The Importance Function Fine Structure and the Central Worth Discrepancy

E. Greenspan
Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08540

Submitted for the 19th Annual Meeting of the American Nuclear Society to be held in June, 1973, at Chicago, Illinois.

This work was supported by U.S. Atomic Energy Commission Contract AT(11-1)-3073.

Calculations of reactivity coefficients are based on perturbation theory formulation that involves bilinear functionals. Group constants in common use for reactivity worth calculations are flux averaged. They can preserve the value of linear functionals, such as reaction rates, but not the value of bilinear functionals.

The possibility that the central worth discrepancy[1, 2, 3] is caused by the use of flux averaged group constants had been the subject of several investigations.[4, 5, 6] Several of these investigations[5, 6] applied the bilinear method for group constants preparation starting from a fine group structure, the group constants for which were flux averaged. These investigations did not resolve the central worth discrepancy. The finest group widths used were too large, however, to account for the fine structure of the importance function. A study[4, 7] that used consistent cross-section averaging starting from the pointwise data considered the GODIVA reactor, in which fine structure effects are very small.

This work investigates the effects the neglect of the importance function fine structure can have on the calculated reactivity worth of resolved resonances. The fine structure of the importance function in the vicinity of a resonance is found, in the NR approximation, to be:

$$\phi^{+}(E) = \frac{\Sigma_s(E)}{\Sigma_t(E)}\phi^{+}_{as} + \frac{\nu\Sigma_f(E)}{\Sigma_t(E)}\phi^{+}_{f} \quad , \qquad (1)$$

where ϕ^{+}_{as} or ϕ^{+}_{f} is the average importance of, respectively, a scattered or a fission born neutron. The amplitude of $\phi^{+}(E)$ can vary strongly in the resonance energies. Its shape depends on

the resonance type; it can vary from a pronounced dip (similar to that of the flux fine structure) in a capture resonance to a peak (as high as $\nu \phi_f^+ / \phi_{as}^+$ times its asymptotic value) in a fission resonance.

The effect of this fine structure on reactivity calculations is investigated by considering the ratio (to be referred to as the C/E ratio) of a reactivity worth calculated with flux averaged constants to the corresponding reactivity worth calculated accurately (i.e., consistent with the continuous energy formulation). The contribution of a single energy group to the C/E ratio of fissile or fertile isotopes is:

$$\frac{C}{E} = \frac{\phi_f^+ \int dE\, \nu\sigma_f(E)\phi(E) - \left(\phi_g^+\right)^\phi \int dE\, \sigma_a(E)\phi(E)}{\phi_f^+ \int dE\, \nu\sigma_f(E)\phi(E) - \int dE\, \phi^+(E)\sigma_a(E)\phi(E)} \qquad (2)$$

where $\left(\phi_g^+\right)^\phi$ is the average group importance function calculated with flux averaged group constants, and the integration is over the group width. This ratio is calculated using a simple model: there is only one resonance in the group; it can be represented by the single level Breit-Wigner formula at $0^\circ K$; it satisfies the NR approximation; except for the resonance cross sections the scattering cross section is constant, and the absorption is zero within the group limits. The calculations are done parametrically; the parameters are the ratio of the total resonance cross section to the mixture potential scattering cross section (R), Γ_a/Γ, Γ_f/Γ_a and the group width. The C/E ratio for a resonance, obtained when the group width is much larger than the practical width of the reso-

nance, is found to be

$$\frac{C}{E} = \left[1 - \frac{\Gamma_a}{2\Gamma} \frac{R}{(1+R)}\right]^{-1} \xrightarrow{R >> 1} \left[1 - \frac{\Gamma_a}{2\Gamma}\right]^{-1} . \quad (3)$$

For absorption resonances $1 < C/E \leq 2$. It does not depend on the nature of the resonance (capture or fission). For comparison, a typical value for calculational/experimental reactivity worth ratio is 1.2.[1,2,3] The effects that the use of flux averaged group constants can have on the accuracy of multigroup calculations of reactivity coefficients are evaluated based on the results of the simple model.

It is concluded that the fine structure of the importance function can contribute significantly to the calculated value of reactivity coefficients. The neglect of this fine structure by the calculational methods in common use can cause the central worth discrepancy.

References

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