NOVEMBER 1974 MATT-1080

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Maximum Power Gains of Radio-Frequency-Driven Two-Energy-Component Tokamak Reactors.*

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ABSTRACT

Two-energy-component fusion reactors in which the suprathermal component (D) is produced by harmonic cyclotron "runaway" of resonant ions are considered. In one ideal case, the fast hydromagnetic wave at $\omega = 2\omega_{cD}$ produces an energy distribution $f(W) \approx$ constant (up to W_{max}) that includes all deuterons, which then thermalize and react with the cold tritons. In another ideal case, $f(W) \approx$ constant is maintained by the fast wave at $\omega = \omega_{cD}$. If one neglects (1) direct rf input to the bulk-plasma electrons and tritons, and (2) the fact that many deuterons are not resonantly accelerated, then the maximum ideal power gain is about 0.85 Q_m in the first case and 1.05 Q_m in the second case, where Q_m is the maximum fusion gain in the beam-injection scheme (e.g., $Q_m = 1.9$ at $T_e = 10$ keV). Because of nonideal effects, the cyclotron runaway phenomenon may find its most practical use in the heating of 50:50 D-T plasmas to ignition.

*Paper E10, presented at the Second Topical Conference on RF Plasma Heating, Lubbock, Texas, 20-22 June 1974.

I. INTRODUCTION

In the two-energy-component reactor scheme,[1] fast deuterium neutrals are injected into a relatively cold tritium plasma. The resulting D ions slow down by Coulomb collisions with the bulk-plasma electrons and tritons, and while thermalizing they produce Q times their injected energy in fusion reactions. For $T_e > 4.4$ keV, $Q > 1$, and Q can be as large as 3.1 for a high-T_e target. The '$n\tau$' of the plasma need be only about 10^{13} sec cm^{-3}, the exact value depending on the ratio $\bar{\Gamma}$ = (suprathermal-ion energy density)/(plasma energy density).

In this paper we consider how a two-component torus (TCT) may be operated with the suprathermal ion component produced entirely in situ by harmonic ion-cyclotron resonant heating that makes use of the fast hydromagnetic wave (magnetosonic wave). In particular, we investigate two simple models that give limiting fusion gains without recourse to the detailed particle motion in the presence of rf fields. These ideal limiting gains are comparable to the gains in the beam-injection scheme, but are bound to be seriously reduced after including rf inputs to the bulk plasma. In any event, rf suprathermal-ion production might find application in non-Maxwellian reactors where both bulk-plasma and beam-plasma fusion reactions contribute to the energy output.[2]

II. EXPERIMENTAL BACKGROUND

Harmonic ion-cyclotron heating experiments have generally demonstrated the formation of long "tails" on the transverse ion-energy distribution.[3,4] In early ST experiments,[3] during heating by the fast hydromagnetic wave at the second harmonic, the tail formed in 1 or 2 msec and was usually Maxwellian. These observations have been explained[5] as the result of single-particle resonance heating of trapped ions; the calculations are summarized in Appendix A. In recent ST experiments at higher power levels (up to 600 kW),[6] the observed tails make up a larger fraction of the total ion population, but they are still essentially Maxwellian. These results suggest that if sufficient rf power is applied, the two-temperature distribution will be replaced with a one-temperature distribution, although the banana orbits of very high energy ions may not be confined.

The Coulomb drag on a fast ion of energy W is

$$\frac{dW}{dt} = c_1 \frac{n_e W \ln \Lambda_e}{T_e^{3/2}} + c_2 \frac{n_i \ln \Lambda_i}{W^{1/2}} \qquad (1)$$

where c_1 and c_2 are constants, $\ln \Lambda_e$ and $\ln \Lambda_i$ are the Coulomb logarithms, and $n_e \approx n_i$ are the bulk-plasma electron and ion densities. Equation (1) is plotted in Fig. 1.

Consider the fast wave at $\omega = 2\omega_{CD}$, where ω_{CD} is the cyclotron frequency of the deuterons. The power input to a deuteron is $dW/dt \propto W$, so that there is a minimum energy W_{min} (W_3 in Fig. 1) at which deuterons will "run away." For cases of interest, W_{min} lies on the ion-drag (low-W) side of Fig. 1, and one can easily show that for those ions that remain resonant with the wave,

$$W_{min} \approx \left(\frac{\sqrt{2} \; c_2 \ln \Lambda_i \, n_e B Z_{eff}}{k_x E_x c} \right)^{2/3} \qquad (2)$$

where E_x is the left-hand component of the transverse electric field, and k_x is the transverse wave number. W_{min} is generally in the range 0.5-2.0 keV, so that for large hot tokamaks, a large fraction of the deuteron population in the resonant zone may "run away."

III. FAST HEATING AND RELAXATION

We first consider a process whereby deuterons are rapidly accelerated to high energies, and then allowed to thermalize with the bulk plasma. The resonant zone for the second harmonic has width $\delta R \simeq 2k_{\parallel} v_{ti} R/\omega$, where R is the major radius of the torus. This width can be made a large fraction of the plasma radius by increasing $k_{\parallel}$. For very large rf power, deuterons will be trapped as their transverse energy becomes many times their initial thermal energy, and a suprathermal distribution formed before there is significant drag, or before the trapped ions can escape from the resonant interaction by scattering from trapped orbits. This limiting distribution will be Maxwellian-like in two dimensions (Appendix A), that is, $f(W) \simeq 1/T_1 \exp(-W/T_1)$, and for very large rf power can be approximated as $f(W) \simeq$ constant, up to a maximum energy W_m which may be that energy, for example, at which the ion banana width becomes comparable to δR.

Once this rectangular distribution is formed, the rf is terminated, and the fast ions thermalize. The fusion energy gain of this cycle (22.4 meV per reaction) is

$$G_1 = \frac{\int_0^{W_m} F(W)\; W\, dW}{\int_0^{W_m} W\, dW} \qquad (3)$$

where $F(W,T_e)$ is the F-factor of Ref. 1. Here we have neglected direct rf input to the bulk plasma. The plasma temperature is maintained provided that $\bar{\Gamma} \gtrsim \langle\tau_s\rangle/\tau_E$, where $\langle\tau_s\rangle$ is the average slowing-down time of the D ions. This relation determines the required deuteron density n_D. As in the beam-injection scheme, n_D/n_e is in the range 0.03 to 0.15. Cyclic operation can be set up by applying rf pulses at intervals of $\langle\tau_s\rangle$.

Figure 2 shows $F(W)$ for $T_e = 10$ keV. If $W_m = W_a$, then $G_1 \simeq (2/3)F_m$, where F_m is the maximum value of $F(W)$. This gain can be improved by increasing W_m. Approximating $F(W)$ by the triangle shown in Fig. 2, one can easily show that G_1 is maximum when $W_m = (W_a^2 W_b)^{1/3} = 360$ keV. Then $G_1 = 0.84\ F_m$. This relation holds rather well throughout the range $T_e = 4\text{-}20$ keV.

IV. ENERGY CLAMPING

Next we consider a process whereby a rectangular energy distribution, $f(W) =$ constant for $W_1 < W < W_2$ (Fig. 1), is maintained by the fast wave at $\omega = \omega_{CD}$.

First harmonic heating applies the same power input to all ions, regardless of their velocity, and this power input can thus be represented as the horizontal line in Fig. 1. If W_1 is not too small, the drag is nearly constant for $W_1 < W < W_2$, so that $f(W)$ will remain fairly constant in this range. This process is a form of energy clamping.[2,7] Actually, the rf essentially scatters the ion energies between W_1 and W_2. Ions tend to run to W_2, but cannot do so, since they do not experience a continuous power input. Ions at $W < W_1$ receive the same power input P_{rf}, but for these ions Coulomb drag is dominant. If τ_h is the lifetime of the fast ions, the maximum energy gain is attained when $\tau_h >> \langle\tau_s\rangle$, or alternatively, if the fast-ion energy can be recovered by magnetic decompression.[7] Thus

$$G_{2max} = \frac{\int_{W_1}^{W_2} P_F(W)\, dW}{\int_{W_1}^{W_2} |dW/dt|\, dW + W_1 P_{rf}} \qquad (4)$$

where P_F is the fusion power output, and P_{rf} is the average rf power input per ion. Note that once W_1 is chosen, the corresponding W_2 must be set by the initial second harmonic heating; see Sec. III.

If W_1 is too small (Fig. 1), most ions will have a strong tendency to run toward W_2, while for maximum gain with $f(W) = \delta(W - W_o)$, W_o should be near the trough of the dW/dt curve.[2,7] Furthermore, the low-energy ions have small fusion reactivities. On the other hand, if W_1 is too large, only a small fraction of the rf-heated ions have large fusion reactivities. Figure 3 shows G_{2max} as a function of W_1. The optimum W_1 is 40 keV, with $G_{2max}/F_M = 1.07$.

V. REDUCTION OF THE IDEAL GAINS

In this section we briefly consider unavoidable phenomena that reduce the ideal gains.

1. In practice the distribution $f(W) \approx$ constant will not include all deuterons in the wave interaction region. Thus, the gain of the cyclic scheme becomes

$$G_1 \frac{n_T}{n_e} = \frac{G_1 n_T}{n_T + n_{Db} + n_{Dp}} \tag{5}$$

where n_{Db} and n_{Dp} are the deuteron densities in the suprathermal distribution and bulk-plasma, respectively. When $n_{Db}/n_e \approx 0.05$, G_1 is not greatly reduced even if only ~1/3 of the deuterons are accelerated. On the other hand, because of the rf input to the cold ions when $\omega = \omega_{CD}$, the gain of the clamped scheme becomes $G_2(n_{Db}/n_D)(n_T/n_e)$, which may be a small fraction of G_2. Here we have ignored the contribution of bulk-plasma D-T reactions, which are important only if the plasma has a very long confinement time.[2]

2. RF energy is supplied directly to plasma electrons. The gains of two-component reactors are derived assuming that all energy to the bulk plasma is supplied by the suprathermal ions.[1,2] Thus, both G_1 and G_2 are reduced by the factor $P_D/(P_e + P_D)$, where P_e and P_D are the rf powers going to electrons and deuterons, respectively. For a one-ion-specie plasma, the magnetosonic wave tends to be damped equally by ions and electrons, but Adam[8] has shown that in two-ion-species plasmas of the type relevant here, with $n_D/n_T \sim 0.1$, the D will be preferentially heated if $\omega = \omega_{CD}$, and P_e/P_D should be as small as 10^{-2}. However, this ratio may be somewhat larger for $\omega = 2\omega_{CD}$.

3. RF energy is supplied directly to tritons, since the tail of the triton distribution tends to be heated resonantly by the fast wave when $\omega = 2\omega_{CD} = 3\omega_{CT}$. This effect reduces G_1 and G_2 by $P_D/(P_T + P_D)$, but may be useful in a reactor with composition 50% D, 50% T.

The above effects may well reduce the gain of an rf-TCT by a factor of 2, thus making energy break-even impractical.

VI. MISCELLANEOUS PROBLEMS

There are other unfavorable features of the rf-TCT that are not found in the beam-injection scheme. These problems will be only briefly mentioned.

1. The deuteron energy distribution is described by $T_{\perp} \gg T_{\parallel}$, unless scattering is very rapid. Scattering of high-W ions should be negligible, however. This distribution is susceptible to a high-frequency shear Alfvén wave instability.[9]

2. Since practically all the rf power goes into transverse ion motion, orbit confinement problems are more severe than in the beam-injection case with tangential injection. Furthermore, the proportion of trapped ions may increase substantially, thus aggravating trapped-ion instabilities.

3. The use of rf coils inside the vacuum vessel may be prohibitive in a neutron environment, may be damaging to plasma purity, and poses serious cooling and insulation problems.

ACKNOWLEDGMENTS

This work was supported by U.S. Atomic Energy Commission Contract AT(11-1)-3073. The author acknowledges helpful conversations with Drs. W. M. Hooke, F. W. Perkins, T. H. Stix, and F. Troyon.

REFERENCES

[1]J. M. Dawson, H. P. Furth, and F. H. Tenney, Phys. Rev. Lett. 26, 1156 (1971); F. Powell, California Research and Development Company Report LWS-24920 (1953) unpublished.

[2]H. P. Furth and D. L. Jassby, Phys. Rev. Lett. 32, 1176 (1974).

[3]W. M. Hooke and J. C. Hosea, Proceedings of the Fifth European Conference on Controlled Fusion and Plasma Physics (Grenoble, France, 1972) I, 107.

[4]D. G. Swanson, et al., Phys. Rev. Lett. 28, 1015 (1972).

[5]W. M. Hooke and D. L. Jassby, Princeton Plasma Physics Laboratory Report TM-270 (1973).

[6]W. M. Hooke and J. C. Hosea, private communication.

[7]D. L. Jassby and H. P. Furth, Princeton Plasma Physics Laboratory Report MATT-1048 (1974).

[8]J. Adam, Symposium on Plasma Heating and Injection (C.N.R. Lab di Fisica del Plasma, Milano, Italy, 1973), 83.

[9]H. L. Berk, private communication.

APPENDIX A. SINGLE-PARTICLE HARMONIC RESONANT HEATING

The equation of motion of an ion in the field of the fast wave at $\omega = 2\omega_C$ is

$$\ddot{x} + \omega_C^2 x \approx \frac{1}{2}\frac{q}{m} k_x E_x a_L \exp\left[i(\omega_C - k_\parallel v_\parallel)t - i\phi\right] \tag{A1}$$

where E_x refers to the left-hand component, a_L is the ion gyroradius, and $k_x a_L << 1$. Considering only ions trapped in the wave field, the solution of Eq. (A1) is

$$\frac{dW}{dt} = \alpha W t \tag{A2}$$

where W is the ion energy and $\alpha = (ck_x E_x/8B)^2$. Equation (A2) preserves the nature of energy distributions in the absence of collisions or escape from the resonance zone. If the initial $f(W)$ is Maxwellian at temperature T_o, then the solution of the equation of continuity in energy space is

$$f(W,t) = [\exp(-0.5\alpha t^2)]/T_o\{\exp[(-W/T_o)\exp(-0.5\alpha t^2)]\} . \tag{A3}$$

Details are given in Ref. 5.

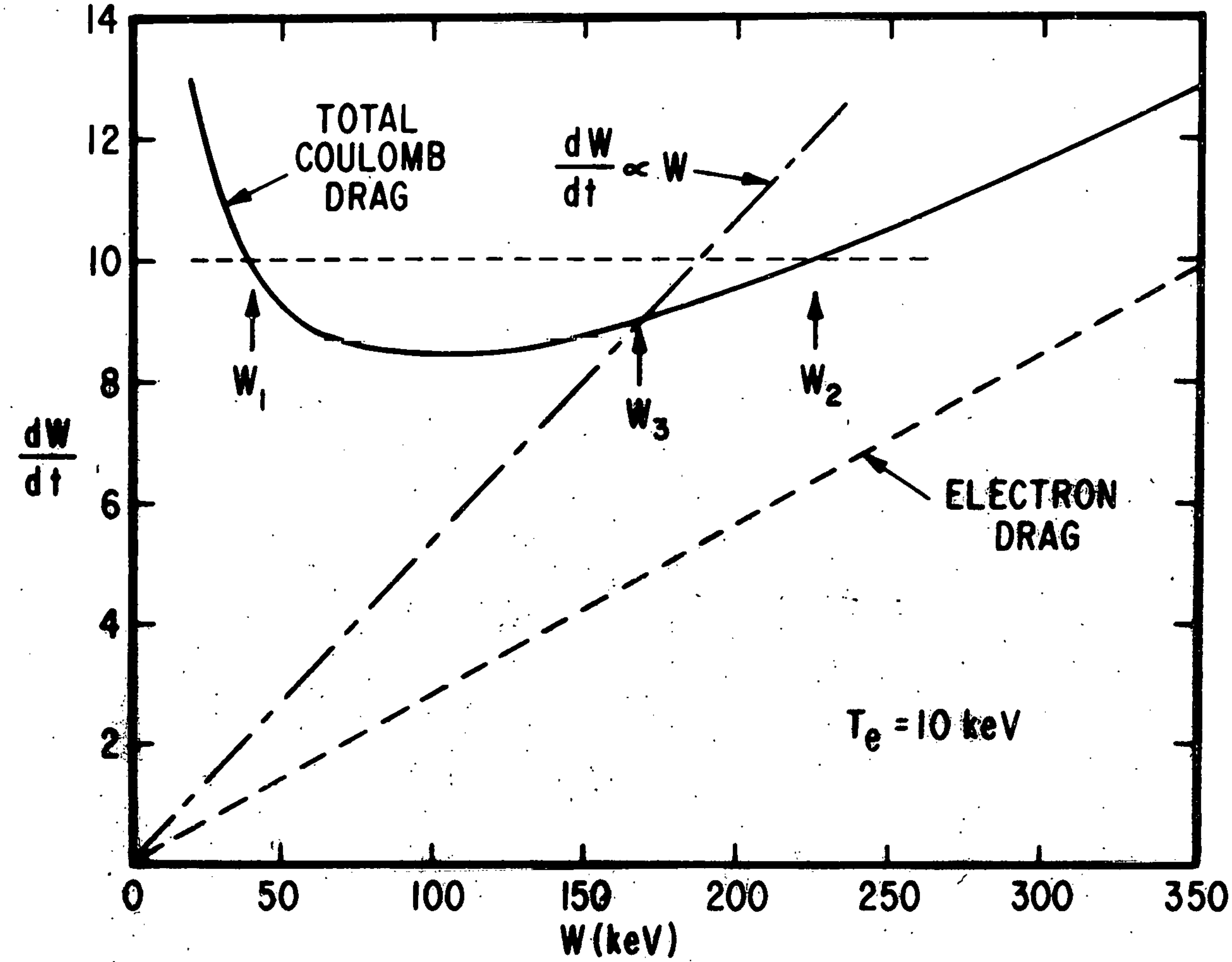

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Fig. 1. Coulomb drag on fast deuterons vs deuteron energy, at T_e = 10 keV. The ordinate units are arbitrary.

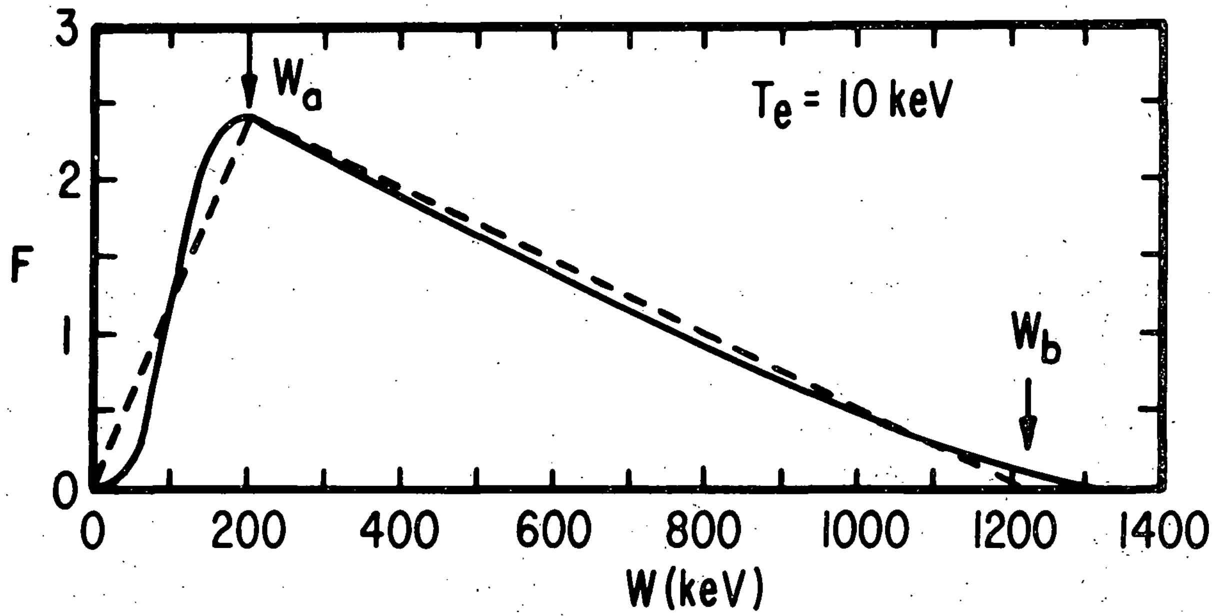

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Fig. 2. Energy multiplication factor F from Ref. 1, for D on T. W is the deuteron energy. $n_e = 10^{14}$ cm^{-3}.

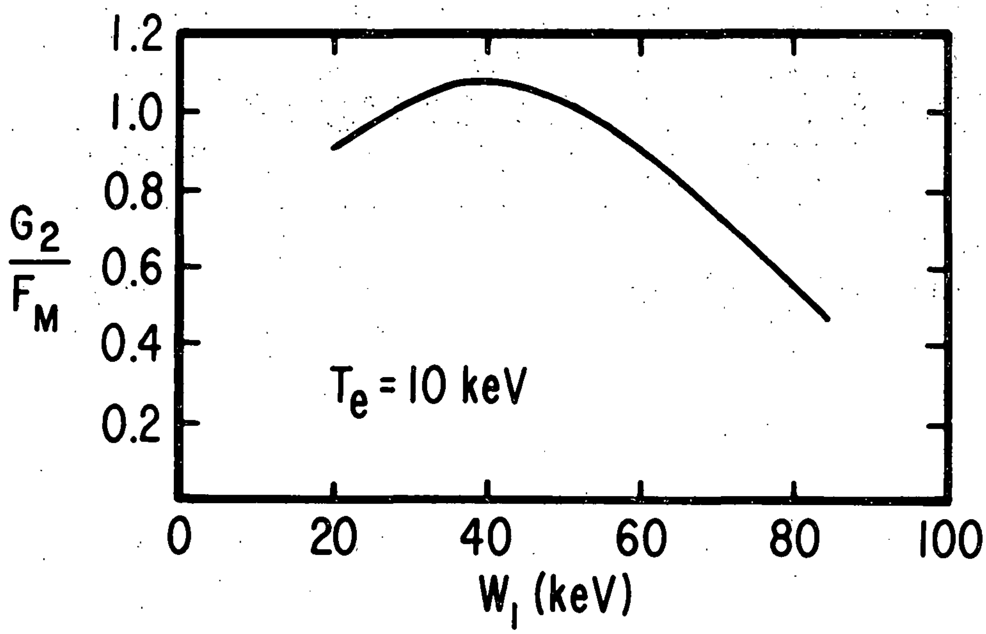

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Fig. 3. Gain of the rf-TCT with a uniform fast deuteron population clamped in the energy range $W_1 < W < W_2$, where W_1 and W_2 have the same Coulomb drag (see Fig. 1). Target plasma is 100% T. F_M is the maximum F-factor at T_e = 10 keV.