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October 20, 2005

Plasma Physics and Controlled Fusion

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Similarity laws for collisionless interaction of superstrong electromagnetic fields with a plasma

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Abstract

Several similarity laws for the collisionless interaction of ultra-intense electromagnetic fields with a plasma of an arbitrary initial shape is presented. Both ultra-relativistic and non-relativistic cases are covered. The ion motion is included. A brief discussion of possible ways of experimental verification of scaling laws is presented. The results can be of interest for experiments and numerical simulations in the areas of particle acceleration, harmonic generation, and Coulomb explosion of clusters.

An advent of ultra-intense lasers based on the chirped-pulse amplification [1] has opened up a possibility of reaching a progress in diverse areas of the experimental physics, including generation of high-energy beams of electrons and ions, creation of extremely strong magnetic fields, generating X-ray harmonics, applying the ultra-intense pulse in a fast ignitor concept, and others. There exists a broad literature on these applications. An interested reader can find a lot of information (as well as further references) in invited papers from the recent EPS Plasma Physics Conference [2-4]. Ultra-intense lasers may have also interesting applications for simulating astrophysical phenomena (see, e.g., a survey [5]).

Interaction of the super-intense light with the matter is a complex problem which does not easily lend itself to an analytical assessment. The approach used to analyze and to guide the experiments in this area is in most cases based on numerical simulations. Despite an increasing computer power and many successful numerical studies of the phenomena involved (see, e.g., a nice summary by Pukhov et al [6]), the numerical approach still has its limitations. Therefore, it might be helpful to supplement the experimental, numerical, and analytical studies by dimensional analyses that are often successfully used in plasma physics and magnetohydrodynamics (e.g., [7-10]). First steps in this direction have been made in a paper by Pukhov et al [6], where a similarity law for the case of strongly relativistic drive and resting ions has been established. [By “relativistic drive” we mean the situation where the electron quiver energy in the incident wave exceeds substantially their rest energy.] We will use an acronym PUGOKIK, derived from the authors’ names on this paper, to designate this similarity.

In our paper we present a more general similarity, not based on the assumption that the drive is ultra-relativistic. After deriving and discussing it, we establish its link to the PUGOKIK similarity. We then make another generalization, to include the ion motion; this similarity can be of a particular interest for the studies of the ion acceleration.

In some cases, in particular in the studies of the Coulomb explosion of clusters (e.g., [11, 12]), the drive can be non-relativistic. We address the limiting case of non-relativistic similarities as well.

We start from the situation of the resting ions. The kinetic equation for collisionless electrons can be presented as:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} - e \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \cdot \frac{\partial f}{\partial \mathbf{p}} = 0; \quad \mathbf{v} = \frac{c\mathbf{p}}{\sqrt{p^2 + m^2 c^2}} \quad (1)$$

We do not make here any assumptions regarding the electron energy. This equation should be solved together with Maxwell equations,

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}; \quad \nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{j}, \quad (2)$$

with the current density being:

$$\mathbf{j} = -e \int \mathbf{v} f d^3 \mathbf{p}. \quad (3)$$

Throughout this paper we use CGS (Gaussian) system of units, with c being the speed of light, m being the electron mass, and e being the elementary charge.

We assume that the initial electrons, before arrival of the main laser pulse, have very small velocities compared to velocities that they acquire early in the pulse. The initial state of the electron gas can then be uniquely characterized by their initial density distribution $n(\mathbf{r})$; the ion charge density in this initial state is assumed to be equal to the electron charge density. We characterize the system with some scale-length L (Fig. 1a); the pulse duration is characterized by its width τ (Fig. 1b). When talking about the length-scale, we mean that it is applied both to the initial size of the plasma object and to the width of the incident beam. In other words, if, in our scaling exercise, we increase the scale-length of the plasma by a factor of 2, the diameter of the incident beam has also to be increased by a factor of 2. This, of course, does not preclude us from studying interaction of a beam with a planar target, whose tangential dimension is much greater than the beam size.

For the most part of this paper we assume that the pulse comprises many wave periods $2\pi/\omega$ (as shown in Fig. 1b); we discuss a similarity for a very short pulses in the paragraph that follows Eq. (23). We designate the maximum value of the electric field during the pulse as E_0 , with the maximum intensity being $I = (c/4\pi)E_0^2$.

For the initially cold, collisionless electrons the flow soon becomes a multi-stream flow, with a very large number of the interpenetrating streams. This is why the description in terms of the kinetic equation is convenient (and efficient).

We now switch to dimensionless variables in space and time:

$$\tilde{\mathbf{r}} = \frac{\mathbf{r}}{L}; \quad \tilde{t} = \frac{t}{\tau}. \quad (4)$$

The initial electron density distribution can be presented as:

$$n = n_0 \hat{n}(\tilde{\mathbf{r}}), \quad (5)$$

where \hat{n} is some dimensionless function of its argument. We introduce the characteristic electron momentum p_0 via the equation

$$p_0 \equiv \frac{eE_0}{\omega}. \quad (6)$$

Note that, depending on the intensity of the wave, p_0 can be non-relativistic or relativistic. We haven't made any assumptions in this regard yet. We normalize the electron momentum (an independent variable in the kinetic equation (1)) to p_0 :

$$\tilde{\mathbf{p}} = \frac{\mathbf{P}}{P_0}. \quad (7)$$

The distribution function can be universally presented in the form:

$$f = \frac{n_0}{P_0^3} \hat{f}(\tilde{\mathbf{p}}, \tilde{\mathbf{r}}, \tilde{t}), \quad (8)$$

where \hat{f} is some dimensionless function of its arguments. The density n and the current density \mathbf{j} can be expressed via the dimensionless distribution function \hat{f} as follows:

$$n = n_0 \int \hat{f}(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}, \tilde{t}) d^3 \tilde{\mathbf{p}}; \quad \mathbf{j} = -en_0 c \int \frac{P\tilde{\mathbf{p}}}{\sqrt{\tilde{p}^2 P^2 + 1}} \hat{f}(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}, \tilde{t}) d^3 \tilde{\mathbf{p}}, \quad (9)$$

where P is the following dimensionless parameter:

$$P \equiv \frac{eE_0}{mc\omega} \quad (10)$$

The parameter P is a measure of the field intensity: it is less than 1 for non-relativistic drive and greater than 1 for the relativistic drive. The electric and magnetic field will be normalized to E_0 :

$$\mathbf{E} = E_0 \hat{\mathbf{E}}(\tilde{\mathbf{r}}, \tilde{t}); \quad \mathbf{B} = E_0 \hat{\mathbf{B}}(\tilde{\mathbf{r}}, \tilde{t}) \quad (11)$$

Inserting the dimensionless variables and dimensionless functions into the set (1)-(2) one can write this set in the dimensionless form:

$$\frac{1}{T} \frac{\partial \hat{f}}{\partial \tilde{t}} + \frac{1}{R} \frac{P\tilde{\mathbf{p}}}{\sqrt{\tilde{p}^2 P^2 + 1}} \cdot \frac{\partial \hat{f}}{\partial \tilde{\mathbf{r}}} - \left(\hat{\mathbf{E}} + \frac{P}{\sqrt{\tilde{p}^2 P^2 + 1}} \tilde{\mathbf{p}} \times \hat{\mathbf{B}} \right) \cdot \frac{\partial \hat{f}}{\partial \tilde{\mathbf{p}}} = 0 \quad (12)$$

$$\tilde{\nabla} \times \hat{\mathbf{E}} = -\frac{R}{T} \frac{\partial \hat{\mathbf{B}}}{\partial \tilde{t}}; \quad \tilde{\nabla} \times \hat{\mathbf{B}} = \frac{R}{T} \frac{\partial \hat{\mathbf{E}}}{\partial \tilde{t}} - RQ \int \frac{\tilde{\mathbf{p}}}{\sqrt{\tilde{p}^2 P^2 + 1}} \hat{f}(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}, \tilde{t}) d^3 \tilde{\mathbf{p}}, \quad (13)$$

where T , R , and Q are dimensionless parameters,

$$T \equiv \omega\tau, \quad R \equiv \omega L/c; \quad Q \equiv \frac{4\pi n_0 e^2}{m\omega^2} \quad (14)$$

The dimensionless equations characterizing two systems will be identical if the four dimensionless parameters, T , R , P , and Q are the same between the two systems. The evolution of the two systems will occur in the identical way (up to the scaling transformation of the functions and variables) if, additionally, the initial density distributions are geometrically similar, and the temporal dependence of the envelopes is similar. The spatial dependence of the incident radiation has also to be geometrically similar between the two systems, with the scaling factor L .

One can easily check that the four dimensionless parameters are independent (i.e., one can vary each of them while keeping the rest of them constant), i.e., the number of the constraints on the parameters of two systems is four. On the other hand, the number of dimensional characteristics of the system is five: ω , τ , L , E_0 and n_0 . This means that, in order to obtain a system which will evolve similarly to the initial one, one can arbitrarily choose one of these parameters and vary the other four in such a way as to keep the four dimensionless parameters, Eqs. (10), (14), constant. We will call the new (similar) system as ‘‘primed’’ system and denote its parameters by the prime.

Assume, for example, that we have changed the intensity of the light in the primed system compared to the initial one. The electric field scales as a square root of the intensity, so that the electric field in the primed system will be

$$E'_0 = E_0 \sqrt{\frac{I'}{I}}. \quad (15)$$

By inspecting the expressions for the dimensionless parameters T , R , P , and Q , one finds that, in order to keep them constant, we have to change the other dimensional characteristics of the primed system in the following way:

$$\omega' = \omega \sqrt{\frac{I'}{I}}; \quad \tau' = \sqrt{\frac{I}{I'}}; \quad L' = L \sqrt{\frac{I}{I'}}; \quad n'_0 = n_0 \frac{I'}{I}. \quad (16)$$

One can easily predict the change of the other parameters in the primed system. The quasi-static magnetic field $\langle B \rangle$ will scale as E_0 , i.e.,

$$\langle B' \rangle = \langle B \rangle \sqrt{\frac{I'}{I}}. \quad (17)$$

The characteristic momentum of the electrons p_0 will remain unchanged,

$$p'_0 = p_0 \quad (18)$$

as well as the shape of the electron distribution function as described by \hat{f} . The reflectivity of the incident light will remain unchanged, as well as a relative amplitude of the higher order harmonics (with respect to the amplitude of the incident wave).

This is a powerful similarity in that it works for an arbitrary drive intensity (including the transitional case of $p_0/mc \sim 1$), for the arbitrary shape of the initial plasma, an arbitrary initial density (both sub-critical and super-critical), and arbitrary polarization of the incident light. If used in a systematic way, in a set of dedicated experiments, it may reveal various limitations on the basic underlying assumptions, e.g., on the negligible role of collisionality.

By making additional assumptions, one can reduce the number of constraints and make the similarity less restrictive. As an example, we can re-derive the aforementioned PUGOKIK similarity [6], which was suggested for the case of an ultra-relativistic drive. In this case, one has $P \gg 1$, and the parameter P drops out of Eq. (12) which becomes:

$$\frac{1}{T} \frac{\partial \hat{f}}{\partial \tilde{t}} + \frac{1}{R} \frac{\tilde{\mathbf{p}}}{\tilde{p}} \cdot \frac{\partial \hat{f}}{\partial \tilde{\mathbf{r}}} - \left(\hat{\mathbf{E}} + \frac{\tilde{\mathbf{p}} \times \hat{\mathbf{B}}}{\tilde{p}} \right) \cdot \frac{\partial \hat{f}}{\partial \tilde{\mathbf{p}}} = 0. \quad (19)$$

In the second of Eq. (13), the parameters Q and P enter the equation only in the combination $Q/P = S$,

$$\tilde{\nabla} \times \hat{\mathbf{E}} = -\frac{R}{T} \frac{\partial \hat{\mathbf{B}}}{\partial \tilde{t}}; \quad \tilde{\nabla} \times \hat{\mathbf{B}} = \frac{R}{T} \frac{\partial \hat{\mathbf{E}}}{\partial \tilde{t}} - RS \int \frac{\tilde{\mathbf{p}}}{\tilde{p}} \hat{f}(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}, \tilde{t}) d^3 \tilde{\mathbf{p}}; \quad (20)$$

$$S \equiv \frac{Q}{P} = \frac{4\pi n_0 e c}{E_0 \omega}; \quad (21)$$

S is the ratio of the plasma density to the relativistically-corrected critical density; this parameter coincides with the one introduced in Ref. [6].

We see that now the number of the independent dimensionless parameters is reduced from 4 to 3: T , R , and S . This means that, in the case of the PUGOKIK similarity, one can arbitrarily choose two of the parameters of the primed system, not one as before. Consider for example that we have chosen some new values of intensity (I') and frequency (ω') in the primed system. The remaining three parameters have to be chosen as:

$$\tau' = \tau \frac{\omega}{\omega'}; \quad L' = L \frac{\omega}{\omega'}; \quad n'_0 = n_0 \frac{\omega'}{\omega} \sqrt{\frac{I'}{I}} \quad (22)$$

The characteristic electron momentum will scale as

$$p'_0 = p_0 \frac{\omega}{\omega'} \sqrt{\frac{I'}{I}} \quad (23)$$

whereas the characteristic magnetic field will scale according to Eq. (17). The reflectivity will remain unchanged. Again, testing these predictions in the set of dedicated experiments would allow one to assess the limits of the underlying model (e.g., that the sub-relativistic electrons do not affect the outcome of an experiment or of a computer run).

For the case where the duration of the electromagnetic pulse τ becomes comparable to the wave period, the pulse can be adequately characterized just by a single parameter of the dimension of time, τ . There is no need to introduce separately the frequency ω . The definition of p_0 can now be changed to $p_0 = eE_0\tau$, and the definition of R to $R = L/c\tau$. The parameter S is changed to $S = 4\pi e c \tau n_0 / E_0$. There are now four parameters characterizing the system, L , τ , E_0 , and n_0 . It is easy to see that in this case the similarity holds if the following two constraints are satisfied: $R=\text{const}$ and $S=\text{const}$.

Another limiting case corresponds to a non-relativistic drive. Such situation may be of some interest for the studies of the Coulomb explosions of clusters (see Ref. [12] for the theory of this process in the non-relativistic limit). At the early stage of the pulse, one can consider the ions as being at rest (we will lift this constraint shortly). As argued in Ref. [12], the interaction of an electromagnetic wave with a cluster with a size much less than the wavelength, can be adequately described in the electrostatic approximation. Therefore, we write that $\mathbf{E} = -\nabla\varphi$, and $\nabla \cdot \mathbf{E} = 4\pi(n_i - n_e)$. In the dimensionless variables, the latter equation becomes (Cf. the second of Eqs. (20)):

$$\tilde{\nabla} \cdot \hat{\mathbf{E}} = S^* \left[\hat{n}(\tilde{\mathbf{r}}) - \int \hat{f}(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}, \tilde{t}) d^3 \tilde{\mathbf{p}} \right] \quad (24)$$

where

$$S^* = \frac{4\pi e n_0 L}{E_0} \quad (25)$$

The kinetic equation for the dimensionless function $\hat{f}(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}, \tilde{t})$ in the limit $P \ll 1$ is:

$$\frac{1}{T} \frac{\partial \hat{f}}{\partial \tilde{t}} + \frac{1}{R^*} \tilde{\mathbf{p}} \cdot \frac{\partial \hat{f}}{\partial \tilde{\mathbf{r}}} - \hat{\mathbf{E}} \cdot \frac{\partial \hat{f}}{\partial \tilde{\mathbf{p}}} = 0 \quad (26)$$

where the dimensionless constant R^* is defined as:

$$R^* = \frac{L m \omega^2}{e E_0} \quad (27)$$

It has a meaning of the ratio of the size of the cluster to the amplitude of the electron oscillations.

We see that, in the problem under consideration, there are three independent dimensionless parameters, T , R^* , and S^* . Their constancy imposes three constraints on five dimensional parameters: ω , τ , L , E_0 and n_0 .

If one deals with the large-enough clusters of the same material, the density n_0 has to be held constant. Then, from the inspection of the conditions that $S^*=\text{const}$ and $R^*=\text{const}$, one finds that ω must be held constant. This, in turn, by virtue of the $T=\text{const}$

constraint, means that, in order to have the primed system scalable to the unprimed one, the pulse duration must be held the same. If we vary the intensity, then the size of the clusters has to vary as

$$L' = L\sqrt{\frac{I'}{I}} \quad (28)$$

The characteristic momentum of the electrons (non-relativistic) will vary as

$$p'_0 = p_0\sqrt{\frac{I'}{I}}, \quad (29)$$

whereas the dimensionless distribution function will remain unchanged.

If one allows for the changes of the composition of the clusters, the density n_0 becomes a free parameter, and the similarity becomes broader. But even in the form mentioned above, it can serve for experimental verification of theory and for code benchmarking.

Let us now include into our analysis the possibility of the ion motion. We assume that it is non-relativistic and collisionless. We consider the ion motion in two systems: that of ultra-relativistic electrons (an extension of the PUGOKIK similarity) and that of non-relativistic electrostatic system.

The non-relativistic ion kinetic equation reads as:

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \frac{\partial f_i}{\partial \mathbf{r}} + \frac{Ze}{M} \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \frac{\partial f_i}{\partial \mathbf{v}} = 0 \quad (30)$$

where Z is the ion charge in the units of e , and M is the ion mass. In the case of the generalized PUGOKIK similarity, we normalize the ion velocity to that of the ion having the energy cp_0 :

$$\tilde{\mathbf{v}} = \mathbf{v} \sqrt{\frac{M}{Zcp_0}} \equiv \mathbf{v} \sqrt{\frac{M\omega}{ZceE_0}} \quad (31)$$

We skip here the factor “2” under the square root. The ion distribution function can be universally presented as

$$f_i = \frac{n_0}{Z} \left(\frac{M}{Zcp_0} \right)^{3/2} \hat{f}_i(\tilde{\mathbf{v}}, \tilde{\mathbf{r}}, \tilde{t}), \quad (32)$$

with $\tilde{\mathbf{r}}$ and \tilde{t} defined according to (4). The dimensionless ion distribution function satisfies the equation:

$$\frac{1}{T} \frac{\partial \hat{f}_i}{\partial \tilde{t}} + \frac{U}{R} \tilde{\mathbf{v}} \cdot \frac{\partial \hat{f}_i}{\partial \tilde{\mathbf{r}}} + U \left(\hat{\mathbf{E}} + U \hat{\mathbf{v}} \times \hat{\mathbf{B}} \right) \frac{\partial \hat{f}_i}{\partial \tilde{\mathbf{v}}} = 0 \quad (33)$$

where U is a new dimensionless parameter,

$$U = \sqrt{\frac{ZeE_0}{M\omega c}} \quad (34)$$

The ion current density is

$$\mathbf{j}_i = en_0 c U \int \tilde{\mathbf{v}} \hat{f}_i(\tilde{\mathbf{v}}, \tilde{\mathbf{r}}, \tilde{t}) d^3 \tilde{\mathbf{v}} \quad (35)$$

so that the second of Eqs. (20) acquires the form:

$$\tilde{\mathbf{v}} \times \hat{\mathbf{B}} = \frac{R}{T} \frac{\partial \hat{\mathbf{E}}}{\partial \tilde{t}} - RS \int \frac{\tilde{\mathbf{p}}}{\tilde{p}} \hat{f}(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}, \tilde{t}) d^3 \tilde{\mathbf{p}} \tilde{p} \mathbf{j}_i + RSU \int \tilde{\mathbf{v}} \hat{f}_i(\tilde{\mathbf{v}}, \tilde{\mathbf{r}}, \tilde{t}) d^3 \tilde{\mathbf{v}} \quad (36)$$

(the first of the equations (20) does not change).

From inspection of the set of dimensionless equations (33) and (36), one sees that now, in order to make this set identical for the two systems, one has to satisfy four dimensionless constraints:

$$T = const, R = const, S = const, U = const. \quad (37)$$

The number of “input” parameters characterizing the system is now 7: ω , τ , L , E_0 , n_0 , Z , and M . In other words, there still remains a substantial flexibility for the scaling. In particular, if one is interested in switching from acceleration of hydrogen (initial system) to acceleration of deuterium (the “primed” system), the mass M increases by a factor of 2, so that, according to condition $U=const$, one has to satisfy the relation:

$$\frac{I'}{\omega'^2} = 4 \frac{I}{\omega^2} \quad (38)$$

The other relations remain the same as for the PUGOKIK similarity, i.e., Eqs. (16) and (17) remain valid. The characteristic ion energy remains unchanged between the two systems.

For the case of the electrostatic model with non-relativistic electrons, we normalize the ion velocity as follows:

$$\tilde{\mathbf{v}} = \frac{\mathbf{v}}{v_0}; \quad v_0 \equiv \sqrt{\frac{Zm}{M} \frac{eE_0}{m\omega}} \quad (39)$$

Then, the dimensionless kinetic equation for the ions acquires the form:

$$\frac{1}{T} \frac{\partial \hat{f}_i}{\partial \tilde{t}} + \frac{1}{R^*} \sqrt{\frac{Zm}{M}} \tilde{\mathbf{v}} \cdot \frac{\partial \hat{f}_i}{\partial \tilde{\mathbf{r}}} + \sqrt{\frac{Zm}{M}} \hat{\mathbf{E}} \frac{\partial \hat{f}_i}{\partial \tilde{\mathbf{v}}} = 0 \quad (40)$$

The dimensionless kinetic equation for the electrons remains Eq. (26). The Poisson equation (24) is replaced by:

$$\tilde{\nabla} \cdot \hat{\mathbf{E}} = S^* \left[\int \hat{f}_i(\tilde{\mathbf{r}}, \tilde{\mathbf{v}}, \tilde{t}) d^3 \tilde{\mathbf{v}} - \int \hat{f}(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}, \tilde{t}) d^3 \tilde{\mathbf{p}} \right] \quad (41)$$

From the set of equations (26), (40), and (41), one sees that, in order to have a scaled transformation between the two systems, one has to use the ions with the same Z/M . Other than that, the constraints on the dimensionless parameters are

$$T=const, R^*=const, S^*=const. \quad (42)$$

In other words, there are three constraints on five parameters, this allowing for a broad variation of parameters of the scaled system

In summary: we have identified several very broad similarities that hold in the collisionless plasmas irradiated by ultra-intense light. The main assumption, aside from the absence of collisions, is that the initial particle velocities are much smaller than the velocities that particle acquire at the very beginning of the main pulse. The similarities hold for an arbitrary shape of the initial plasma, for an arbitrary polarization of the incident light, and for an arbitrary density of the initial plasma. In some cases, one can arbitrarily choose two of the characteristic parameters of the “primed” system and vary the remaining three parameters according to the rules provided in this paper, to obtain a similar system. This provides a significant experimental flexibility. A consistent experimental test of the limits of the similarities identified in this paper would provide an important information regarding the underlying assumptions. In a number of cases, these similarities could be used as a predictive tool. They would also be useful for benchmarking codes and verifying prediction of the analytical theory.

This work was performed under the auspices of the U.S. Department of Energy by University of California Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.

References

1. D. Strickland and G. Mourou. *Opt. Commun.*, **56**, 219 (1985); M.D. Perry, G. Mourou. *Science*, **264**, 917 (1994); G.A. Mourou, C.P.J. Barty. M.D. Perry; *Phys. Today*, **51**, 22-28 (1998); M.D. Perry, et al. *Opt. Lett.*, **24**, 160-162 (1999).
2. P.A. Norreys, K.M. Krushelnick, and M. Zepf. *Plasma Phys. Contr. Fus.*, **46**, B13-21 (2004)
3. T. Katsouleas. *Plasma Phys. Contr. Fus.*, **46**, B575-582 (2004)
4. K.A. Tanaka, R. Kodama, Y. Kitagawa, et al. *Plasma Phys. Contr. Fus.*, **46**, B41-50 (2004)
5. B.A. Remington. *Plasma Physics & Controlled Fusion*, **47**, A191 (2005).
6. A. Pukhov, S. Gordienko, S. Kiselev, I. Kostyukov. *Plasma Phys. Contr. Fus.*, **46**, B179 (2004).
7. J. Lacina. *Similarity Rules in Plasma Physics. Plasma Physics*, **13**, 303 (1970).
8. B.B. Kadomtsev. *Tokamaks and dimensional analysis. Sov. J. Plasma Phys.*, **1**, 296 (1975).
9. J.W. Connor, J.B. Taylor. *Scaling laws for plasma confinement. Nucl. Fusion*, **17**, 1047 (1977).
10. Ryutov D D, Drake R P, Kane J O, Liang E, Remington B A, and Wood-Vasey W M. *Astrophysical Journal*, **518**, 821 (1999); Ryutov D D, Remington B A, Robey H F, Drake R P. *Phys. Plasmas*, **8**, 1804 (2001); D.D. Ryutov, B.A. Remington. *Physics of Plasmas*, **10**, 2629, 2003.
11. T. Ditmire, S. Bless, G. Dyer, et al. *Radiation Physics & Chemistry*, **70**, no.4-5, 535-52 (2004).
12. B.N. Breizman, A.V. Arefiev, M.V. Fomyts'kyi. *Physics of Plasmas*, **12**, .56706-1-9 (2005).

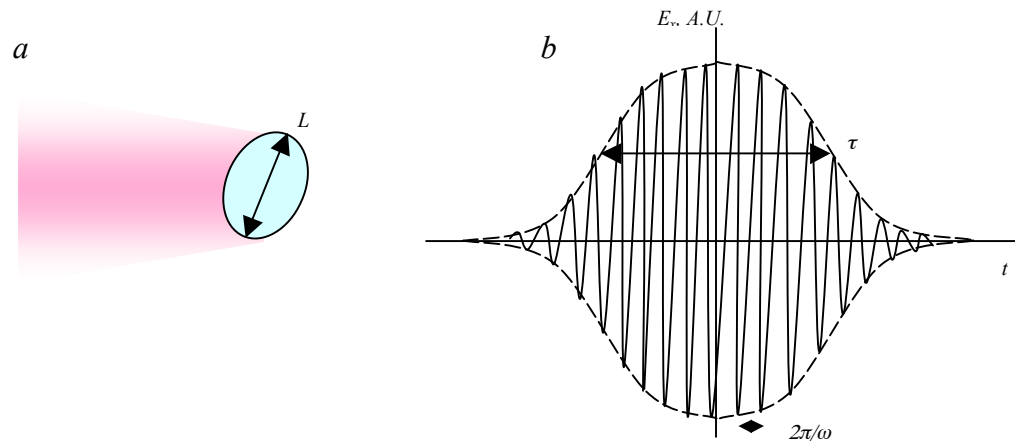


Fig. 1. *a* - Interaction of a laser beam with a non-spherically-symmetric plasma of a characteristic size L ; *b* - The shape of the incident laser pulse with a duration τ substantially exceeding the wave period $2\pi/\omega$; the shape of the envelope must be similar between the two systems.