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Black Hole Attractors and Pure Spinors

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Abstract

We construct black hole attractor solutions for a wide class of $\mathcal{N} = 2$ compactifications. The analysis is carried out in ten dimensions and makes crucial use of pure spinor techniques. This formalism can accommodate non-Kähler manifolds as well as compactifications with flux, in addition to the usual Calabi–Yau case. At the attractor point, the charges fix the moduli according to $\sum f_k = \mathrm{Im}\,(C\Phi)$, where Φ is a pure spinor of odd (even) chirality in IIB (A). For IIB on a Calabi–Yau, $\Phi = \Omega$ and the equation reduces to the usual one. Methods in generalized complex geometry can be used to study solutions to the attractor equation.

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1 Introduction

The attractor mechanism is a general feature of black hole solutions to four dimensional $\mathcal{N} = 2$ supergravity [1–3]. It states that near the horizon of a supersymmetric black hole the vector multiplet moduli flow to special values which only depend on the charge of the black hole and not on the asymptotic values of the moduli. The simplest application of the attractor mechanism is to compactifications of type II string theory on a Calabi-Yau manifold Y. In this case the ten dimensional action of string theory reduces, in the low energy limit, to an effective $\mathcal{N} = 2$ supergravity theory in four dimensions, whose field content and action depend on the choice of Y. The attractor mechanism has an elegant interpretation in terms of the geometry of Y: for type IIA (IIB), the Kähler (complex) structure of Y flows to an attractor fixed point at the horizon. The attractor mechanism has also been shown to occur for some non-supersymmetric but extremal black holes [4–6].

In this paper we will study supersymmetric black hole attractors for a broader class of compactifications which preserve $\mathcal{N} = 2$ supersymmetry but are not necessarily Calabi-Yau. This class includes both non-Kähler compactifications as well as compactifications with non-trivial background flux. Examples of such $\mathcal{N} = 2$ compactifications have been constructed using T-duality [7]. Some geometrically more interesting non–Kähler vacua have also been provided recently in [8], but they involve $g_s \neq 0$ and hence cannot be used as supergravity solutions. Although from the four dimensional perspective the resulting black hole solutions are exactly as in [1–3], the geometric description is less clear than in the Calabi-Yau case. For example, there is no general description of the vector multiplet moduli space of these compactifications in terms of geometric quantities.

For this reason, we will study these configurations as solutions to the full ten dimensional equations of motion, rather than the low energy effective theory in four dimensions. From the ten dimensional point of view, these black holes are simply special classes of solutions with flux, to which we can apply the pure spinor techniques of [9]. For example, the near horizon geometry of a BPS black hole is just a particular flux compactification whose four dimensional geometry is $AdS_2 \times S^2$.[4]

The ten dimensional gravitino variations yield a new form of the attractor equation, phrased in the language of pure spinors. These pure spinors play a central role in the theory of generalized complex manifolds [14–16], and have recently found several applications in supergravity, in the study of compactifications on six [9, 17, 18] and seven [18, 19] dimen-

[4] This fact has led to a fruitful interplay between between the study of flux compactifications and extremal black holes; see e.g. [10–13].

sional manifolds. We give a brief introduction to pure spinors in appendix B. For practical purposes, a pure spinor Φ may be thought of as formal sum of differential forms of different rank.

To describe $\mathcal{N}=2$ compactifications in terms of pure spinors, we will follow the approach of [17]. These authors classified $\mathcal{N}=1$ vacua using a pair of pure spinors $\Phi_{\pm}$, which determine the metric on the internal manifold. For type II string theory on a Calabi-Yau, these two pure spinors have a simple interpretation. One of them is the holomorphic three form Ω, which fixes the complex structure of the Calabi-Yau, and the other is e^{iJ} where J is the Kähler form. In general, an $\mathcal{N}=2$ vacua is characterized by two pairs of pure spinors, along with the constraint that each pair determines the same metric on the internal manifold. The BPS black holes under consideration break the $\mathcal{N}=2$ supersymmetry of a background down to $\mathcal{N}=1$.

The attractor equations describe how the geometry of the internal manifold changes as a function of radius. At every value of r, the internal manifold satisfies the equations for an $\mathcal{N}=2$ vacuum in four dimensions. However, one linear combination Φ of the pure spinors flows as a function of radius. So as r changes, the internal manifold flows through the moduli space of $\mathcal{N}=2$ compactifications. At the horizon, this pure spinor approaches a fixed value determined only by the charge of the black hole – it obeys an equation of the form

$$\sum_k f_k = \mathrm{Im}\,(\bar{C}\Phi)$$

where f_k is a k-form flux. This equation can be solved in simple geometric terms, using a theorem of Hitchin [14]. (Since Φ is related to pure spinors describing the vacua, it also obeys an extra differential condition, whose general solution is more complicated, as we will see.) This theorem involves the construction of a function, whose integral – known as the Hitchin functional – can be interpreted as the entropy of the associated black hole.[5] Our construction may be thought of as a physical implementation of this theorem; the attractor equations admit a solutions precisely when the associated black hole has a finite area horizon.

The approach described above has several advantages, which are relevant even for standard Calabi-Yau compactifications. First, because we have solved the full ten dimensional equations of motion, the solutions apply in cases where the four dimensional supergravity equations are no longer valid. In particular, they can describe configurations where the Kaluza-Klein length scale of the compactification manifold is not small compared to the length scales of the four dimensional solution. It may therefore prove useful in the study

[5]This relation between the Hitchin functional and the black hole entropy has been noted by [20,21]. The Hitchin functional has also found use in other closely related physical contexts, see e.g. [20,22,23].

of small black holes, where the radius of curvature of the black horizon can be of order the Kaluza-Klein scale (see, e.g. [25–29]). In addition, this derivation demonstrates explicitly that BPS black hole solutions can be consistently lifted to solutions of the full ten dimensional supergravity.

Our hope is that the universal attractor behavior described in this paper may play a role in a better understanding of the dynamics and definition of string theory in these backgrounds. Recently, it has been proposed that such black holes provide a non–perturbative definition of topological string theory in the Calabi-Yau case [24]. It is therefore natural to expect that the black hole attractors described in this paper are related to topological string theory on non-Calabi-Yau compactifications.[6]

This paper is organized as follows. In the next section we will describe the attractor equations in terms of pure spinors, and discuss the general properties of these solutions. In section 3 we will consider a few simple examples. Appendix A describes our spinor conventions, and Appendix B contains a brief introduction to the pure spinor constructions used in the text. Appendix C reviews a few features of the four dimensional attractor equations which are necessary to make contact with the pure spinor formulation.

2 Attractor Black Holes in Ten Dimensions

In this section we will derive the attractor equations for a wide class of BPS black holes, using ten dimensional supergravity. These equations describe the radial flow of a pure spinor on the internal manifold. The derivation given below requires some technical manipulations, but the main results are rather simple to state. For each of the backgrounds under consideration, one can construct eight pure spinors, which we will call $\Phi_\pm^{13}$, $\Phi_\pm^{24}$, $\Phi_\pm^{14}$ and $\Phi_\pm^{23}$. These pure spinors are constructed from the supersymmetry variations. The first two of these pure spinors obey the constraints required for a compactification to an $\mathcal{N}=2$ Minkowski vacuum. The other two obey a first order differential equation, which describes how the internal geometry flows in the moduli space of $\mathcal{N}=2$ vacua as a function of radius. These equations are the attractor equations for this background; from the four dimensional point of view, they describe the radial flow of the vector multiplet moduli. The explicit equations describing this flow are written down at the end of section 2.3.

In section 2.1 we describe the basic form of the backgrounds under consideration, in

[6] A recent paper [21] has discussed a generalization of the conjecture of [24] in this context, although in absence of RR fields; see also [22].

section 2.2 we write down the fermion variations, and in section 2.3 we rewrite the BPS conditions in terms of pure spinors. Section 2.4 contains a brief discussion of the solutions of these equations, using a theorem of Hitchin's.

2.1 The Background

We will start by describing the background under consideration.

We are interested in BPS solutions of type II supergravity that describe a four dimensional black hole geometry times an internal six-manifold Y. The ten dimensional metric will be of the form

$$ds^2 = e^{2B(y)}\left(-e^{2U(r)}dt^2 + e^{-2U(r)}(dr^2 + r^2(d\theta^2 + \cos\theta^2 d\phi^2))\right) + g_{mn}(r,y)dy^m dy^n. \tag{2.1}$$

The (t, r, θ, ϕ) components of the metric describe an extremal black hole solution in four dimensions, whose geometry depends on the function $U(r)$. The metric $g_{mn}dy^m dy^n$ on Y is a function of radius as well as the internal coordinates, and we have explicitly included a warp factor $B(y)$. Although in principal we could dimensionally reduce on Y to obtain an effective supergravity in $D = 4$, it turns out to be much easier to study these black hole solutions by working directly with ten dimensional quantities.

The spin–connection following from this metric has the form $D_M = \partial_M + \frac{1}{4}\Omega_M^{AB}\Gamma_{AB}$, where M is a curved 10-dimensional index and A, B are flat indices. The components of the spin connection are[7]

$$\begin{gathered}
{\Omega_t}^{01} = e^{2U}U', \quad {\Omega_\theta}^{12} = -1 + rU', \quad {\Omega_\phi}^{13} = \cos\theta(-1 + rU'), \quad {\Omega_\phi}^{23} = \sin\theta \\
{\Omega_r}^{ab} = e^{m[a}e_m^{b]}{}' = 0, \quad {\Omega_m}^{1a} = -\tfrac{1}{2}e^{-B+U}e^{na}g'_{nm}, \quad {\Omega_m}^{ab} = {\omega_m}^{ab} \\
\Omega_t^{0a} = e^{B+U}e^{am}\partial_m B\ , \quad \Omega_r^{1a} = e^{B-U}e^{am}\partial_m B\ , \quad \Omega_\theta^{2a} = re^{B-U}e^{am}\partial_m B\ , \quad \Omega_\phi^{3a} = re^{B-U}\cos\theta e^{am}\partial_m B\ .
\end{gathered} \tag{2.2}$$

where m, n are curved indices on Y, and a, b the associated flat indices. The 6-bein e_a^m on Y obeys $e_a^m e_b^n g_{mn} = \delta_{ab}$. We have chosen our local frame to obey $({e_a}^m)' = {\beta^m}_n {e_a}^n$, where $\beta_{mn} = -\frac{1}{2}g'_{mn}$ is symmetric in mn. This is why ${\Omega_r}^{ab} = 0$.

In addition to the metric described above, the backgrounds under consideration will include flux. The R-R fluxes can be decomposed as

$$F_{2n}^{(10)} = \mathrm{vol}_A \wedge f_{2n-2}^A + \mathrm{vol}_S \wedge f_{2n-2}^S + F_{2n}^i + \mathrm{vol}_A \wedge \mathrm{vol}_S \wedge F_{2n-4}^e \tag{2.3}$$

[7]Here $'$ denotes derivative with respect to r.

where $\mathrm{vol}_A = (e^{2U}/r^2)dt\wedge dr$ and $\mathrm{vol}_S = \cos\theta d\theta\wedge d\phi$. Here f^A, f^S, F^i and F^e are differential forms on Y. A subscript on a form indicates its rank; in the discussion below we will often drop these subscripts when they are not necessary. The first two terms in (2.3) are the gauge field produced by the charged black hole; if we were to dimensionally reduce to four dimensions, they would describe electric and magnetic fluxes sourced by a configuration of branes wrapped on Y. The last two terms describe purely internal and external $2n$-form flux. In type IIA, the index n runs over $0, 2, 4, 6, 8, 10$ while in type IIB n runs over $1/2, 3/2, 5/2, 7/2, 9/2$. The R-R fluxes described above contain both field strengths and their duals, so we must impose the self-duality relations[8]

$$F^{(10)}_{2n} = (-1)^{Int[n]} *_{10} F_{10-2n} \ . \tag{2.4}$$

This relates f^A to f^S and F^i to F^e, so from now on we will write our expressions involving R-R fluxes in terms of $F \equiv F^i$ and $f \equiv f^S$.

We will also consider NS-NS fluxes of the form

$$H^{(10)} = H_3 + dr\wedge b'_2 \tag{2.5}$$

where H and b are differential forms on Y. The second term in this expression arises because we are allowing the internal NS-NS two form b to depend on r.

The Bianchi identities and source-free equations of motion for the R-R fields take the form $(d - H^{(10)}\wedge)F^{(10)} = 0$. For the fluxes described above, this is

$$\begin{array}{lll} (d-H\wedge)F = 0 \ , & (d+H\wedge)(e^{4B} * F) = 0 \ , & dH = 0 \ ; \\ \partial_r(e^{-b\wedge}F) = 0 \ , & d(e^{2B} * b) = 0 \ , & d(e^{4B} * H) = \dfrac{e^{2(U+B)}}{r^2}\partial_r(r^2 * b') \ , \\ \partial_r(e^{-b\wedge}f) = 0 \ , & (d-H\wedge)f = 0 \ , & (d+H\wedge) * f = 0 \ . \end{array} \tag{2.6}$$

Here $*$ is the hodge star on Y, and we are omitting the n indices used above. These identities, together with BPS equations written below, imply the full ten dimensional equations of motion.

2.2 The supersymmetry variations

The gravitino and dilatino variations in ten-dimensional type II supergravity are

$$\begin{array}{rcl} \delta\psi_M & = & (D_M + \frac{1}{4}H_M\mathcal{P})\epsilon + \frac{e^\phi}{16}\sum_n \not{F}_{2n}\Gamma_M\mathcal{P}_n\epsilon \\ \delta\lambda & = & (\not{\partial}\phi + \frac{1}{2}\not{H}\mathcal{P})\epsilon + \frac{e^\phi}{16}\sum_n \Gamma^M\not{F}_{2n}\Gamma_M\mathcal{P}_n\epsilon. \end{array} \tag{2.7}$$

[8] $Int[n]$ denotes the integer part of n and $*_{10}$ the ten dimensional Hodge star.

We have not written the spinor indices explicitly. Our gamma matrix conventions are described in Appendix A. We have also suppressed the doublet indices $i = 1, 2$ on the gravitino ψ_M, dilatino λ, and supersymmetry parameter ϵ. For example, $\epsilon = (\epsilon^1, \epsilon^2)$ is a doublet of ten-dimensional Majorana-Weyl spinors. The $\mathcal{P}$ matrices act on these doublet indices, as $\mathcal{P} = \Gamma_{11}$ and $\mathcal{P}_n = \Gamma_{11}^n \sigma^1$ in type IIA, and as $\mathcal{P} = -\sigma^3$, $\mathcal{P}_n = \sigma^1$ for $(n + 1/2)$ even and $\mathcal{P}_n = i\sigma^2$ for $(n + 1/2)$ odd in type IIB.

Using the self-duality relation (2.4), and putting in the doublet indices explicitly, the the gravitino equation can be written as

$$\begin{aligned} \delta\psi_M^1 &= (D_M \pm \tfrac{1}{4}H_M)\epsilon^1 \mp \tfrac{e^\phi}{8}\Gamma^{01}\tfrac{e^{2U}}{r^2}\not{f}\Gamma_M\epsilon^2 + \not{F}\Gamma_M\epsilon^2 \\ \delta\psi_M^2 &= (D_M \mp \tfrac{1}{4}H_M)\epsilon^2 + \tfrac{e^\phi}{8}\Gamma^{01}\tfrac{e^{2U}}{r^2}\not{f}^\dagger\Gamma_M\epsilon^1 \pm \not{F}^\dagger\Gamma_M\epsilon^2. \end{aligned} \tag{2.8}$$

The upper sign is for type IIA and the lower sign for IIB. We have defined

$$\begin{aligned} \not{f} &= \not{f}_0^A - \not{f}_2^A + \not{f}_4^A - \not{f}_6^A \\ \not{F} &= \tfrac{e^\phi}{8}\left(\not{F}_0^i - \not{F}_2^i + \not{F}_4^i - \not{F}_6^i\right). \end{aligned} \tag{2.9}$$

for type IIA, and

$$\begin{aligned} \not{f} &= \not{f}_1^A + \not{f}_3^A + \not{f}_5^A \\ \not{F} &= \tfrac{e^\phi}{8}\left(\not{F}_1^i + \not{F}_3^i + \not{F}_5^i\right) \end{aligned} \tag{2.10}$$

for type IIB. In the IIB case, $\not{f}_3$ is anti-hermitian while $\not{f}_{1,5}$ are hermitian. In IIA, $\not{f}_{0,4}$ are hermitian while $\not{f}_{2,6}$ are anti-hermitian.

Using the expression for the spin connection, we can write out the components of the gravitino variations in their full glory. For example,

$$\begin{aligned} \delta\psi_t^1 &= e^{2U}\Gamma^{01}\left(-\tfrac{1}{2}U'\epsilon^1 \mp A(r)\not{f}\Gamma_0\epsilon^2\right) + \Gamma_t(\pm\not{F}\epsilon^2 + \tfrac{1}{2}\not{\partial}B\epsilon^1) \\ \delta\psi_r^1 &= \partial_r\epsilon^1 \pm A(r)\not{f}\Gamma_0\epsilon^2 + \Gamma_r(\pm\not{F}\epsilon^2 + \tfrac{1}{2}\not{\partial}B\epsilon^1) \pm \tfrac{1}{4}\not{b}'\epsilon^1 \\ \delta\psi_m^1 &= (D_m \pm \tfrac{1}{4}H_m)\epsilon^1 + \not{F}\Gamma_m\epsilon^2 + \Gamma^r\left(\tfrac{1}{4}\Gamma^n(-g \pm b)'_{mn}\epsilon^1 \pm A(r)\Gamma^0\not{f}\Gamma_m\epsilon^2\right), \end{aligned} \tag{2.11}$$

where $A(r) = e^{B+U+\phi}/8r^2$. The expressions for $\delta\psi_M^2$ are identical, but with $\not{f} \to \mp\not{f}^\dagger$, $\not{F} \to \pm\not{F}^\dagger$ and $H \to -H$. The angular components of $\delta\psi$ are similar, so we will not write them down explicitly.

In a supersymmetric background these fermion variations vanish. We are looking for solutions that preserve half of the four dimensional supersymmetry, so only one linear combination of the supersymmetry parameters ϵ^1 and ϵ^2 will be preserved by the background. It turns out that the correct linear relation between ϵ^1 and ϵ^2 includes the action of Γ^0, but not the action of any other four–dimensional gamma matrices.[9] This implies that the terms

[9] This is the standard situation for branes in $\mathbb{R}^4$ that extend in time but not in any other spatial directions.

in the gravitino variation containing Γ^1 must vanish separately from those that do not. So the $\delta\psi_t = 0$ and $\delta\psi_m = 0$ equations become

$$\begin{aligned} 0 &= -\tfrac{1}{2}U'\epsilon^1 \mp A(r)\not{f}\Gamma_0\epsilon^2 \\ 0 &= \tfrac{1}{4}\Gamma^n(-g \pm b)'_{mn}\epsilon^1 \pm A(r)\Gamma^0\not{f}\Gamma_m\epsilon^2 \end{aligned} \tag{2.12}$$

and

$$\begin{aligned} 0 &= \pm\not{F}\epsilon^2 + \tfrac{1}{2}\not{\partial}B\epsilon^1 \\ 0 &= (D_m \pm \tfrac{1}{4}H_m)\epsilon^1 + \not{F}\Gamma_m\epsilon^2 \ . \end{aligned} \tag{2.13}$$

Using (2.13) and (2.12), we can eliminate the R-R dependence in the $\delta\lambda = 0$ and $\delta\psi_r = 0$ equations. The dilatino variation becomes

$$0 = (\not{D} + \not{\partial}(2B - \phi))\epsilon^1 \pm \frac{1}{4}\not{H}\epsilon^2 \ ; \qquad (\frac{1}{4}g^{mn}g'_{mn})\epsilon^1 = 0 \tag{2.14}$$

while the radial gravitino equation becomes

$$0 = \partial_r\epsilon^1 - \frac{1}{2}U'\epsilon^1 - \frac{1}{4}\not{b}'\epsilon^1 \ . \tag{2.15}$$

The final term in this equation is the only one which depends explicitly on the internal coordinates; we will take it to vanish separately from the other two terms.

Similarly, one can eliminate the R-R dependence from the $\delta\psi^1_{\theta,\phi} = 0$ equations. The result is

$$D_\alpha\epsilon^1 + \frac{1}{2}\gamma_\alpha\Gamma^1\epsilon^1 = 0 \ , \qquad \alpha = (\theta, \phi) \ . \tag{2.16}$$

where D and γ_α denote the spin connection and gamma matrices on a unit S^2.

We can now integrate (2.15) and (2.16) to determine the spatial dependence of ϵ. The radial equation implies that

$$\epsilon^i(r, \theta, \phi, y) = e^{-\frac{1}{2}U(r)}\epsilon^i_0(\theta, \phi, y) \ , \tag{2.17}$$

where ϵ_0 is independent of radius. We will not need to write down the explicit dependence of ϵ_0 on (θ, ϕ), but it is straightforward to do so using methods similar to those described in [30].

We can now decompose the radially independent, ten-dimensional spinors $\epsilon_0^{1,2}$ in terms of four and six dimensional spinors, as

$$\begin{aligned} \epsilon^1_0 &= \zeta^1_+(\theta, \phi) \otimes \eta^1_+(y) + \zeta^2_+(\theta, \phi) \otimes \eta^2_+(y) + \text{c. c.} \\ \epsilon^2_0 &= \zeta^1_+(\theta, \phi) \otimes \eta^3_\mp(y) + \zeta^2_+(\theta, \phi) \otimes \eta^4_\mp(y) + \text{c. c.} \end{aligned} \tag{2.18}$$

Here ζ^i and η^i denote four and six dimensional spinors, respectively; a subscript on a spinor denotes its chirality. The type IIA (IIB) case is given by the upper (lower) sign, where $\epsilon^{1,2}$

have the opposite (same) chirality. Physically, $\zeta^{1,2}$ can be thought of as the two supercharges in four dimensions that would be preserved if the black hole were not present. The Calabi–Yau case involves taking $\eta^2 = \eta^3 = 0$ and $\eta^1 = \eta^4$ to be the single globally defined spinor.

We can now insert (2.18) into (2.12) and collect terms of the same four-dimensional chirality. These equations imply that $\Gamma_0 \zeta^2_- = \alpha(r)\zeta^1_+$, where $\alpha(r)$ is an r-dependent phase that will be determined. This relationship can be thought of as breaking the $\mathcal{N} = 2$ supersymmetry that would have been preserved in four dimensions down to a single linear combination.

Equation (2.12) becomes

$$\begin{aligned}
-\tfrac{1}{2}U'\eta^1_+ \mp \alpha A(r)\not{f}\eta^4_\pm &= 0\ , & \tfrac{1}{4}(-g \pm b)'_{mn}\gamma^n\eta^1_+ \mp \alpha A(r)\not{f}\gamma_m\eta^4_\pm &= 0\ ,\\
-\tfrac{1}{2}U'\eta^2_+ \pm \alpha A(r)\not{f}\eta^3_\pm &= 0\ , & \tfrac{1}{4}(-g \pm b)'_{mn}\gamma^n\eta^2_+ \pm \alpha A(r)\not{f}\gamma_m\eta^3_\pm &= 0\ ,\\
-\tfrac{1}{2}U'\eta^4_+ - \alpha A(r)\not{f}^\dagger\eta^1_\pm &= 0\ , & \tfrac{1}{4}(-g \pm b)'_{mn}\gamma^n\eta^4_+ - \alpha A(r)\not{f}^\dagger\gamma_m\eta^1_\pm &= 0\ ,\\
-\tfrac{1}{2}U'\eta^3_+ + \alpha A(r)\not{f}^\dagger\eta^2_\pm &= 0\ , & \tfrac{1}{4}(-g \pm b)'_{mn}\gamma^n\eta^3_+ + \alpha A(r)\not{f}^\dagger\gamma_m\eta^2_\pm &= 0\ .
\end{aligned} \tag{2.19}$$

Equations (2.13) and (2.14) are precisely the equations that arise in the classification of Minkowski vacua in [17]. In the next section we will follow the analysis of [17] to study these equations.

2.3 From variations to the attractor

In this section we will analyze the spinor equations described above in terms of geometrical quantities. In doing so, it will be convenient to use the pure spinor formalism of [14–16], which is reviewed briefly in appendix B. We will focus on the IIB case, and use the lower sign in (2.19). The analysis for IIA is almost identical – we will simply quote the IIA results at the end of this section.

We will start by reviewing the various structures defined by our spinors on the internal manifold Y – this is described in greater detail in appendix B. In six dimensions, a single spinor η with no zeros defines an SU(3) structure on the tangent bundle T of Y. This is simply the statement that one can form from this spinor two non-vanishing differential forms, a two form J and three form Ω, which obey $J \wedge \Omega = 0$ and $J^3 = \frac{3}{4} i\Omega \wedge \bar{\Omega}$. These forms are associated to two elements $\not{\Omega}$ and $e^{\not{i}J}$ of the Clifford algebra, which are given by exterior products of the original spinor: $\not{\Omega} = \eta_+ \otimes \eta_-$ and $e^{\not{i}J} = \eta_+ \otimes \eta_+$.

For a pair of spinors, say (η^1, η^3), the structure induced on the tangent bundle of Y is more complicated – it depends on the relative orientation of η^1 and η^3. If the spinors are always parallel they define an SU(3) structure. If they are orthogonal they define what is known as an SU(2) structure. If they are neither parallel nor orthogonal, they define what is sometimes known as a “dynamic SU(2) structure.”

So far we have discussed the structures defined on the tangent bundle, but it is more useful to discuss the structure defined on the sum of the tangent and cotangent bundles, $T \oplus T^*$. In fact, all of the cases described above define an SU(3)×SU(3) structure on $T \oplus T^*$. To see this, first note that the bispinor $\not{\Phi}^{13}_\pm = \eta^1_+ \otimes \eta^{3\dagger}_\pm$ defines a pair of SU(3) structures, via its annihilators from the left and from the right. These two SU(3) structures live on $T \oplus T^*$, because $\not{\Phi}^{13}_\pm$ can be mapped via the Clifford map to the bundle of differential forms, which is a representation of the Clifford algebra on $T \oplus T^*$. These bispinors are known as pure spinors, because they are annihilated by half of the elements of the algebra Clifford(6,6).

When we have four spinors $\eta^1_+, \ldots, \eta^4_+$, the structures are even more complicated. On the tangent bundle T, they can define anything from an SU(3) structure to a trivial structure (meaning that T is trivial and the manifold is parallelizable). The structure defined on the sum $T \oplus T^*$ can range from SU(3)×SU(3) to SU(2)×SU(2). The reason is that this time, even for the classification of vacua we need more pure spinors: not just $\Phi^{13}_\pm$, but also $\Phi^{24}_\pm$, as we will see shortly.

We now turn to the analysis of the equations. We will start with (2.13) and (2.14), which are the same as the ones found in the classification Minkowski vacua. In particular, the pairs (η^1, η^3) and (η^2, η^4) each *separately* satisfy the equations for a Minkowski vacuum. It is straightforward to write these equations in terms of pure spinors. In type IIB, one gets [17]

$$e^{-2B+\phi}(d - H\wedge)(e^{2B-\phi}\Phi^{13,24}_+) = dB \wedge \bar{\Phi}^{13,24}_+ + i\frac{e^\phi}{8}|\eta_{1,2}|^2 * \sigma(F) \, , \tag{2.20}$$

$$e^{-2B+\phi}(d - H\wedge)(e^{2B-\phi}\Phi^{13,24}_-) = 0 \tag{2.21}$$

where $\sigma(F_k) = (-)^{[k/2]}F_k$ as in [15]. In addition, we also have $d\log|\eta_{1,2}|^2 = dB$. We have used here the fact that, to have a supergravity vacuum with fluxes, one needs an orientifold action; this relates ϵ^1 to ϵ^2 in such a way that $|\eta^1|^2 = |\eta^2|^2$ and $|\eta^3|^2 = |\eta^4|^2$ [17]. In fact, we will see shortly that all four of these norms are equal.

To summarize, we have found that for any r the internal manifold must support an $\mathcal{N} = 2$ vacuum. In other words, the radial flow moves us through the moduli space of $\mathcal{N} = 2$ Minkowski vacua, just as we would expect.

We now turn to the main computation of this paper: analyzing the equations for radial evolution through the moduli space of $\mathcal{N} = 2$ vacua. We first look at the evolution of U.

The first equation in the first column of (2.19) implies that

$$U' = 2\alpha A|\eta|^2 \frac{(\sigma(f) \wedge \bar{\Phi}_-^{14})_{\text{top}}}{(\sigma(\Phi_-^{14}) \wedge \bar{\Phi}_-^{14})_{\text{top}}} = \frac{e^U}{r^2}\left(c\alpha(r) \frac{\int e^{2B+\phi}\sigma(f) \wedge \bar{\Phi}_-^{14}}{4\int \sigma(\Phi_-^{14}) \wedge \bar{\Phi}_-^{14}} \right) . \tag{2.22}$$

In the first step we have used $\text{Tr}(\gamma \slashed{A}\slashed{B}) = \frac{8i}{\sqrt{g}}(A \wedge B)_{\text{top}}$, and in the second step we have used the fact that U' is constant in the internal directions. The appearance of σ in this formula might seem unfamiliar; the pairing $(\sigma(A) \wedge B)_{\text{top}}$ between differential forms A and B, often denoted (A, B), is known as the *Mukai pairing* (see for example [15]). The term in the parenthesis in (2.22) may be thought of as the absolute value of the central charge of the black hole, $|Z|$, which typically arises in the black hole attractor equations – a brief review of these equations is contained in Appendix C. Note that the phase α is determined by (2.22), since U is real.

If we had instead used the third equation in the first column of (2.19), we would have obtained the same equation with $|\eta_4|^2$ replaced by $|\eta_1|^2$. This implies that $|\eta_4|^2 = |\eta_1|^2$. We can derive similar equations for η^2 and η^3, from which it follows that all of the spinor norms are equal. These spinor norms are just given by $|\eta|^2 = ce^B$ where c is an integration constant (see the comment after (2.21)). We have used this fact in writing (2.22).

Since we have already factored the radial dependence out of ϵ_0, the bispinors $\slashed{\Phi}_\pm^{14}$ are independent of r: $\partial_r \slashed{\Phi}_\pm^{14} = 0$. However, this does not mean that the differential forms $\Phi_\pm$ (related to the bispinors by the Clifford map) are independent of radius. This is because the internal metric g_{mn}, and hence the gamma matrices γ_m, depend on the radius. For an odd bispinor $\slashed{C}$,

$$\partial_r(\slashed{C}) = \quad \slashed{(\partial_r C + \beta^m{}_n dx^n \wedge \iota_m C)} = \quad (\slashed{\partial_r C}) + \frac{1}{2}\beta_{mn}\left(g^{mn}\slashed{C} + \frac{1}{2}\gamma^m \slashed{C}\gamma^n \right) \tag{2.23}$$

where, as described in section 2.1, $\beta_{mn} = -\frac{1}{2}g'_{mn}$. In the second step we have used (B.1). The resulting equation describes the variation of an odd bispinor $\slashed{C}$ due only to the variation of the components $C_{m_1 \ldots m_k}$, after removing the contribution from the gamma matrices. The formula for even bispinors differs from (2.23) by some signs.

Let us consider the case where $\slashed{C} = \slashed{\Phi}_-^{14} = \eta_+^1 \eta_-^{4\dagger}$. First, note that the $g'_{mn}g^{mn}$ term in (2.23) vanishes due to (2.14). We are left with a term of the form $g_{mn}\gamma^n \slashed{\Phi}_-^{14}\gamma^m$, which by (2.19), can be written as the sum of two terms, one proportional to $b'_{mn}\gamma^m \slashed{\Phi}_-^{14}\gamma^n$ and the other proportional to $\slashed{f}\gamma_m \slashed{\Phi}_-^{14}\gamma^m$. We will now describe how to massage these two terms.

We will start with the $b'_{mn}\gamma^m \slashed{\Phi}_-^{14}\gamma^n$ term. The m, n indices are antisymmetrized, so we can use the fact that $\gamma^{[m}(\cdot)\gamma^{n]} = -dx^m \wedge dx^n \wedge + \iota^{mn}$. Since $\slashed{b}'\eta_\pm^i = 0$, (see the discussion after (2.15)), it follows that $b'_{mn}[\gamma^{mn}, \slashed{\Phi}_-^{14}] = 0$; this can be rewritten as $b'_{mn}(dx^m \wedge dx^n + \iota_{mn})\Phi_-^{14} = 0$. So we are just left with $2b' \wedge \Phi_-^{14}$.

We can now attack the $\not{f}\gamma_m \not{\Phi}_-^{14}\gamma^m$ term, which is more interesting. The manipulation we will describe is similar to one used in [9, 17].[10] Since $\not{\Phi}_-^{14} = \eta_+^1 \eta_-^{4\,\dagger}$, we can use the first equation in the second column of (2.19) to get $\not{f}\gamma_m \eta_-^4 \eta_-^{4\,\dagger}\gamma^m$. We can then use the fact that $\frac{1+\gamma}{2} = \eta_+\eta_+^\dagger + \frac{1}{2}\gamma^m\eta_-\eta_-^\dagger\gamma^m$ for any η; this is just the expansion of the operator $\frac{1+\gamma}{2}$ in the chiral basis η_+, $\gamma^m\eta_-$. So the term under consideration can be written as a linear combination of $\not{f}(1+\gamma)$ and $\not{f}\eta_+^4\eta_+^{4\,\dagger}$. Using the third equation in the first column of (2.19), this second term is just $U'\not{\Phi}_+^{14}$.

One can similarly manipulate $\partial_r \Phi_+^{14}$. The only difference is that this time the f contribution looks like $\not{f}\gamma_m \eta_-^4 \eta_+^{4\,\dagger}\gamma^m$. This vanishes, because $\eta_-^4 \eta_+^{4\,\dagger}$ is (the slash of) a three–form and $\gamma_m \not{C}_k \gamma^m = (-)^k(6-2k)\not{C}_k$.

If we put this all together, we get two equations involving bispinors $\not{\Phi}_\pm^{14}$. We can write these equations in terms of differential forms as

$$e^{b\wedge}\partial_r\left(e^{-b\wedge}\Phi_+^{14}\right) = 0 \tag{2.24}$$

$$e^{b\wedge}\partial_r\left(e^{-b\wedge}\Phi_-^{14}\right) = \alpha A|\eta|^2(f + i\sigma(*f)) - \alpha^2 U'\bar{\Phi}_-^{14} \tag{2.25}$$

where again $\sigma(f_k) = (-)^{[k/2]}f_k$. It is interesting to note that these formulas are quite similar to ones describing vacua. This resemblance would be even more explicit if we had, e.g. considered a non-compact Y – the norms of the spinors would not necessarily be equal, and we would have obtained an F term in addition to $*F$. Finally, we should note that there is a similar pair of equations for $\Phi_\pm^{23}$, which are found by taking $^{14} \to {}^{23}$ and $\alpha \to -\alpha$. We will focus on only the $\Phi_\pm^{14}$ equations for the rest of this subsection.

These formulae, together with equation (2.22) for U', describe how the geometry of the black hole and the internal manifold varies with radius. They are the generalizations of the attractor equations for this background, and one of the main results of this paper. Equation (2.24) says that the four dimensional hypermultiplet moduli do not flow with radius. Equation (2.25) describes how the four dimensional vector moduli flow. To compare these to the usual attractor equations it is useful to write them in a slightly different form. Taking the real part of (2.25) gives

$$\mathrm{Re}\,\frac{\partial_r\left(e^{-b\wedge}\Phi_-^{14}\right)}{A\alpha|\eta|^2} = e^{-b\wedge}\left[f - \mathrm{Re}\left(\Phi_-^{14}\frac{U'}{\alpha A|\eta|^2}\right)\right] \tag{2.26}$$

which resembles the standard attractor flow equations.

[10] From the description given in the text, it is not clear that we have extracted all of the information from (2.19). To show that this is indeed the case, one can expand f in terms of the pure Hodge diamond basis used in [15, 17].

Near the horizon of the black hole at $r = 0$ the geometry of Y approaches an attractor fixed point. At this fixed point, the pure spinor obeys a generalized stabilization equation

$$f_1 + f_3 + f_5 = 2\mathrm{Im}(\bar{C}\Phi_-^{14}) \ , \qquad \bar{C} = \frac{i \int e^{2B+\phi}\sigma(f) \wedge \bar{\Phi}_-^{14}}{e^{2B+\phi} \int \sigma(\Phi_-^{14}) \wedge \bar{\Phi}_-^{14}} \ . \tag{2.27}$$

Thus the charges of the black hole, in terms of the fluxes f_1, f_3, f_5, fix $\not{\Phi}_-^{14}$ on the internal manifold. This is the generalization of the statement that, in type IIB, the holomorphic three form of a Calabi-Yau is fixed by the charge of a BPS black hole. We will demonstrate that this equation can indeed be solved in the following section, using a theorem of Hitchin's.

We will simply quote the corresponding results for type IIA. The function U' obeys

$$U' = \frac{e^U}{r^2}\left(c\alpha(r)\frac{\int e^{2B+\phi}\sigma(f) \wedge \bar{\Phi}_+^{14}}{4\int \sigma(\Phi_+^{14}) \wedge \bar{\Phi}_+^{14}}\right) . \tag{2.28}$$

The attractor equations obeyed by the pure spinors are

$$e^{-b\wedge}\partial_r\left(e^{b\wedge}\Phi_-^{14}\right) \ = \ 0 \tag{2.29}$$

$$e^{-b\wedge}\partial_r\left(e^{b\wedge}\Phi_+^{14}\right) \ = \ \alpha A|\eta|^2(f - i\sigma(*f)) + \alpha^2 U'\bar{\Phi}_+^{14} \ . \tag{2.30}$$

Again, from the four dimensional point of view this says that the vector multiplet moduli flow as a function of r. Taking the real part of (2.30) gives

$$\mathrm{Re}\,\frac{\partial_r\left(e^{b\wedge}\Phi_+^{14}\right)}{A\alpha|\eta|^2} = e^{b\wedge}\left[f + \mathrm{Re}\left(\Phi_+^{14}\frac{U'}{\alpha A|\eta|^2}\right)\right] . \tag{2.31}$$

At the attractor point this gives the stabilization equation

$$f_0 + f_2 + f_4 + f_6 = 2\mathrm{Im}\left(\bar{C}\Phi_+^{14}\right) \ , \qquad \bar{C} = \frac{i\int e^{2B+\phi}\sigma(f) \wedge \bar{\Phi}_+^{14}}{e^{2B+\phi}\int \sigma(\Phi_+^{14}) \wedge \bar{\Phi}_+^{14}} \ . \tag{2.32}$$

As in the IIB case, the constant can be determined from (2.31) and (2.28), or by wedging both sides with Φ_+ and integrating.

2.4 Solving the attractor equation

Equations (2.27) are a new version of the usual attractor equations, phrased in the language of pure spinors. We can now use mathematical results concerning pure spinors, such as those of Hitchin [14, 31, 32] to describe the solutions to these equations.[11] These results determine

[11] See [23] for a review of this mathematics in the context of four–dimensional effective theories.

exactly when a sum of differential forms can be the imaginary part of a pure spinor, in terms of a stability condition.

Before discussing this theorem, let us make one comment about the attractor equation (2.27). First, note that the Bianchi identity (2.6) implies that $\partial_r(e^{-b\wedge} f) = 0$. So f depends on r, as one would expect since the geometry of Y changes as a function of radius. The value of f at $r = \infty$ is related to the value at $r = 0$ by $f_\infty = f_{\rm att} e^{\Delta b}$, where $\Delta b = b_\infty - b_{\rm att}$. The flux f appearing in (2.27) is evaluated at the attractor fixed point, $f = f_{\rm att}$. So the attractor equation we are trying to solve is, when written in terms of the flux at infinity, $f_\infty = 2{\rm Im}\,(\bar{C} e^{\Delta b} \Phi_-^{14})$.

We are now in a position to state Hitchin's theorem; the ideas behind it are explained briefly in appendix B and in the references. Given a sum of forms f, define

$$q(f) = {\rm Tr}(\mathcal{J}^2)\ , \quad \mathcal{J}_{\Lambda\Sigma} \equiv \frac{(\sigma(f)\wedge\Gamma_{\Lambda\Sigma} f)_{\rm top}}{\rm vol}\ . \tag{2.33}$$

Here Λ and Σ are indices on $T\oplus T^*$, as explained in the appendix. Then, f is the imaginary part of a pure spinor Φ if and only if $q(f) < 0$ everywhere (the quotient is understood pointwise). If this condition is satisfied, the pure spinor Φ is determined explicitly, as

$$e^{\Delta b}\bar{C}\Phi_-^{14} = i\, f_\infty - \frac{\mathcal{J}_{\Lambda\Sigma}}{\sqrt{-q/12}} \Gamma^{\Lambda\Sigma} f_\infty\ . \tag{2.34}$$

This is precisely the same pure spinor $\cancel{\Phi}_-^{14} = \eta_+^1 \eta_-^{4\,\dagger}$ that appeared on the right hand side of (2.27). We should note that Hitchin's theorem describes a *pointwise* obstruction to solving the attractor equation.[12]

The function q described above is related to the Bekenstein-Hawking entropy of the black hole. This entropy is given by the area of the horizon in four dimensions, which depends on $U(r)$, and can be determined in terms of f via (2.22). Plugging (2.34) into (2.22) gives an expression for the entropy in terms of the pure spinor Φ_-^{14} evaluated at the attractor fixed point: it is essentially the square of the central charge $|Z|^2$, which is $|\int \sigma(f)\wedge\Phi_-^{14}|^2$ times an appropriate normalization factor. In fact, this entropy can be written succinctly in terms of $q(f)$ as

$$S \sim \int e^{4B+2\phi}\sqrt{-q(f)}\ . \tag{2.35}$$

This relation between the entropy and $q(f)$ has been noted already by [20, 21]. We should emphasize that this construction gives a nice physical interpretation to Hitchin's theorem:

[12] However, we should note that the sign of $f\wedge\Phi \sim \bar{\Phi}\wedge\Phi$ at one point in the internal manifold determines the sign at every other point, since $|\eta|^2 = ce^B$.

one can solve the attractor equation precisely when the corresponding solution has positive (and real) entropy, i.e. when the black hole has a non-vanishing horizon.

There is one additional subtlety we have not yet discussed. The pure spinor Φ_-^{14} fixed by the attractor equations is implicitly related to the pure spinors describing the vacuum, $\Phi_\pm^{13,24}$, since they are both built out of the same spinors. In particular, equations (2.20) and (2.21) can be expressed as a rather complicated differential constraint on Φ_-^{14}. We expect that this constraint can be solved by changing $f \to f + dc$ for a suitable choice of c. This is the approach used in [14], for the case where Φ^{14} is closed. There, the existence of a suitable c is reduced to a variational problem for the integral of $q(f)$. This gives a moduli space of solutions as an open set in the appropriate de Rham cohomology. In our case, this can be applied directly when Y is a Calabi–Yau. For example, when Y is a torus case all spinors are covariantly constant and the differential constraints are trivial. More generally, if Φ^{14} is not closed, one would need to modify the Hitchin functional. We hope to be able to describe the general solutions of these constraints in the future.

3 Examples

We will now consider a few particular cases of the general equations constructed above. In section 3.1 we will describe how these equations reduce to the standard form in the Calabi-Yau case, before considering the explicit example of T^6 in section 3.2. We should emphasize that the examples considered in this section are meant to be illustrative of the techniques involved in solving the equations, but are probably not representative.

3.1 The Calabi-Yau Case

The four dimensional attractor equations in this case are well known; they are reviewed in appendix C. Here our approach differs from existing ones only in that it is formulated in ten dimensions, rather than in terms of a low energy $\mathcal{N} = 2$ supergravity in four dimensions.

When the internal manifold Y is Calabi-Yau, it admits only one globally defined spinor η. The ten-dimensional spinor ansatz is given by (2.18), with $\eta^2 = \eta^3 = 0$ and $\eta^1 = \eta^4 = \eta$. The pure spinors are related to the holomorphic three form and Kahler form on Y, by $\slashed{\Phi}_-^{14} = \frac{i}{8}\slashed{\Omega}$ and $\slashed{\Phi}_+^{14} = \slashed{e}^{iJ}$. All other pure spinors vanish.

We will first consider the IIB case. The stabilization equation for the pure spinor at

$r = 0$, (2.27), becomes

$$f_3 = 2\mathrm{Im}\,\left(\bar{C}\Omega\right), \qquad \bar{C} = \frac{i\int f_3 \wedge \bar{\Omega}}{\int \Omega \wedge \bar{\Omega}}\,. \tag{3.1}$$

This is the usual stabilization equation (see, e.g. [33]). In addition, one can verify that the radial flow in complex structure moduli space is precisely that described by (2.27). The attractor equations at finite r are typically written in terms of a symplectic periods (X^I, F_I) rather than directly in terms of the holomorphic three form Ω. For this reason, we have included in appendix C a discussion of the finite r attractor equation, formulated as a differential equation for Ω. It is straightforward to verify that this equation is just (2.27).

The expressions in type IIA are identical, except that the two pure spinors Φ_-^{14} and Φ_+^{14} are exchanged. For example, the stabilization equation becomes

$$f_0 + f_2 + f_4 + f_6 = 2\mathrm{Im}\,(\bar{C}e^{iJ}) \tag{3.2}$$

where the constant is fixed by

$$\bar{C} = \frac{i\int \sigma(f) \wedge e^{iJ}}{\int \sigma(e^{iJ}) \wedge e^{iJ}} = \frac{2\int (f_0 - f_2 + f_4 - f_6) \wedge e^{iJ}}{\int J \wedge J \wedge J}\,. \tag{3.3}$$

3.2 IIB on T^6

Consider type IIB string theory compactified on T^6. For most of this section we will not consider orientifolds of T^6; they will be discussed briefly at the end of the section. Without orientifolds or flux, type IIB on T^6 gives an $\mathcal{N} = 8$ supergravity in $d = 4$. The field content is a single $\mathcal{N} = 8$ gravity multiplet, which contains 70 real scalar fields and 28 vector fields. There are 56 objects charged under these gauge fields:

gauge field	electric object	magnetic object	number
$C_{\mu abc}$	D3	D3	20
$C_{\mu a}$	D1	D5	12
$B_{\mu a}$	F1	NS5	12
$g_{\mu a}$	KK momentum	KK monopole	12

The attractor mechanism for black holes made out of D3 branes on the torus is a special case of the usual Calabi-Yau attractor equations, and is described nicely in [33]. Using the pure spinor formulation, it is straightforward to write down analogous attractor equations for black holes made out of D1 and D5 branes as well. This provides a simple illustration of the power of pure spinor techniques.

First, we must decide the form of the spinor ansatz (2.18). There are many possibilities. The simplest is to consider the torus as a Calabi–Yau, which means taking $\eta^2 = \eta^3 = 0$. We will use this ansatz in what follows, because it is the simplest: in general, η^2 and η^3 will not be zero, and one will have to solve the extra equations for the resulting pure spinor Φ^{23}.

We will now describe the pure spinor $\not{\Phi}_-^{14} = \eta_+^1 \eta_-^{4\dagger}$ on T^6. If η^1 and η^4 are equal (as in the Calabi-Yau case, where there is only one globally defined spinor), the one form and five form pieces of Φ_-^{14} vanish. This can be seen easily by using a basis where the γ_m are antisymmetric. The attractor equations in this case reduce to those described by [33].

In general, however, η^4 will not be proportional to η^1. The pure spinor $\eta_+^1 \otimes \eta_-^{4\dagger}$ will be of the form [18]

$$\Phi_-^{14} = \Omega + e^{ij} \wedge v \ . \tag{3.4}$$

The first term is due to the component of η^4 parallel to η^1, and the second term is due to the component perpendicular to η^1. So the attractor equation for a configuration of D1, D3 and D5 branes on a torus is

$$f_1 + f_3 + f_5 = 2\mathrm{Im}\ \left(\bar{C}(\Omega + e^{ij} \wedge v)\right) \ . \tag{3.5}$$

As we saw in section 2.4, we can determine whether this equation has a solution by looking at the charges. However, in order to illustrate the existence of new solutions, we can proceed in the opposite direction; first we choose a pure spinor, and take f to be its imaginary part. For example, we may choose the pure spinor to be $dz^1(dz^2dz^3 + e^{(1/2i)(dz^2 d\bar{z}^2 + dz^3 d\bar{z}^3})$. This leads to the charges $f_1 = dx^1$, $f_3 = dx^1dx^2dx^3 - dy^1dy^2dx^3 - dy^1dx^2dy^3 - dx^1dy^2dy^3 + dx^1dx^2dy^2 + dx^1dx^3dy^3$, and $f_5 = dx^1dx^2dy^2dx^3dy^3$. This choice leads to a square torus (with all $\tau = i$) and a finite value of the black hole area and entropy.

Finally, we can discuss more complicated cases, where the T^6 is orientifolded. Consider the orientifold that reverses all the coordinates on T^6, which generates 2^6 O3 planes. This projection leaves invariant only the $C_{a\mu}$ and $B_{a\mu}$ gauge bosons. We must now choose a spinor ansatz that is compatible with the orientifold action. This constrains $\eta_+^1 = i\eta_+^3$ and $\eta_+^2 = i\eta_+^4$, so that $\Phi_-^{23} = \sigma(\Phi_-^{14})$. This is compatible with the fact that only f_1 and f_5 charges are allowed, since the three–form parts of $\Phi_-^{14,23}$ vanish. One can easily prove in this case that no such pure spinors exist. This can be seen by noting that if a pure spinor starts with a one form v, it will necessarily be of the form ve^{form_2}, by a theorem in [15]. It can also be proven using the theorem in section 2.4. (Remember that there are no differential constraints in this case, since the spinors are all covariantly constant.) If we call $\tilde{f}_5$ the *vector* dual to the form f_5 (so that we do not need to use the metric), then $q(f_1 + f_5) = 6(f_1\tilde{f}_5)^2 \geq 0$. This shows that these charges lead to no solution.

The previous discussion assumed that the charge of the orientifold is balanced by D3 branes. One could ask what would happen if there are $H \wedge F_3$ terms as well – this is perhaps the simplest example of a flux compactification. In addition to the problem described above, an additional constraint arises from the Bianchi identities. In particular, the H flux generates new terms of the form $H \wedge f_1$ and $H \wedge * f_5$. Canceling both would require either taking $H = 0$ (as we did above) or $f_1 = f_5 = 0$.

From the four–dimensional point of view, the vectors coming from the R-R sector alone are not enough to have a non–singular solution in the orientifold case. One would have to mix the charges with those coming from the NS-NS sector. It would be interesting to extend the present work to incorporate these charges.[13] Similar considerations apply to the simple non–Kähler vacua introduced in [7] by acting with T–duality on the torus with F_3 and H. Indeed, we would expect that one could write down simple attractor equations in these cases, as the $\mathcal{N} = 2$ examples constructed in [7] are dual to $\mathcal{N} = 2$ Calabi–Yau compactifications [36].

Finally, we should mention that many of the considerations in this subsection apply easily to $K3 \times T^2$. This is $\mathcal{N} = 4$ rather than $\mathcal{N} = 8$, so the choice of internal spinors is more limited. In this case we obtain an attractor equation of the form (3.5), where ω and j are members of the triplet of covariantly constant two–forms on $K3$.

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A Gamma matrix conventions

Our conventions for the four and six dimensional gamma matrices are

$$\begin{aligned} \gamma_0 &= \gamma_0^* = -\gamma_0^\dagger \\ \gamma_i &= \gamma_i^* = \gamma_i^\dagger \\ \gamma_m &= -\gamma_m^* = \gamma_m^\dagger \end{aligned} \tag{A.1}$$

where the four dimensional indices $\mu = (0, i)$ are raised and lowered with the flat Lorentzian metric and the six dimensional indices m are raised and lowered with the flat Euclidean

[13] For example, we may consider a solution with NS-NS charge $H_3 \sim \mathrm{vol}_A \wedge h^A + \mathrm{vol}_S \wedge h^S$. This leads to stabilization equation $0 = \mathrm{Im}\,(C(\iota_h \Phi_+ + h \wedge \Phi_+))$ where $h = h^A + i h^S$.

metric. With these definitions, the chirality matrices are

$$\begin{array}{rcccccc} \gamma_5 & = & i\gamma_0\ldots\gamma_3 & = & -\gamma_5^* & = & \gamma_5^\dagger \\ \gamma_7 & = & -i\gamma_4\ldots\gamma_9 & = & -\gamma_7^* & = & \gamma_7^\dagger \\ . & & & & & & \end{array} \tag{A.2}$$

Note that with our conventions γ_5 is pure imaginary. The ten dimensional gamma matrices are

$$\Gamma_\mu = \gamma_\mu \otimes 1, \qquad \Gamma_m = \gamma_5 \otimes \gamma_m \tag{A.3}$$

and the ten dimensional chirality matrices are

$$\Gamma_5 = \gamma_5 \otimes 1 \qquad \Gamma_7 = 1 \otimes \gamma_7 \qquad \Gamma_{11} = \gamma_5 \otimes \gamma_7 = \Gamma_{11}^\dagger . \tag{A.4}$$

B Pure spinors

The objects Φ and $\slashed{\Phi}$ considered in this paper have geometrical significance. The first lives in the bundle of differential forms, and the second in the space of bispinors. The two are related by the Clifford map, which sends a form $dx^{m_1\ldots m_k}$ to $\gamma^{m_1\ldots m_k}$. In this paper we denote the bispinor corresponding to a differential form C by $\slashed{C}$.

The space of bispinors can be viewed as the representation space for two Clifford(6) algebras, which act by left and right multiplication. The space of differential forms can be viewed as the representation space for an algebra generated by wedging with one forms, $dx^m\wedge$, and contracting with vectors, $\iota_m \equiv \iota_{\partial_m}$. This algebra is call Clifford(6,6). It is generated by *twelve* gamma matrices (identified with $dx^m\wedge$ and ι_m) and has an indefinite metric (given by the pairing between vectors and one–forms). It is sometimes useful to denote these twelve gamma matrices collectively as Γ^Λ.

The action of Clifford(6)×Clifford(6) on the space of bispinors is related to the action of Clifford(6,6) of differential forms. For an even (odd) differential form $C_\pm$,

$$\gamma^m\, \slashed{C}_\pm = \slashed{\left[(dx^m \wedge + g^{mn}\iota_n)C_\pm\right]}\ , \qquad \slashed{C}_\pm\, \gamma^m = \pm\ \slashed{\left[(dx^m \wedge - g^{mn}\iota_n)C_\pm\right]}\ . \tag{B.1}$$

The observation that this Clifford product is represented by a combination of wedging and contracting is an old one, see e.g. [34]. This relation is also apparent in the identities used to manipulate products of antisymmetrized gamma matrices $\gamma^{m_1\ldots m_k}$, as in e.g. the appendix of [35]. Recently, this fact has been used in the context of generalized complex geometry [9, 16–18].

A pure spinor Φ is annihilated by a dimension six subspace of Clifford(6,6) – i.e. by six linear combinations of $dx^m\wedge$ and ι_m. To see why this definition is useful, consider the sum of the tangent and cotangent bundles $T\oplus T^*$. In general, the structure group of this bundle is O(6,6). However, the existence of a pure spinor Φ allows us to restrict this structure group to the subgroup of $O(6,6)$ that leaves Φ invariant. This turns out to reduce the structure group to SU(3,3).

To see how this works, consider the space of annihilators of Φ, which has dimension six. This space can be viewed as the $(1,0)$ space of an almost complex structure $\mathcal{J}$ on $T\oplus T^*$, which restricts the structure group to U(3,3). This $\mathcal{J}$ is known as a generalized (almost) complex structure because it lives on $T\oplus T^*$ rather than T. This complex structure can be computed explicitly – it is the expression given in equation (2.33). To understand the origin of this formula, remember that an ordinary almost complex structure can be defined from an ordinary spinor η_+ as $iJ_{mn}=\eta_+\gamma_{mn}\eta_+=\mathrm{Re}\,(\eta_+)\gamma_{mn}\mathrm{Re}\,(\eta_+)$. The expression in (2.33) is the same, but with $T\oplus T^*$ indices. It is now clear why Hitchin's criterion is necessary: since $\mathcal{J}$ is an almost complex structure, it must obey (with appropriate normalization) $\mathrm{Tr}(\mathcal{J}^2)=-12$.[14]

We can now ask what happens if the geometry admits a *pair* of pure spinors $\Phi_\pm$. This pair allows us to reduce the structure group on $T\oplus T^*$ to $SU(3)\times SU(3)$. For the geometries described in this paper, these pairs appear as exterior products of ordinary spinors, of the form $\Phi^{13}_\pm=\eta^1_+\eta^3_\pm$. Of course, not all pairs of pure spinors can be written in this product form; they must obey a compatibility condition. This compatibility condition implies, for example, that the intersection of the two spaces of annihilators has dimension 3. A pair of pure spinors $\Phi_\pm$ can be used to define a metric [15]

$$g^{mn}=\mathcal{J}_{+}{}^{m}{}_{p}\mathcal{J}_{-}^{pn}+\mathcal{J}_{+}^{mp}\mathcal{J}_{-}{}^{n}{}_{p}\ ,\quad \mathcal{J}_{\pm\,\Lambda\Sigma}\equiv\frac{(\sigma(\mathrm{Re}(\Phi_\pm))\wedge\Gamma_{\Lambda\Sigma}\mathrm{Re}\,(\Phi_\pm))_{\mathrm{top}}}{\mathrm{vol}}\ . \tag{B.2}$$

Given this metric, one can use (B.1) to map the annihilators of the $\Phi_\pm$ to a subspace of dimension 3 of the left Clifford(6) action – this subspace is precisely the annihilator of the left η^1_+. Likewise, once can also construct the annihilators of the right $\eta^3_\pm$. These spaces of left and right annihilators define an SU(3)×SU(3) structure on $T\oplus T^*$.

In the main text, the vacua under consideration are characterized by *two* pairs of pure spinors, $\Phi^{13}_\pm$ and $\Phi^{24}_\pm$. Of course, these two pairs are not independent: they must define the same metric.

We should also mention an important special case, where the two spinors that define

[14] For spinors on T we usually do not discuss criteria of this form. This is because in six dimensions all Clifford(6) spinors are pure.

$\not{\Phi}_\pm = \eta_+ \eta_\pm$ are equal. In this case we obtain an SU(3) structure on T. In general, one can compute the explicit form of $\not{\Phi}_\pm$ using Fierz identities. For the case at hand, it turns out that $\not{\Phi}_- \equiv \frac{i}{8}\not{\Omega}$ for a complex three–form Ω, and $\not{\Phi}_+ = \frac{1}{8}e^{\not{i}J}$ for a real two–form J. In this case, the compatibility condition between the pure spinors is that $J \wedge \Omega = 0$ (i.e. J is a $(1,1)$ form), and that $J^3 = \frac{3}{4} i \Omega \bar{\Omega}$. Together, J and Ω provide an equivalent way of characterizing the SU(3) structure of T. They also define a positive signature metric, $g_{i\bar{j}} = i J_{i\bar{j}}$.

Finally, we should mention Calabi–Yau case. Here, the spinor is covariantly constant ($D_m \eta = 0$) and the differential forms are closed ($dJ = 0 = d\Omega$). These two conditions are equivalent.

C The four dimensional attractor equations

In this appendix we will review the four dimensional attractor equations (see also [10] for a review), and demonstrate that they are equivalent to the form described in the text.

For a Calabi–Yau compactification of type II supergravity, the low energy theory is D=4, $\mathcal{N} = 2$ supergravity with some number of vector and hyper multiplets. The low energy theory includes BPS black hole solutions, whose metric is of the form

$$ds^2 = -e^{2U(r)} dt^2 + e^{-2U(r)} (dr^2 + r^2 (d\theta^2 + \cos^2\theta d\phi^2)) \ . \tag{C.1}$$

The metric factor $U(r)$ and the vector multiplet moduli $t^a(r)$ are functions of radius. The BPS equations for this background reduce to a set of linear differential equations for U and t^a,

$$\dot{U} = \frac{e^U}{r^2}|Z|, \quad \dot{t}^a = \frac{e^U}{r^2|Z|} g^{a\bar{b}} \partial_{\bar{b}} |Z|^2. \tag{C.2}$$

Here $\cdot$ denotes d/dr, $Z(t^a(r))$ is the central charge and $g_{a\bar{b}}$ is the metric on vector multiplet moduli space. For the rest of this appendix, we will describe using geometric language the attractor equations in type IIB.

For type IIB on a Calabi-Yau the moduli t^a describe deformations of the complex structure, which is related to the holomorphic three form Ω. The charge of the black hole is parameterized by an element F of H^3. The metric is Kähler,

$$g_{a\bar{b}} = K_{a\bar{b}}, \quad e^{-K} = i\big\langle \Omega, \bar{\Omega} \big\rangle \tag{C.3}$$

and the central charge is

$$Z = e^{K/2} \langle \Omega, F \rangle \ . \tag{C.4}$$

The subscripts ${}_{a,\bar{b}}$ denotes derivatives, and we have defined $\langle \alpha, \beta \rangle = \int_Y \alpha \wedge \beta$. Since we are considering only the four dimensional effective theory, we may regard a three form on Y not as a full differential three form but rather as (the harmonic representative of) an element of H^3. So $\langle \, , \, \rangle$ may be thought of as the symplectic inner product on a finite dimensional vector space.

Now, since Ω and $\bar{\Omega}$ are basis elements of $H^{3,0}$ and $H^{0,3}$, it is useful to define projection operators

$$P^{3,0}\alpha = \frac{\langle \alpha, \bar{\Omega} \rangle}{\langle \Omega, \bar{\Omega} \rangle}\Omega, \quad P^{0,3}\alpha = \frac{\langle \Omega, \alpha \rangle}{\langle \Omega, \bar{\Omega} \rangle}\bar{\Omega}. \tag{C.5}$$

We will denote projection operators onto the transverse space by $P_\perp^{3,0} = 1 - P^{3,0}$ and $P_\perp^{0,3} = 1 - P^{0,3}$. The derivatives Ω_a and $\bar{\Omega}_{\bar{a}}$ are in $H^{3,0} \oplus H^{2,1}$ and $H^{0,3} \oplus H^{1,2}$, respectively, so we can define projection operators onto $H^{2,1}$ and $H^{1,2}$ by

$$P^{2,1}\alpha = -g^{a\bar{b}} \frac{\langle \alpha, P_\perp^{0,3}\bar{\Omega}_{\bar{b}} \rangle}{\langle \Omega, \bar{\Omega} \rangle} P_\perp^{3,0}\Omega_a, \quad P^{1,2}\alpha = -g^{a\bar{b}} \frac{\langle P_\perp^{3,0}\Omega_a, \alpha \rangle}{\langle \Omega, \bar{\Omega} \rangle} P_\perp^{0,3}\bar{\Omega}_{\bar{b}}. \tag{C.6}$$

It is straightforward to verify that all of these projection operators obey $P^2 = P$, commute with one another, and are adjoints with respect to the symplectic inner product:

$$\langle \alpha, P^{0,3}\beta \rangle = \langle P^{3,0}\alpha, \beta \rangle, \quad \langle \alpha, P^{1,2}\beta \rangle = \langle P^{2,1}\alpha, \beta \rangle. \tag{C.7}$$

To show this, it is useful to use the explicit form of the metric

$$g_{a\bar{b}} = -\frac{\langle P_\perp^{3,0}\Omega_a, \bar{\Omega}_{\bar{b}} \rangle}{\langle \Omega, \bar{\Omega} \rangle} = -\frac{\langle \Omega_a, P_\perp^{0,3}\bar{\Omega}_{\bar{b}} \rangle}{\langle \Omega, \bar{\Omega} \rangle}. \tag{C.8}$$

Moreover, the operators described above form a complete basis of H^3, so

$$P^{3,0} + P^{2,1} + P^{1,2} + P^{0,3} = 1. \tag{C.9}$$

It is straightforward to show that

$$\partial_a |Z|^2 = i\frac{\langle F, \bar{\Omega} \rangle}{\langle \Omega, \bar{\Omega} \rangle} \langle P_\perp^{3,0}\Omega_a, F \rangle, \quad \partial_{\bar{a}} |Z|^2 = i\frac{\langle \Omega, F \rangle}{\langle \Omega, \bar{\Omega} \rangle} \langle F, P_\perp^{0,3}\bar{\Omega}_{\bar{a}} \rangle. \tag{C.10}$$

Using the fact that $\dot{\Omega} = \dot{t}^a\Omega_a$, we can multiply both sides of the second attractor equation by $g_{a\bar{b}}$ to get

$$\left\langle \dot{\Omega} + i\frac{e^U}{r^2}\frac{\langle \Omega, F \rangle}{|Z|} F, P_\perp^{0,3}\bar{\Omega}_{\bar{a}} \right\rangle = 0, \quad \left\langle P_\perp^{3,0}\Omega_a, \dot{\bar{\Omega}} + i\frac{e^U}{r^2}\frac{\langle F, \bar{\Omega} \rangle}{|Z|} F \right\rangle = 0. \tag{C.11}$$

These equations fix the components of $\dot{\Omega}$ and $\dot{\bar{\Omega}}$ in $H^{2,1}$ and $H^{1,2}$, respectively, so that

$$P^{2,1}\dot{\Omega} = -i\frac{e^U}{r^2}\frac{\langle \Omega, F\rangle}{|Z|}P^{2,1}F, \quad P^{1,2}\dot{\bar{\Omega}} = -i\frac{e^U}{r^2}\frac{\langle F, \bar{\Omega}\rangle}{|Z|}P^{1,2}F. \tag{C.12}$$

The $H^{3,0}$ and $H^{0,3}$ components are left unfixed, so we can write

$$\dot{\Omega} = -i\frac{e^U}{r^2}\frac{\langle \Omega, F\rangle}{|Z|}P^{2,1}F + \chi\Omega, \quad \dot{\bar{\Omega}} = -i\frac{e^U}{r^2}\frac{\langle F, \bar{\Omega}\rangle}{|Z|}P^{1,2}F + \bar{\chi}\bar{\Omega} \tag{C.13}$$

for an arbitrary function $\chi(r)$. These are unphysical components of $\dot{\Omega}$ which can be absorbed into Kähler transformations on moduli space. Recall that the moduli space metric, and indeed the entire low energy action, are invariant under the Kähler transformation

$$\Omega \to e^{f(t^a)}\Omega, \quad \bar{\Omega} \to e^{\bar{f}(\bar{t^a})}\bar{\Omega} \tag{C.14}$$

for an arbitrary holomorphic function $f(t^a)$ on moduli space. This transformation takes $K \to K - (f(t^a) + \bar{f}(\bar{t^a}))$ and

$$\chi \to \chi + \dot{f} = \chi + f_a\dot{t}^a. \tag{C.15}$$

So, by judicious choice of $f(t^a)$, we may set $\chi(r)$ to be whatever we like.

One natural choice is $\chi = 0$. In this case $\left\langle \dot{\Omega}, \bar{\Omega}\right\rangle = \left\langle \Omega, \dot{\bar{\Omega}}\right\rangle = 0$, and the Kähler potential is independent of radius. Another simple choice is to take

$$\dot{\Omega} = -i\frac{e^U}{r^2}\frac{\langle \Omega, F\rangle}{|Z|}(P^{3,0} + P^{2,1})F, \quad \dot{\bar{\Omega}} = -i\frac{e^U}{r^2}\frac{\langle F, \bar{\Omega}\rangle}{|Z|}(P^{0,3} + P^{1,2})F. \tag{C.16}$$

In this case

$$\left\langle \dot{\Omega}, \bar{\Omega}\right\rangle = \left\langle \Omega, \dot{\bar{\Omega}}\right\rangle = i\frac{e^U}{r^2}|Z|e^{-K} \tag{C.17}$$

is pure imaginary, so

$$\dot{K} = 2\frac{e^U|Z|}{r^2} = 2\dot{U}. \tag{C.18}$$

This equation can be integrated to give the relation of [1] between the spacetime and moduli space metrics: $2U(r) = K(r) - K(\infty)$.

However, it is useful to have a more explicit form for the attractor equations that does not require fixing Kähler gauge invariance. First, note that

$$\dot{K} = -(\chi + \bar{\chi}), \quad \partial_r\left(\ln\frac{Z}{\bar{Z}}\right) = \chi - \bar{\chi}. \tag{C.19}$$

So $\chi = -\dot{\Theta}/\Theta$, where

$$\Theta = \sqrt{\frac{\bar{Z}}{Z}}e^{K/2}. \tag{C.20}$$

With a little algebra, the attractor equations become

$$\partial_r(\Theta\Omega) = -i\frac{e^U}{r^2}P^{2,1}F, \quad \partial_r(\bar{\Theta}\bar{\Omega}) = i\frac{e^U}{r^2}P^{1,2}F. \tag{C.21}$$

Using the completeness relation for projection operators, these are equivalent to the single equation

$$\mathrm{Im}\,\{\partial_r\,(\Theta\Omega)\} = -\frac{e^U}{2r^2}\left(F - \mathrm{Im}\,2e^{K/2}\bar{Z}\Omega\right) \tag{C.22}$$

where Θ is defined above. Note that under Kähler transformations $\Theta \to e^{-f}\Theta$, so both sides of this equation are invariant. It is straightforward to show that the equations in this form are equivalent to the pure spinor equation (2.27) described in the text.

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