Nearly Conformal QCD and AdS/CFT[*]

Guy F. de Téramond[†] and Stanley J. Brodsky[‡]

Universidad de Costa Rica, San José, Costa Rica

Stanford Linear Accelerator Center,
Stanford University, Stanford, California 94309, USA

Abstract

The AdS/CFT correspondence is a powerful tool to study the properties of conformal QCD at strong coupling in terms of a higher dimensional dual gravity theory. The power-law falloff of scattering amplitudes in the non-perturbative regime and calculable hadron spectra follow from holographic models dual to QCD with conformal behavior at short distances and confinement at large distances. String modes and fluctuations about the AdS background are identified with QCD degrees of freedom and orbital excitations at the AdS boundary limit. A description of form factors in space and time-like regions and the behavior of light-front wave functions can also be understood in terms of a dual gravity description in the interior of AdS.

Presented at
First Workshop on Quark-Hadron Duality and the Transition to pQCD
6-8 June 2005
Frascati, Italy

The correspondence [1], between string theory in a warped 10-dimensional space and conformal Yang-Mills theories defined at its four dimensional space-time boundary at infinity has led to important insights into the properties of QCD at strong coupling. The applications include the nonperturbative derivation [2] of dimensional counting rules [3] for hard exclusive glueball scattering, the description of deep inelastic structure functions [4] and the power falloff of hadronic light-front wave functions (LFWF) including orbital angular momentum [5]. In the original correspondence [1], the low energy supergravity approximation to type IIB string compactified on $AdS_5 \times S^5$, is dual to the $\mathcal{N} = 4$ super Yang-Mills (SYM) theory at large N_C.

QCD is fundamentally different from SYM theories where all of the matter fields transform in adjoint multiplets of $SU(N_C)$. Its string dual is unknown. We assume however that such a string exists and that, in principle, it can be defined in terms of the QCD degrees of freedom at infinity. In practice, we can deduce some of the dual string properties by studying its boundary ultraviolet limit, $r \to \infty$, as well as the small-r infrared region of AdS space, characteristic of strings dual to confining gauge theories. This approach, which can be described as a bottom-up approach, has been successful in obtaining general properties of the low-lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD [6], in addition to the scattering results described in [2, 4, 5]. A different approach, a top-bottom approach, consists in studying the full supergravity equations to compute the glueball spectrum [7]. The addition of D3/D7 branes [8] or the gravitational fluctuations of a thick brane in Minkowski space [9] leads to a calculable meson spectrum. Other aspects of high energy scattering in warped spaces have been addressed in [10].

It is remarkable that dimensional scaling for exclusive processes works so well at relatively low energies where higher twist effects are expected to be dominant [11]. Counting rules can be understood if QCD resembles a strongly coupled theory at moderate energies. The isomorphism of the group $SO(2, 4)$, which act as the group of conformal symmetries at the AdS boundary in the limit of massless quarks and vanishing β function, with the group of isometries of AdS, $x^\mu \to \lambda x^\mu$, $r \to r/\lambda$, maps scale transformations into the holographic coordinate r. Consequently, the string mode in r is the extension of the hadron wave function into the fifth dimension. In particular, the $r \to 0$ boundary corresponds to the zero separation limit between quarks. Conversely, color confinement implies that there is a maximum separation of quarks and a minimum value $r_0 = \Lambda_{\text{QCD}} R^2$, where the string modes can propagate. The cutoff at r_0 is dual to the introduction of a mass gap Λ_{QCD}, it breaks the conformal invariance and is responsible for the generation of a spectrum of color singlet hadronic states.

The duality between a gravity theory on AdS_{d+1} space and the strong coupling

limit of a conformal gauge theory at its d-dimensional boundary, is given in terms of the $d+1$ partition function in the bulk $Z_{grav}[\Phi(x,z)] = \int \mathcal{D}[\Phi]e^{iS_{grav}[\Phi]}$, and the d-dimensional functional integral over quarks q and gluons G in presence of an external source,

$$Z_{QCD}[\Phi_o(x)] = \int [\mathcal{D}G][\mathcal{D}q] \exp\left\{iS_{QCD}[G,q] + i\int d^d x \Phi_o \mathcal{O}\right\}, \qquad (1)$$

with conditions [12] $Z_{grav}[\Phi(x,z)_{z=0} = \Phi_o(x)] = Z_{QCD}[\Phi_o]$, where $z = R^2/r$ and R is the AdS radius. Near the AdS boundary, $z \to 0$, $\Phi(x,z)$ behaves as $\Phi(x,z) \to z^\Delta \Phi_+(x) + z^{d-\Delta}\Phi_-(x)$, where $\Phi_-(x)$ is the boundary source, $\Phi_- = \Phi_o$, and $\Phi_+(x)$ is the normalizable solution with conformal dimension Δ. The physical string modes $\Phi(x,z) \sim e^{-iP\cdot x}f(r)$, are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$. For large-$r$, $f(r) \sim r^{-\Delta}$. The dimension Δ of the string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P|\mathcal{O}|0\rangle \neq 0$.

QCD degrees of freedom are defined at the AdS boundary at infinity. Quarks and gluons also propagate in the AdS interior. In the limit where the linearized equations for spin 0, $\frac{1}{2}$, 1 and $\frac{3}{2}$ on $AdS_5 \times S^5$ have no interactions, only color singlet states of dimension 3, 4 and $\frac{9}{2}$ have dual string modes and a physical spectrum. Consequently, only the hadronic states (dimension-3) $J^P = 0^-, 1^-$ pseudoscalar and vector mesons, the (dimension-$\frac{9}{2}$) $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ baryons, and the (dimension-4) $J^P = 0^+$ glueball states, corresponding exactly to the lowest-mass physical hadronic states can be derived in the classical holographic limit [13]. Hadrons fluctuate in particle number, but there are also orbital angular momentum fluctuations. A major difficulty in describing the hadron spectrum with AdS/CFT arises from the nature of the string solutions, since duality cannot be established for spin > 2, where the conformal dimensions become very large. Higher Fock components are manifestations of the quantum fluctuations of QCD; metric fluctuations of the bulk geometry about the fixed AdS background should correspond to quantum fluctuations of Fock states above the valence state. Indeed, for large Lorentz spin, orbital excitations in the boundary correspond to quantum fluctuations about the AdS metric [14]. We identify the higher spin hadrons with the fluctuations around the spin 0, $\frac{1}{2}$, 1 and $\frac{3}{2}$ classical string solutions on the AdS_5 sector [13].

As a specific example, consider the twist (dimension minus spin) two glueball interpolating operator $\mathcal{O}_{4+L}^{\ell_1 \cdots \ell_m} = F D_{\{\ell_1} \ldots D_{\ell_m\}} F$ with total internal space-time orbital momentum $L = \sum_{i=1}^m \ell_i$ and conformal dimension $\Delta_L = 4 + L$. We match the large r asymptotic behavior of each string mode to the corresponding conformal dimension of the boundary operators of each hadronic state while maintaining conformal invariance. In the conformal limit, an L quantum, which is identified with a quantum

3

fluctuation about the AdS geometry, corresponds to an effective five-dimensional mass μ in the bulk side. The allowed values of μ are uniquely determined by requiring that asymptotically the dimensions become spaced by integers, according to the spectral relation $(\mu R)^2 = \Delta_L(\Delta_L - 4)$ [13]. The interaction term in Eq. 1 for a state with orbital L at the asymptotic boundary results in the effective coupling

$$S_{int} = \int d^4x \; \partial_{x_{\ell_1}} \cdots \partial_{x_{\ell_m}} \Phi(x,z)|_{z=0} \; \mathcal{O}^{\ell_1 \cdots \ell_m}. \tag{2}$$

The string modes $\Phi^{\ell_1 \dots \ell_m}(x,z) = \partial_{x_{\ell_1}} \cdots \partial_{x_{\ell_m}} \Phi(x,z)$ for a given eigenvalue μR, with $\Phi(x,z) = Ce^{-iP \cdot x} z^2 J_\alpha(z\beta_{\alpha,k}\Lambda_{QCD})$, $\alpha = \Delta_L - 2$, have the correct Lorentz structure at the AdS boundary, since each ∂_{x^μ} pulls down a P_μ from the exponential factor of the string mode, leaving intact the holographic z-dependence which is determined by the conformal dimension Δ_L.

The four-dimensional mass spectrum follows from the Dirichlet boundary condition $\Phi(x, z_o) = 0$, $z_0 = 1/\Lambda_{\mathrm{QCD}}$, on the AdS string modes for the different wave functions corresponding to spin < 2 and is given in terms of the zeros of Bessel functions, $\beta_{\alpha,k}$. In the case of mesons the predicted spectrum is shown in Figure 1 for $\Lambda_{\mathrm{QCD}} = 0.263$ GeV, the only parameter in the model. The baryon spectrum is discussed in [13].
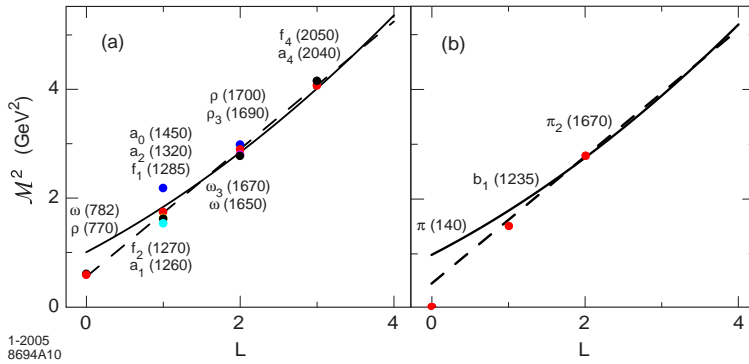

Figure 1: Light meson orbital states for $\Lambda_{\mathrm{QCD}} = 0.263$ GeV: (a) vector mesons and (b) pseudoscalar mesons. The dashed line is a linear Regge trajectory with slope 1.16 GeV^2.

The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron Φ_I and Φ_F and the non-normalizable mode $J(Q,z)$, dual to the external source

$$F(Q^2)_{I \to F} \simeq R^{3+2\sigma} \int_0^{z_o} \frac{dz}{z^{3+2\sigma}} \; \Phi_F(z) \; J(Q,z) \; \Phi_I(z), \tag{3}$$

where $\sigma_n = \sum_{i=1}^n \sigma_i$ is the spin of the interpolating operator $\mathcal{O}_n$, which creates an n-Fock state $|n\rangle$ at the AdS boundary. $J(Q,z)$ has the value 1 at zero momentum

transfer, and as boundary limit the external current, thus $A^\mu(x, z) = \epsilon^\mu e^{iQ \cdot x} J(Q, z)$. The solution to the AdS wave equation subject to boundary conditions at $Q = 0$ and $z \to 0$ is [4] $J(Q, z) = zQK_1(zQ)$. At large enough $Q \sim r/R^2$, the important contribution to (3) is from the region near $z \sim 1/Q$. At small z, the n-mode $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$, and we recover the power law scaling [3], $F(Q^2) \to [1/Q^2]^{\tau_n-1}$, where the twist $\tau_n = \Delta_n - \sigma_n$, is equal to the number of partons, $\tau_n = n$. A numerical computation for the pion form factor [15] gives the results shown in Figure 2, where the resonant structure in the time-like region from the AdS cavity modes is apparent.
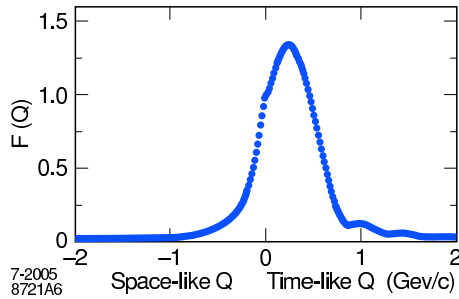

Figure 2: Space and time-like structure for the pion form factor in AdS/QCD.

The AdS/QCD correspondence provides also a simple description of hadrons at the amplitude level by mapping string modes to the impact space representation of LFWF. In terms of partonic variables $x_i \vec{r}_{\perp i} = \vec{R}_\perp + \vec{b}_{\perp i}$, where $\vec{r}_{\perp i}$ are the physical position coordinates, $\vec{b}_{\perp i}$ frame-independent internal coordinates, $\sum_i \vec{b}_{\perp i} = 0$, and $\vec{R}_\perp$ the hadron transverse center of momentum $\vec{R}_\perp = \sum_i x_i \vec{r}_{\perp i}$, $\sum_i x_i = 1$, we find for a two-parton LFWF to first approximation

$$\psi_{n=2,\ell,k}(x, \vec{b}_\perp) \sim x(1-x) J_{n+\ell-1}\left(|\vec{b}_\perp| \beta_{n-1,k} \Lambda_{QCD}\right) / |\vec{b}_\perp|.$$

The first eigenmodes are depicted in the figure below.

The holographic model is quite successful in describing the known light hadron spectrum. The only mass scale is Λ_{QCD}. The model incorporates confinement and conformal symmetry. Only dimension 3, $\frac{9}{2}$ and 4 states $\bar{q}q$, qqq, and gg appear in the duality at the classical level. Non-zero orbital angular momentum and higher Fock-states require the introduction of quantum fluctuations. The model gives a simple description of the structure of hadronic form factors and LFWFs. It explains the suppression of the odderon. The dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model.
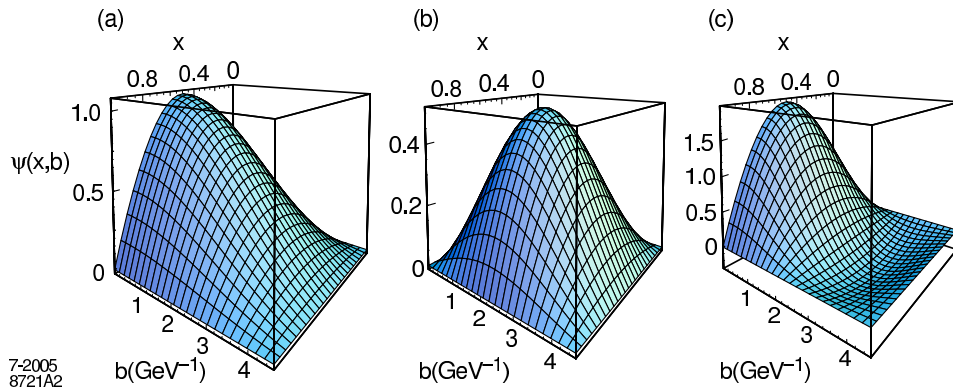
5

Figure 3: Two-parton LFWF $\psi(x, b = |\vec{b}_\perp|)$: (a) ground state $\ell = 0$, $k = 1$, (b) first orbital excited state $\ell = 1$, $k = 1$, (c) first radial exited state $\ell = 0$, $k = 2$.

References

[1] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].

[2] J. Polchinski and M. J. Strassler, "Hard scattering and gauge/string duality," Phys. Rev. Lett. **88**, 031601 (2002) [arXiv:hep-th/0109174].

[3] S. J. Brodsky and G. R. Farrar, "Scaling Laws At Large Transverse Momentum," Phys. Rev. Lett. **31**, 1153 (1973); V. A. Matveev, R. M. Muradian and A. N. Tavkhelidze, "Automodellism In The Large - Angle Elastic Scattering And Structure Of Hadrons," Lett. Nuovo Cimento **7**, 719 (1973).

[4] J. Polchinski and M. J. Strassler, "Deep inelastic scattering and gauge/string duality," JHEP **0305**, 012 (2003) [arXiv:hep-th/0209211].

[5] S. J. Brodsky and G. F. de Téramond, "Light-front hadron dynamics and AdS/CFT correspondence," Phys. Lett. B **582**, 211 (2004) [arXiv:hep-th/0310227].

[6] H. Boschi-Filho and N. R. F. Braga, "QCD / string holographic mapping and glueball mass spectrum," Eur. Phys. J. C **32**, 529 (2004) [arXiv:hep-th/0209080]; "Gauge / string duality and scalar glueball mass ratios," JHEP **0305**, 009 (2003) [arXiv:hep-th/0212207]; G. F. de Teramond and S. J. Brodsky, "Baryonic states in QCD from gauge / string duality at large N(c)," arXiv:hep-th/0409074; J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, "QCD and a holographic model of hadrons," arXiv:hep-ph/0501128; S. Hong, S. Yoon and M. J. Strassler, "On the couplings of the rho meson in AdS/QCD," arXiv:hep-ph/0501197; L. Da Rold and A. Pomarol, "Chiral symmetry breaking from five dimensional spaces," arXiv:hep-ph/0501218; H. Boschi-Filho, N. R. F. Braga and H. L. Carrion, "Glueball Regge trajectories from gauge / string duality and the pomeron," arXiv:hep-th/0507063.

[7] C. Csaki, H. Ooguri, Y. Oz and J. Terning, "Glueball mass spectrum from supergravity," JHEP **9901**, 017 (1999) [arXiv:hep-th/9806021]; R. de Mello Koch, A. Jevicki, M. Mihailescu and J. P. Nunes, "Evaluation of glueball masses from supergravity," Phys. Rev. D **58**, 105009 (1998) [arXiv:hep-th/9806125]; C. Csaki, Y. Oz, J. Russo and J. Terning, "Large N QCD from rotating branes," Phys. Rev. D **59**, 065012 (1999) [arXiv:hep-th/9810186]; J. A. Minahan, "Glueball mass spectra and other issues for supergravity duals of QCD models," JHEP **9901**, 020 (1999) [arXiv:hep-th/9811156]; R. C. Brower, S. D. Mathur and C. I. Tan, "Glueball spectrum for QCD from AdS supergravity duality," Nucl. Phys. B **587**, 249 (2000) [arXiv:hep-th/0003115]; E. Caceres and C. Nunez, "Glueballs of super Yang-Mills from wrapped branes," arXiv:hep-th/0506051.

[8] A. Karch, E. Katz and N. Weiner, "Hadron masses and screening from AdS Wilson loops," Phys. Rev. Lett. **90**, 091601 (2003) [arXiv:hep-th/0211107]; M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, "Meson spectroscopy in AdS/CFT with flavour," JHEP **0307**, 049 (2003) [arXiv:hep-th/0304032]; "Towards a holographic dual of large-N(c) QCD," JHEP **0405**, 041 (2004) [arXiv:hep-th/0311270]; T. Sakai and J. Sonnenschein, "Probing flavored mesons of confining gauge theories by supergravity," JHEP **0309**, 047 (2003) [arXiv:hep-th/0305049]; J. Babington, J. Erdmenger, N. J. Evans, Z. Guralnik and I. Kirsch, "A gravity dual of chiral symmetry breaking," Fortsch. Phys. **52**, 578 (2004) [arXiv:hep-th/0312263]; C. Nunez, A. Paredes and A. V. Ramallo, "Flavoring the gravity dual of N = 1 Yang-Mills with probes," JHEP **0312**, 024 (2003) [arXiv:hep-th/0311201]; S. Hong, S. Yoon and M. J. Strassler, "Quarkonium from the fifth dimension," JHEP **0404**, 046 (2004) [arXiv:hep-th/0312071]; "On the couplings of vector mesons in AdS/QCD," arXiv:hep-th/0409118; M. Kruczenski, L. A. P. Zayas, J. Sonnenschein and D. Vaman, "Regge trajectories for mesons in the holographic dual of large-N(c) QCD," arXiv:hep-th/0410035; T. Sakai and S. Sugimoto, "Low energy hadron physics in holographic QCD," Prog. Theor. Phys. **113**, 843 (2005) [arXiv:hep-th/0412141]; "More on a holographic dual of QCD," arXiv:hep-th/0507073; A. Paredes and P. Talavera, "Multiflavour excited mesons from the fifth dimension," Nucl. Phys. B **713**, 438 (2005) [arXiv:hep-th/0412260]; I. Kirsch and D. Vaman, "The D3/D7 background and flavor dependence of Regge trajectories," arXiv:hep-th/0505164.

[9] G. Siopsis, "Correspondence principle for a brane in Minkowski space and vector mesons," arXiv:hep-th/0503245.

[10] S. B. Giddings, "High energy QCD scattering, the shape of gravity on an IR brane, and the Froissart bound," Phys. Rev. D **67**, 126001 (2003) [arXiv:hep-th/0203004]; O. Andreev and W. Siegel, "Quantized tension: Stringy amplitudes with Regge poles and parton behavior," Phys. Rev. D **71**, 086001 (2005) [arXiv:hep-th/0410131]; K. Kang and H. Nastase, "High energy QCD from Planckian scattering in AdS and the Froissart bound," arXiv:hep-th/0410173; H. Nastase, "The soft pomeron from AdS-CFT," arXiv:hep-th/0501039; "The RHIC fireball as a dual black hole," arXiv:hep-th/0501068.

[11] See for example the contributions of A. Mirazita, H. Gao and A. Deur to these proceedings.

[12] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109]; E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].

[13] G. F. de Teramond and S. J. Brodsky, "The hadronic spectrum of a holographic dual of QCD," Phys. Rev. Lett. **94**, 201601 (2005) [arXiv:hep-th/0501022].

[14] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "A semi-classical limit of the gauge/string correspondence," Nucl. Phys. B **636**, 99 (2002) [arXiv:hep-th/0204051].

[15] S. J. Brodsky and G. F. de Téramond, in preparation.