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Effect of Delayed Link Failure on Probability of Loss of Assured Safety in Temperature-Dependent Systems with Multiple Weak and Strong Links

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Effect of Delayed Link Failure on Probability of Loss of Assured Safety in Temperature-Dependent Systems with Multiple Weak and Strong Links

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Abstract

Weak link (WL)/strong link (SL) systems constitute important parts of the overall operational design of high consequence systems, with the SL system designed to permit operation of the system only under intended conditions and the WL system designed to prevent the unintended operation of the system under accident conditions. Degradation of the system under accident conditions into a state in which the WLs have not deactivated the system and the SLs have failed in the sense that they are in a configuration that could permit operation of the system is referred to as loss of assured safety. The probability of such degradation conditional on a specific set of accident conditions is referred to as probability of loss of assured safety (PLOAS). Previous work has developed computational procedures for the calculation of PLOAS under fire conditions for a system involving multiple WLs and SLs and with the assumption that a link fails instantly when it reaches its failure temperature. Extensions of these procedures are obtained for systems in which there is a temperature-dependent delay between the time at which a link reaches its failure temperature and the time at which that link actually fails.

Key Words: Aleatory uncertainty, Competing failure, Competing risk, Delayed failure, Fire environment, High consequence system, Probability of loss of assured safety, Reliability, Strong link, Weak link

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Acronyms

PLOAS	probability of loss of assured safety
SL	strong link
SNL	Sandia National Laboratories
WL	weak link

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1. Introduction

Weak link (WL)/strong link (SL) systems constitute important parts of the operational design of high-consequence systems.[1-6] In such designs, the SL system is very robust and is intended to permit operation of the entire system under, and only under, intended conditions (e.g., by transmitting a command to activate the system). In contrast, the WL system is intended to fail in a predictable and irreversible manner under accident conditions (e.g., in the event of a fire) and render the entire system inoperational before an accidental operation of the SL system. Possible configurations of a WL/SL system with one WL and one SL are illustrated in Fig. 1 of Ref. [7].

An important property associated with WL/SL systems is the probability of loss of assured safety (PLOAS). Specifically, PLOAS is the probability conditional on a specific accident (e.g., a fire with well-defined properties) that the WL system fails to deactivate the entire system before the SL system fails in a manner that could allow an unintended operation of the entire system. A previous presentation has developed representations for PLOAS for accidents involving fire for a variety of WL/SL configurations.[7] Further, two related presentations consider the verification of calculations to determine PLOAS.[8, 9]

A fundamental assumption in the representations for PLOAS studied in Refs. [7-10] is that a link fails instantly when it reaches its failure temperature. The purpose of this presentation is to study representations for PLOAS obtained with the assumption that there is a delay between the time when a link reaches its failure temperature and the time at which the link actually fails.

The presentation is organized as follows. First, results obtained in Ref. [7] for PLOAS when there is no delay in link failure are briefly reviewed (Sect. 2). Then, results with constant delays in link failure are presented for systems with one WL and one SL (Sect. 3) and more generally for systems with nWL WLs and nSL SLs (Sect. 4). Next, the numerical calculation of PLOAS is illustrated with both quadrature-based and sampling-based procedures (Sect. 5). Then, the representation of PLOAS for systems with temperature-dependent delays in link failure is described for systems with one WL and one SL (Sect. 6) and also for systems with nWL WLs and nSL SLs (Sect. 7), and the numerical calculation of PLOAS for such systems is illustrated (Sect. 8). Next, the verification of PLOAS calculations is discussed and illustrated (Sect. 9). Through Sect. 9, loss of assured safety is assumed to correspond to the failure of all SLs before the failure of any WL. The calculation of PLOAS for other definitions of loss of assured safety is discussed and illustrated in Sect. 10. Finally, the presentation ends with a brief summary (Sect. 11).

The determination of PLOAS for WL/SL systems falls in the broader area of study for engineered systems known as competing risk analysis or, equivalently, competing failure analysis.[11-14]

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2. No Failure Delay

The simplest WL/SL configuration considered in Ref. [7] is one WL and one SL. For this configuration, the value *pF* for PLOAS under fire conditions has several equivalent integral representations (Table 1). The representations for *pF* in Table 1 are based on the assumptions that (i) a single fire giving rise to the time-temperature functions $TMPWL(t)$ and $TMPSL(t)$ is under consideration, (ii) the functions $TMPWL(t)$ and $TMPSL(t)$ are nondecreasing, (iii) the density functions $fWL(T_{SL})$ and $fSL(T_{SL})$ characterize uncertainty (e.g., variability in a population of WL/SL systems) in WL and SL failure temperatures, (iv) a link fails instantly when it reaches its failure temperature, and (v) PLOAS corresponds to the SL failing before the WL.

The representations for *pF* in Table 1 are derived in Sect. 2.1 of Ref. [7]. Specifically, the first integral in Table 1 represents *pF* with a Stieltjes integral involving time (i.e., an integral of the form $\int_a^b f(t)\mathrm{d}g(t)$; see Sect. 2.9, Ref. [15]); the second integral represents *pF* with the corresponding Riemann integral on time (i.e., an integral of the form $\int_a^b f(t)g'(t)\mathrm{d}t$; see Theorem 29.8, p. 200, Ref. [8]); the third integral represents *pF* with a Riemann integral on SL failure temperature that is obtained from the second integral through a change of variables; and the final integral involving $G(T_{SL})$ provides a representation for *pF* that facilitates the description and implementation of a quadrature-based approximation to *pF*.

A more complex WL/SL configuration considered in Ref. [7] involves *nWL* WLs and *nSL* SLs with loss of assured safety occurring when all SLs fail before any WL fails. Similarly to the representations for *pF* for the one WL, one SL configuration in Table 1, the value *pF* for PLOAS under fire conditions for this configuration has several equivalent integral representations (Table 2). The representations for *pF* in Table 2 are based on the assumptions that (i) a single fire giving rise to the time-temperature functions $TMPWL_j(t)$ and $TMPSL_k(t)$ is under consideration, (ii) the functions $TMPWL_j(t)$ and $TMPSL_k(t)$ are nondecreasing, (iii) the density functions $fWL_j(T_{WL})$ and $fSL_k(T_{SL})$ characterize uncertainty in WL and SL failure temperatures, (iv) a link fails instantly when it reaches its failure temperature, and (v) PLOAS corresponds to all SLs failing before any WL fails.

Table 1. Representation of Value *pF* for PLOAS Under Fire Conditions for a WL/SL System With One WL, One SL and the Assumptions that (i) a Link Fails Instantly When it Reaches its Failure Temperature and (ii) Loss of Assured Safety Corresponds to the SL Failing before the WL

$$pF = \int_{tMIN}^{tMAX} \left\{ fSL\left[TMPSL(t) \right] \right\} \left\{ I\left[TMPWL(t), \infty, fWL \right] \right\} \mathrm{d}TMPSL(t)$$

$$= \int_{tMIN}^{tMAX} \left\{ fSL\left[TMPSL(t) \right] \right\} \left\{ \mathrm{d}TMPSL(t)/\mathrm{d}t \right\} \left\{ I\left[TMPWL(t), \infty, fWL \right] \right\} \mathrm{d}t$$

$$= \int_{TMNSL}^{TMXSL} \left\{ fSL\left[T_{SL} \right] \right\} \left\{ I\left[TMPWL\left[TMPSL^{-1}(T_{SL}) \right], \infty, fWL \right] \right\} \mathrm{d}T_{SL}$$

$$= \int_{TMNSL}^{TMXSL} G(T_{SL})\, \mathrm{d}T_{SL},$$

where

$$I[a, b, f] = \int_a^b f(T)\, \mathrm{d}T$$

$fSL(T_{SL})$	=	density function (°C^{-1}) for SL failure temperature,
$fWL(T_{WL})$	=	density function (°C^{-1}) for WL failure temperature,
$TMPSL(t)$	=	SL temperature (°C) at time t for $tMIN \le t \le tMAX$,
$TMPWL(t)$	=	WL temperature (°C) at time t for $tMIN \le t \le tMAX$,
$TMNSL$	=	$TMPSL\,(tMIN)$,
$TMXSL$	=	$TMPSL\,(tMAX)$,
$G(T_{SL})$	=	$fSL(T_{SL})\, I\left[TMPWL\left[TMPSL^{-1}(T_{SL}) \right], \infty, fWL \right]$

Table 2. Representation of Value *pF* for PLOAS Under Fire Conditions for a WL/SL System With *nWL* WLs, *nSL* SLs, and the Assumptions that (i) a Link Fails Instantly When it Reaches its Failure Temperature and (ii) Loss of Assured Safety Corresponds to all SLs Failing Before Any WL Fails (adapted from Table 2, Ref. [9])

$$
\begin{aligned}
pF &= \sum_{k=1}^{nSL}\left(\int_{tMIN}^{tMAX}\left\{fSL_k\left[TMPSL_k(t)\right]\right\}\left\{\prod_{\substack{l=1\\ l\neq k}}^{nSL} I\left[-\infty, TMPSL_l(t), fSL_l\right]\right\}\right.\\
&\qquad\qquad \left.\times\left\{\prod_{j=1}^{nWL} I\left[TMPWL_j(t), \infty, fWL_j\right]\right\}\ \mathrm{d}TMPSL_k(t)\right)\\
&= \int_{tMIN}^{tMAX}\left(\sum_{k=1}^{nSL}\left\{fSL_k\left[TMPSL_k(t)\right]\right\}\left\{\prod_{\substack{l=1\\ l\neq k}}^{nSL} I\left[-\infty, TMPSL_l(t), fSL_l\right]\right\}\left\{\mathrm{d}TMPSL_k(t)/\mathrm{d}t\right\}\right)\\
&\qquad\qquad \times\left(\prod_{j=1}^{nWL} I\left[TMPWL_j(t), \infty, fWL_j\right]\right)\mathrm{d}t\\
&= \sum_{k=1}^{nSL}\int_{TMNSL_k}^{TMXSL_k}\left(\left\{fSL_k\left(T_{SL}\right)\right\}\left\{\prod_{\substack{l=1\\ l\neq k}}^{nSL} I\left[-\infty, TMPSL_l\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], fSL_l\right]\right\}\right.\\
&\qquad\qquad \left.\times\left\{\prod_{j=1}^{nWL} I\left[TMPWL_j\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], \infty, fWL_j\right]\right\}\right)\mathrm{d}T_{SL}\\
&= \int_{TMNSL}^{TMXSL} G\left(T_{SL}\right)\mathrm{d}T_{SL},
\end{aligned}
$$

where

$$I\left[a, b, f\right] = \int_a^b f\left(T\right)dT$$

$fWL_j\left(T_{WL}\right)$ = density function (°C^{-1}) for failure temperature of WL j,

$fSL_k\left(T_{SL}\right)$ = density function (°C^{-1}) for failure temperature of SL k,

$TMPWL_j\left(t\right)$ = temperature (°C) of WL j at time t for $tMIN \le t \le tMAX$,

$TMPSL_k\left(t\right)$ = temperature (°C) of SL k at time t for $tMIN \le t \le tMAX$,

Table 2. Representation of Value *pF* for PLOAS Under Fire Conditions for a WL/SL System With *nWL* WLs, *nSL* SLs, and the Assumptions that (i) a Link Fails Instantly When it Reaches its Failure Temperature and (ii) Loss of Assured Safety Corresponds to all SLs Failing Before Any WL Fails (adapted from Table 2, Ref. [9]) (Continued)

$$
\begin{aligned}
TMNSL_k &= TMPSL_k\,(tMIN), \\
TMXSL_k &= TMPSL_k\,(tMAX), \\
TMNSL &= \min\left\{TMNSL_k\,,\, k=1,2,\ldots,nSL\right\}, \\
TMXSL &= \max\left\{TMXSL_k\,,\, k=1,2,\ldots,nSL\right\}, \\
G_k\left(T_{SL}\right) &= fSL_k\left(T_{SL}\right)\left\{\prod_{\substack{l=1\\ l\neq k}}^{nSL} I\left[-\infty, TMPSL_l\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], fSL_l\right]\right\} \\
&\quad \times\left\{\prod_{j=1}^{nWL} I\left[TMPWL_j\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], \infty, fWL_j\right]\right\} \text{ for } TMNSL_k \le T_{SL} \le TMXSL_k \\
&= 0 \qquad \text{otherwise} \\
G\left(T_{SL}\right) &= \sum_{k=1}^{nSL} G_k\left(T_{SL}\right)
\end{aligned}
$$

3. One WL, One SL, Constant Failure Delays

The results for PLOAS in Table 1 for one WL and one SL are derived with the assumption that a link fails instantly when it reaches its failure temperature. These results are now rederived with the assumption that there exists a constant delay time between when a link reaches its failure temperature and when it actually fails. Specifically, the delay times are represented by

$\Delta WL_0 =$ difference (min) between time when WL fails and time when WL reaches its failure temperature (3.1)

and

$\Delta SL_0 =$ difference (min) between time when SL fails and time when SL reaches its failure temperature. (3.2)

Further, the assumption is made that failure does not occur before the failure temperature is reached; as a result, the inequalities $\Delta WL_0 \geq 0$ and $\Delta SL_0 \geq 0$ hold.

The first integral representation for pF in Table 1 is based on the approximation

$$pF \cong \sum_{i=1}^{nTM} \left\{ fSL\left[TMPSL\left(t_i\right)\right]\left[TMPSL\left(t_i\right) - TMPSL\left(t_{i-1}\right)\right]\right\}_1 \left\{ I\left[TMPSL\left(t_i\right), \infty, fWL\right]\right\}_2 , \tag{3.3}$$

where $tMIN = t_0 < t_1 < \ldots < t_{nTM} = tMAX$, $\{\sim\}_1$ is an approximation to the probability that the SL fails in the time interval $[t_{i-1}, t_i]$, and $\{\sim\}_2$ is the probability that the WL fails after t_i. The first integral in Table 1 is then produced in the limit as $\Delta t_i \to 0$.

A similar approach leads to the representations for pF with the delay times ΔWL_0 and ΔSL_0. For an initial demonstration, the assumption is made that $\Delta WL_0 = 0$. Then,

$$pF \cong \sum_{i=1}^{nTM} \left\{ fSL\left[TMPSL\left(t_i\right)\right]\left[TMPSL\left(t_i\right) - TMPSL\left(t_{i-1}\right)\right]\right\}_1 \left\{ I\left[TMPWL\left(t_i + \Delta SL_0\right), \infty, fWL\right]\right\}_3 , \tag{3.4}$$

where the t_i's and $\{\sim\}_1$ have the same properties as in Eq. (3.3) and $\{\sim\}_3$ is the probability that the WL fails after $t_i + \Delta SL_0$ (i.e., the SL fails at time $t_i + \Delta SL_0$ given that the SL reached its failure temperature at time t_i). The representation

$$pF = \int_{tMIN}^{tMAX} \left\{ fSL\left[TMPSL(t)\right]\right\} \left\{ I\left[TMPWL\left(t + \Delta SL_0\right), \infty, fWL\right]\right\} \mathrm{d}TMPSL(t) \tag{3.5}$$

is then produced in the limit as $\Delta t_i \to 0$.

Similarly with the assumption that both ΔWL_0 and ΔSL_0 could be nonzero,

$$pF \cong \sum_{i=1}^{nTM} \left\{ fSL\left[TMPSL\left(t_i\right)\right]\left[TMPSL\left(t_i\right) - TMPSL\left(t_{i-1}\right)\right]\right\}_1 \left\{ I\left[TMPWL\left(t_i + \Delta SL_0 - \Delta WL_0\right), \infty, fWL\right]\right\}_4 , \tag{3.6}$$

where the t_i's and $\{\sim\}_1$ have the same properties as in Eq. (3.3) and $\{\sim\}_4$ is the probability that the WL fails after time $t_i + \Delta SL_0$ (i.e., for the WL to fail after the time $t_i + \Delta SL_0$ at which the SL fails, the WL must reach its failure temperature by time $t_i + \Delta SL_0 - \Delta WL_0$). The assignment

$$t_i + \Delta SL_0 - \Delta WL_0 = tMIN \quad \text{if } t_i + \Delta SL_0 - \Delta WL_0 < tMIN \tag{3.7}$$

is tacitly assumed to be made in Eq. (3.6) if necessary. The representation

$$pF = \int_{tMIN}^{tMAX} \left\{ fSL\left[TMPSL(t) \right] \right\} \left\{ I\left[TMPWL(t + \Delta SL_0 - \Delta WL_0), \infty, fWL \right] \right\} \mathrm{d}TMPSL(t) \tag{3.8}$$

is then produced in the limit as $\Delta t_i \rightarrow 0$.

The resultant representations for pF are summarized in Table 3. Specifically, the first integral in Table 3 is the representation for pF with the Stieltjes integral involving time in Eq. (3.8); the second integral represents pF with the corresponding Riemann integral on time; the third integral represents pF with a Riemann integral on SL failure temperature that is obtained from the second integral through a change of variables; and the final integral involving $G(T_{SL})$ provides a representation for pF that facilitates the description and implementation of a quadrature-based approximation to pF.

Table 3. Representation of Value pF for PLOAS Under Fire Conditions for a WL/SL System With One WL, One SL and the Assumptions that (i) a Constant Delay Exists Between When a Link Reaches its Failure Temperature and the Time at Which the Link Fails and (ii) Loss of Assured Safety Corresponds to the SL Failing Before the WL

$$\begin{aligned}
pF &= \int_{tMIN}^{tMAX} \left\{ fSL\left[TMPSL(t) \right] \right\} \left\{ I\left[TMPWL(t + \Delta SL_0 - \Delta WL_0), \infty, fWL \right] \right\} \mathrm{d}TMPSL(t) \\
&= \int_{tMIN}^{tMAX} \left\{ fSL\left[TMPSL(t) \right] \right\} \left\{ \mathrm{d}TMPSL(t)/\mathrm{d}t \right\} \left\{ I\left[TMPWL(t + \Delta SL_0 - \Delta WL_0), \infty, fWL \right] \right\} \mathrm{d}t \\
&= \int_{TMNSL}^{TMXSL} \left\{ fSL\left[T_{SL} \right] \right\} \left\{ I\left[F(T_{SL}), \infty, fWL \right] \right\} \mathrm{d}T_{SL} \\
&= \int_{TMNSL}^{TMXSL} G(T_{SL})\, \mathrm{d}T_{SL},
\end{aligned}$$

where

$$F(T_{SL}) = TMPWL\left[TMPSL^{-1}(T_{SL}) + \Delta SL_0 - \Delta WL_0 \right],$$

$$G(T_{SL}) = fSL(T_{SL})\, I\left[F(T_{SL}), \infty, fWL \right],$$

the delay times ΔWL_0 and ΔSL_0 are defined in Eqs. (3.1) and (3.2), and the remaining terms are defined in Table 2.

4. *nWL* WLs, *nSL* SLs, Constant Failure Delays

The results for PLOAS in Table 2 for *nWL* WLs and *nSL* SLs are derived with the assumption that a link fails instantly when it reaches its failure temperature. These results are now rederived with the assumption that there exists a constant delay time between when a link reaches its failure temperature and when it actually fails. Specifically, the delay times are represented by

ΔWL_{0j} = difference (min) between time when WL j fails and time when WL j reaches its failure temperature (4.1)

and

ΔSL_{0k} = difference (min) between time when SL k fails and time when SL k reaches its failure temperature. (4.2)

Further, ΔWL_{0j} and ΔSL_{0k} are assumed to be nonnegative.

The first integral representation for pF in Table 2 is based on the approximation

$$pF \cong \sum_{k=1}^{nSL} \left(\sum_{i=1}^{nTM} \left\{ fSL_k \left[TMPSL_k\left(t_i\right)\right]\left[TMPSL_k\left(t_i\right) - TMPSL_k\left(t_{i-1}\right)\right]\right\}_1 \times \left\{ \prod_{\substack{l=1 \\ l\neq k}}^{nSL} I\left[-\infty, TMPSL_l\left(t_{i-1}\right), fSL_l\right]\right\}_2 \left\{ \prod_{j=1}^{nWL} I\left[TMPWL_j\left(t_i\right), \infty, fWL_j\right]\right\}_3 \right), \tag{4.3}$$

where $tMIN = t_0 < t_1 < \ldots < t_{nTM} = tMAX$, $\{\sim\}_1$ is an approximation to the probability that SL k fails in the time interval $[t_{i-1}, t_i]$, $\{\sim\}_2$ is the probability that all SLs except SL k have failed by time t_{i-1}, and $\{\sim\}_3$ is the probability that all WLs fail after time t_i. The first integral in Table 2 is then produced in the limit at $\Delta t_i \rightarrow 0$.

An approach similar to that shown in Eq. (4.3) and analogous to the overall approach used in Sect. 3 leads to the representation for pF with the delay times ΔWL_{0j} and ΔSL_{0k}. Specifically,

$$pF \cong \sum_{k=1}^{nSL} \left(\sum_{i=1}^{nTM} \left\{ fSL_k \left[TMPSL_k\left(t_i\right)\right]\left[TMPSL_k\left(t_i\right) - TMPSL_k\left(t_{i-1}\right)\right]\right\}_1 \times \left\{ \prod_{\substack{l=1 \\ l\neq k}}^{nSL} I\left[-\infty, TMPSL_l\left(t_{i-1} + \Delta SL_{0k} - \Delta SL_{0l}\right), fSL_l\right]\right\}_4 \times \left\{ \prod_{j=1}^{nWL} I\left[TMPWL_j\left(t_i + \Delta SL_{0k} - \Delta WL_{0j}\right), \infty, fWL_j\right]\right\}_5 \right), \tag{4.4}$$

where the t_i's and $\{\sim\}_1$ are the same as in Eq. (4.3), $\{\sim\}_4$ is the probability that all SLs except SL k fail before time $t_{i-1} + \Delta SL_{0k}$, and $\{\sim\}_5$ is the probability that all WLs fail after time $t_i + \Delta SL_{0k}$. Given that SL k reaches its failure temperature in the time interval $[t_{i-1}, t_i]$, $\{\sim\}_4$ is an approximation to the probability that all other SLs have failed by the time SL k fails and $\{\sim\}_5$ is an approximation to the probability that none of the WLs have failed by the time SL k fails. The representation

$$pF \cong \sum_{k=1}^{nSL} \left(\int_{tMIN}^{tMAX} \left\{ fSL_k \left[TMPSL_k \left(t \right) \right] \right\} \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} I \left[-\infty, TMPSL_l \left(t + SL_{0k} - \Delta SL_{0l} \right), fSL_l \right] \right\} \right.$$

$$\left. \times \left\{ \prod_{j=1}^{nWL} I \left[TMPWL_j \left(t + \Delta SL_{0k} - \Delta WL_{0j} \right), \infty, fWL_j \right] \right\} \mathrm{d}TMPSL_k \left(t \right) \right) \tag{4.5}$$

is then produced in the limit as $\Delta t_i \rightarrow 0$. The resultant integral representations for pF are summarized in Table 4.

Table 4. Representation of Value *pF* for PLOAS Under Fire Conditions for a WL/SL System With *nWL* WLs, *nSL* SLs, and the Assumptions that (i) a Constant Delay Exists Between When a Link Reaches its Failure Temperature and the Time at Which the Link Fails and (ii) Loss of Assured Safety Corresponds to all SLs Failing Before Any WL Fails

---

$$
\begin{aligned}
pF = \sum_{k=1}^{nSL} \Bigg(& \int_{tMIN}^{tMAX} \left\{ fSL_k \left[TMPSL_k(t) \right] \right\} \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} I\left[-\infty, TMPSL_l\left(t + \Delta SL_{0k} - \Delta SL_{0l}\right), fSL_l \right] \right\} \\
& \times \left\{ \prod_{j=1}^{nWL} I\left[TMPWL_j\left(t + \Delta SL_{0k} - \Delta WL_{0j}\right), \infty, fWL_j \right] \right\} \mathrm{d}TMPSL_k(t) \Bigg) \\
= \int_{tMIN}^{tMAX} \Bigg(& \sum_{k=1}^{nSL} \left\{ fSL_k \left[TMPSL_k(t) \right] \right\} \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} I\left[-\infty, TMPSL_l\left(t + \Delta SL_{0k} - \Delta SL_{0l}\right), fSL_l \right] \right\} \left\{ \mathrm{d}TMPSL_k(t)/\mathrm{d}t \right\} \\
& \times \left(\prod_{j=1}^{nWL} I\left[TMPWL_j\left(t + \Delta SL_{0k} - \Delta WL_{0j}\right), \infty, fWL_j \right] \right) \Bigg) \mathrm{d}t \\
= \sum_{k=1}^{nSL} \int_{TMNSL_k}^{TMXSL_k} \Bigg(& \left\{ fSL_k\left(T_{SL}\right) \right\} \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} I\left[-\infty, TMPSL_l\left[TMPSL_k^{-1}\left(T_{SL}\right) + \Delta SL_{0k} - \Delta SL_{0l} \right], fSL_l \right] \right\} \\
& \times \left\{ \prod_{j=1}^{nWL} I\left[TMPWL_j\left[TMPSL_k^{-1}\left(T_{SL}\right) + \Delta SL_{0k} - \Delta WL_{0j} \right], \infty, fWL_j \right] \right\} \Bigg) \mathrm{d}T_{SL} \\
= \int_{TMNSL}^{TMXSL} & G\left(T_{SL}\right) \mathrm{d}T_{SL},
\end{aligned}
$$

where

$$
\begin{aligned}
G_k\left(T_{SL}\right) &= fSL_k\left(T_{SL}\right) \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} I\left[-\infty, TMPSL_l\left[TMPSL_k^{-1}\left(T_{SL}\right) + \Delta SL_{0k} - \Delta SL_{0l} \right], fSL_l \right] \right\} \\
& \quad \times \left\{ \prod_{j=1}^{nWL} I\left[TMPWL_j\left[TMPSL_k^{-1}\left(T_{SL}\right) + \Delta SL_{0k} - \Delta WL_{0j} \right], \infty, fWL_j \right] \right\} \\
& \qquad \text{for } TMNSL_k \le T_{SL} \le TMXSL_k \\
&= 0 \qquad \text{otherwise,} \\
G\left(T_{SL}\right) &= \sum_{k=1}^{nSL} G_k\left(T_{SL}\right),
\end{aligned}
$$

the delay times ΔWL_{0j} and ΔSL_{0k} are defined in Eqs. (4.1) and (4.2), and the remaining terms are defined in Table 2.

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5. Numerical Approximation with Constant Failure Delays

Two quadrature-based approaches (i.e., trapezoidal method and Simpson's method) and two sampling-based approaches (i.e., simple random sampling and importance sampling) to the approximation of pF for instantaneous link failure are described in Ref. [7]. The same procedures can also be used to approximate pF as defined in Tables 3 and 4 for delayed link failures. The only differences are the end points for the integrals that define SL failure probability and the starting points for the integrals that define WL failure probabilities. Otherwise, the numerical implementation is the same as described in Ref. [7].

A modified version of the example involving two WLs and two SLs presented in Sect. 3.4 of Ref. [7] is used for illustration. In this example, the temperature curves are defined by

$$TMPWL_j(t) = c_1 + \left[c_2 + c_{3j}\exp\left(-c_{4j}t\right)\sin\left(c_{5j}t\right)\right]\tanh\left(c_{6j}t\right) \tag{5.1}$$

for j = 1, 2, and

$$TMPSL_k(t) = c_1 + c_2\tanh\left[c_{65}\left(1 + c_{7k}\right)t\right] \tag{5.2}$$

for k = 1, 2, with c_1 = 10 °C, c_2 =900 °C, c_{31} = –900 °C, c_{32} = –1100 °C, c_{41} = 0.25 min^{-1}, c_{42} = 0.3 min^{-1}, c_{51} = 0.12 min^{-1}, c_{52} = 0.18 min^{-1}, c_{61} = 0.02 min^{-1}, c_{62} = 0.04 min^{-1}, c_{71} = 0.5 and c_{72} = 0.8 (Fig. 1). Although hypothetical, the preceding temperature curves are defined to mimic the shape of temperature curves observed in numerical simulations of WL/SL systems in a fire. Further, the density functions for failure temperatures are defined by

$$fWL_j\left(T_{WL}\right) = \left(1/c_9\sqrt{2\pi}\right)\exp\left[-\left(T_{WL} - c_8\right)^2/2c_9^2\right] \tag{5.3}$$

for j = 1, 2 and

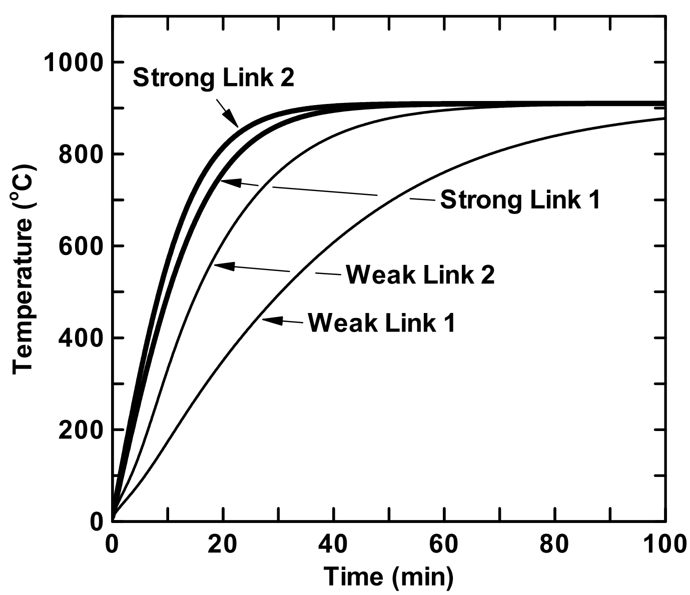

Fig. 1. Temperature curves $TMPWL_j(t)$, j = 1, 2, and $TMPSL_k(t)$, k = 1, 2, defined in Eqs. (5.1) and (5.2) (corrected form of Fig. 6, Ref. [7], which had identifying labels for the two WLs reversed and also the identifying labels for the two SLs reversed).

$$fSL_k\left(T_{SL}\right)=\left(1/c_{11}\sqrt{2\pi}\right)\exp\left[-\left(T_{SL}-c_{10}\right)^2/2c_{11}^2\right] \tag{5.4}$$

for k = 1, 2, with c_8 = 310 °C, c_9 = 8 °C, c_{10} = 560 °C, c_{11} = 18 °C and the additional assumption that the distributions for the individual links are independent.

In the original example, a link is assumed to fail instantly when its failure temperature is reached. To illustrate the calculation of PLOAS with delays in time of link failure, the example is modified by assuming that there is a delay of

$$\Delta WL_{01}=\Delta WL_{02}=1 \text{ min} \tag{5.5}$$

between when a WL reaches its failure temperature and when it actually fails, and that there is a delay of

$$\Delta SL_{01}=\Delta SL_{02}=2 \text{ min} \tag{5.6}$$

between when a SL reaches its failure temperature and when it actually fails.

For comparison, the value for *pF* obtained in the original analysis with no delay in link failure (Table 4, Ref. [7]) and the values for *pF* obtained with the delays indicated in Eqs. (5.5) and (5.6) are presented (Table 5). With the indicated delays, the value for *pF* decreases from the value obtained with no delay. In general, the effect of the delays on *pF* will depend on the particular characteristics of the problem under consideration, with these characteristics having the potential to either increase or decrease *pF*. Three of the four numerical procedures resulted in similar values for *pF* when implemented with the indicated delays; however, as a result of the small value for *pF*, the procedure based on simple random sampling was not effective.

In the examples, no attempt has been made to optimize the distributions used for importance sampling. In general, the distributions used for importance sampling have the potential, depending on the particular properties of the analysis, to either increase or decrease the rate of convergence in a Monte Carlo approximation of an integral.

For additional perspective, the analysis was also run with

$$\Delta WL_{01}=\Delta WL_{02}=2 \text{ min} \tag{5.7}$$

and

$$\Delta SL_{01}=\Delta SL_{02}=1 \text{ min}, \tag{5.8}$$

which is a reversal of the WL and SL delay times indicated in Eqs. (5.5) and (5.6). The result of this reversal is to significantly increase the value for *pF* (Table 5). As indicated by the results in Table 5, altering the values for delay times can have a major impact on the value for *pF*.

Table 5. Approximation of Failure Probability *pF* for System with Two WLs, Two SLs, Normal Distributions for WL and SL Failure Temperatures, and Failure of Both SLs Before Either WL Constituting Loss of Assured Safety[a]

N[b]	Trapezoidal Rule[c]	Simpson's Rule[d]	N[e]	Random Sampling[f]	Importance Sampling[g]
		No Delay in Link Failure (Table 4, Ref. [7])			
17	1.058E-07	7.183E-08	1E3	0.000E+00	2.185E-13
33	2.052E-06	2.700E-06	1E4	0.000E+00	5.521E-07
65	1.567E-06	1.405E-06	1E5	0.000E+00	2.226E-06
129	1.557E-06	1.553E-06	1E6	2.000E-06	1.968E-06
257	1.557E-06	1.557E-06	1E7	1.500E-06	1.573E-06
513	----	1.557E-06	1E8	1.670E-06	1.594E-06
		Delays in Link Failure: $\Delta WL_{01} = \Delta WL_{02} = 1$ min, $\Delta SL_{01} = \Delta SL_{02} = 2$ min (see Eqs. (5.5), (5.6))			
17	4.189E-12	2.973E-12	1E3	0.000E+0000	1.188E-61
33	2.258E-12	1.614E-12	1E4	0.000E+0000	4.774E-37
65	1.850E-12	1.715E-12	1E5	0.000E+0000	1.400E-17
129	1.841E-12	1.838E-12	1E6	0.000E+0000	1.847E-17
257	----	1.841E-12	1E7	0.000E+0000	1.877E-13
513	----	1.841E-12	1E8	0.000E+0000	1.575E-12
		Delays in Link Failure: $\Delta WL_{01} = \Delta WL_{02} = 2$ min, $\Delta SL_{01} = \Delta SL_{02} = 1$ min (see Eqs. (5.7), (5.8))			
17	1.106E-02	1.449E-02	1E3	4.000E-03	2.412E-30
33	5.653E-03	3.851E-03	1E4	3.200E-03	2.509E-09
65	4.370E-03	3.943E-03	1E5	4.090E-03	2.073E-03
129	4.355E-03	4.350E-03	1E6	4.341E-03	6.044E-03
257	4.355E-03	4.355E-03	1E7	4.369E-03	4.028E-03
513	----	4.355E-03	1E8	4.352E-03	4.302E-03

[a] Calculations performed with a modification of the CPLOAS program (App. III, Ref.[10])
[b] Number of evaluations of $G(T_{SL})$ (see Tables 2 and 4) with trapezoidal rule and Simpson's rule, which corresponds to the interval [*TMNSL*, *TMXSL*] being divided into N–1 subintervals.
[c] Approximations to *pF* obtained with trapezoidal rule (see Eq. (2.49), Ref. [7]).
[d] Approximations to *pF* obtained with Simpson's rule (see Eq. (2.50), Ref. [7]).
[e] Number of evaluations of $\delta(\mathbf{T}_{WL}, \mathbf{T}_{SL}) = \delta(T_{WL,1}, T_{WL,2}, T_{SL,1}, T_{SL,2})$ (see Eqs. (4.13) – (4.15), Ref. [7]) for random sampling and importance sampling with appropriate modifications to $\delta(\mathbf{T}_{WL,}, \mathbf{T}_{SL,})$ for delays in link failure.
[f] Approximations to *pF* obtained with random sampling (see Eq. (3.18), Ref. [7]).
[g] Approximation to *pF* obtained with importance sampling with uniform distributions for $T_{WL,1}$, $T_{WL,2}$, $T_{SL,1}$ and $T_{SL,2}$, (see Eqs. (3.20) and (3.21) of Ref. [7], with $fIWL_j(T_{WL,j})$ and $fISL_k(T_{SL,k})$ defined as indicated in Eqs. (2.70) and (2.71) of Ref. [7], respectively).

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6. One WL, One SL, Temperature-Dependent Failure Delays

A step up in complexity for a system with one WL and one SL is to assume that the delay times ΔWL and ΔSL are functions of the failure temperatures for the corresponding links. In this case, ΔWL and ΔSL are functions of the form

$\Delta WL(T_{WL})$ = difference (min) between time when WL fails and time when WL reaches a failure temperature (°C) of T_{WL} (6.1)

and

$\Delta SL(T_{SL})$ = difference (min) between time when SL fails and time when SL reaches a failure temperature (°C) of T_{SL}. (6.2)

As for ΔWL_0 and ΔSL_0 in Eqs. (3.1) and (3.2), $\Delta WL(T_{WL})$ and $\Delta SL(T_{SL})$ are assumed to be nonnegative.

The special case in which $\Delta WL(T_{WL}) = \Delta WL_0$ is a constant as indicated in Eq. (3.1) and $\Delta SL(T_{SL})$ in a function of SL failure temperature is considered first. In this case, the approximation to pF in Eq. (3.6) takes the form

$$pF \cong \sum_{i=1}^{nTM} \left\{ fSL\left[TMPSL(t_i)\right]\left[TMPSL(t_i) - TMPSL(t_{i-1})\right]\right\}_1 \times \left\{ I\left[TMPWL\left(t_i + \Delta SL\left[TMPSL(t_i)\right] - \Delta WL_0\right), \infty, fWL\right]\right\}_2, \tag{6.3}$$

where the t_i's and $\{\sim\}_1$ have the same properties as in Eq. (3.3) and $\{\sim\}_2$ is the probability that the WL fails after time $t_i + \Delta SL[TMPSL(t_i)]$. The representation

$$pF = \int_{tMIN}^{tMAX} \left\{ fSL\left[TMPSL(t)\right]\right\}\left\{ I\left[TMPWL\left(t + \Delta SL\left[TMPSL(t)\right] - \Delta WL_0\right), \infty, fWL\right]\right\} \mathrm{d}TMPSL(t) \tag{6.4}$$

is then produced as $\Delta t_i \to 0$.

The general case in which the delay $\Delta WL(T_{WL})$ is a function of WL failure temperature is more complex because the delay could be shortening, or possibly increasing, as WL failure temperature increases. In turn, this complicates the integral that defines the probability that the WL fails after the strong link because, for a fixed SL failure time t_{FSL} and two preceding times $t_1 < t_2$, it is possible that reaching the WL failure temperature at t_1 could result in the WL failing after t_{FSL} (i.e., at time $t_1 + \Delta WL(t_1) > t_{FSL}$) while reaching the WL failure temperature at time t_2 could result in the WL failing before t_{FSL} (i.e., at time $t_2 + \Delta WL(t_2) < t_{FSL}$). The reverse is also possible if $\Delta WL(T_{WL})$ increases rather than decreases with increasing values for T_{WL}.

Because of the preceding possibilities, the approximation to pF in Eq. (3.6) now takes the form

$$pF \cong \sum_{i=1}^{nTM} \left\{ fSL\left[TMPSL(t_i)\right]\left[TMPSL(t_i) - TMPSL(t_{i-1})\right]\right\}_1 \times \left\{ I_1\left(TMPWL\left[tMN(t_i)\right], TMPWL\left[tMX(t_i)\right], \bar{\delta}\left(TMPWL^{-1}, t_i\right) fWL\right) + I_2\left(TMPWL\left[tMX(t_i)\right], \infty, fWL\right)\right\}_3, \tag{6.5}$$

where (i) the t_i's and $\{\sim\}_1$ have the same properties as in Eq. (3.3), (ii) the times $tMN(t_i)$ and $tMX(t_i)$ and an associated set $\mathcal{WL}(t_i)$ are defined by

$$\mathcal{WL}(t_i) = \left\{t : tMIN \le t \le tMAX \text{ and } t \le t_i + \Delta SL\left[TMPSL(t_i)\right] \le t + \Delta WL\left[TMPWL(t)\right]\right\}$$

$$tMN(t_i) = \begin{cases} \inf\left\{t : t \in \mathcal{WL}(t_i)\right\} & \text{if } \mathcal{WL}(t_i) \ne \varnothing \\ tMAX & \text{if } \mathcal{WL}(t_i) = \varnothing \end{cases}$$

and

$$tMX(t_i) = \begin{cases} \sup\left\{t : t \in \mathcal{WL}(t_i)\right\} & \text{if } \mathcal{WL}(t_i) \ne \varnothing \\ tMAX & \text{if } \mathcal{WL}(t_i) = \varnothing \end{cases}$$

(Note: Because $\Delta WL(T_{WL})$ is nonnegative, $tMAX < t_i + \Delta SL[TMPSL(t_i)]$ is a necessary but not sufficient condition for $\mathcal{WL}(t_i) = \varnothing$. Further, $tMX(i) = t_i + \Delta SL[TMPSL(t_i)]$ unless the inequality $tMAX < t_i + \Delta SL[TMPSL(t_i)]$ holds, and $[tMN(t_i), tMX(t_i)]$ is the smallest interval that contains all values of t with the property that, if the WL reaches its failure temperature at time t, then it fails after the SL fails at time $t_i + \Delta SL[TMPSL(t_i)]$; however, all times contained in $[tMN(t_i), tMX(t_i)]$ do not necessarily have this property), (iii) the indicator function $\bar{\delta}(\tau_1, \tau_2)$ is defined by

$$\bar{\delta}(\tau_1, \tau_2) = \begin{cases} 1 & \text{if } \tau_2 + \Delta SL\left[TMPSL(\tau_2)\right] < \tau_1 + \Delta WL\left[TMPWL(\tau_1)\right] \\ 0 & \text{otherwise} \end{cases}$$

for $(\tau_1, \tau_2) \in [tMIN, tMAX] \times [tMIN, tMAX]$, and (iv) $\{\sim\}_3$ is the probability that the WL fails after the time $t_i + \Delta SL(TMPSL(t_i)]$ at which the SL fails.

The integrals $I_1(\sim)$ and $I_2(\sim)$ that constitute $\{\sim\}_3$ are now considered in more detail. The integral

$$I_1(\sim) = \int_{TMN(t_i)}^{TMX(t_i)} \bar{\delta}\left[TMPWL^{-1}(T_{WL}), t_i\right] fWL(T_{WL})\, \mathrm{d}T_{WL}, \tag{6.6}$$

with $TMN(t_i) = TMPWL[tMN(t_i)]$ and $TMX(t_i) = TMPWL[tMX(t_i)]$ used for notational convenience, equals the probability of the set

$$\begin{aligned} \mathcal{T}_1(t_i) = \Big\{T_{WL} : T_{WL} = TMPWL(t),\ & tMN(t_i) \le t \le tMX(t_i), \text{ and} \\ & t_i + \Delta SL\left[TMPSL(t_i)\right] < t + \Delta WL\left[TMPWL(t)\right]\Big\}. \end{aligned} \tag{6.7}$$

Specifically, the integral $I_1(\sim)$ is over the set

$$\mathcal{U}(t_i) = \left\{T_{WL} : TMN(t_i) \le T_{WL} \le TMX(t_i)\right\}, \tag{6.8}$$

with the indicator function $\bar{\delta}[TMPWL^{-1}(T_{WL}), t_i]$ picking out the elements of $\mathcal{U}(t_i)$ that are also elements of $\mathcal{T}_1(t_i)$. As a result, $I_1(\sim)$ is the probability of set $\mathcal{T}_1(t_i)$ or, more precisely, the probability that loss of assured safety would result from the existence of a WL failure temperature between $TMN(t_i)$ and $TMX(t_i)$ given that the SL reaches its failure temperature at time t_i.

The integral

$$I_2(\sim) = \int_{TMX(t_i)}^{\infty} fWL(T_{WL})\, \mathrm{d}T_{WL} \tag{6.9}$$

equals the probability of the set

$$\mathcal{T}_2(t_i) = \{T_{WL} : TMX(t_i) < T_{WL} < \infty\}, \tag{6.10}$$

or, more precisely, the probability that loss of assured safety would result from the existence of a WL failure temperature above $TMX(t_i)$ given that the SL reaches its failure temperature at time t_i. As a result, because $\mathcal{T}_1(t_i)$ and $\mathcal{T}_2(t_i)$ are disjoint and contain all relevant WL failure temperatures, the sum $I_1(\sim) + I_2(\sim)$ associated with $\{\sim\}_3$ in Eq. (6.5) is the PLOAS given that the SL fails at time t_i.

In turn, the approximation in Eq. (6.5) leads to the representation

$$\begin{aligned} pF = \int_{tMIN}^{tMAX} fSL\left[TMPSL(t)\right]\Big\{ & I\left[TMN(t), TMX(t), \bar{\delta}\left(TMPWL^{-1}, t\right) fWL\right] \\ & + I\left[TMX(t), \infty, fWL\right]\Big\}\, \mathrm{d}TMPSL(t) \end{aligned} \tag{6.11}$$

as $\Delta t_i \rightarrow 0$. Additional representations for pF are summarized in Table 6, which has the same organization as Table 3.

If $t + \Delta WL[TMPWL(t)]$ is a nondecreasing function of t, then

$$\bar{\delta}\left[TMPWL^{-1}(T_{WL}), t\right] = 1 \tag{6.12}$$

for $TMN(t) \le T_{WL} \le TMX(t)$, and as a consequence, the representation for pF in Eq. (6.11) simplifies to

$$pF = \int_{tMIN}^{tMAX} fSL\left[TMPSL(t)\right] I\left[TMN(t), \infty, fWL\right] \mathrm{d}TMPSL(t). \tag{6.13}$$

If $\Delta WL(T_{WL})$ has a constant value ΔWL_0, then

$$TMN(t) = TMPWL\left(t + \Delta SL\left[TMPSL(t)\right] - \Delta WL_0\right) \tag{6.14}$$

and the representation for pF in Eq. (6.11) further simplifies to the representation for pF in Eq. (6.4). Finally, if $\Delta WL(T_{WL})$ and $\Delta SL(T_{SL})$ have constant values ΔWL_0 and ΔSL_0, respectively, then

$$TMN(t) = TMPWL(t + \Delta SL_0 - \Delta WL_0) \tag{6.15}$$

and the representation for pF in Eq. (6.11) simplifies to the representation for pF in Eq. (3.8). A further simplification to the representation for pF in Table 1 takes place if ΔWL_0 and ΔSL_0 both equal zero.

Table 6. Representation of Value *pF* for PLOAS Under Fire Conditions for WL/SL System With One WL, One SL and the Assumption that a Temperature-Dependent Delay Exists Between When a Link Reaches its Failure Temperature and the Time at Which the Link Fails

$$pF = \int_{tMIN}^{tMAX} fSL\left[TMPSL(t)\right]\left\{I\left[TMN(t), TMX(t), \bar{\delta}\left(TMPWL^{-1}, t\right) fWL\right] + I\left[TMX(t), \infty, fWL\right]\right\} \mathrm{d}TMPSL(t)$$

$$= \int_{tMIN}^{tMAX} fSL\left[TMPSL(t)\right]\left\{\mathrm{d}TMPSL(t)/\mathrm{d}t\right\}$$

$$\times\left\{I\left[TMN(t), TMX(t), \bar{\delta}\left(TMPWL^{-1}, t\right) fWL\right] + I\left[TMX(t), \infty, fWL\right]\right\}\mathrm{d}t$$

$$= \int_{TMNSL}^{TMXSL} fSL(T_{SL})\Big(I\Big\{TMN\left[TMPSL^{-1}(T_{SL})\right], TMX\left[TMPSL^{-1}(T_{SL})\right], \bar{\delta}\left[TMPWL^{-1}, TMPSL^{-1}(T_{SL})\right] fWL\Big\}$$

$$+ I\Big\{TMX\left[TMPSL^{-1}(T_{SL})\right], \infty, fWL\Big\}\Big)\mathrm{d}T_{SL}$$

$$= \int_{TMNSL}^{TMXSL} G(T_{SL})\, \mathrm{d}T_{SL}$$

where (i)

$$G(T_{SL}) = fSL(T_{SL})\Big(I\Big\{TMN\left[TMPSL^{-1}(T_{SL})\right], TMX\left[TMPSL^{-1}(T_{SL})\right], \bar{\delta}\left[TMPWL^{-1}, TMPSL^{-1}(T_{SL})\right] fWL\Big\}$$

$$+ I\Big\{TMX\left[TMPSL^{-1}(T_{SL})\right], \infty, fWL\Big\}\Big),$$

(ii) the delay times $\Delta WL(T_{WL})$ and $\Delta SL(T_{SL})$ are defined in Eqs. (6.1) and (6.2), (iii) the indicator function $\bar{\delta}(\tau_1, \tau_2)$ is defined in conjunction with Eq. (6.5), (iv) the temperatures $TMN(t)$ and $TMX(t)$ are defined in conjunction with Eq. (6.6), and (v) the remaining terms are defined in Table 1.

7. *nWL* WLs, *nSL* SLs, Temperature-Dependent Failure Delays

A step up in complexity for *nWL* WLs and *nSL* SLs is to assume that the delay times ΔWL_j and ΔSL_k are functions of the failure temperatures for the corresponding links. In this case, ΔWL_j and ΔSL_k are functions of the form

$\Delta WL_j(T_{WL})$ = difference (min) between time when WL j fails and time when WL j reaches a failure temperature (°C) of T_{WL} (7.1)

and

$\Delta SL_k(T_{SL})$ = difference (min) between time when SL k fails and time when SL k reaches a failure temperature (°C) of T_{SL}. (7.2)

As for ΔWL_{0j} and ΔSL_{0k} in Eqs. (4.1) and (4.2), $\Delta WL_j(T_{WL})$ and $\Delta SL_k(T_{SL})$ are assumed to be nonnegative.

The development of *pF* when ΔWL_j and ΔSL_k are functions of failure temperature is similar to, but more complex than, the derivation leading to Eq. (6.11) for the one WL, one SL case. Specifically, the appropriate modification of Eq. (4.3), which is analogous to the modification of Eq. (3.6) used in the derivation of Eq. (6.11), is

$$\begin{aligned} pF = \sum_{k=1}^{nSL}\Bigg(\sum_{i=1}^{nTM} & \left\{ fSL_k\left[TMPSL_k\left(t_i\right)\right]\left[TMPSL_k\left(t_i\right) - TMPSL_k\left(t_{i-1}\right)\right]\right\}_1 \\ \times & \left\{ \prod_{j=1}^{nWL} \left(I_1\left[TMPWL_j\left[tMNWL_{jk}\left(t_i\right)\right], TMPWL_j\left[tMXWL_{jk}\left(t_i\right)\right], \bar{\delta}_{jk}\left(TMPWL_j^{-1}, t_i\right) fWL_j\right]\right.\right. \\ & \left.\left. + I_2\left[TMPWL_j\left[tMXWL_{jk}\left(t_i\right)\right], \infty, fWL_j\right]\right)_{WL,j}\right\}_2 \\ \times & \left\{ \prod_{\substack{l=1 \\ l\neq k}}^{nSL} \left(I_3\left[-\infty, TMPSL_l\left[tMNSL_{kl}\left(t_i\right)\right], fSL_l\right]\right.\right. \\ & \left.\left. + I_4\left[TMPSL_j\left[tMNSL_{kl}\left(t_i\right)\right], tMXSL_{kl}\left(t_i\right), \underline{\delta}_{kl}\left(TMPSL_l^{-1}, t_i\right) fSL_l\right]\right)_{SL,l}\right\}_3 , \end{aligned} \tag{7.3}$$

where (i) the t_i's and $\{\sim\}_1$ are the same as in Eq. (4.3), (ii) the times $tMNWL_{jk}(t_i)$, $tMXWL_{jk}(t_i)$, $tMNSL_{kl}(t_i)$, $tMXSL_{kl}(t_i)$ and associated sets $\mathcal{WL}_{jk}(t_i)$, $\mathcal{SL}_{kl}(t_i)$ are defined by

$$\mathcal{WL}_{jk}\left(t_i\right) = \left\{t : tMIN \le t \le tMAX \text{ and } t \le t_i + \Delta SL_k\left[TMPSL_k\left(t_i\right)\right] \le t + \Delta WL_j\left[TMPWL_j\left(t\right)\right]\right\},$$

$$\mathcal{SL}_{kl}\left(t_i\right) = \left\{t : tMIN \le t \le tMAX \text{ and } t \le t_i + \Delta SL_k\left[TMPSL_k\left(t_i\right)\right] \le t + \Delta SL_j\left[TMPSL_l\left(t\right)\right]\right\},$$

$$tMNWL_{jk}\left(t_i\right) = \begin{cases} \inf\left\{t : t \in \mathcal{WL}_{jk}\left(t_i\right)\right\} & \text{if } \mathcal{WL}_{jk}\left(t_i\right) \neq \varnothing \\ tMAX & \text{if } \mathcal{WL}_{jk}\left(t_i\right) = \varnothing, \end{cases}$$

$$tMXWL_{jk}\left(t_i\right) = \begin{cases} \sup\left\{t : t \in \mathcal{WL}_{jk}\left(t_i\right)\right\} & \text{if } \mathcal{WL}_{jk}\left(t_i\right) \neq \varnothing \\ tMAX & \text{if } \mathcal{WL}_{jk}\left(t_i\right) = \varnothing, \end{cases}$$

$$tMNSL_{kl}(t_i) = \begin{cases} \inf\{t : t \in \mathcal{SL}_{kl}(t_i)\} & \text{if } \mathcal{SL}_{kl}(t_i) \neq \varnothing \\ tMAX & \text{if } \mathcal{SL}_{kl}(t_i) = \varnothing, \end{cases}$$

and

$$tMXSL_{kl}(t_i) = \begin{cases} \sup\{t : t \in \mathcal{SL}_{kl}(t_i)\} & \text{if } \mathcal{SL}_{kl}(t_i) \neq \varnothing \\ tMAX & \text{if } \mathcal{SL}_{kl}(t_i) = \varnothing, \end{cases}$$

(iii) the indicator functions $\bar{\delta}_{jk}(\tau_1, \tau_2)$ and $\underline{\delta}_{kl}(\tau_1, \tau_2)$ are defined by

$$\bar{\delta}_{jk}(\tau_1, \tau_2) = \begin{cases} 1 & \text{if } \tau_2 + \Delta SL_k\left[TMPSL_k(\tau_2)\right] < \tau_1 + \Delta WL_j\left[TMPWL_j(\tau_1)\right] \\ 0 & \text{otherwise} \end{cases}$$

and

$$\underline{\delta}_{kl}(\tau_1, \tau_2) = \begin{cases} 1 & \text{if } \tau_1 + \Delta SL_l\left[TMPSL_l(\tau_1)\right] < \tau_2 + \Delta SL_k\left[TMPSL_k(\tau_2)\right] \\ 0 & \text{otherwise} \end{cases}$$

for $(\tau_1, \tau_2) \in [tMIN, tMAX] \times [tMIN, tMAX]$, (iv) $\{\sim\}_2$ is the probability that all WLs fail after time $t_i + \Delta SL_k[TMPSL_k(t_i)]$, and (v) $\{\sim\}_3$ is the probability that all SLs except SL_k have failed before time $t_i + \Delta SL_k[TMPSL_k(t_i)]$.

The expressions $\{\sim\}_2$ and $\{\sim\}_3$ in Eq. (7.3) are now considered in more detail. The expression $\{\sim\}_2$ is the product of the expressions $(\sim)_{WL,j}$, where $(\sim)_{WL,j}$ is the probability that WL j fails after SL k fails at time $t_i + \Delta SL_k[TMPSL_k(t_i)]$. In turn, $(\sim)_{WL,j}$ is the sum of the expressions $I_1(\sim)$ and $I_2(\sim)$, where $I_1(\sim)$ and $I_2(\sim)$ are the probabilities for two distinct possibilities for the failure of WL_j after time $t_i + \Delta SL_k[TMPSL_k(t_i)]$. Specifically, $I_1(\sim)$ is the probability that a failure temperature for WL_j less than $TMPWL_j[tMXWL_{jk}(t_i)]$ results in the failure of WL_j after time $t_i + \Delta SL_k[TMPSL_k(t_i)]$, and $I_2(\sim)$ is the probability that a failure temperature for WL_j greater than $TMPWL_j[tMXWL_{jk}(t_i)]$ results in the failure of WL_j after time $t_i + \Delta SL_k[TMPSL_k(t_i)]$. The expressions $I_1(\sim)$ and $I_2(\sim)$ in Eq. (7.3) are analogous to the expressions $I_1(\sim)$ and $I_2(\sim)$ in Eq. (6.5).

Similarly, the expression $\{\sim\}_3$ is the product of the expressions $(\sim)_{SL,l}$, where $(\sim)_{SL,l}$ is the probability that SL l fails before SL k fails at time $t_i + \Delta SL_k[TMPSL_k(t_i)]$. In turn, $(\sim)_{SL,l}$ is the sum of the expressions $I_3(\sim)$ and $I_4(\sim)$, where $I_3(\sim)$ and $I_4(\sim)$ are the probabilities for two distinct possibilities for the failure of SL l before time $t_i + \Delta SL_k[TMPSL_k(t_i)]$. Specifically, $I_3(\sim)$ is the probability that a failure temperature for SL l less than $TMPSL_l[tMNSL_{kl}(t_i)]$ results in the failure of SL l before time $t_i + \Delta SL_k[TMPSL_k(t_i)]$, and $I_4(\sim)$ is the probability that a failure temperature for SL l greater than $TMPSL_l[tMNSL_{kl}(t_i)]$ results in the failure of SL l before time $t_i + \Delta SL_k[TMPSL_k(t_i)]$.

Given the approximation to pF in Eq. (7.3), the representation

$$pF = \sum_{k=1}^{nSL}\Bigg(\int_{tMIN}^{tMAX}\left\{fSL_k\left[TMPSL_k(t)\right]\right\}$$

$$\times\left\{\prod_{j=1}^{nWL}\left(I\left[TMNWL_{jk}(t), TMXWL_{jk}(t), \bar{\delta}_{jk}\left(TMPWL_j^{-1}, t\right) fWL_j\right] + I\left[TMXWL_{jk}(t), \infty, fWL_j\right]\right)\right\}$$

$$\times\left\{\prod_{\substack{l=1\\l\neq k}}^{nSL}\left(I\left[-\infty, TMNSL_{kl}(t), fSL_l\right] + I\left[TMNSL_{kl}(t), TMXSL_{kl}(t), \underline{\delta}_{kl}\left(TMPSL_l^{-1}, t\right) fSL_l\right]\right)\right\} \mathrm{d}TMPSL_k(t)\Bigg) \tag{7.4}$$

results as $\Delta t_i \rightarrow 0$, where

$$TMNWL_{jk}(t) = TMPWL_j\left[tMNWL_{jk}(t)\right], TMXWL_{jk}(t) = TMPWL_j\left[tMXWL_{jk}(t)\right]$$

$$TMNSL_{kl}(t) = TMPSL_l\left[tMNSL_{kl}(t)\right], TMXSL_{kl}(t) = TMPSL_l\left[tMXSL_{kl}(t)\right]$$

are introduced for notational simplification and for consistency with the use of $TMN(t)$ and $TMX(t)$ in Eq. (6.11). The resultant integral representations for pF are summarized in Table 7.

If $t + \Delta WL_j[TMPWL_j(t)]$, $j = 1, 2, \ldots, nWL$, and $t + \Delta SL_l[TMPSL_l(t)]$, $l = 1, 2, \ldots, nSL$, are nondescreasing functions of t, then considerable simplifications to the representation for pF in Eq. (7.4) are possible. With the indicated assumptions,

$$\bar{\delta}_{jk}\left[TMPWL_j^{-1}(T_{WL}), t\right] = 1 \tag{7.5}$$

for $TMNWL_{jk}(t) \leq T_{WL} \leq TMXWL_{jk}(t)$ and

$$\underline{\delta}_{kl}\left[TMPSL_l^{-1}(T_{SL}), t\right] = 0 \tag{7.6}$$

for $TMNSL_{kl}(t) \leq T_{SL} \leq TMXSL_{kl}(t)$. As a result, the representation for pF in Eq. (7.4) simplifies to

$$pF = \sum_{k=1}^{nSL}\left(\int_{tMIN}^{tMAX}\left\{fSL_k\left[TMPSL_k(t)\right]\right\}\left\{\prod_{j=1}^{nWL} I\left[TMNWL_{jk}(t), \infty, fWL_j\right]\right\}\right.$$

$$\left.\times\left\{\prod_{\substack{l=1\\l\neq k}}^{nSL} I\left[-\infty, TMNSL_{kl}(t), fSL_l\right]\right\}\mathrm{d}TMPSL_k(t)\right). \tag{7.7}$$

Further, if $\Delta WL_j(T_{WL})$ and $\Delta SL_l(T_{SL})$ have constant values ΔWL_{0j} and ΔSL_{0l} for $j = 1, 2, \ldots, nWL$ and $l = 1, 2, \ldots, nSL$, then

$$TMNWL_{jk}(t) = TMPWL_j\left(t + \Delta SL_{0k} - \Delta WL_{0j}\right) \tag{7.8}$$

Table 7. Representation of Value *pF* for PLOAS Under Fire Conditions for WL/SL System With *nWL* WLs, *nSL* SLs and the Assumptions That (i) a Temperature-Dependent Delay Exists Between When a Link Reaches its Failure Temperature and the Time at Which the Link Fails and (ii) Loss of Assured Safety Corresponds to All SLs Failing Before Any WL Fails

$$
\begin{aligned}
pF = \sum_{k=1}^{nSL} \Bigg(& \int_{tMIN}^{tMAX} \left\{ fSL_k \left[TMPSL_k(t) \right] \right\} \\
& \times \left\{ \prod_{j=1}^{nWL} \left(I\left[TMNWL_{jk}(t), TMXWL_{jk}(t), \bar{\delta}_{jk}\left(TMPWL_j^{-1}, t\right) fWL_j \right] + I\left[TMXWL_{jk}(t), \infty, fWL_j \right] \right) \right\} \\
& \times \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} \left(I\left[-\infty, TMNSL_{kl}(t),\, fSL_l \right] + I\left[TMNSL_{kl}(t), TMXSL_{kl}(t), \underline{\delta}_{kl}\left(TMPSL_l^{-1}, t\right) fSL_l \right] \right) \right\} \mathrm{d}TMPSL_k(t) \Bigg) \\
= \int_{tMIN}^{tMAX} \Bigg(& \sum_{k=1}^{nSL} \left\{ fSL_k \left[TMPSL_k(t) \right] \right\} \\
& \times \left\{ \prod_{j=1}^{nWL} \left(I\left[TMNWL_{jk}(t), TMXWL_{jk}(t), \bar{\delta}_{jk}\left(TMPWL_j^{-1}, t\right) fWL_j \right] + I\left[TMXWL_{jk}(t), \infty, fWL_j \right] \right) \right\} \\
& \times \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} \left(I\left[-\infty, TMNSL_{kl}(t),\, fSL_l \right] + I\left[TMNSL_{kl}(t), TMXSL_{kl}(t), \underline{\delta}_{kl}\left(TMPSL_l^{-1}, t\right) fSL_l \right] \right) \right\} \left\{ \mathrm{d}TMPSL_k(t)/\mathrm{d}t \right\} \Bigg) \mathrm{d}t \\
= \sum_{k=1}^{nSL} \Bigg(& \int_{TMNSL_k}^{TMXSL_k} \left\{ fSL_k(T_{SL}) \right\} \\
& \times \left\{ \prod_{j=1}^{nWL} \left(I\left[TMNWL_{jk}\left[TMPSL_k^{-1}(T_{SL}) \right], TMXWL_j\left[TMPSL_k^{-1}(T_{SL}) \right], \bar{\delta}_{jk}\left[TMPSL_l^{-1}, TMPSL_k^{-1}(T_{SL}) \right] fWL_j \right] \right. \right. \\
& \qquad \left. \left. + I\left[TMXWL_{jk}\left[TMPSL_k^{-1}(T_{SL}) \right], \infty, fWL_j \right] \right) \right\}
\end{aligned}
$$

Table 7. Representation of Value *pF* for PLOAS Under Fire Conditions for WL/SL System With *nWL* WLs, *nSL* SLs and the Assumptions That (i) a Temperature-Dependent Delay Exists Between When a Link Reaches its Failure Temperature and the Time at Which the Link Fails and (ii) Loss of Assured Safety Corresponds to All SLs Failing Before Any WL Fails (Continued)

$$\times\left\{\prod_{\substack{l=1\\l\neq k}}^{nSL}\left(I\left[-\infty, TMNSL_{kl}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], fSL_l\right]\right.\right.$$

$$\left.\left.+I\left[TMNSL_{kl}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], TMXSL_{kl}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], \underline{\delta}_{kl}\left[TMPSL_l^{-1}, TMPSL_k^{-1}\left(T_{SL}\right)\right]fSL_l\right]\right)\right\}\ \mathrm{d}T_{SL}\Bigg)$$

$$=\int_{TMNSL}^{TMXSL} G\left(T_{SL}\right)\,\mathrm{d}T_{SL}$$

where

$$G_k\left(T_{SL}\right)=\left\{fSL_k\left(T_{SL}\right)\right\}$$

$$\times\left\{\prod_{j=1}^{nWL}\left(I\left[TMNWL_{jk}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], TMXWL_j\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], \overline{\delta}_{jk}\left[TMPSL_l^{-1}, TMPSL_k^{-1}\left(T_{SL}\right)\right]fWL_j\right]\right.\right.$$

$$\left.\left.+I\left[TMXWL_{jk}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], \infty, fWL_j\right]\right)\right\}$$

$$\times\left\{\prod_{\substack{l=1\\l\neq k}}^{nSL}\left(I\left[-\infty, TMNSL_{kl}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], fSL_l\right]\right.\right.$$

$$\left.\left.+I\left[TMNSL_{kl}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], TMXSL_{kl}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right], \underline{\delta}_{kl}\left[TMPSL_k^{-1}, TMPSL_k^{-1}\left(T_{SL}\right)\right]fSL_l\right]\right)\right\}$$

$$\text{for } TMNSL_k \le T_{SL} \le TMXSL_k$$

$$= 0 \qquad \text{otherwise.}$$

$$G\left(T_{SL}\right)=\sum_{k=1}^{nSL} G_k\left(T_{SL}\right),$$

the delays times $\Delta WL_j(T_{WL})$ and $\Delta SL_k(T_{SL})$ are defined in Eqs. (7.1) and (7.2), the indicator functions $\overline{\delta}_{jk}(\tau_1, \tau_2)$ and $\underline{\delta}_{kl}(\tau_1, \tau_2)$ are defined in conjunction with Eq. (7.3), the functions $TMNWL_{jk}(t)$, $TMXWL_{jk}(t)$, $TMNSL_{kl}(t)$ and $TMXSL_{kl}(t)$ are defined in conjunction with Eq. (7.4), and the remaining symbols are defined in Table 3.

---

and

$$TMNSL_{kl}\left(t\right) = TMPSL_{l}\left(t + \Delta SL_{0k} - \Delta SL_{0l}\right), \tag{7.9}$$

with the result that the representation for *pF* in Eq. (7.4) reduces to the representation for *pF* in Eq. (4.5).

The conditions that $t + \Delta WL_j[TMPWL_j(t)]$, $j = 1, 2, \ldots, nWL$, and $t + \Delta SL_l[TMPSL_l(t)]$, $l = 1, 2, \ldots, nSL$, be nondecreasing functions of t are equivalent to the requirement that the inequalities

$$-1 \le \left.\frac{\mathrm{d}\Delta WL_j\left(T_{WL}\right)}{\mathrm{d}T_{WL}}\right|_{T_{WL}=TMPWL_j(t)} \frac{\mathrm{d}TMPWL_j\left(t\right)}{\mathrm{d}t} \tag{7.10}$$

and

$$-1 \le \left.\frac{\mathrm{d}\Delta SL_l\left(T_{SL}\right)}{\mathrm{d}T_{SL}}\right|_{T_{SL}=TMPSL_l(t)} \frac{\mathrm{d}TMPSL_l\left(t\right)}{\mathrm{d}t} \tag{7.11}$$

hold, provided the indicated derivatives exist. Thus, because the derivatives $\mathrm{d}TMPWL_j(t)/\mathrm{d}t$ and $\mathrm{d}TMPSL_l(\mathrm{t})/\mathrm{d}t$ are nonnegative from the assumption that $TMPWL_j(t)$ and $TMPSL_l(t)$ are nondecreasing functions of t, the indicated conditions imply that either (i) $\Delta WL_j(T_{WL})$ and $\Delta SL_l(T_{SL})$ are nondecreasing functions of T_{WL} and T_{SL} or (ii) at the minimum, the maximum rates at which $\Delta WL_j(T_{WL})$ and $\Delta SL_l(T_{SL})$ can decrease are determined by the rates at which $TMPWL_j(t)$ and $TMPSL_l(t)$ are increasing.

8. Numerical Approximation with Temperature-Dependent Failure Delays

As previously indicated, two quadrature-based approaches and two sampling-based approaches to the approximation of pF for instantaneous link failure are described in Sect. 2.3 of Ref. [7]. The modification of the sampling-based approaches for temperature-dependent delays in link failure is straightforward as all that is involved is changing the time of link failure in the implementation procedures.

The implementation for the quadrature-based procedures is not as straightforward. The quadrature-based approaches described in Ref. [7] involve numerically integrating the function $G(T_{SL})$ defined in Table 7. These approaches take advantage of the existence of computationally efficient ways to evaluate integrals that constitute internal parts of $G(T_{SL})$ (i.e., when $G(T_{SL})$ has the simpler forms appearing in Tables 2 and 4).

With temperature-dependent failure delays, the four integrals contained within the definitions of $G(T_{SL})$ have the form

$$I_1[\sim] = I\left[a_{jk}, b_{jk}, \overline{\delta}_{jk}\left[TMPWL_j^{-1}, TMPSL_k^{-1}(T_{SL})\right] fWL_j\right]$$

$$= \int_{a_{jk}}^{b_{jk}} \overline{\delta}_{jk}\left[TMPWL_j^{-1}(T_{WL}), TMPSL_k^{-1}(T_{SL})\right] fWL_j(T_{WL})\, \mathrm{d}T_{WL}, \tag{8.1}$$

$$I_2[\sim] = I\left[b_{jk}, \infty, fWL_j\right] = \int_{b_{jk}}^{\infty} fWL_j(T_{WL})\, \mathrm{d}T_{WL}, \tag{8.2}$$

$$I_3[\sim] = I\left[-\infty, c_{kl}, fSL_{kl}\right] = \int_{-\infty}^{c_{kl}} fSL_{kl}(T_{SL})\, \mathrm{d}T_{SL} \tag{8.3}$$

and

$$I_4[\sim] = I\left[c_{kl}, d_{kl}, \underline{\delta}_{kl}\left[TMPSL_l^{-1}, TMPSL_k^{-1}(T_{SL})\right] fSL_l\right]$$

$$= \int_{c_{kl}}^{d_{kl}} \underline{\delta}_{kl}\left[TMPSL_l^{-1}(T_{SL}), TMPSL_k^{-1}(T_{SL})\right] fSL_l(T_{SL})\, \mathrm{d}T_{SL}, \tag{8.4}$$

where

$$a_{jk} = TMNWL_{jk}\left[TMPSL_k^{-1}(T_{SL})\right], b_{jk} = TMXWL_{jk}\left[TMPSL_k^{-1}(T_{SL})\right]$$

$$c_{kl} = TMNSL_{kl}\left[TMPSL_k^{-1}(T_{SL})\right], d_{kl} = TMXSL_{kl}\left[TMPSL_k^{-1}(T_{SL})\right].$$

The integrals $I_2[\sim]$ and $I_3[\sim]$ can be efficiently evaluated as described in Sect. 2.3 of Ref. [7] as they simply involve the integration of the density functions fWL_j and fSL_l. However, in general, the integrals $I_1[\sim]$ and $I_4[\sim]$ cannot be evaluated in this efficient manner. In particular, the integrands in $I_1[\sim]$ and $I_4[\sim]$ can have a complex structure that results from the indicator functions $\overline{\delta}_{jk}$ and $\underline{\delta}_{kl}$ having values that switch between 0 and 1 in possibly complicated ways. As a result, the integrand can potentially be more complicated than a simple density function as is the case for $I_2[\sim]$ and $I_3[\sim]$.

A possible numerical strategy for the evaluation of pF with temperature-dependent failure delays is to use a quadrature-based approach with $G(T_{SL})$ as described in Sect. 2.3 of Ref. [7] but, as appropriate, alter the manner in which the integrals $I_1[\sim]$, $I_2[\sim]$, $I_3[\sim]$ and $I_4[\sim]$ associated with each evaluation of $G(T_{SL})$ are approximated. Specifically, $I_2[\sim]$ and $I_3[\sim]$ can still be efficiently evaluated as described in Sect. 2.3 of Ref. [7]. However, additional procedures are needed for the evaluation of $I_1[\sim]$ and $I_4[\sim]$. One possibility is to simply use a quadrature procedure (e.g., trapezoidal or Simpson's as described in Sect. 2.3 of Ref. [7]) to evaluate $I_1[\sim]$ and $I_4[\sim]$. As a reminder, $I_1[\sim]$ and $I_4[\sim]$ must be evaluated for each evaluation of $G(T_{SL})$ in the overall numerical integration. Thus, a significant number of evaluations of the integrals $I_1[\sim]$ and $I_4[\sim]$ are required. However, the intervals $[a_{jk}, b_{jk}]$ and $[c_{kl}, d_{kl}]$ over which $I_1[\sim]$ and $I_4[\sim]$ must be evaluated are likely to be short, and as a result, the computational cost will probably be reasonable. It is also possible that the integrals in $I_1[\sim]$ and $I_4[\sim]$ may have some special properties that can be taken advantage of to achieve computational efficiencies. For example, it may be possible to subdivide each interval $[a_{jk}, b_{jk}]$ and $[c_{kl}, d_{kl}]$ into a small number of subintervals on which the indicator functions $\overline{\delta}_{jk}$ and $\underline{\delta}_{kl}$ are either always 0 or always 1. However, such computational strategies are likely to be very analysis-specific.

Significant numerical simplification takes place if $t + \Delta WL_j[TMPWL_j(t)]$, $j = 1, 2, \ldots, nWL$, and $t + \Delta SL_k[TMPSL_k(t)]$, $k = 1, 2, \ldots, nSL$, are nondescreasing functions of t as indicated in Eq. (7.7). In this case, the representation for $G(T_{SL})$ in Table 7 becomes

$$G(T_{SL}) = \sum_{k=1}^{nSL} G_k(T_{SL}), \tag{8.5}$$

with

$$\begin{aligned} G_k(T_{SL}) &= fSL_k(T_{SL}) \left\{ \prod_{j=1}^{nWL} I\left[TMNWL_{jk}\left[TMPSL_k^{-1}(T_{SL})\right], \infty, fWL_j \right] \right\} \\ &\qquad \times \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} I\left[-\infty, TMNSL_{kl}\left[TMPSL_k^{-1}(T_{SL})\right], fSL_l \right] \right\} \\ &= fSL_k(T_{SL}) \left\{ \prod_{j=1}^{nWL} I\left[a_{jk}, \infty, fWL_j \right] \right\} \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} I\left[-\infty, c_{jk}, fSL_l \right] \right\} \end{aligned}$$

for $TMNSL_k \le T_{SL} \le TMXSL_k$ and $G_k(T_{SL}) = 0$ otherwise. In this situation, the integrands in the integrals appearing in the definition of $G_k(T_{SL})$, and hence in the definition of $G(T_{SL})$, involve only the density functions fWL_j and fSL_l. As a result, these integrals can be evaluated with the efficient procedures described in Sect. 2.3 of Ref. [7] and pF can be calculated with quadrature procedures as described in Sect. 2.3 of Ref. [7].

A modified version of the example involving two WLs and two SLs used in Sect. 5 to illustrate the determination of pF with constant failure delays (see Eqs. (5.1) – (5.8)) is now used to illustrate the calculation of pF with temperature-dependent delays. Specifically, rather than assuming that the failure delays are constant, the constant delays used in the two examples in Sect. 5 are assumed to be valid at the initial system temperature (i.e., 10 °C) and to decrease linearly to a delay of 0 min at the asymptotic temperature for the system (i.e., 910 °C). As a result, the constant temperature delays in Eqs. (5.5) and (5.6) give rise to the temperature-dependent delays

$$\Delta WL_1(T_{WL}) = \Delta WL_2(T_{WL}) = (1 \text{ min})\left(910\ ^\circ\text{C} - T_{WL}\right) \Big/ \left(900\ ^\circ\text{C}\right) \tag{8.6}$$

and

$$\Delta SL_1(T_{SL}) = \Delta SL_2(T_{SL}) = (2 \text{ min})\left(910\ ^\circ\text{C} - T_{SL}\right) \Big/ \left(900\ ^\circ\text{C}\right), \tag{8.7}$$

and the constant temperature delays in Eqs. (5.3) and (5.4) give rise to the temperature-dependent delays

$$\Delta WL_1\left(T_{WL}\right)=\Delta WL_2\left(T_{WL}\right)=\left(2\text{ min}\right)\left(910\ ^\circ\text{C}-T_{WL}\right)\Big/\left(900\ ^\circ\text{C}\right) \tag{8.8}$$

and

$$\Delta SL_1\left(T_{SL}\right)=\Delta SL_2\left(T_{SL}\right)=\left(1\text{ min}\right)\left(910\ ^\circ\text{C}-T_{SL}\right)\Big/\left(900\ ^\circ\text{C}\right). \tag{8.9}$$

All other properties of the system are the same as described in Sect. 3.4 of Ref. [7] and previously used in Sect. 5.

The temperature-dependent delays defined in Eqs. (8.6) – (8.9) result in $t + \Delta WL_j[TMPWL_j(t)]$, $j = 1, 2$, and $t + \Delta SL_k[TMPSL_k(t)]$, $k = 1, 2$, being nondecreasing functions of t. As a result, $G(T_{SL})$ has the form described in Eq. (8.5) and pF can be approximated with the quadrature procedures described in Sect. 2.3 of Ref. [7] (Table 8). As comparison with Table 5 shows, changing from the constant delays in Eqs. (5.5) and (5.6) to the temperature-dependent delays in Eqs. (8.6) and (8.7) has a small (i.e., factor of 2) effect on pF; however, changing from the constant delays in Eqs. (5.7) and (5.8) to the temperature-dependent delays in Eqs. (8.8) and (8.9) has a large (i.e., factor of 3×10^5) effect on pF. In general, such effects will be dependent on the particular properties of the system under consideration.

Table 8. Approximation of Failure Probability pF for System with (i) Two WLs, Two SLs, (ii) Normal Distributions for WL and SL Failure Temperatures, (iii) Failure of Both SLs Before Either WL Constituting Loss of Assured Safety, and (iv) Temperature-Dependent Delays in Link Failure[a]

N[b]	Trapezoidal Rule[c]	Simpson's Rule[d]	N[e]	Random Sampling[f]	Importance Sampling[g]
$\Delta WL_1 = \Delta WL_2$: Linear 1 min to 0 min; $\Delta SL_1 = \Delta SL_2$: Linear 2 min to 0 min (see Eqs. (8.6) and (8.7))					
17	1.154E-08	7.692E-09	1E3	0.000E-00	1.188E-61
33	4.9895E-08	6.276E-08	1E4	0.000E-00	4.774E-37
65	6.401E-08	6.869E-08	1E5	0.000E-00	1.832E-08
129	6.415E-08	6.419E-08	1E6	0.000E-00	3.512E-09
257	6.415E-08	6.415E-08	1E7	0.000E-00	6.795E-08
513	6.415E-08	6.415E-08	1E8	7.000E-08	6.234E-08
$\Delta WL_1 = \Delta WL_2$: Linear 2 min to 0 min; $\Delta SL_1 = \Delta SL_2$: Linear 1 min to 0 min (see Eqs. (8.8) and (8.9))					
17	4.084E-03	5.318E-03	1E3	2.000E-03	2.412E-30
33	2.167E-03	1.528E-03	1E4	1.700E-03	2.509E-09
65	2.086E-03	2.059E-03	1E5	1.950E-03	4.635E-06
129	2.095E-03	2.098E-03	1E6	2.013E-03	2.913E-03
257	2.095E-03	2.095E-03	1E7	2.084E-03	2.929E-03
513	----	2.095-E-03	1E8	2.090E-03	2.029E-03

[a – g] Same as in Table 5.

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9. Verification of Numerical Procedures

Verification of conceptual and computational correctness is an essential part of any analysis.[16-25] Consistent with this importance, a procedure for verifying the conceptual development and numerical implementation of integration algorithms used to determine *pF* with temperature-dependent delays is now described. This procedure also provides the basis for an alternative approach to the numerical approximation of *pF*.

Model verification and model validation are two related, but different, concepts. Two widely used definitions are (Ref. [23], p. 3):

> **Verification:** The process of determing that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model.
>
> **Validaiton:** The process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model.

Thus, verification relates to assessing the correctness of the development and implementation of a model. It is in this sense that verification is used in this presentation. In contrast, validation relates to assessing the degree to which a model represents the actual behavior of the processes under consideration. In general, validation involves the comparison of model predictions with experimental results. Such comparisons are not considered in this presentation.

The verification procedure is based on the use of the cumulative distributions $CDF_{WL,j}(t)$ and $CDF_{SL,k}(t)$ of failure time for the individual links. Specifically,

$$\begin{aligned}
CDF_{WL,j}(t) &= \text{probability that WL } j \text{ has failed by time } t \\
&= \int_0^t fWL_j\left[TMPWL_j(\tau)\right]\delta\left\{t-\tau-\Delta WL_j\left[TMPWL_j(\tau)\right]\right\}\mathrm{d}TMPWL_j(\tau) \\
&= \int_0^t fWL_j\left[TMPWL_j(\tau)\right]\delta\left\{t-\tau-\Delta WL_j\left[TMPWL_j(\tau)\right]\right\}\left[\mathrm{d}TMPWL_j(\tau)/\mathrm{d}\tau\right]\mathrm{d}\tau \\
&= \int_{TMPWL_j(0)}^{TMPWL_j(t)} fWL_j(T_{WL})\,\delta\left\{t-TMPWL_j^{-1}(T_{WL})-\Delta WL_j(T_{WL})\right\}\mathrm{d}T_{WL} \qquad (9.1)
\end{aligned}$$

and

$$\begin{aligned}
CDF_{SL,k}(t) &= \text{probability that SL } k \text{ has failed by time } t \\
&= \int_0^t fSL_k\left[TMPSL_k(\tau)\right]\delta\left\{t-\tau-\Delta SL_k\left[TMPSL_k(\tau)\right]\right\}\mathrm{d}TMPSL_k(\tau) \\
&= \int_0^t fSL_k\left[TMPSL_k(\tau)\right]\delta\left\{t-\tau-\Delta SL_k\left[TMPSL_k(\tau)\right]\right\}\left[\mathrm{d}TMPSL_k(\tau)/\mathrm{d}\tau\right]\mathrm{d}\tau \\
&= \int_{TMPSL_k(0)}^{TMPSL_k(t)} fSL_k(T_{SL})\,\delta\left\{t-TMPSL_k^{-1}(T_{SL})-\Delta SL_k(T_{SL})\right\}\mathrm{d}T_{SL}, \qquad (9.2)
\end{aligned}$$

where

$$\delta\{d\} = \begin{cases} 1 & \text{if } d \ge 0 \\ 0 & \text{if } d < 0 \end{cases}$$

and an initial time of $tMIN = 0$ is used for notational convenience. The role of the indicator function $\delta\{\sim\}$ in Eqs. (9.1) and (9.2) is to identify the failure times (i.e., the τ's) that result in link failure by time t.

In turn, $CDF_{WL,j}(t)$ and $CDF_{SL,k}(t)$ can be used to obtain the following representation for PLOAS as a function of time:

$$\begin{aligned} pF(t) &= \sum_{k=1}^{nSL}\left(\int_0^t \left\{\prod_{\substack{l=1\\l\neq k}}^{nSL} CDF_{SL,j}(\tau)\right\}\left\{\prod_{j=1}^{nWL}\left[1-CDF_{WL,j}(\tau)\right]\right\} \mathrm{d}CDF_{SL,k}(\tau)\right) \\ &= \sum_{k=1}^{nSL}\left(\int_0^t \left\{\prod_{\substack{l=1\\l\neq k}}^{nSL} CDF_{SL,l}(\tau)\right\}\left\{\prod_{j=1}^{nWL}\left[1-CDF_{WL,j}(\tau)\right]\right\}\left\{\mathrm{d}CDF_{SL,k}(\tau)/\mathrm{d}\tau\right\}\mathrm{d}\tau\right), \end{aligned} \tag{9.3}$$

where $pF(t)$ is the probability that all SLs fail before time t and all WLs fail after time t (Eq. (2.1), Ref. [9]). Further, if $CDF_{WL,j}(t)$ and $CDF_{SL,k}(t)$ correspond to the same function $p(t)$ for j = 1, 2, …, nWL and k = 1, 2, …, nSL with $p(t) \rightarrow 1$ as $t \rightarrow \infty$, then

$$pF(\infty) = \lim_{t\to\infty} pF(t) = nSL!\,nWL!/(nSL + nWL)! \tag{9.4}$$

as shown in Eq. (2.5) of Ref. [9].

The representation for $pF(\infty)$ in Eq. (9.4) provides a way to verify the correctness of numerical procedures to estimate PLOAS. Specifically, if all links are assigned identical properties and ultimately fail as t increases, then $pF(\infty)$ will have the form shown in Eq. (9.4). This provides a way to specify a problem with a known solution that fully exercises the numerical calculation of PLOAS.

As an example, the time-temperature curve for WL 1 in Fig. 1 and the associated density function for failure temperature are used for illustration. Specifically, this results in temperature curves defined by

$$TMPWL_j(t) = TMPSL_k(t) = c_1 + \left[c_2 + c_3 \exp(-c_4 t)\sin(c_5 t)\right]\tanh(c_6 t) \tag{9.4}$$

for $0 \le t \le 100$ min, c_1 = 10 °C, c_2 = 900 °C, c_3 = –900 °C, c_4 = 0.25 min^{-1}, c_5 = 0.12 min^{-1}, and c_6 = 0.02 min^{-1}, and failure temperature density functions defined by

$$fWL_j(T) = fSL_k(T) = \left(1/c_9\sqrt{2\pi}\right)\exp\left[-(T-c_8)^2/2c_9^2\right] \tag{9.5}$$

for c_8 = 310 °C and c_9 = 8 °C. Further, the temperature-dependent failure delay defined in Eq. (8.6) is assumed to hold for all links.

Use of the preceding properties for all links should yield the results shown in Table 9 for different numbers of weak and strong links. When applied to systems with different numbers of weak and strong links with all links assumed to have the properties described in the preceding paragraph, all of the numerical procedures illustrated in Table 8 (i.e., trapezoidal rule, Simpson's rule, random sampling, and importance sampling) produced results very close to those shown in Table 9, which gives a strong indication that the implementation of the procedures leading to the results in Table 8 is correct.

Table 9. Values for PLOAS (i.e., $pF(\infty)$ in Eq. (9.4)) for Different Numbers of WLs and SLs Predicated on the Assumptions that (i) Loss of Assured Safety Corresponds to Failure of All SLs Before Failure of Any WL, (ii) The Failures of the Individual Links are Independent, and (iii) All Links Have the Same Distribution for Failure Time (adapted from Table 1, Ref. [9])

Integer Ratio Representation					
nSL/nWL	**1**	**2**	**3**	**4**	**5**
1	1/2	1/3	1/4	1/5	1/6
2	1/3	1/6	1/10	1/15	1/21
3	1/4	1/10	1/20	1/35	1/56
4	1/5	1/15	1/35	1/70	1/126
5	1/6	1/21	1/56	1/126	1/252
Decimal Fraction Representation					
nSL/nWL	**1**	**2**	**3**	**4**	**5**
1	0.50000	0.33333	0.25000	0.20000	0.16667
2	0.33333	0.16667	0.10000	0.06667	0.04762
3	0.25000	0.10000	0.05000	0.02857	0.01786
4	0.20000	0.06667	0.02857	0.01429	0.00794
5	0.16667	0.04762	0.01786	0.00794	0.00397

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10. Alternate PLOAS Definitions and Numerical Procedures

The representation for PLOAS in Eq. (9.3) provides an alternative to the numerical procedures described in Sect. 8. Specifically, $CDF_{WL,j}(t)$ and $CDF_{SL,k}(t)$ can be precalculated and then used in a numerical approximation to the representation for $pF(t)$ in Eq. (9.3)

The representation for PLOAS in Eq. (9.3) is predicated on the assumption that loss of assured safety corresponds to all SLs failing before any WL fails. Representations for PLOAS of the form given in Eq. (9.3) are also presented in Ref. [9] for several additional definitions of loss of assured safety. These representations and their associated verification tests are summarized in Table 10. Once $CDF_{WL,j}(t)$ and $CDF_{SL,k}(t)$ are determined as indicated in Eqs. (9.1) and (9.2), these representations can be used to determine PLOAS for the specified definitions of loss of assured safety.

A complication, but a surmountable complication, in the determination of $CDF_{WL,j}(t)$ and $CDF_{SL,k}(t)$ is that the upper limit of integration t also appears as a term in the function being integrated (see Eqs. (9.1) and (9.2)). As is the case for the numerical procedures described in Sect. 8, the characteristics of specific problems can be used to obtain simplifications in the numerical evaluation of $CDF_{WL,j}(t)$ and $CDF_{SL,k}(t)$.

If $t + \Delta WL_j[TMPWL_j(t)]$, $j = 1, 2, \ldots, nWL$, is a nondecreasing function of t, then

$$\begin{aligned}CDF_{WL,j}(t) &= \int_0^{WL_j(t)} fWL_j\left[TMPWL_j(\tau)\right] \mathrm{d}TMPWL_j(\tau) \\ &= \int_0^{WL_j(t)} fWL_j\left[TMPWL_j(\tau)\right]\left[\mathrm{d}TMPWL_j(\tau)/\mathrm{d}\tau\right]\mathrm{d}\tau \\ &= \int_{TWL_j(0)}^{TWL_j(t)} fWL_j(T_{WL})\,\mathrm{d}T_{WL},\end{aligned} \tag{10.1}$$

where

$$TWL_j(0) = TMPWL_j(0),\; TWL_j(t) = TMPWL_j\left[WL_j(t)\right]$$

and

$$WL_j(t) = \inf\left\{\tau : 0 \le t - \tau - \Delta WL_j\left[TMPWL_j(\tau)\right]\right\}.$$

Similarly, if $t + \Delta SL_k[TMPSL_k(t)]$, $k = 1, 2, \ldots, nWL$, is a nondecreasing function of t, then

$$\begin{aligned}CDF_{SL,k}(t) &= \int_0^{SL_k(t)} fSL_k\left[TMPSL_k(\tau)\right] \mathrm{d}TMPSL_k(\tau) \\ &= \int_0^{SL_k(t)} fSL_k\left[TMPSL_k(\tau)\right]\left[\mathrm{d}TMPSL_k(\tau)/\mathrm{d}\tau\right]\mathrm{d}\tau \\ &= \int_{TSL_k(0)}^{TSL_k(t)} fSL_k(T_{SL})\,\mathrm{d}T_{SL},\end{aligned} \tag{10.2}$$

where

$$TSL_k(0) = TMPSL_k(0),\; TSL_k(t) = TMPSL_k\left[SL_k(t)\right]$$

and

Table 10. Representation of Value $pF(t)$ for PLOAS for a WL/SL System with nWL WLs and nSL SLs under Fire Conditions and Associated Verification Test for Alternate Definitions of Loss of Assured Safety

Failure of all SLs before failure of any WL (Eqs. (2.1) and (2.5), Ref. [9])

$$pF(t) = \sum_{k=1}^{nSL} \left(\int_0^t \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} CDF_{SL,l}(\tau) \right\} \left\{ \prod_{j=1}^{nWL} \left[1 - CDF_{WL,j}(\tau) \right] \right\} \mathrm{d}CDF_{SL,k}(\tau) \right)$$

Verification test: $pF(\infty) = nSL!\,nWL!/(nSL + nWL)!$

Failure of any SL before failure of any WL (Eqs. (3.1) and (3.4), Ref. [9])

$$pF(t) = \sum_{k=1}^{nSL} \left(\int_0^t \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} \left[1 - CDF_{SL,l}(\tau) \right] \right\} \left\{ \prod_{j=1}^{nWL} \left[1 - CDF_{WL,j}(\tau) \right] \right\} \mathrm{d}CDF_{SL,k}(\tau) \right)$$

Verification test: $pF(\infty) = nSL/(nWL + nSL)$

Failure of all SLs before failure of all WLs (Eqs. (4.1) and (4.4), Ref. [9])

$$pF(t) = \sum_{k=1}^{nSL} \left(\int_0^t \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} CDF_{SL,l}(\tau) \right\} \left\{ 1 - \prod_{j=1}^{nWL} CDF_{WL,j}(\tau) \right\} \mathrm{d}CDF_{SL,k}(\tau) \right)$$

Verification test: $pF(\infty) = nWL/(nWL + nSL)$

Failure of any SL before failure of all WLs (Eqs. (5.1) and (5.4), Ref. [9])

$$pF(t) = \sum_{k=1}^{nSL} \left(\int_0^t \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} \left[1 - CDF_{SL,l}(\tau) \right] \right\} \left\{ 1 - \prod_{j=1}^{nWL} CDF_{WL,j}(\tau) \right\} \mathrm{d}CDF_{SL,k}(\tau) \right)$$

Verification test: $pF(\infty) = 1 - \left[nWL!\,nSL!/(nWL + nSL)! \right]$

$$SL_k\left(t\right)=\inf\left\{\tau:0\le t-\tau-\Delta SL_k\left[TMPSL_k\left(\tau\right)\right]\right\}.$$

Because $t + \Delta WL_j[TMPWL_j(t)]$ and $t + \Delta SL_k[TMPSL_k(t)]$ are nondecreasing functions of t, the functions $WL_j(t)$ and $SL_k(t)$ exist and are also nondecreasing functions of t. Once $WL_j(t)$ and $SL_k(t)$ are determined, the evaluation of $CDF_{WL,j}(t)$ and $CDF_{SL,k}(t)$ with the final integral representations in Eqs. (10.1) and (10.2) is straightforward.

Representations for PLOAS analogous to the representation discussed in Sects. 7 and 8 can also be developed for the three additional definitions of loss of assured safety considered in Table 10 (Table 11). The representations for PLOAS in Table 11 are predicated on the assumption that $\tau + \Delta WL_j[TMPWL_j(\tau)]$, $j = 1, 2, \ldots, nWL$, and $\tau + \Delta SL_k[TMPSL_k(\tau)]$, $k = 1, 2, \ldots, nSL$, are nondecreasing functions of τ. General representations for PLOAS without this assumption can be obtained with the integral constructions introduced in Sect. 7 and shown in Table 7. Specifically, representations for PLOAS in Table 11 can be converted to general representations by replacing the integral $I[a_{jk}(T_{SL}), \infty, fWL_j]$ with the sum

$$\begin{aligned}&I\left[TMNWL_{jk}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right],TMXWL_j\left[TMPSL_k^{-1}\left(T_{SL}\right)\right],\bar{\delta}_{jk}\left[TMPSL_l^{-1},TMPSL_k^{-1}\left(T_{SL}\right)\right]fWL_j\right]\\&\quad+I\left[TMXWL_{jk}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right],\infty,fWL_j\right]\end{aligned}\tag{10.3}$$

and replacing the integral $I[-\infty, c_{kl}(T_{SL}), fSL_l]$ with the sum

$$\begin{aligned}&I\left[-\infty,TMNSL_{kl}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right],fSL_l\right]\\&\quad+I\left[TMNSL_{kl}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right],TMXSL_{kl}\left[TMPSL_k^{-1}\left(T_{SL}\right)\right],\underline{\delta}_{kl}\left[TMPSL_l^{-1},TMPSL_k^{-1}\left(T_{SL}\right)\right]fSL_l\right].\end{aligned}\tag{10.4}$$

With this substitution, the first representation for PLOAS in Table 11 is the same as the final representation for PLOAS in Table 7. Further, the remaining three representations for PLOAS in Table 11 provide analogous representations for other definitions of loss of assured safety.

The calculation of PLOAS with the cumulative probabilities $CDF_{WL,j}(t)$ and $CDF_{SL,k}(t)$ is now illustrated for (i) the four definitions of loss of assured safety indicated in Table 10, (ii) a slight modification of the WL/SL system previously considered in conjunction with Tables 5 and 8, and (iii) the temperature-dependent failure delays defined in Eqs. (8.6) and (8.7). Specifically, the temperature curves defined in Eqs. (5.1) and (5.2) and illustrated in Fig. 1 are modified by changing the definitions of c_{61}, c_{71} and c_{72} to $c_{61} = 0.036\ \mathrm{min}^{-1}$, $c_{71} = 0.3$ and $c_{72} = 0.6$; all other properties remain as previously defined in conjunction with Eqs. (5.1) and (5.2). The effect of these changes is to move the WL and SL temperature curves closer together (Fig. 2). This change is made so that PLOAS will be less than one for all definitions of loss of assured safety in the following example. The resultant values for $CDF_{WL,1}(t)$, $CDF_{WL,2}(t)$, $CDF_{SL,1}(t)$ and $CDF_{SL,2}(t)$ for the four representations of the delay in link failure under consideration are shown in Fig. 3.

For this example, the Stieltjes integrals appearing in each definition of $pF(t)$ in Table 10 are numerically evaluated. Specifically, approximations of the form

$$\int_0^{100}f\left(\tau\right)\mathrm{d}g\left(\tau\right)\cong\sum_{i=1}^{N}\left[f\left(\tau_{i-1}\right)+f\left(\tau_i\right)\right]\left[g\left(\tau_i\right)-g\left(\tau_{i-1}\right)\right]/2\tag{10.3}$$

are used for individual Stieltjes integrals with

$$\tau_i=0+\left[\left(100-0\right)/N\right]i$$

Table 11. Representation of value $pF(t)$ for PLOAS for a WL/SL system with nWL WLs and nSL SLs under fire conditions with the assumption that $\tau + \Delta WL_j$ [$TMPWL_j(\tau)$, j = 1, 2, …, nWL, and $\tau + \Delta SL_k$ [$TMPSL_k(\tau)$, k = 1, 2, …, nSL, are nondecreasing functions of τ for alternate definitions of loss of assured safety

Failure of all SLs before failure of any WL (see Eq. (8.5))
$pF(t) = \sum_{k=1}^{nSL} \int_0^{TMPSL_k(t)} fSL_k(T_{SL}) \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} I\left[-\infty, c_{kl}(T_{SL}), fSL_l\right] \right\} \left\{ \prod_{j=1}^{nWL} I\left[a_{jk}(T_{SL}), \infty, fWL_j\right] \right\} \mathrm{d}T_{SL}$
Failure of any SL before failure of any WL
$pF(t) = \sum_{k=1}^{nSL} \int_0^{TMPSL_k(t)} fSL_k(T_{SL}) \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} \left(1 - I\left[-\infty, c_{kl}(T_{SL}), fSL_l\right]\right) \right\} \left\{ \prod_{j=1}^{nWL} I\left[a_{jk}(T_{SL}), \infty, fWL_j\right] \right\} \mathrm{d}T_{SL}$
Failure of all SLs before failure of all WLs
$pF(t) = \sum_{k=1}^{nSL} \int_0^{TMPSL_k(t)} fSL_k(T_{SL}) \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} I\left[-\infty, c_{kl}(T_{SL}), fSL_l\right] \right\} \left\{ \prod_{j=1}^{nWL} \left(1 - I\left[a_{jk}(T_{SL}), \infty, fWL_j\right]\right) \right\} \mathrm{d}T_{SL}$
Failure of any SL before failure of all WLs
$pF(t) = \sum_{k=1}^{nSL} \int_0^{TMPSL_k(t)} fSL_k(T_{SL}) \left\{ \prod_{\substack{l=1 \\ l \neq k}}^{nSL} \left(1 - I\left[-\infty, c_{kl}(T_{SL}), fSL_l\right]\right) \right\} \left\{ \prod_{j=1}^{nWL} \left(1 - I\left[a_{jk}(T_{SL}), \infty, fWL_j\right]\right) \right\} \mathrm{d}T_{SL}$
Note: $a_{jk}(T_{SL})$ and $c_{kl}(T_{SL})$ are defined in conjunction with Eqs. (8.1) – (8.4) with the dependence on T_{SL} added for notational completeness.

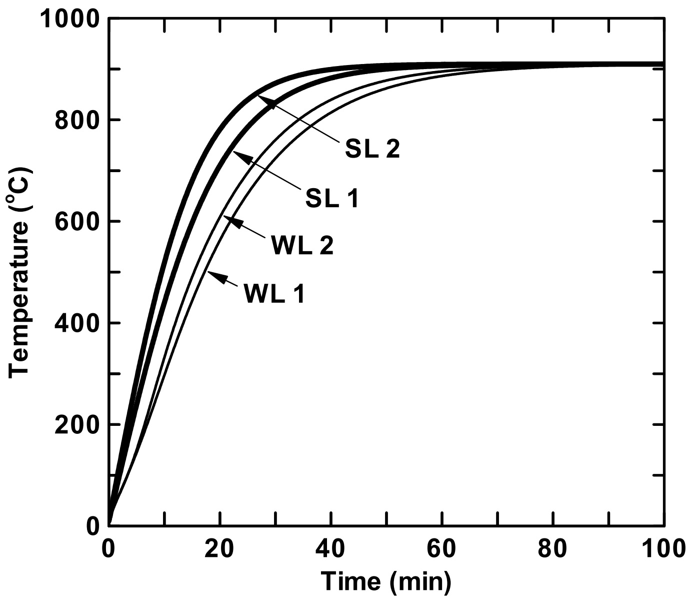

Fig. 2. Temperature curves $TMPWL_j(t)$, j = 1, 2, and $TMPSL_k(t)$, k = 1, 2, defined in Eqs. (5.1) and (5.2) with modifications $c_{61} = 0.036$, $c_{71} = 0.3$ and $c_{72} = 0.6$.

for $i = 0, 1, \ldots, N$. As part of the numerical evaluation, $TWL_1(\tau_i)$, $TWL_2(\tau_i)$, $TSL_1(\tau_i)$ and $TSL_2(\tau_i)$ are determined for each τ_i and used in the evaluation of $CDF_{WL,1}(\tau_i)$, $CDF_{WL,2}(\tau_i)$, $CDF_{SL,1}(\tau_i)$ and $CDF_{SL,2}(\tau_i)$. As a reminder, the functions $f(\tau)$ and $g(\tau)$ in Eq. (10.3) represent expressions involving $CDF_{WL,1}(\tau)$, $CDF_{WL,2}(\tau)$, $CDF_{SL,1}(\tau)$ and $CDF_{SL,2}(\tau)$ (see Table 10).

The resultant numerical evaluations of pF for the indicated example and the four definitions of loss of assured safety in Table 10 are illustrated in Table 12. For comparison, results obtained with Monte Carlo procedures and quadrature procedures based on the representations in Table 11 are also presented. As should be the case, the evaluations of PLOAS for the Stieltjes integral representations in Table 10 based on cumulative distribution functions for failure time and the quadrature approximations for the self-contained representations based on Riemann integrals in Table 11 are essentially identical. Within their limitations, the two sampling procedures also gave similar results. However, simple random sampling did not accurately predict small values for PLOAS because of a poor representation of the extreme tails of the failure temperature distributions, and importance sampling did not accurately predict large values for PLOAS because the importance sampling distribution in use emphasized sampling from the extreme tails of the failure temperature distributions.

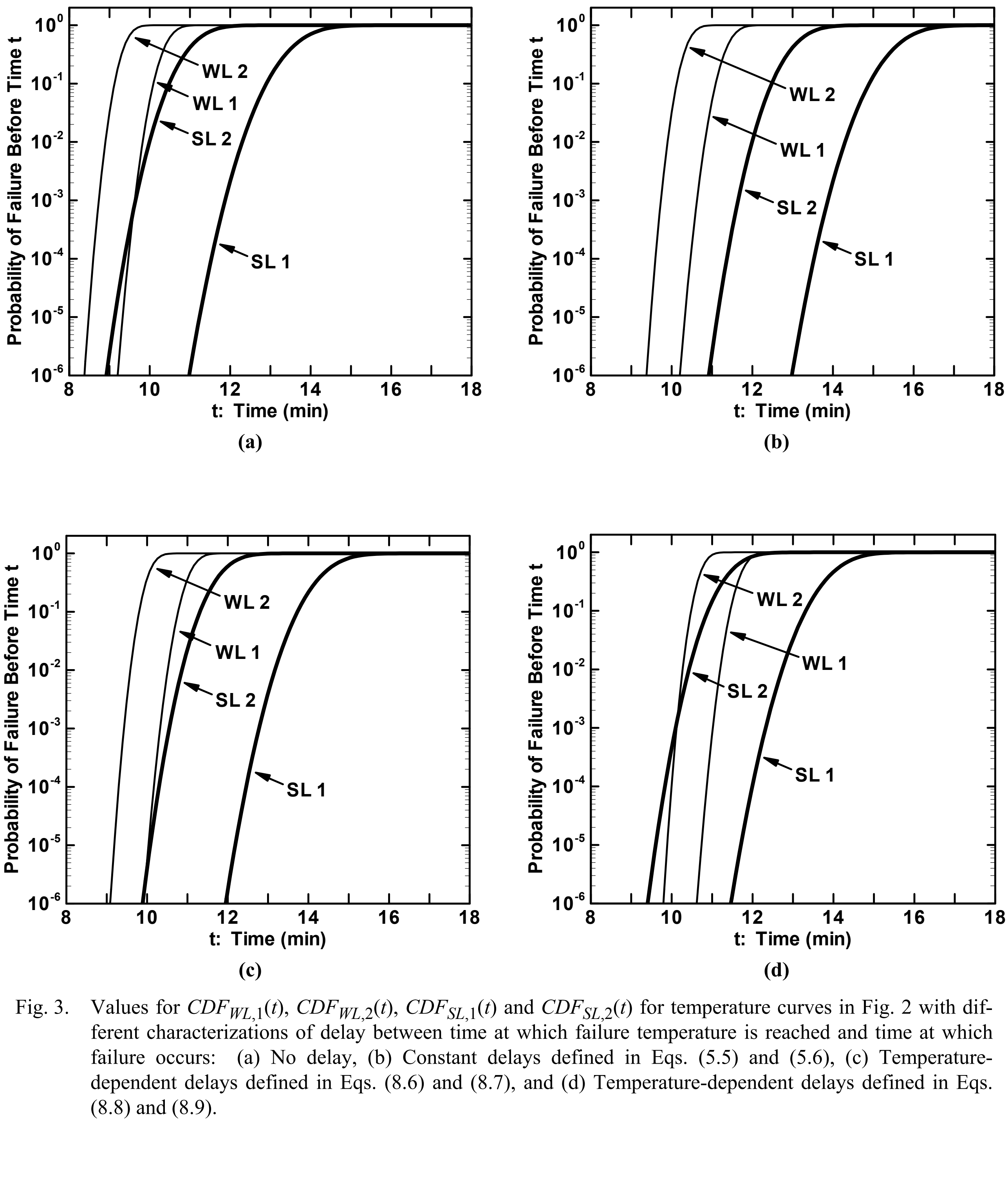

Fig. 3. Values for $CDF_{WL,1}(t)$, $CDF_{WL,2}(t)$, $CDF_{SL,1}(t)$ and $CDF_{SL,2}(t)$ for temperature curves in Fig. 2 with different characterizations of delay between time at which failure temperature is reached and time at which failure occurs: (a) No delay, (b) Constant delays defined in Eqs. (5.5) and (5.6), (c) Temperature-dependent delays defined in Eqs. (8.6) and (8.7), and (d) Temperature-dependent delays defined in Eqs. (8.8) and (8.9).

Table 12. Approximation of Stieltjes Integral Representation of Failure Probability *pF* with Different Definitions of Loss of Assured Safety for System with (i) Two WLs, Two SLs, (ii) Normal Distributions for WL and SL Failure Temperatures, and (iii) Temperature-Dependent Delays in Link Failure[a]

N[b]	Trapezoidal Rule[c]	Simpson's Rule[d]	N[e]	Random Sampling[f]	Importance Sampling[g]	N[h]	Stieltjes Integral[i]
Failure of All SLs before Failure of Any WL: No delay							
17	3.466E-15	4.027E-15	1E3	0.000E+00	1.717E-72	1E2	8.450E-16
33	2.440E-15	2.098E-15	1E4	0.000E+00	4.774E-37	1E3	3.414E-15
65	5.520E-15	6.547E-15	1E5	0.000E+00	4.072E-23	1E4	5.119E-15
129	5.386E-15	5.341E-15	1E6	0.000E+00	3.548E-15	1E5	5.358E-15
257	5.386E-15	5.386E-15	1E7	0.000E+00	5.441E-16	5E5	5.380E-15
513		5.386E-15	1E8	0.000E+00	5.800E-15	1E6	5.383E-15
Failure of All SLs before Failure of Any WL: $\Delta WL_1 = \Delta WL_2$ with a linear decrease from 1 min to 0 min and $\Delta SL_1 = \Delta SL_2$ with a linear decrease from 2 min to 0 min (see Eqs. (8.6) and (8.7))							
17	4.417E-18	5.887E-18	1E3	0.000E+00	1.717E-72	1E2	8.439E-19
33	8.477E-18	9.830E-18	1E4	0.000E+00	1.281E-38	1E3	3.317E-18
65	5.603E-18	4.645E-18	1E5	0.000E+00	4.072E-23	1E4	5.299E-18
129	5.625E-18	5.633E-18	1E6	0.000E+00	3.668E-18	1E5	5.593E-18
257	5.625E-18	5.625E-18	1E7	0.000E+00	9.867E-19	5E5	5.621E-18
513	5.627E-18	5.628E-18	1E8	0.000E+00	6.285E-18	1E6	5.624E-18
Failure of All SLs before Failure of Any WL: $\Delta WL_1 = \Delta WL_2$ with a linear decrease from 2 min to 0 min and $\Delta SL_1 = \Delta SL_2$ with a linear decrease from 1 min to 0 min (see Eqs. (8.8) and (8.9))							
17	2.316E-09	1.544E-09	1E3	0.000E+00	1.717E-72	1E2	2.462E-11
33	1.475E-09	1.195E-09	1E4	0.000E+00	4.774E-37	1E3	7.776E-10
65	1.122E-09	1.004E-09	1E5	0.000E+00	2.750E-10	1E4	1.097E-09
129	1.145E-09	1.153E-09	1E6	0.000E+00	3.503E-11	1E5	1.140E-09
257	1.145E-09	1.145E-09	1E7	0.000E+00	4.748E-10	5E5	1.144E-09
513		1.145E-09	1E8	0.000E+00	8.148E-10	1E6	1.145E-09
Failure of Any SL before Failure of Any WL: No delay							
17	4.320E-04	5.758E-04	1E3	2.000E-03	2.412E-30	1E2	3.554E-05
33	3.792E-04	3.617E-04	1E4	6.000E-04	4.342E-03	1E3	3.844E-04
65	5.393E-04	5.926E-04	1E5	5.400E-04	4.366E-04	1E4	5.204E-04
129	5.390E-04	5.389E-04	1E6	5.480E-04	9.964E-05	1E5	5.371E-04
257	5.390E-04	5.390E-04	1E7	5.510E-04	5.189E-04	5E5	5.387E-04
513		5.390E-04	1E8	5.383E-04	4.779E-04	1E6	5.388E-04
Failure of Any SL before Failure of Any WL: $\Delta WL_1 = \Delta WL_2$ with a linear decrease from 1 min to 0 min and $\Delta SL_1 = \Delta SL_2$ with a linear decrease from 2 min to 0 min (see Eqs. (8.6) and (8.7))							
17	1.149E-05	1.516E-05	1E3	0.000E+00	2.412E-30	1E2	2.341E-06
33	6.778E-05	8.654E-05	1E4	1.000E-04	4.342E-03	1E3	4.906E-05
65	7.383E-05	7.585E-05	1E5	8.000E-05	4.366E-04	1E4	7.072E-05
129	7.385E-05	7.385E-05	1E6	8.400E-05	9.873E-05	1E5	7.352E-05
257	7.385E-05	7.385E-05	1E7	7.640E-05	6.495E-05	5E5	7.378E-05
513		7.384E-05	1E8	7.467E-05	7.131E-05	1E6	7.381E-05

Table 12. Approximation of Stieltjes Integral Representation of Failure Probability *pF* with Different Definitions of Loss of Assured Safety for System with (i) Two WLs, Two SLs, (ii) Normal Distributions for WL and SL Failure Temperatures, and (iii) Temperature-Dependent Delays in Link Failure [a] (Continued)

N[b]	Trapezoidal Rule[c]	Simpson's Rule[d]	N[e]	Random Sampling[f]	Importance Sampling[g]	N[h]	Stieltjes Integral[i]
Failure of Any SL before Failure of Any WL: $\Delta WL_1 = \Delta WL_2$ with a linear decrease from 2 min to 0 min and $\Delta SL_1 = \Delta SL_2$ with a linear decrease from 1 min to 0 min (see Eqs. (8.8) and (8.9))							
17	6.247E-02	8.330E-02	1E3	6.900E-02	2.412E-30	1E2	2.572E-02
33	4.926E-02	4.486E-02	1E4	6.870E-02	4.342E-03	1E3	5.575E-02
65	6.752E-02	7.361E-02	1E5	6.743E-02	6.879E-02	1E4	6.616E-02
129	6.743E-02	6.740E-02	1E6	6.809E-02	9.710E-02	1E5	6.730E-02
257	6.743E-02	6.743E-02	1E7	6.759E-02	7.553E-02	5E5	6.740E-02
513		6.743E-02	1E8	6.742E-02	6.826E-02	1E6	6.742E-02
All SLs Fail before All WLs Fail: No delay							
17	1.159E-08	7.724E-09	1E3	0.000E+00	1.109E-33	1E2	1.267E-08
33	2.579E-08	3.052E-08	1E4	0.000E+00	5.229E-25	1E3	2.933E-08
65	4.114E-08	4.626E-08	1E5	0.000E+00	1.168E-07	1E4	3.959E-08
129	4.107E-08	4.105E-08	1E6	0.000E+00	8.366E-08	1E5	4.092E-08
257	4.107E-08	4.107E-08	1E7	1.000E-07	2.076E-08	5E5	4.104E-08
513		4.107E-08	1E8	6.000E-08	4.325E-08	1E6	4.106E-08
All SLs Fail before All WLs Fail: $\Delta WL_1 = \Delta WL_2$ with a linear decrease from 1 min to 0 min and $\Delta SL_1 = \Delta SL_2$ with a linear decrease from 2 min to 0 min (see Eqs. (8.6) and (8.7))							
17	1.742E-09	1.161E-09	1E3	0.000E+00	1.109E-33	1E2	3.851E-10
33	1.091E-09	8.736E-10	1E4	0.000E+00	1.114E-34	1E3	9.318E-10
65	1.376E-09	1.471E-09	1E5	0.000E+00	1.242E-15	1E4	1.320E-09
129	1.378E-09	1.379E-09	1E6	0.000E+00	6.349E-10	1E5	1.373E-09
257	1.378E-09	1.378E-09	1E7	1.000E-07	6.323E-10	5E5	1.377E-09
513		1.379E-09	1E8	1.000E-08	1.405E-09	1E6	1.378E-09
All SLs Fail before All WLs Fail: $\Delta WL_1 = \Delta WL_2$ with a linear decrease from 2 min to 0 min and $\Delta SL_1 = \Delta SL_2$ with a linear decrease from 1 min to 0 min (see Eqs. (8.8) and (8.9))							
17	1.466E-05	1.277E-05	1E3	0.000E+00	1.109E-33	1E2	1.041E-05
33	6.402E-05	8.047E-05	1E4	0.000E+00	6.402E-23	1E3	4.948E-05
65	6.571E-05	6.628E-05	1E5	8.000E-05	1.176E-07	1E4	6.403E-05
129	6.603E-05	6.613E-05	1E6	5.800E-05	9.670E-05	1E5	6.581E-05
257	6.603E-05	6.603E-05	1E7	6.600E-05	1.070E-04	5E5	6.597E-05
513		6.601E-05	1E8	6.524E-05	6.905E-05	1E6	6.599E-05
Failure of Any SL before Failure of All WLs: No delay							
17	6.293E-02	8.389E-02	1E3	1.140E-01	2.412E-30	1E2	1.505E-02
33	9.962E-02	1.119E-01	1E4	1.054E-01	4.342E-03	1E3	9.161E-02
65	1.073E-01	1.099E-01	1E5	1.058E-01	1.040E-01	1E4	1.056E-01
129	1.073E-01	1.073E-01	1E6	1.075E-01	1.118E-01	1E5	1.072E-01
257	1.073E-01	1.073E-01	1E7	1.074E-01	8.523E-02	5E5	1.073E-01
513		1.073E-01	1E8	1.074E-01	1.016E-01	1E6	1.073E-01

Table 12. Approximation of Stieltjes Integral Representation of Failure Probability *pF* with Different Definitions of Loss of Assured Safety for System with (i) Two WLs, Two SLs, (ii) Normal Distributions for WL and SL Failure Temperatures, and (iii) Temperature-Dependent Delays in Link Failure [a] (Continued)

N[b]	Trapezoidal Rule[c]	Simpson's Rule[d]	N[e]	Random Sampling[f]	Importance Sampling[g]	N[h]	Stieltjes Integral[i]
Failure of Any SL before Failure of All WLs: $\Delta WL_1 = \Delta WL_2$ with a linear decrease from 1 min to 0 min and $\Delta SL_1 = \Delta SL_2$ with a linear decrease from 2 min to 0 min (see Eqs. (8.6) and (8.7))							
17	5.580E-02	7.440E-02	1E3	7.000E-02	2.412E-30	1E2	6.545E-03
33	5.078E-02	4.910E-02	1E4	5.830E-02	4.342E-03	1E3	4.904E-02
65	5.951E-02	6.243E-02	1E5	5.827E-02	1.040E-01	1E4	5.836E-02
129	5.951E-02	5.951E-02	1E6	5.968E-02	1.083E-01	1E5	5.939E-02
257	5.951E-02	5.951E-02	1E7	5.959E-02	6.969E-02	5E5	5.948E-02
513		5.951E-02	1E8	5.955E-02	5.643E-02	1E6	5.950E-02
Failure of Any SL before Failure of All WLs: $\Delta WL_1 = \Delta WL_2$ with a linear decrease from 2 min to 0 min and $\Delta SL_1 = \Delta SL_2$ with a linear decrease from 1 min to 0 min (see Eqs. (8.8) and (8.9))							
17	4.317E-01	3.311E-01	1E3	6.910E-01	4.498E-12	1E2	3.789E-01
33	6.426E-01	7.130E-01	1E4	6.809E-01	4.342E-03	1E3	6.513E-01
65	6.841E-01	6.979E-01	1E5	6.857E-01	1.040E-01	1E4	6.810E-01
129	6.842E-01	6.843E-01	1E6	6.849E-01	3.407E-01	1E5	6.839E-01
257	6.842E-01	6.842E-01	1E7	6.845E-01	5.777E-01	5E5	6.842E-01
513		6.843E-01	1E8	6.843E-01	6.533E-01	1E6	6.842E-01

[a – g] Same as in Table 5 with appropriate modifications to the definition of $\delta(\mathbf{T}_{WL}, \mathbf{T}_{SL})$ in Footnote e for the case under consideration.
[h] Number of equal length intervals used in evaluation of Stieltjes integral representation for *pF*(100) as indicated in Eq. (10.3).
[g] Approximation to Stieltjes' integral representation for *pF*(100) in Table 10.

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11. Summary

Three previous presentations have considered the determination of PLOAS for WL/SL systems in which there is no delay between the time at which a link reaches its failure temperature and the time at which the link actually fails.[7-9] This presentation extends previous results by developing representations for PLOAS for the situation in which there is a delay between the time at which a link reaches its failure temperature and the time at which the link actually fails.

When the delays between the times when individual links reach their failure temperatures and the times at which the links fail are constant or have monotonic properties, the representations for PLOAS and the associated numerical evaluations are reasonably straightforward modifications of the no delay case. However, when the delays have a complex pattern of nonmonotonic behavior, the representations for PLOAS and the associated numerical evaluations can become very involved. Formal representations for PLOAS for both situations are presented.

Three general approaches to the numerical evaluations of PLOAS are presented: (i) quadrature-based evaluation of a Riemann integral defining PLOAS with the trapezoidal rule or Simpson's rule, (ii) sampling-based evaluation of PLOAS with simple random sampling or importance sampling, and (iii) numerical evaluation of a Stieltjes integral defining PLOAS on the basis of cumulative distributions for link failure times.

The quadrature-based approaches indicated in (i) appear to be the most numerically efficient of the indicated approaches but are only applicable when the delay times are well behaved.

The sampling-based approaches indicated in (ii) are applicable to all patterns of delay but can require a very large sample size when PLOAS is small. The sample size can be reduced by using an appropriately selected importance sampling procedure; in turn, this selection requires a certain amount *a priori* knowledge with respect to which subsets of the failure temperature ranges should be emphasized in the importance sampling.

The Stieltjes integral approach indicated in (iii) is applicable to all patterns of delay and has the desirable feature of dividing the numerical evaluation of PLOAS into two separate parts. Specifically, the cumulative distribution functions for link failure time can be calculated individually for each link before these distributions are brought together in the numerical evaluation of the Stieltjes integral that defines PLOAS. When the individual links have complex patterns of delay between the time at which a link reaches its failure temperature and the time at which the link actually fails, separation of the calculation of PLOAS into the two indicated parts can significantly simplify the overall calculation. In contrast, the quadrature-based approach involving Riemann integrals indicated in (i) combines everything into a single calculation, which may not be practicable in the presence of complex patterns of temperature-dependent delay.

For completeness, four different definitions of loss of assured safety are considered: failure of all SLs before failure of any WL, failure of any SL before failure of any WL, failure of all SLs before failure of all WLs, and failure of any SL before failure of all WLs. Formal definitions of PLOAS are given and numerically illustrated for each definition. As shown, changing the definition of loss of assured safety can significantly change the value for PLOAS.

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