FINAL SCIENTIFIC/TECHNICAL REPORT

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2. N/A.

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8.

1. Award #: DE-FG02-92ER54184;
   Awardee: George M. Zaslavsky;
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   Principal Investigator: George M. Zaslavsky;
   Teaming Members: Mark Edelman and visitors.

2. N/A.

3. Chaotic dynamics can be considered as a physical phenomenon that bridges the regular evolution of systems with the random one. These two alternative states of physical processes are, typically, described by the corresponding alternative methods: quasiperiodic or other regular functions in the first case, and kinetic or other probabilistic equations in the second case. What kind of kinetics should be for chaotic dynamics that is intermediate between completely regular (integrable) and completely random (noisy) cases? What features of the dynamics and in what way should they be represented in the kinetics of chaos? These are the subjects of the research under our project, where the new concept of fractional kinetics is reviewed for systems with Hamiltonian chaos. Particularly, we show how the notions of dynamical quasi-traps, Poincare recurrences, Levy flights, exit time distributions, phase space topology prove to be important in the construction of kinetics. The concept of fractional kinetics enters a different area of applications, such as particle dynamics in different potentials, particle advection in fluids, plasma physics and fusion devices, quantum optics, and many others. New characteristics of the kinetics are involved to fractional kinetics and the most important are anomalous transport, superdiffusion, weak mixing, and others. The fractional kinetics does not look as the usual one since some moments of the distribution function are infinite and fluctuations from the equilibrium state do not have any finite time of relaxation. Different important physical phenomena: cooling of particles and signals, particle and wave traps, Maxwell's Demon, etc. represent some domains where fractional kinetics proves to be valuable.

4. All proposed goals were completed and achieved.

5. A new theory of anomalous transport was developed and applied to tokamak plasma. The materials were published in a book and **23** papers. Here is formulation of these results.

5a. In Hamiltonian dynamics chaotic trajectories can be characterized by a non-zero Lyapunov exponent. In general case of random dynamics the Lyapunov exponent can be close to zero because of the stickiness, or simply zero, as in the case of pseudochaos. Kinetic description of such situations is based on scaling properties of the dynamics in both space and time. It is shown for different models including the ergodic layer and magnetic surfaces, that the ergodic theorem cannot be applied for the observed data, and that weak mixing leads to unusual macroscopic behavior. Such phenomenon as Maxwell's Demon obtains a natural realization as a persistent fluctuation that does not decay in an exponential way as in the kinetics of the Gaussian type .

5b. We describe a wide class of systems, which corresponds to the random non-chaotic dynamics with zero Lyapunov exponents. We call this type of dynamics pseudochaos and show that the corresponding kinetic description of such systems can be developed in the frame of the so-called fractional kinetics with space-time self-similarity. Numerous simulations and some analytical predictions display unusual features of the pseudochaos.

5c. In dynamical systems with a zero Lyapunov exponent, weak mixing can be governed by a specific topological structure of some surfaces that are invariant with respect to particle dynamics. In particular, when the genus of the invariant surfaces is more than one, they may have weak mixing and the corresponding fractional kinetics. This possibility is demonstrated by using a typical example from plasma physics, a three-dimensional resistive pressure-gradient-driven turbulence model. In a toroidal geometry and with a low-pressure gradient, this model shows the emergence of quasicoherent structures. In this situation, the isosurfaces of the velocity stream function have a web structure with filamentary surfaces emerging from the outer region of the torus and covering the inner region. The filamentary surfaces can result in stochastic jets of particles that cause a "topological instability." In such a situation, particle transport along the surfaces is of the anomalous superdiffusion type].

5d. Advection properties of passive particles in flows generated by point vortices are considered. Transport properties are anomalous with characteristic transport exponent μ similar to 1.5. This behavior is linked back to the presence of coherent fractal structures within the flow. A fractional kinetic analysis allows to link the characteristic transport exponent it to the trapping time exponent $\gamma = 1 + \mu$. The quantitative agreement is found for different systems of vortices investigated and a clear signature is obtained of the fractional nature of transport in these flows.

5e. The goal of this part is to develop an approach convenient to study dynamics along the magnetic surfaces and in the stochastic layers for the cases when the Lyapunov exponent is very small and the transport is anomalous. New notions of the complexity function $C(\varepsilon;t,s)$ and entropy function $S(\varepsilon;t,s)$ are introduced to describe systems with nonzero or zero Lyapunov exponents or systems that exhibit strong intermittent behavior with "flights," trappings, weak mixing, etc. The important part of the new notions is the first appearance of ε-separation of initially close trajectories. The complexity function is similar to the propagator p(t(0),x(0);t,x) with a replacement of x by the natural lengths s of trajectories, and its introduction does not assume of the space-time independence in the process of evolution of the system. A special stress is done on the choice of variables and the replacement $t \to \eta = \ln t$, $s \to \xi = \ln s$ makes it possible to consider time-algebraic and space-algebraic complexity and some mixed cases. It is shown that for typical cases the entropy function $S(\varepsilon;\xi,\eta)$ possesses invariants (α,β) that describe the fractal dimensions of the space-time structures of trajectories. The invariants (α,β) can be linked to the transport properties of the system, from one side, and to the Riemann invariants for simple waves, from the other side. This analog provides a new meaning for the transport exponent mu that can be considered as the speed of a Riemann wave in the log-phase space of the log-space-time variables. Some other applications of new notions are considered and numerical examples are presented .

5f. A family of the billiard-type systems with zero Lyapunov exponent is considered as an example of dynamics which is between the regular one and chaotic mixing. This type of dynamics is called ``pseudochaos''. We demonstrate how the fractional kinetic equation can be introduced for the pseudochaos and how the main critical exponents of the fractional kinetics can be evaluated from the dynamics. Problems related to pseudochaos are discussed: Poincaré recurrences, continued fractions, log-periodicity, rhombic billiards, and others. Pseudochaotic dynamics and fractional kinetics can be applied to streamlines or magnetic field lines behavior .

5g. In Hamiltonian dynamics chaotic trajectories can be characterized by a non-zero Lyapunov exponent. In general case of random dynamics the Lyapunov exponent can be close to zero because of the stickiness, or simply zero, as in the case of pseudochaos. Kinetic description of such situations is based on scaling properties of the dynamics in both space and time and it reveals an equation with fractional derivatives in space and time. It is shown for different models that the ergodic theorem cannot be applied for the observed data, and that weak mixing leads to unusual macroscopic behavior with persistent fluctuations that do not decay in an exponential way as in the kinetics of the Gaussian type].

5h. We present two observations related to the application of linear (LFE) and nonlinear fractional equations (NFE). First, we give the comparison and estimates of the role of the fractional derivative term to the normal diffusion term in a LFE. The transition of the solution from normal to anomalous transport is demonstrated and the dominant role of the power tails in the long time asymptotics is shown. Second, wave propagation or kinetics in a nonlinear media with fractal properties is considered. A corresponding fractional generalization of the Ginzburg–Landau and nonlinear Schrödinger equations is proposed .

6. The results of the project’s research were presented in the following publications:

a. **Books, Reviews, and Proceedings**

1. G. M. Zaslavsky**, Hamiltonian Chaos & Fractional Dynamics**,
   Oxford University Press, New York, 2005.
2. G. M. Zaslavsky and S. Benkadda, editors, **Chaotic Transport and Complexity in Classical and Quantum Dynamics**, Commun. Nonlin. Sci. Numer. Simul., Special Issue
   8, No. 3-4, Elsevier, Amsterdam, 2003.
3. G. M. Zaslavsky, Chaos, Fractional Kinetics, and Anomalous Transport
   Phys. Rep. **371**, 461-580, 2002.

Articles

4. Zaslavsky G.M., Edelman M., Stickiness of trajectories in a perturbed Anosov system Regular & Chaotic Dynamics 11 (2): 329-336 2006
5. Zaslavsky G.M., Stanislavsky A.A., Edelman M., Chaotic and pseudochaotic attractors of perturbed fractional oscillator, Chaos 16 (1): Art. No. 013102, 2006
6. Zaslavsky G.M., Edelman M., Polynomial dispersion of trajectories in sticky dynamics, Phys. Rev. E 72 (3): Art. No. 036204, 2005
7. Leoncini X., Agullo O., Benkadda S., Zaslavsky G.M., Anomalous transport in Charney-Hasegawa-Mima flows , Phys. Rev. E 72 (2): Art. No. 026218, 2005
8. Zaslavsky G.M., Carreras B.A., Lynch V.E., et al., Topological instability along invariant surfaces and pseudochaotic transport, Phys. Rev. E 72 (2): Art. No. 026227, 2005
9. Tarasov V.E., Zaslavsky G.M., Fractional Ginzburg-Landau equation for fractal media, Physica A, 354: 249-261, 2005
10. Zaslavsky G.M., Long way from the FPU-problem to chaos, Chaos 15 (1): Art. No. 015103, 2005
11. Zaslavsky G.M., Edelman M.A., Fractional kinetics: from pseudochaotic dynamics to Maxwell's Demon, Physica D, 193 (1-4): 128-147, 2004.
12. Lyubomudrov O., Edelman M., Zaslavsky G.M., Pseudochaotic systems and their fractional kinetics, Int. J. Mod. Phys. B 17 (22-24): 4149-4167, Part 1, 2003.
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15. Leoncini X., Kuznetsov L., Zaslavsky G.M., Evidence of fractional transport in point vortex flow, Chaos Solitons & Fractals 19 (2): 259-273, 2004.
16. Zaslavsky G.M., Edelman M., “Pseudochaos”, in Perspectives and Problems in Nonlinear Science, Eds: E. Kaplan, J.E. Marsden, R.S. Katepalli, 421-443, Springer, New York, 2003.
17. Afraimovich V., Zaslavsky G.M., Space-time complexity in Hamiltonian dynamics, Chaos, 13 (2): 519-532, 2003.
18. Kuznetsov L., Zaslavsky G.M., Scaling invariance of the homoclinic tangle, Phys. Rev. E 66 (4): Art. No. 046212, 2002

19. Prants S.V., Edelman M., Zaslavsky G.M., Chaos and flights in the atom-photon interaction in cavity QED, Phys. Rev E 66 (4): Art. No. 046222, 2002.
20. Iomin A., Zaslavsky G.M., Quantum manifestation of Levy-type flights in a chaotic system, Chem. Phys., 284 (1-2): 3-11, 2002.
21. Zaslavsky G.M., Dynamical traps, Physica D, 168: 292-304, 1 2002.
22. Leoncini X., Zaslavsky G.M., Jets, stickiness, and anomalous transport, Phys. Rev. E 65 (4): Art. No. 046216, 2002.
23. Iomin A., Fishman S., Zaslavsky G.M., Quantum localization for a kicked rotor with accelerator mode islands, Phys. Rev. E 65 (3): Art. No. 036215, 2002.

b. George M. Zaslavsky's home page: http://physics.nyu.edu/people/zaslavsky.george.html
c. N/A.
d. N/A.

7. N/A.