

$$\omega_c^2 = \frac{K_{fl}}{qS_{A\,min}}$$

S_{Amin} is determined by the desired acceleration noise spectra and the available input noise spectra of the digitizer (or following amplifier). The physical components m, $\bar{r}$, q, and r_m are determined by keeping the thermal noise below the desired acceleration noise spectra. E_m is set by the maximum peak acceleration (i.e. dynamic range) required, and K_{fl} determines the bandwidth of the instrument.

The design of K_{fl} requires a careful evaluation of the various noise levels and gains of its component entities to ensure a minimum of electronic noise in the instrument. These noise inputs are explored in section 8.

8 NOISE ANALYSIS OF CONTROL LOOP

Several noise sources are shown in the diagram of Figure 8-1. The effect of a noise source can be determined in a manner similar to that used to define S_a. For example,

Equation 8-1
$$\frac{\overline{E}_o^2}{\overline{E}_{pan}^2} = \left(\frac{K_{pa} K_{LD}}{1 + K_s K_B K_{pa} K_{LD} \frac{K_m}{m\bar{r}} \left(\frac{1}{R_K} + C_K s \right)} \right)^2 ,$$

where $\overline{E}_o^2$ and $\overline{E}_{pan}^2$ are the respective noise spectral densities. Making the same substitutions that were made for Equation 6-4 and Equation 6-5 this equation becomes

Equation 8-2
$$\frac{\overline{E}_o^2}{\overline{E}_{pan}^2} = \left(\frac{K_{pa} K_{LD} \left(s^2 + 2\lambda_m \omega_m + \omega_m^2 \right)}{s^2 + 2\lambda_c \omega_c s + \omega_c^2} \right)^2 .$$

A similar equation can be written for each of the noise sources in the forward section of the loop circuit. The noise referred to the input of the seismometer can be obtained by dividing by S_a^2 to give, for example

Equation 8-3
$$\frac{\ddot{y}^2}{\overline{E}_{pan}^2} = \left(\frac{q\left(s^2 + 2\lambda_m \omega_m + \omega_m^2 \right)}{K_B} \right)^2 .$$

If we assume that the gain of the preamp K_{pa} is such that its noise at its output is greater than $\overline{E}_{LD}^2$, then the preamp noise will predominate. This assumption must be verified for the actual circuits used.

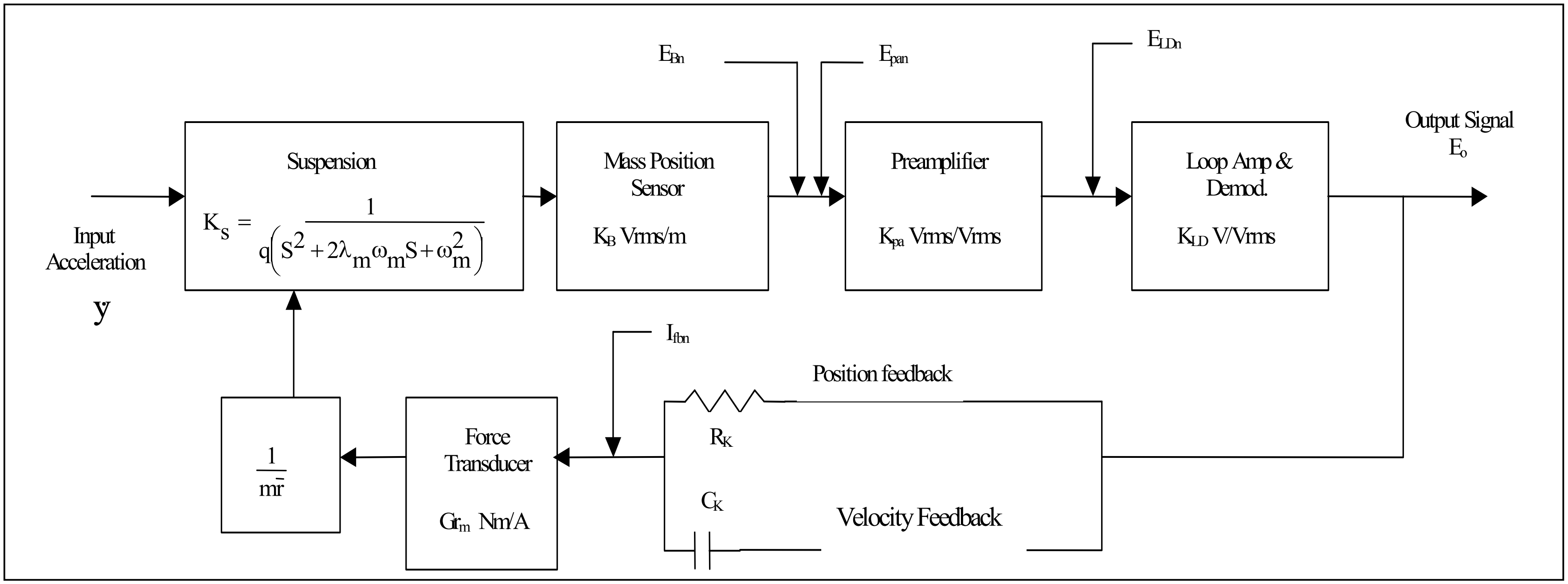


Figure 8-1 Various noise inputs to loop circuit

The effect of noise generated in the feedback loop can be derived in a similar manner, but the results are different.

$$\text{Equation 8-4} \quad \frac{\overline{E}_o^2}{\overline{I}_{fbn}^2} = \left[\frac{\dfrac{Gr_m}{m\bar{r}} K_s K_B K_{pa} K_{LD}}{1 + K_s K_B K_{pa} K_{LD} \dfrac{K_m}{m\bar{r}} \left(\dfrac{1}{R_K} + C_K s \right)} \right]^2$$

$$= \left[\frac{\dfrac{Gr_m}{m\bar{r}q} K_B K_{pa} K_{LD}}{s^2 + 2\lambda_c \omega_c s + \omega_c^2} \right]^2 .$$

The feedback noise referred to the input is

$$\text{Equation 8-5} \quad \frac{\overline{\ddot{y}}^2}{I_{fbn}^2} = \left(\frac{Gr_m}{m\bar{r}q} \right)^2 .$$

9 FIRST PROTOTYPE MODEL

Based on the requirements set forth in the preceding sections, a prototype model was made of proposed seismometer. This model is shown in Figure 9-1. The basic seismometer is 3.5 inches (102 mm) in diameter and approximately 96 inches in length including accessories. One of the requirements may be to install the new seismometer in existing KS54000 boreholes. Larger diameter accessories are used at both ends to allow installation in larger boreholes. Without these accessories the seismometer will fit into a 5 3/4-inch casing or possibly a 5 inch casing.

Figure 9-1 First prototype Model.

. Figure 9-2 is a CAD drawing of the internal seismometer showing the three modules and the electronics.

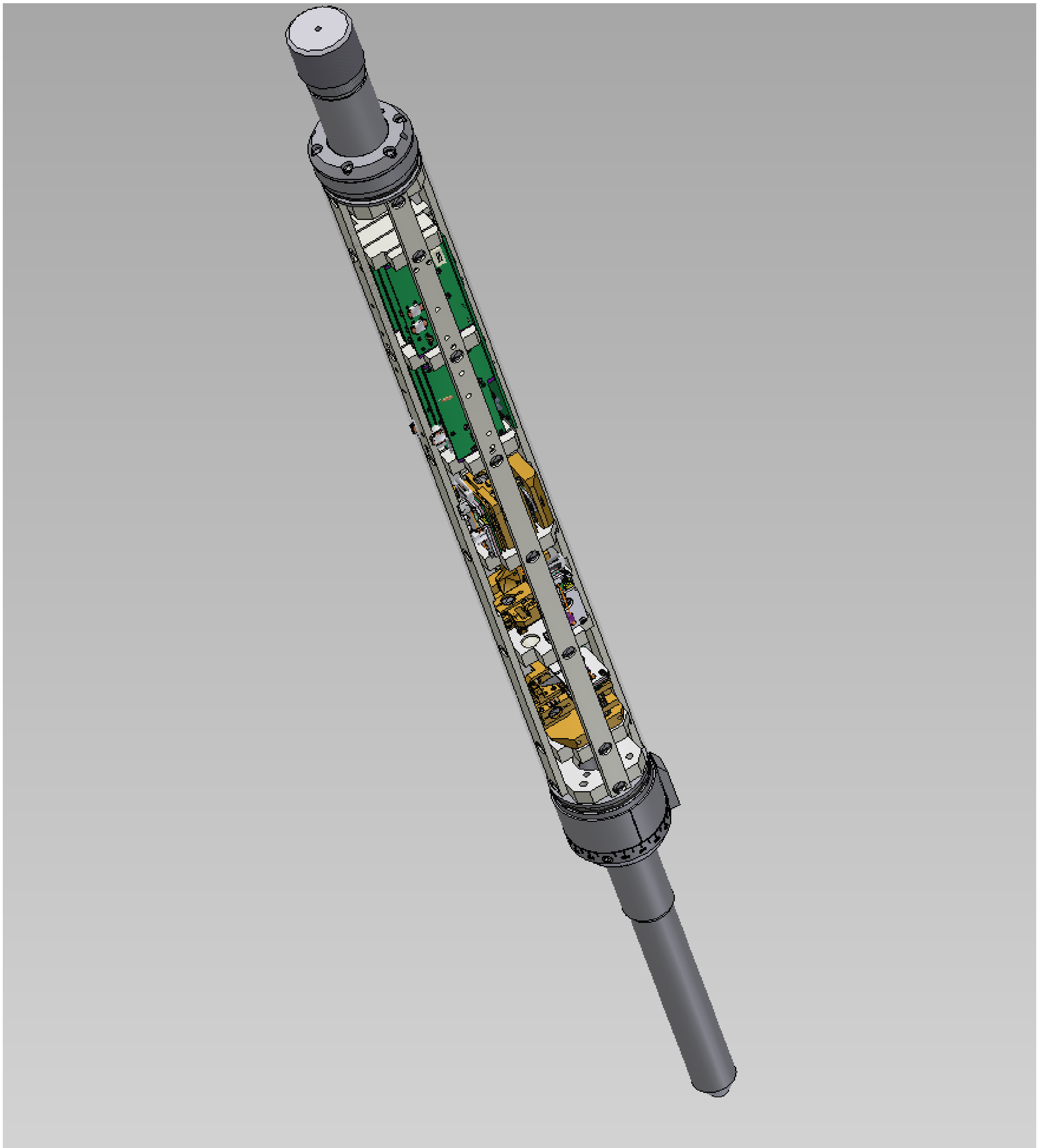


Figure 9-2 Internal view of seismometer

10 CONCLUSIONS

One of the questions to be considered in this preliminary study was to determine if it was feasible to achieve a seismometer self noise level at least 6 dB below the USGS low noise model over the frequency range of 0.2 to 16 Hz. A second question was could it be done with one instrument or would two instruments be required. Based on this preliminary theoretical study, only one instrument will be required. The requirements for the electronics, suspension, and basic sensors can be achieved with current technology based on the experience with and the history of the KS36000, KS54000 and similar designs.