# FINDING THE MAGNETIC CENTER OF A QUADRUPOLE TO HIGH RESOLUTION 

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## 1. Introduction

In a companion proposal ${ }^{[1]}$ it is proposed to align quadrupoles of a transport line to within transverse tolerances of 5 to 10 micrometers. Such a proposal is meaningful only if the effective magnetic center of such lenses can in fact be repeatably located with respect to some external mechanical tooling to comparable accuracy. It is the purpose of this note to describe some new methods and procedures that will accomplish this aim. It will be shown that these methods are capable of yielding greater sensitivity than the more traditional methods used in the past. ${ }^{[2[3]}$ The notion of the "nodal" point is exploited.

## 2. Concept

Consider the arrangement depicted in Figure 1. A fine wire is suspended through a quadrupolar field, fixed at each end at coordinates $x_{1}, y_{1}, z_{1}$ and $x_{2}, y_{2}, z_{2}$. The tension in the wire is effected by the hanging weight $W=m g$. An infinitesimal disturbance $\delta x$ is introduced at coordinate 1 . The wire will execute resonant oscillations at modal frequencies n given by:

$$
f_{n}=\frac{n}{2 D} \sqrt{T / \mu}
$$

in which $D$ is the length of the suspension and $\mu$ is the mass per unit length.

[^0]If the two ends of the wire are connected to form a loop (the return wire being stationary with respect to the magnetic field) a voltage will be developed across the terminals that is characteristic only of the induction that the wire is cutting. For the purposes of a simple calculation, let $x_{1}=x_{2}=x$ and $y_{1}=y_{2} \approx 0$. For D >> $L$ we can write:

$$
\begin{equation*}
x=x_{o}+a \sin (2 \pi f t) \tag{2.1}
\end{equation*}
$$

in which $x_{0}$ is a constant offset from the axis and $a$ is the amplitude of oscillation. The voltage then developed will be:

$$
\begin{equation*}
E=-d \Phi / d t=-d B A / d t=-d(k x)(L x) / d t \tag{2.2}
\end{equation*}
$$

in which k is the gradient of the magnet such that $B=k x$ and L is its effective magnetic length. Substituting for x and differentiating we obtain two terms for the voltage:

$$
\begin{equation*}
E=2 k L a \omega\left[x_{o} \cos \omega t+a \sin 2 \omega t\right] \tag{2.3}
\end{equation*}
$$

the first at the first harmonic of the driving frequency proportional to the offset, the second a term independent of the offset at the second harmonic. As expected, this equation demonstrates the benefit on signal strength of being able to use relatively high oscillating frequencies.

## 3. Some experimentally relevant numerical parameters

### 3.1. The wire

For a simple test we have chosen the length of the wire to be 100 cm . Using a $1.5 \mathrm{mil}(38 \mu)$ gold plated tungsten wire, density say $19 \mathrm{gr} / \mathrm{cc}$, the calculated frequency of the fundamental, with a mass $\mathrm{m}=25 \mathrm{gr}$ providing the tension is 47 Hz . The observed frequency was 49 Hz .

### 3.2. Signal Strength

The magnet was 20 cm long, had a bore diameter of 1 cm and a pole tip field of 1.2 Tesla. Hence the integrated gradient strength kL was 48 Tesla. It is not possible to measure the amplitude "a" but we can judge the sensitivity to offset as follows. If the second harmonic term is equal to the first in peak voltage then $a=x_{0}$. Let "a" be 30 microns, then we may expect to see rms voltages of about 19 microvolts. Put slightly differently the sensitivity would be $0.63 \mu \mathrm{~V} /$ micron.

## 4. Some experimental results

A photograph of the apparatus is shown in Figure 2. The pulley transferring the weight W into tension was at first a balanced razor blade. It was necessary to take great care that the mechanical shaker did not introduce unwanted mechanical motions or generate a common mode field that could be picked up by the flux loop. It. was found that a 4 inch loud speaker pushing on a fiber rod, pushing on the wire mount. (or better yet on the wire directly) produced stable, non-interfering oscillations of the wire.

The very first plot of signal versus off-set (one end only) yielded a resolution of detecting the null point and repeatability of the micrometer setting of 0.000 l inches.

Improved wire shaking techniques led to the data taken one week later which is shown in Figure 3a. The ability to find the intercept can be inferred from Figure 3b. It is about 1 micron. This translates to 0.5 micron at the quadrupole.

In order to achieve a good null it was useful to adjust the orthogonal coordinate by iterating. The sensitivity to the coordinate at right angles to the motion is not severe. This is apparent when one examines the flux lines of a quadrupole. The flux lines are parallel to the motion are not cut.

The signal was also processed by a lock-in amplifier ${ }^{[4]}$ with which the phase of the signal could be unambiguously seen with respect to the driving signal. This
detector has several more decades of gain. Attempts to suppress common mode noise continue. We believe the method holds promise of even higher resolution. Best results are obtained after 4 pm and the building environment to mechanical disturbances improves. Footsteps and loud speech currently limit improvements!

## 5. The "N odal" point concept.

In the foregoing simple sample calculation we have set $x_{1}=x_{2}$. In the experimental world we have no a-priori knowledge that the wire is parallel to the real magnetic axis (whatever this means). What the wire coordinates measure is a family of lines all of whom satisfy the condition that the net flux cut is zero. This is precisely the condition we wish to achieve for the trajectories of the beam particles. To first order then, we have accomplished the task. Setting $x_{1}=x_{2}$ is not critical therefore and can be accomplished by more traditional means (jig-transit) if one chooses.

The aforementioned family of lines should all intersect at a common point. We call this position $z_{0}$ the "nodal" point. A round this point we can rotate the magnet ( in yaw and pitch ) without deflecting the beam. In practice this point may not lie in the exact axial center of the steel because, as we have observed in earlier studies, coil configurations on the lead and non-lead ends of the magnet are not identical.

To verify this belief, studies were carried out in the following way: coordinates for a null condition were found, then the wire at one end of the magnet was moved a given distance ( say 60 microns ) and the motion required at the other end to restore the null condition recorded. The results were, as expected, that. the motion at other end was equal and opposite within the resolution of the set up.

## 6. Field Use of Centering Data

It is not sufficient to find the "center" of a lens in the laboratory. One must be able to reproduce the coordinates in the field. Experience with magnets in the past has shown that they cannot be taken apart and reassembled if one wishes to achieve micron reproducibility. This fact leads to the notion that they must be measured with vacuum chamber in place. This immediately leads to the notion that the same tooling used to position and reference the wire remains permanently attached to the magnet. In fact it becomes the same tooling to which the beam position monitor must be referenced. (This notion of carrying part of the calibrating apparatus into the field is not at all unusual in magnetic measurement technique. For example, if one truly wants to reproduce the strength versus current relationship in the field, one often uses the very same current measuring device in the field as that which was used in the lab).

One conceivable (and very simple) readout is shown in Figure 4. Very accurately machined flanges are permanently fixed to the quad bore by clamping the bore tube to the pole pieces. A ring guage with three bearing pads can be rotated in the plane perpendicular to the quad axis. One of the three pads is spring loaded so that the gauge can be mounted on the flange. The micrometer post is run in so that the wire barely touches it. This condition is observed when the oscillating wire shorts to ground periodically. The micrometer reading is recorded. The ring gauge is now rotated by 180 degrees and the process is repeated. The wire must have been half-way between the readings with respect to the edges of the flange. The process is repeated with the wire oscillating in the other plane. The flange, and its mate at the other end of the quadrupole, are the tooling to which the field survey is referenced. Experimental tests of wire touching resolution show repeatability at the 0.2 micron level.

## 7. Proposal

We intend to develop the method outlined above (we have only scratched the surface of many possibilities that come to mind) and propose that the resulting apparatus and tooling be used in connection with the construction of the Final Focus Test Beam in the coming year.

## 8. Acknowledgements

We gratefully acknowledge Ed Garwin for lending us the lock-in detector and Dan Jones for setting up the driver electronics.

## 9. References

1. Final Focus Test Beam Alignment, R.E.Ruland and H.E.Fischer. February 1989 SLAC-TN-89-02 2nd this workshop.
2. Magnetic center Iocation in Multipolar Fields, J.K.Cobb and J.J. Murray. Nuc.Instr.Methods Vo1.45, pl (1967)
3. The most common method used in the past is to identify the magnetic center of a lens with its mechanical center of construction; often "defined" to be centered with respect to its iron pole surfaces, or better, its mechanical split planes.
4. Model HR-8 Princeton Applied Research


Figure 1.


Figure 2.


Figure 3a.


Figure 3b.


Figure 4.


[^0]:    * Work supported by the Department of Energy, contract DCAC02-76SF00515.

