

Fermion Number in Supersymmetric Models*

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Abstract

The two known methods for introducing a conserved fermion number into supersymmetric models are discussed. While the introduction of a conserved fermion number often requires that the Lagrangian be massless or that bosons carry fermion number, we discuss a model in which masses can be introduced via spontaneous symmetry breaking and fermion number is conserved at all stages without assigning fermion number to bosons.

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A major problem encountered when attempting to construct realistic supersymmetric Lagrangians is that they are often not invariant under a phase transformation which can be associated with conservation of fermion number.^{1,2} Some Lagrangians are only invariant under phase transformations which, when identified with fermion number conservation, require scalars to have fermion number,³ a somewhat unattractive situation from a physical point of view. In several known models^{1,2} fermion number can be conserved without assigning fermion number to bosons, but these theories are massless and cannot be broken spontaneously.

We study the circumstances under which one can introduce a phase transformation that leads to a conserved fermion number, so that the physical fields can be identified in supersymmetric Lagrangians, and discuss the two known methods of introducing fermion number within the context of several published supersymmetric Lagrangians which have a conserved fermion number, i.e., a conserved fermion number carried only by fermions. Then we present a supersymmetric Abelian gauge invariant model in which fermion number is conserved.⁴ Masses can be introduced in this model by spontaneously breaking either the supersymmetry of the vacuum or the Abelian invariance of the vacuum without destroying conservation of fermion number.

At first sight it would seem trivial to conserve fermion number in supersymmetric Lagrangians since fermions come in pairs. The problem of fermion number conservation arises primarily because the naturally occurring fermion fields in supersymmetric Lagrangians are neutral Majorana fields which cannot transform with a phase. The reason that Majorana spinors appear instead of Dirac spinors in the Lagrangians results from the facility of formulating the supersymmetric algebra in terms of neutral Majorana spinors. When a Majorana

spinor ψ is written in terms of a two component spinor ψ_β and its conjugate $\bar{\psi}^\beta$ as

$$\psi = \begin{pmatrix} \psi_\beta \\ \bar{\psi}^\beta \end{pmatrix}, \quad (1)$$

we see that the transformation law

$$\psi \rightarrow e^{i\alpha} \psi, \quad (2)$$

implies the two relations

$$\psi_\beta \rightarrow e^{i\alpha} \psi_\beta, \quad (3a)$$

$$\bar{\psi}^\beta \rightarrow e^{i\alpha} \bar{\psi}^\beta, \quad (3b)$$

which are mathematically inconsistent for $\alpha \neq 0$. It is, however, possible for a neutral Majorana spinor to transform according to the relation²

$$\psi \rightarrow e^{-\gamma_5 \alpha} \psi. \quad (4)$$

The helicity method of defining fermion number is associated with the invariance of a supersymmetric Lagrangian under (4). The eigenstates of fermion number are the helicity eigenstates ψ_L and ψ_R defined by⁵

$$\psi_L \equiv L \psi \equiv \frac{1}{2} (1 - i\gamma_5) \psi \rightarrow \frac{1}{2} (1 - i\gamma_5) e^{-\gamma_5 \alpha} \psi = e^{-i\alpha} \psi_L, \quad (5a)$$

$$\psi_R \equiv R \psi \equiv \frac{1}{2} (1 + i\gamma_5) \psi \rightarrow \frac{1}{2} (1 + i\gamma_5) e^{-\gamma_5 \alpha} \psi = e^{i\alpha} \psi_R. \quad (5b)$$

If the Lagrangian contains two neutral Majorana spinors and is invariant under the transformation

$$\psi \rightarrow e^{-\gamma_5 \alpha} \psi, \quad \lambda \rightarrow e^{\gamma_5 \alpha} \lambda, \quad (6)$$

we can form the fermion number eigenstates

$$\eta_1 = \psi_L + \delta \lambda_R, \quad (7a)$$

$$\eta_2 = \psi_R + \delta^* \lambda_L, \quad \delta = \text{phase}, \quad (7b)$$

which have fermion number -1 and 1, respectively, and are not eigenstates of helicity. If the spinors were massive or became massive via spontaneous symmetry breaking, one would have to use the procedure (7) instead of (5) for introducing fermion number.

A second method¹ for introducing fermion number exists. If a Lagrangian is a function of two neutral Majorana spinors ψ, λ we can write it in terms of the complex Majorana spinors

$$\chi_1 = (\lambda + i\psi)/\sqrt{2}, \quad \chi_2 = (\lambda - i\psi)/\sqrt{2}. \quad (8)$$

When written in terms of χ_1 and χ_2 , the Lagrangian is invariant under the phase transformation

$$\chi_1 \rightarrow e^{-id} \chi_1, \quad \chi_2 \rightarrow e^{id} \chi_2, \quad (9)$$

provided there are no terms involving $\bar{\chi}_1$ and χ_2 or $\bar{\chi}_2$ and χ_1 .

We now illustrate the two methods of defining the fermion number in the SU(N) Lagrangian,¹

$$\begin{aligned} \mathcal{L} = \text{Tr} \left\{ -\frac{1}{4} V_{\mu\nu}^2 - \frac{i}{2} \bar{\lambda} \gamma^\mu D_\mu \lambda + \frac{1}{2} D^2 - \frac{1}{2} (D_\mu A)^2 \right. \\ \left. - \frac{1}{2} (D_\mu B)^2 - \frac{i}{2} \bar{\Psi} \gamma^\mu D_\mu \Psi + g \bar{\lambda} [A + \gamma_5 B, \Psi] + ig D[A, B] \right\}, \end{aligned} \quad (10)$$

where

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + ig [V_\mu, V_\nu], \quad (11)$$

$$D_\mu \lambda = \partial_\mu \lambda + ig [V_\mu, \lambda] \quad \text{etc.} \quad (12)$$

Lagrangian (10) is invariant under the global transformation (6).

In order to implement the helicity method, we define the fermion eigenstates

$$\begin{aligned} \eta_1 &= \Psi_L + \lambda_R, \\ \eta_2 &= \Psi_R + \lambda_L. \end{aligned} \quad (13)$$

Under the transformation (6), the states (13) transform as

$$\begin{aligned}\eta_1 &\rightarrow e^{-i\alpha} \eta_1, \\ \eta_2 &\rightarrow e^{i\alpha} \eta_2.\end{aligned}\tag{14}$$

In terms of the fermion eigenstates, the spinor-scalar interaction term in (10) is expressible as

$$\begin{aligned}g \text{Tr} \{ \bar{\eta}_1 [A+iB, L\eta_1] + \bar{\eta}_2 [A-iB, R\eta_2] \} \\ = \sqrt{2}g \text{Tr} \{ \bar{\eta}_1 [\phi^+, L\eta_1] + \bar{\eta}_2 [\phi^-, R\eta_2] \}.\end{aligned}\tag{15}$$

To construct fermion eigenstates using the second method, we define as in (8)

$$\chi_1 = (\lambda + i\psi)/\sqrt{2}, \quad \chi_2 = (\lambda - i\psi)/\sqrt{2}.$$

When the Lagrangian (10) is expressed in terms of χ_1 and χ_2 , it is invariant under the global phase transformation

$$\chi_1 \rightarrow e^{-i\alpha} \chi_1, \quad \chi_2 \rightarrow e^{i\alpha} \chi_2,$$

and the spinor-scalar interaction term is

$$\begin{aligned}-ig \text{Tr} \{ \bar{\chi}_1 [A + \gamma_5 B, \chi_1] + \bar{\chi}_2 [A + \gamma_5 B, \chi_2] \} \\ = -\sqrt{2}ig \text{Tr} \{ \bar{\chi}_1 [\phi^+, L\chi_1] + \bar{\chi}_1 [\phi^-, R\chi_1] + \bar{\chi}_2 [\phi^+, L\chi_2] + \bar{\chi}_2 [\phi^-, R\chi_2] \}.\end{aligned}\tag{16}$$

We note that depending on the method chosen to define the eigenstates of

fermion number, one can obtain different scattering processes. The mass term of Lagrangian (10) has been set equal to zero in order to conserve fermion number, and the Lagrangian cannot become massive via spontaneous symmetry breaking.

The fermion number or a more general phase transformation can sometimes be introduced directly through the superfields. When the scalar and vector superfields transform respectively as

$$S(x, \theta) \rightarrow e^{in\alpha} S(x, \theta e^{-i\alpha}), \quad (17)$$

$$V(x, \theta, \bar{\theta}) \rightarrow V(x, \theta e^{-i\alpha}, \bar{\theta} e^{i\alpha}). \quad (18)$$

then the ordinary fields contained in the superfields transform as

$$\begin{aligned} \phi^\pm &= (A \pm iB)/\sqrt{2} \rightarrow e^{\mp in\alpha} \phi^\pm, & H^\pm &= (F \pm iG)/\sqrt{2} \rightarrow e^{\pm i(n-2)\alpha} H^\pm, \\ \psi_L &\rightarrow e^{i(n-1)\alpha} \psi_L, & \psi_R &\rightarrow e^{-i(n-1)\alpha} \psi_R, \\ \lambda_L &\rightarrow e^{i\alpha} \lambda_L, & \lambda_R &\rightarrow e^{-i\alpha} \lambda_R, \\ V_\mu &\rightarrow V_\mu, & D &\rightarrow D. \end{aligned} \quad (19)$$

When a Lagrangian is expressed in terms of superfields, its invariance properties under (17) and (18) are manifest. Invariance of Lagrangian (10) under the transformation (6) corresponds to the case $n=0$ of (19).

We now discuss a model which is initially massless but becomes massive through spontaneous symmetry breaking while conserving fermion number at all stages. The Abelian supersymmetric Lagrangian is⁴

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} (D_\mu A)^2 - \frac{1}{2} (D_\mu B)^2 - \frac{i}{2} \bar{\Psi} \gamma^\mu D_\mu \Psi \\ & + \frac{1}{2} F^2 + \frac{1}{2} G^2 + \frac{g}{2} (A^2 + B^2) D - ig \bar{\lambda} (B - \gamma_5 A) \Psi \\ & - \frac{1}{4} V_{\mu\nu}^2 - \frac{i}{2} \bar{\lambda} \gamma^\mu \partial_\mu \lambda + \frac{1}{2} D_s^2 \end{aligned} \quad (20)$$

where

$$\begin{aligned} D_\mu A &= \partial_\mu A + g V_\mu B, & D_\mu B &= \partial_\mu B - g V_\mu A, \\ D_\mu \Psi &= \partial_\mu \Psi + g V_\mu \gamma_5 \Psi, & V_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu. \end{aligned} \quad (21)$$

The Lagrangian (20) is invariant under (19). Therefore, one can use the helicity method to write the following eigenstates⁶ of fermion number when $n=0$:

$$\eta_1 = \Psi_L + i \lambda_R, \quad \eta_2 = \Psi_R - i \lambda_L. \quad (22)$$

As a result of (19), the Lagrangian is invariant under

$$\eta_1 \rightarrow e^{-id} \eta_1, \quad \eta_2 \rightarrow e^{id} \eta_2. \quad (23)$$

The term $\frac{1}{2}(F^2 + G^2) = H^+ H^-$ is invariant and does not introduce scalar particles with fermion number because it decouples from the Lagrangian (20).

We now proceed to break the supersymmetry of the vacuum by adding to the Lagrangian given by (20) the term

$$\mathcal{L}_B = -\xi g D. \quad (24)$$

The Lagrangian $\mathcal{L} + \mathcal{L}_B$ is expressible in terms of η_1 and η_2 of Eq. (22) as⁴

$$\begin{aligned} \mathcal{L} + \mathcal{L}_B = & -\frac{1}{2} (D_\mu A)^2 - \frac{1}{2} (D_\mu B)^2 - \frac{i}{2} \bar{\eta}_1 \gamma^\mu D_\mu \eta_1 - \frac{i}{2} \bar{\eta}_2 \gamma^\mu D_\mu \eta_2 \\ & - \frac{1}{4} V_{\mu\nu}^2 - ig [\bar{\eta}_1 (A + iB) L \eta_1 + \bar{\eta}_2 (A - iB) \eta_2] \\ & - \frac{g^2}{8} (A^2 + B^2)^2 - \frac{1}{2} (-\xi g^2) (A^2 + B^2), \end{aligned} \quad (25)$$

where

$$\begin{aligned} D_\mu \eta_1 &= \partial_\mu \eta_1 + ig L \eta_1, \\ D_\mu \eta_2 &= \partial_\mu \eta_2 - ig R \eta_2. \end{aligned} \quad (26)$$

The scalar particles A and B have each gained a mass m ($m^2 = -\xi g^2$, $\xi < 0$) while fermion number is still conserved.

We can spontaneously break the Abelian symmetry by making the substitution

$$A \rightarrow A + a, \quad a^2 = 2\xi, \quad \xi > 0, \quad (27)$$

in which case the scalar B becomes massless and all other particles obtain a mass $m = ga$. Fermion number remains unbroken. One cannot use the second method of introducing a conserved fermion number in (20) due to the fact that when the Lagrangian is expressed in terms of χ_1 and χ_2 in (8), non-invariant terms such as $\bar{\chi}_1 \chi_2$ appear.

The helicity method is widely applicable because one can often express the interaction terms in a form that is diagonal with respect to fields of the type η_1 and η_2 in Eq.(13), thus conserving fermion number. The mass terms do not conserve fermion number so that one must require the Lagrangian to be massless initially.⁷ In order to create masses through spontaneous symmetry breaking in a supersymmetric SU(N) gauge invariant model, one may use the method of Fayet and Iliopoulos⁸ and construct a corresponding SU(N) \times U(1) model. If the restrictions of supersymmetry and gauge invariance allow the model to have a mass term, fermion number conservation requires that the mass be set equal to zero and it is not possible to introduce a mass term by spontaneous symmetry breaking, because the U(1) mass term is needed to obtain real masses.⁹ If the U(1) model does not permit a mass term, then it is possible to obtain mass terms by spontaneous symmetry breaking. In order to obtain a Lagrangian with a conserved fermion number and massive particles, we speculate that it is necessary to start with a supersymmetric U(1) Lagrangian that does not permit a mass term, and then obtain the masses by a spontaneous symmetry breaking mechanism.

The second method of introducing fermion number is available when the Lagrangian has SU(N) symmetry. This option is associated with F-type coupling of spinors and scalar fields where $\bar{\lambda}\psi$ and $\bar{\psi}\lambda$ terms are expressible in terms of the type $\bar{\chi}_1 \chi_1$ and $\bar{\chi}_2 \chi_2$ where χ_1 and χ_2 are defined in (8).

We next consider the phase transformation exemplified by (19) which can be regarded as a generalization of the transformation associated with fermion number conservation. It is remarkable that the vector models^{3,4} are invariant under the transformation (19) for arbitrary n and $mass = 0$. On the other hand, the scalar model¹⁰ with $m = 0$ is invariant for $n = 2/3$. It appears that phase transformation (19) imposes as much restriction on the form of the Lagrangian as the conservation of fermion number which corresponds to $n = 0$, because all the available supersymmetric massless Lagrangians respect this phase transformation.

After completion of this work, a paper entitled "Supersymmetry and Fermion-Number Conservation" by A. Salam and J. Strathdee, Nucl. Phys. B87, 85 (1975) was called to our attention. The authors conclude that it is often necessary to tolerate some bosons with fermion number two in order to have conservation of fermion number.

Footnotes and References

1. A. Salam and J. Strathdee, IC/74/36; S. Ferrara and B. Zumino, Nucl. Phys. B79, 413 (1974).
2. R. Delbourgo, A. Salam and J. Strathdee, IC/74/45.
3. P. Fayet, PTEN 74/7, to be published in Nucl. Phys. B.
4. G. B. Mainland and K. Tanaka, Ohio State University Report C00-1545-152, 1975 (unpublished).
5. For massless leptons, for example, ψ_L and ψ_R may be associated with opposite fermion number.
6. When we compare Eq.(7) and (22), $\delta = i$. This choice is taken so that when we break the Abelian invariance of the vacuum via $A \rightarrow A + a$ as in Ref.4, the usual mass terms appear.
7. In scalar models, it is possible to have mass terms that conserve fermion number but the bosons also have to carry a fermion number.
8. P. Fayet and J. Iliopoulos, Phys. Lett. 51B, 461 (1974).
9. It is possible to spontaneously break the Lagrangian and conserve fermion number if bosons are permitted to carry fermion number.
10. J. Iliopoulos and B. Zumino, Nucl. Phys. B76, 310 (1974).