

# FLANGE: A Computer Program for the Analysis of Flanged Joints with Ring-Type Gaskets

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Reactor Division

**FLANGE: A COMPUTER PROGRAM FOR THE ANALYSIS  
OF FLANGED JOINTS WITH RING-TYPE GASKETS**

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## FOREWORD

The work reported here was performed at Oak Ridge National Laboratory and at Battelle-Columbus Laboratories under Union Carbide Corp., Nuclear Division, Subcontract No. 291, as part of the ORNL Design Criteria for Piping and Nozzles Program, S. E. Moore, Manager. This program is funded by the Division of Reactor Safety Research (RSR) of the U.S. Nuclear Regulatory Commission as part of a cooperative effort with industry to develop and verify analytical methods for assessing the safety of pressure-vessel and piping-system design. The cognizant RSR project engineer is E. K. Lynn. The cooperative effort is coordinated through the Pressure Vessel Research Committee of the Welding Research Council under the Subcommittee on Piping, Pumps, and Valves.

The study described in this report was conducted under the general direction of W. L. Greenstreet and S. E. Moore, Solid Mechanics Department, Reactor Division, ORNL, and is a continuation of work supported in prior years by the Division of Reactor Research and Development, U.S. Energy Research and Development Administration (formerly the USAEC).

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## 1. INTRODUCTION

### Purpose and Scope

The *ASME Boiler and Pressure Vessel Code*<sup>1</sup> gives rules for designing bolted flange connections with ring-type gaskets based on a stress analysis developed by Waters et al.<sup>2</sup> These rules give formulas and graphs for calculating stresses due to a moment applied to the flange ring. The Code rules, however, do not require that stresses due to internal pressure be taken into account, although Ref. 2 briefly discusses such stresses.

The computer program FLANGE was written to calculate not only the stresses due to moment loads on the flange ring but also stresses due to internal pressure; stresses due to a temperature difference between the hub and ring; and stresses due to the variations in bolt load that result from pressure, hub-ring temperature gradient, and/or bolt-ring temperature difference. The program FLANGE is applicable to tapered-hub, straight, and blind flanges. The analysis method is based on the differential equations for thin plates and shells rather than on the strain-energy method used by Waters et al.<sup>2</sup> The stresses due to moment loading calculated by the two methods are essentially identical for identical boundary conditions. The analysis provided herein also includes a different, and perhaps more realistic, set of boundary conditions than those used in Ref. 2.

The nomenclature used in this report is identified in the remainder of this chapter. In Chapter 2 a description of the general model of flanges used in the theoretical development of the computer code is provided. The actual mathematical expressions for calculating stresses and displacements due to moment and pressure loads are derived in Chapters 3, 4, and 5 for tapered-hub, straight hub, and blind flanges, respectively. In Chapters 6 and 7, these expressions are extended to include the effects of thermal gradients and variations in bolt loads. The computer program FLANGE is described in the last chapter of this report. Example calculations, listings, and flowcharts of the program and its subroutines are included as appendices.

Nomenclature

- $a$  = outside radius of ring  
 $A = 2a$  = outside diameter of ring  
 $A_b$  = cross-sectional bolt area  
 $A_g$  = gasket area  
 $b$  = inside radius of ring and mean radius of pipe  
 $B = 2b$  = inside diameter of ring  
 $b_n$  = Bessel function of  $n$   
 $c$  = bolt-circle radius  
 $C = 2c$  = bolt-circle diameter  
 $C_i$  = constant of integration  
 $C_i' = C_i/b$   
 $D = Et^3/12(1 - \nu^2)$   
 $D_{ij}$  = constants of integration (blind-flange analysis)  
 $E = E_f$  = modulus of elasticity of flange material  
 $E_b$  = modulus of elasticity of bolt material  
 $E_g$  = modulus of elasticity of gasket material  
 $f$  = ASME Code design parameter  
 $F$  = ASME Code design parameter  
 $g_0$  = wall thickness of pipe  
 $g_1$  = wall thickness of hub at intersection with ring  
 $g$  = gasket centerline radius  
 $G = 2g$  = gasket centerline diameter  
 $h$  = length of tapered-wall hub  
 $k = a/b = A/B$   
 $l_0$  = bolt length  
 $M$  = total moment applied to ring, in.-lb  
 $M_i$  or  $M_{ij}$  = moment resultants, in.-lb/in.  
 $p$  = internal pressure  
 $P_i$  = shear resultants, lb/in.  
 $p^* = \frac{[1 - (\nu/2)]bp}{g_0E}$  = nondimensional pressure parameter  
 $r$  = radial coordinate, ring

- $t$  = ring thickness  
 $t_x$  = hub thickness  
 $u$  = radial displacement, hub  
 $u_i$  = radial displacement, pipe  
 $u_r$  = radial displacement, ring  
 $V$  = ASME Code design parameter  
 $v_0$  = undeformed gasket thickness  
 $w$  = axial displacement, ring  
 $W_1$  = initial bolt load, lb  
 $W_2$  = residual bolt load, lb  
 $x$  = axial coordinate, hub  
 $x_i$  = axial coordinate, pipe  
 $\alpha = (g_1 - g_0)/g_0 = \rho - 1$  = nondimensional wall-thickness parameter  
 $\beta = [3(1 - \nu^2)/b^2 g_0^2]^{1/4}$  = dimensional parameter used in the analysis  
 $\gamma = [12(1 - \nu^2)/b^2 g_0^2]^{1/4} (h)$  = dimensional parameter used in the analysis  
 $\Delta$  = temperature difference between hub/pipe and ring  
 $\delta_i$  = axial displacement of ring  
 $\epsilon_f$  = coefficient of thermal expansion, flange material  
 $\epsilon_b$  = coefficient of thermal expansion, bolt material  
 $\epsilon_g$  = coefficient of thermal expansion, gasket material  
 $\eta = 2\gamma(\psi/a)^{1/2}$  = nondimensional argument of the modified Bessel functions  
 $\nu$  = Poisson's ratio (0.3 used herein)  
 $\xi = x/h$  = nondimensional distance parameter  
 $\rho = g_1/g_0$  = nondimensional wall-thickness parameter  
 $\sigma$  = stress, with subscripts:  
      $l$  = longitudinal (pipe or hub)  
      $c$  = circumferential (pipe or hub)  
      $t$  = tangential (ring)  
      $r$  = radial (ring)  
      $b$  = bending  
      $m$  = membrane  
      $o$  = outside surface of the pipe or hub on the hub side of ring  
      $i$  = inside surface of the pipe or hub on the gasket-face side of ring  
 $\psi = \xi + (1/\alpha)$  = nondimensional parameter

## 2. GENERAL DESCRIPTION OF THE ANALYSIS

The model used for the analysis of tapered-hub flanges is shown in Fig. 1. The three parts involved are the pipe, hub, and ring, respectively. The analysis presented here is based on the theory of thin plates and shells. The pipe is considered to be a uniform-wall-thickness cylindrical shell with midsurface radius  $b$ . The hub is considered to be a linearly variable-wall-thickness cylindrical shell with midsurface radius  $i$ . The ring is considered to be a flat annular plate with constant thickness  $t$ , inside radius  $b$ , and outside radius  $a$ . The effects of the bolt holes are neglected.

Three different types of loadings on bolted flanges are considered:

1. Bolt load, represented by  $W$  in Fig. 1. In application, the moment  $M$  applied to the flange ring is converted into an equivalent bolt load by the relationship  $W(a - b) = M$ . This is the same approach used in the ASME Code calculation method.<sup>1</sup>

2. Internal pressure, acting radially on the pipe, hub, and ring and axially on an (assumed remote) end closure on the pipe.

3. A temperature difference between the pipe and the ring. The pipe and the hub are assumed to be at the same uniform temperature. The ring is also assumed to be at a uniform temperature, which may be different from that of the pipe or hub.

Upon integration of the shell and plate differential equations, algebraic equations in terms of dimensions, materials properties and loadings, and 12 integration constants are obtained, 4 for each part. These constants are evaluated by the usual discontinuity analysis method of writing continuity equations at the junctures of the parts and at the boundaries. After numerical values are determined for the constants, the algebraic equations provide the means for computing the stresses and deflections. In the development of the equations for stresses, the assumption is made that the bolt load  $W$  does not change with pressure or temperature. Later the analysis is modified to include changes in  $W$  as a function of these loadings. Because the relations are linear, it is possible to determine the stresses (or stress range) due to combinations

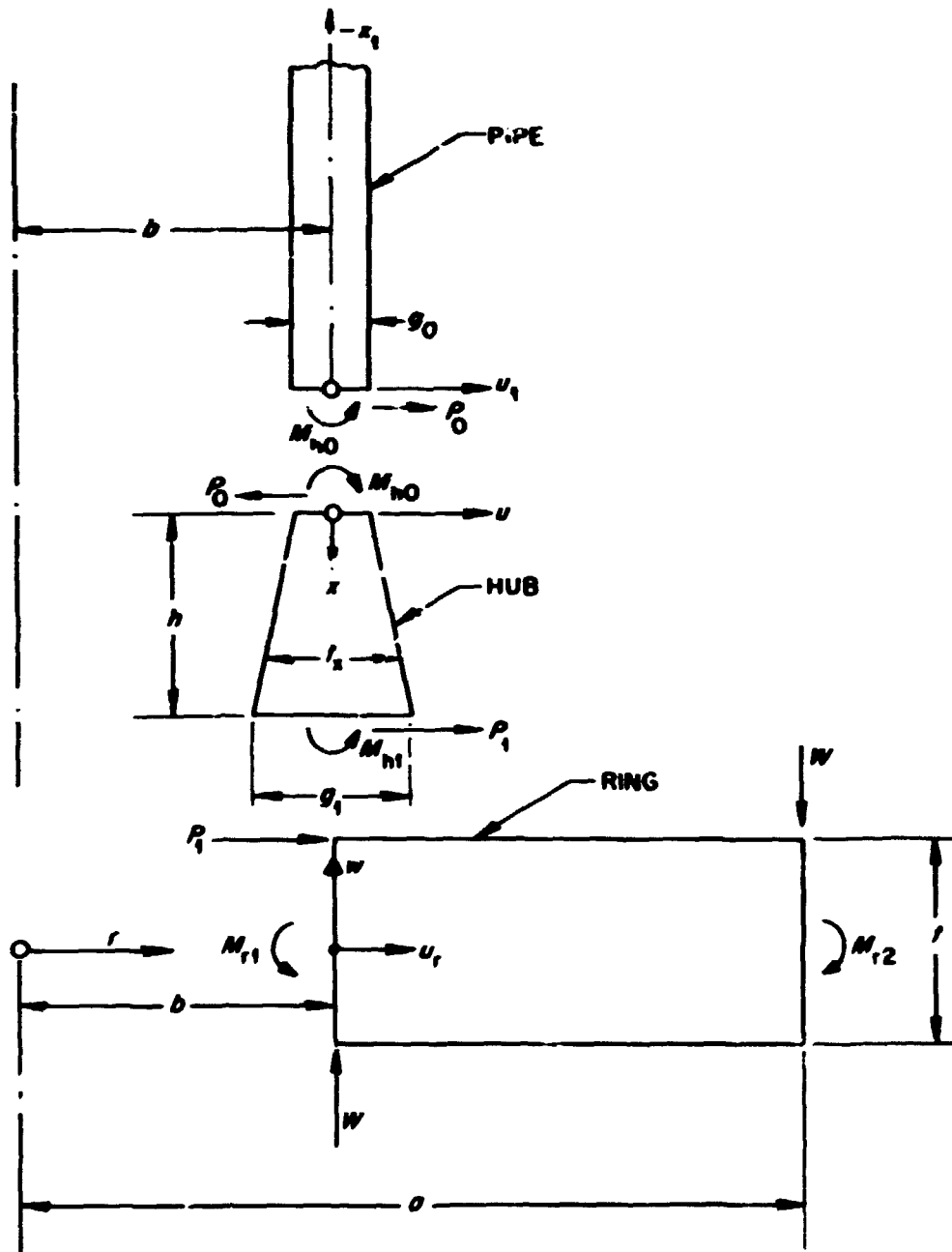


Fig. 1. Analysis model of a tapered-hub flange.



of initial bolt loading, pressure, and temperature change. The model used for straight-hub flanges is a simplification of the tapered-hub case in that only two parts are involved, the pipe and the ring.

In common with all shell-type analyses, the analysis gives anomalous results at points of abrupt thickness change or meridional direction change. In particular, the stresses at the juncture of the hub to the ring represent only the gross loading effect; detailed local stresses are not determined by the theory. Displacements, however, are represented fairly accurately.

### 3. FLANGE WITH A TAPERED-WALL HUB

The first step in deriving the stress equations is to state the basic shell/plate equations for the ring, the hub, and the pipe. We then inspect the boundary conditions, compute the constants, and calculate the stresses and displacements.

#### Equations for the Annular Ring

The basic differential equation for the displacement  $w$  of a circular plate given by Timoshenko<sup>3</sup> is

$$\frac{1}{r} \frac{d}{dr} \left\{ r \frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} \left( r \frac{dw}{dr} \right) \right] \right\} = \frac{q}{D}, \quad (1)$$

where the coordinate  $r$  and displacement  $w$  are illustrated in Fig. 1 and  $q =$  a uniformly distributed lateral load on the plate,  $D = Et^3/12(1 - \nu^2) =$  the flexural rigidity of the plate,  $E =$  modulus of elasticity of the flange material,  $t =$  plate thickness, and  $\nu =$  Poisson's ratio. Equation (1) can be integrated to give a relation for the displacement in terms of arbitrary constants:

$$w = C_7 r^2 \ln r + C_8 r^2 + C_9 \ln r + C_{10} + \frac{r^4 q}{64D}, \quad (2)$$

where numerical values for the constants  $C_7, \dots, C_{10}$  are established from boundary conditions. Derivatives of  $w$ , required in the subsequent analysis, are:

$$\frac{dw}{dr} = C_7(2r \ln r + r) + 2C_8 r + \frac{C_9}{r} + \frac{r^3 q}{16D}, \quad (3)$$

$$\frac{d^2 w}{dr^2} = C_7(2 \ln r + 3) + 2C_8 - \frac{C_9}{r^2} + \frac{3r^2 q}{16D}, \quad (4)$$

and

$$\frac{d^3w}{dr^3} = C_7 \left( \frac{2}{r} \right) + \frac{2C_3}{r^3} + \frac{3rq}{8D} \quad (5)$$

In the subsequent analysis the distributed load  $q$  is taken as zero.

The radial and tangential moments are given<sup>3</sup> by the equations:

$$M_r = -D \left( \frac{d^2w}{dr^2} + \frac{\nu}{r} \frac{dw}{dr} \right) \quad (6)$$

and

$$M_t = -D \left( \frac{1}{r} \frac{dw}{dr} + \nu \frac{d^2w}{dr^2} \right) \quad (7)$$

Using Eq.: (3) and (4), these moments can be expressed as

$$M_r = -D \left\{ C_7 [2(1 + \nu) \ln r + (3 + \nu)] + C_8 [2(1 + \nu)] - C_9 \left( \frac{1 - \nu}{r^2} \right) \right\} \quad (8)$$

and

$$M_t = -D \left\{ C_7 [2(1 + \nu) \ln r + (1 + 3\nu)] + C_8 [2(1 + \nu)] + C_9 \left( \frac{1 - \nu}{r^2} \right) \right\} \quad (9)$$

#### Equations for the Tapered Hub

The basic differential equation for the radial displacement  $u$  of a cylindrical shell with a linearly variable wall thickness  $t_x$  is given by Timoshenko<sup>3</sup> as

$$\frac{d^2}{dx^2} \left( \tau^3 \frac{d^2 u}{dx^2} \right) + \frac{12(1 - \nu^2) \tau_x u}{b^2} - \frac{12(1 - \nu^2) [1 - (\nu/2)] p}{E} = 0 \quad (10)$$

The solution of Eq. (10) can be shown\* to be:

$$u = \frac{b}{\psi^{1/2}} (C_1 b_1 + C_2 b_2 + C_3 b_3 + C_4 b_4) + \frac{b P^*}{1 + \alpha \xi} \quad (11)$$

where  $P^* = [1 - (\nu/2)] bp/g_0 E$ . Derivatives of  $u$ , required in the subsequent analysis, are

$$u' = \frac{du}{dx} = \frac{b}{2\psi^{3/2} h} (C_1 b_5 + C_2 b_6 + C_3 b_7 + C_4 b_8) - \frac{b \alpha P^*}{h(1 + \alpha \xi)^2} \quad (12)$$

$$u'' = \frac{d^2 u}{dx^2} = \frac{b}{4\psi^{5/2} h^2} (C_1 b_9 + C_2 b_{10} + C_3 b_{11} + C_4 b_{12}) + \frac{2b \alpha^2 P^*}{h^2 (1 + \alpha \xi)^3} \quad (13)$$

and

$$u''' = \frac{d^3 u}{dx^3} = \frac{b}{8\psi^{7/2} h^3} (C_1 b_{13} + C_2 b_{14} + C_3 b_{15} + C_4 b_{16}) - \frac{6b \alpha^3 P^*}{h^3 (1 + \alpha \xi)^4} \quad (14)$$

The  $b_n$ 's used in Eqs. (11) through (14) are modified Bessel functions of argument  $n = 2\gamma(\psi/\alpha)^{1/2}$  defined in Table 1, which gives equations for  $n = 1$  through 20;  $\psi$ ,  $\alpha$ , and  $\xi$  are defined in the nomenclature.

---

\* A solution to an equation that is essentially the same as Eq. (10) is given by Timoshenko,<sup>3</sup> who credits the original solution to G. Kirchoff in 1879.

Table 1. Modified Bessel functions of argument  $\eta^a$ 

$$b_1 = \text{ber}' \eta$$

$$b_2 = \text{bei}' \eta$$

$$b_3 = \text{ker}' \eta$$

$$b_4 = \text{kei}' \eta$$

$$b_5 = -\eta \text{bei} \eta - 2 \text{ber}' \eta$$

$$b_6 = \eta \text{ber} \eta - 2 \text{bei}' \eta$$

$$b_7 = -\eta \text{kei} \eta - 2 \text{ker}' \eta$$

$$b_8 = \eta \text{ker} \eta - 2 \text{kei}' \eta$$

$$b_9 = 4\eta \text{bei} \eta + 8 \text{ber}' \eta - \eta^2 \text{bei}' \eta$$

$$b_{10} = -4\eta \text{ber} \eta + 8 \text{bei}' \eta + \eta^2 \text{ber}' \eta$$

$$b_{11} = 4\eta \text{kei} \eta + 8 \text{ker}' \eta - \eta^2 \text{kei}' \eta$$

$$b_{12} = -4\eta \text{ker} \eta + 8 \text{kei}' \eta + \eta^2 \text{ker}' \eta$$

$$b_{13} = -\eta^3 \text{ber} \eta - 24\eta \text{bei} \eta - 48 \text{ber}' \eta + 8\eta^2 \text{bei}' \eta$$

$$b_{14} = -\eta^3 \text{bei} \eta + 24\eta \text{ber} \eta - 48 \text{bei}' \eta - 8\eta^2 \text{ber}' \eta$$

$$b_{15} = -\eta^3 \text{ker} \eta - 24\eta \text{kei} \eta - 48 \text{ker}' \eta + 8\eta^2 \text{kei}' \eta$$

$$b_{16} = -\eta^3 \text{kei} \eta + 24\eta \text{ker} \eta - 48 \text{kei}' \eta - 8\eta^2 \text{ker}' \eta$$

$$b_{17} = -\eta \text{ber} \eta + 2 \text{bei}' \eta$$

$$b_{18} = -\eta \text{bei} \eta - 2 \text{ber}' \eta$$

$$b_{19} = -\eta \text{ker} \eta + 2 \text{kei}' \eta$$

$$b_{20} = -\eta \text{kei} \eta - 2 \text{ker}' \eta$$

<sup>a</sup>The argument  $\eta = 2\gamma(\psi/\alpha)^{1/2}$ , where  $\gamma = [12(1 - \nu^2)/b^2 g_0^2]^{1/4}(h)$ ,  $\psi = \xi + (1/\alpha)$ ,  $\xi = x/h$ , and  $\alpha = (z_1 - g_0)/g_0$ .

Equations for the Pipe

The basic differential equation for the radial displacement  $u_1$  of a cylindrical shell with uniform wall thickness is:

$$\beta_0^3 \frac{d^4 u_1}{dx_1^4} + \frac{12(1 - \nu^2)g_0}{b^2} u_1 - \frac{12(1 - \nu^2)[1 - (\nu/2)]p}{E} = 0. \quad (15)$$

The solution of Eq. (15) is:

$$u_1 = e^{-\beta x_1} (C_{11} \sin \beta x_1 + C_{12} \cos \beta x_1) \\ + e^{\beta x_1} (C_5 \sin \beta x_1 + C_6 \cos \beta x_1) + bP^*. \quad (16)$$

For large negative values of  $x_1$ ,  $u_1 = bP^*$ . Hence,  $C_{11} = C_{12} = 0$ . Derivatives of  $u_1$  needed in the subsequent analysis are

$$u_1' = \frac{du_1}{dx_1} = \beta e^{\beta x_1} [C_5 (\sin \beta x_1 + \cos \beta x_1) \\ + C_6 (\cos \beta x_1 - \sin \beta x_1)], \quad (17)$$

$$u_1'' = \frac{d^2 u_1}{dx_1^2} = 2\beta^2 e^{\beta x_1} [C_5 \cos \beta x_1 - C_6 \sin \beta x_1], \quad (18)$$

and

$$u_1''' = \frac{d^3 u_1}{dx_1^3} = -2\beta^3 e^{\beta x_1} [C_5 (\sin \beta x_1 - \cos \beta x_1) \\ + C_6 (\sin \beta x_1 + \cos \beta x_1)]. \quad (19)$$

Boundary Conditions

The equations listed above involve ten unknown constants:  $C_1, C_2, \dots, C_{10}$ . These can be determined from the ten boundary-condition

equations shown in Table 2 [Eq. (20)]. The ASME Code stress-calculation method<sup>1</sup> is based on the assumption that the radial displacement at the hub-to-ring juncture is zero. A more realistic assumption (particularly for internal pressure loading) is that the displacement of the hub equals the displacement of the surface of the ring where it joins the hub. Boundary-condition equations for both of these alternatives are provided in Table 2. [See Eqs. (20-5).] In Eq. (20-5b) a positive  $dw/dr$  gives a negative radial displacement at the surface of the ring adjacent to the hub. Also in Eq. (20-5b),  $u_r$  is the radial expansion of the ring due to internal pressure as given by Lamé's equation:

$$u_r = \frac{b}{E} \left[ \frac{(1 + \nu)k^2 + (1 - \nu)}{k^2 - 1} \right] \left( p - \frac{P_1}{t} \right), \quad (21)$$

where  $k = a/b$ . In this expression, it is assumed that in addition to internal pressure  $p$ , the shear resultant  $P_1$  is uniformly distributed around the inner edge of the ring.

#### Boundary Equations

When the equations in Table 2 are satisfied simultaneously, they establish the values of the ten constants ( $C_1, C_2, \dots, C_{10}$ ) in terms of the dimensions, Poisson's ratio, and the loads (total bolt load  $W$  and internal pressure  $p$ ). After algebraic manipulation, the equations are reduced to the forms shown in Table 3. This table provides the elements for the matrix equation  $[A]|C| + |B| = 0$ , where the terms in the coefficient matrix  $[A]$  are given under the headings of the corresponding constants in the column matrix  $|C|$ . The loading parameters constitute the column matrix  $|B|$ .

To derive numerical values for the constants, three items should be noted.

1. It is convenient to define two new constants,  $C'_5 = C_5/b$  and  $C'_6 = C_6/b$ .
2. The radial expansion of the ring  $u_r$  is defined in Eq. (21).

Table 2. Equations for the boundary conditions for a tapered-hub flange

	Hub-to-pipe juncture		Hub-to-ring juncture		Ring	
	Equation	Eq. No.	Equation	Eq. No.	Equation	Eq. No.
Displacements <sup>d</sup>	$(u)_{x=0} = (u_1)_{x_1=0}$	(20-1)	$(u)_{x=h} = 0$ (20-5a) $(u)_{x=h} = \left(u_r \cdot \frac{1}{2} \frac{dr}{dr}\right)_{r=h}$ (20-5b)		$(w)_{r=h} = 0$ (20-8) (Footnote 1)	
Rotations	$(u')_{x=0} = (u'_1)_{x_1=0}$	(20-2)	$(u')_{x=h} = \left(\frac{dw}{dr}\right)_{r=h}$	(20-6)		
Moments <sup>c</sup>	$(u'')_{x=0} = (u''_1)_{x_1=0}$	(20-3)	$M_{h1} = -M_{r1} + \frac{1}{2} P_1 t$ (Footnote 1)	(20-7)	$M_{r2} = 0$	(20-9)
Shears	$\left(\frac{3a}{h} u'' + u'''\right)_{x=0} = (u'''_1)_{x_1=0}$	(20-4)			$Q = -\frac{dw}{dr} + \frac{M_1}{r} - \frac{M_r}{r} = \frac{W}{2a r}$	(20-10)

<sup>a</sup>Radial for hub-to-pipe and hub-to-ring junctures and axial for the ring.

<sup>b</sup>Setting  $(w)_{r=h}$  equal to zero provides a reference point for all other axial displacements.

<sup>c</sup>Radial for ring.

<sup>d</sup>The assumption is that the shear  $P_1$  of the hub on the ring produces an additional moment on the ring.



Table 3. Matrix coefficients of the differential equations for a string with a tapered wall web

Eq. No.	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$	$C_8$	$C_9$	$C_{10}$	Leading parameters
(20-1) <sup>2</sup>	$b_1^0$	$b_2^0$	$b_3^0$	$b_4^0$	$b_5^0$	$b_6^0$	$b_7^0$	$b_8^0$	$b_9^0$	$b_{10}^0$	$\nu$
(20-2)	$b_1^0$	$b_2^0$	$b_3^0$	$b_4^0$	$b_5^0$	$b_6^0$	$b_7^0$	$b_8^0$	$b_9^0$	$b_{10}^0$	$\nu$
(20-3)	$b_1^0$	$b_2^0$	$b_3^0$	$b_4^0$	$b_5^0$	$b_6^0$	$b_7^0$	$b_8^0$	$b_9^0$	$b_{10}^0$	$\nu$
(20-4)	$b_1^0$	$b_2^0$	$b_3^0$	$b_4^0$	$b_5^0$	$b_6^0$	$b_7^0$	$b_8^0$	$b_9^0$	$b_{10}^0$	$\nu$
(20-5a) <sup>3</sup>	$b_1^0$	$b_2^0$	$b_3^0$	$b_4^0$	$b_5^0$	$b_6^0$	$b_7^0$	$b_8^0$	$b_9^0$	$b_{10}^0$	$\nu$
(20-5b) <sup>3</sup>	$b_1^0 + u_1^0 b_1^0$	$b_2^0 + u_2^0 b_2^0$	$b_3^0 + u_3^0 b_3^0$	$b_4^0 + u_4^0 b_4^0$	$b_5^0 + u_5^0 b_5^0$	$b_6^0 + u_6^0 b_6^0$	$b_7^0 + u_7^0 b_7^0$	$b_8^0 + u_8^0 b_8^0$	$b_9^0 + u_9^0 b_9^0$	$b_{10}^0 + u_{10}^0 b_{10}^0$	$\nu$
(20-6)	$b_1^0$	$b_2^0$	$b_3^0$	$b_4^0$	$b_5^0$	$b_6^0$	$b_7^0$	$b_8^0$	$b_9^0$	$b_{10}^0$	$\nu$
(20-7) <sup>4</sup>	$b_1^0 + u_1^0 b_1^0$	$b_2^0 + u_2^0 b_2^0$	$b_3^0 + u_3^0 b_3^0$	$b_4^0 + u_4^0 b_4^0$	$b_5^0 + u_5^0 b_5^0$	$b_6^0 + u_6^0 b_6^0$	$b_7^0 + u_7^0 b_7^0$	$b_8^0 + u_8^0 b_8^0$	$b_9^0 + u_9^0 b_9^0$	$b_{10}^0 + u_{10}^0 b_{10}^0$	$\nu$
(20-8)	$b_1^0$	$b_2^0$	$b_3^0$	$b_4^0$	$b_5^0$	$b_6^0$	$b_7^0$	$b_8^0$	$b_9^0$	$b_{10}^0$	$\nu$
(20-9)	$b_1^0$	$b_2^0$	$b_3^0$	$b_4^0$	$b_5^0$	$b_6^0$	$b_7^0$	$b_8^0$	$b_9^0$	$b_{10}^0$	$\nu$
(20-10)	$b_1^0$	$b_2^0$	$b_3^0$	$b_4^0$	$b_5^0$	$b_6^0$	$b_7^0$	$b_8^0$	$b_9^0$	$b_{10}^0$	$\nu$

<sup>2</sup>These equations are in the form  $[A]U' + [B]U = 0$ , where  $[A]$  is the coefficient matrix,  $[B]$  is the column matrix of unknown constants, and  $U$  is the column matrix of leading parameters.

<sup>3</sup>A superscript "a" on the  $b$ 's indicates that the Bessel function is to be evaluated at  $x = b_1, b_2, \dots, b_{10}$ .

<sup>4</sup>A prime (') on the  $b$ 's indicates that the Bessel function is to be evaluated at  $x = b_1, b_2, \dots, b_{10}$ .

$b_1 = c/\alpha_1 \nu$ ;  $b_2 = \sqrt{c_2^2/\alpha_2^2 \nu^2 + c_3^2/\alpha_3^2 \nu^2}$ ;  $b_3 = (b_2/\alpha_3) \sqrt{1 + \alpha_3^2/\alpha_2^2}$ ;  $b_4 = (b_2/\alpha_3) \sqrt{1 + \alpha_3^2/\alpha_2^2}$ ;  $b_5 = (b_2/\alpha_3) \sqrt{1 + \alpha_3^2/\alpha_2^2}$ ;  $b_6 = (b_2/\alpha_3) \sqrt{1 + \alpha_3^2/\alpha_2^2}$ ;  $b_7 = (b_2/\alpha_3) \sqrt{1 + \alpha_3^2/\alpha_2^2}$ ;  $b_8 = (b_2/\alpha_3) \sqrt{1 + \alpha_3^2/\alpha_2^2}$ ;  $b_9 = (b_2/\alpha_3) \sqrt{1 + \alpha_3^2/\alpha_2^2}$ ;  $b_{10} = (b_2/\alpha_3) \sqrt{1 + \alpha_3^2/\alpha_2^2}$ .

5. The ASME Code stress-calculation method uses a moment  $M$ , applied to the flange ring, rather than a bolt load  $W$ , where the correlation between  $M$  and  $W$  is  $M = W(a - b)$ . In the present analysis, however, Eq. (20-10) from Table 2 is used with the loading parameter  $M$ , rather than  $W$ .

### Stresses

After having solved the set of equations in Table 3 for the constants  $C_1, \dots, C_{10}$ , the stresses can be obtained anywhere in the structure. The equations for these stresses, used in other reports<sup>4,5</sup> in this series, are given in Table 4 [Eqs. (22)–(45)] for the same locations as those given by the ASME Code stress-calculation method; these are (1) at the hub-to-pipe juncture, (2) in the hub at the hub-to-ring juncture, and (3) at the inside edge of the ring ( $r = b$ ).

### Displacements

In Chapter 7 the displacements  $w$  of the flange ring are used. The equations for these displacements (with  $w$  arbitrarily set to zero at  $r = b$ ) are:

$$w_g = C_7 g^2 \ln g + C_8 g^2 + C_9 \ln g + C_{10} \quad (46)$$

at the gasket centerline radius,  $g = G/2$ ; and

$$w_c = C_7 c^2 \ln c + C_8 c^2 + C_9 \ln c + C_{10} \quad (47)$$

at the bolt-circle radius,  $c = C/2$ .

Table 4. Equations for the stresses in a tapered hub flange

Type	Hub-to-pipe junction, longitudinal and circumferential	Hub-to-ring junction, longitudinal and circumferential	Inside edges of ring, tangential and radial	
Equation	Eq. No.	Equation	Equation	
Longitudinal or tangential bending	$(\sigma_r)_h = \frac{14p_0}{2(1-\nu^2)} \frac{14p_0}{(2b-10c)} \frac{14p_0}{2(1-\nu^2)} \frac{14p_0}{(2b-10c)}$ (22)	$(\sigma_r)_h = \frac{14p_0}{2(1-\nu^2)} \frac{14p_0}{(2b-10c)} \frac{14p_0}{2(1-\nu^2)} \frac{14p_0}{(2b-10c)}$ (23)	$(\sigma_r)_h = (6p_0/10) \frac{14p_0}{(2b-10c)} \frac{14p_0}{2(1-\nu^2)} \frac{14p_0}{(2b-10c)}$ (24)	(30)
Membrane	$(\sigma_r)_m = pb/2g_0$ (25)	$(\sigma_r)_m = pb/2g_0$ (26)	$(\sigma_r)_m = pb/2g_0$ (27)	(31)
Outside	$(\sigma_r)_o = pb/2g_0 + 1.5166c^2$ (28)	$(\sigma_r)_o = pb/2g_0 + 1.5166c^2$ (29)	$(\sigma_r)_o = pb/2g_0 + 1.5166c^2$ (30)	(32)
Inside	$(\sigma_r)_i = pb/2g_0 + 1.5166c^2$ (31)	$(\sigma_r)_i = pb/2g_0 + 1.5166c^2$ (32)	$(\sigma_r)_i = pb/2g_0 + 1.5166c^2$ (33)	(33)
Circumferential or radial bending	$(\sigma_r)_b = \nu(\sigma_r)_h$ (34)	$(\sigma_r)_b = \nu(\sigma_r)_h$ (35)	$(\sigma_r)_b = \nu(\sigma_r)_h$ (36)	(34)
Membrane	$(\sigma_r)_m = (E\sigma_0/b) + \nu(pb/2g_0)$ (37)	$(\sigma_r)_m = (E\sigma_0/b) + \nu(pb/2g_0)$ (38)	$(\sigma_r)_m = (E\sigma_0/b) + \nu(pb/2g_0)$ (39)	(35)
Outside	$(\sigma_r)_o = (E\sigma_0/b) + \nu(\sigma_r)_o$ (40)	$(\sigma_r)_o = (E\sigma_0/b) + \nu(\sigma_r)_o$ (41)	$(\sigma_r)_o = (E\sigma_0/b) + \nu(\sigma_r)_o$ (42)	(36)
Inside	$(\sigma_r)_i = (E\sigma_0/b) + \nu(\sigma_r)_i$ (43)	$(\sigma_r)_i = (E\sigma_0/b) + \nu(\sigma_r)_i$ (44)	$(\sigma_r)_i = (E\sigma_0/b) + \nu(\sigma_r)_i$ (45)	(37)

where,  $k = a/b$ , and  $\frac{1}{1-\nu^2} = \frac{E\sigma_0^2}{(1-\nu^2) 2b\sigma_0^2/2}$  ( $C_1b_1^2 + C_2b_2^2 + C_3b_3^2 + C_4b_4^2$ ).

<sup>a</sup>Hub-side surface of ring.

<sup>b</sup>Gasket-side surface of ring.

<sup>c</sup> $\sigma_0 = b(C_1 + p)$ .

<sup>d</sup> $\sigma_0 = \frac{b}{\sqrt{12}} (C_1b_1^2 + C_2b_2^2 + C_3b_3^2 + C_4b_4^2) + bp/(1-\nu)$ .

#### 4. FLANGE WITH A STRAIGHT HUB

Although the mathematical expressions for the straight hub can be obtained by letting  $g_0 = g_1$ , this would result in indeterminate quantities in the computer program. Therefore, the direct solution to the ring with a straight hub was obtained by using the previously given basic equations for only the pipe and the ring. There are six constants of integration to be established; the boundary-condition equations are displayed in Table 5 [Eq. (48)].

After algebraic manipulation, the equations displayed in Table 5 are reduced to the matrix-equation form  $[A]|C| + |B| = 0$ , where the terms in the coefficient matrix  $[A]$  are given in Table 6 under the headings of the corresponding constants in the column matrix  $|C|$ . Solving this set of equations for the six constants ( $C_5^i, C_6^i, C_7, C_8, C_9$ , and  $C_{10}$ ) allows calculation of the stresses in the structure. The equations for the stresses in the pipe at the pipe-to-ring juncture and in the ring at the inner edge ( $r = b$ ) are analogous to those previously derived for the flange with a tapered hub (see Table 4).

One can calculate the displacements  $w_g$  and  $w_c$  for a straight-hub flange from Eqs. (46) and (47), respectively, using the constants  $C_7, \dots, C_{10}$ , identified in Table 6.

Table 5. Equations for the boundary conditions for a straight-hub flange

	Hub-to-ring juncture		Ring	
	Equation	Eq. No.	Equation	Eq. No.
Displacements	$(u_1)_{x_1=0} = 0$	(48-1a) <sup>a,b</sup>	$(w)_{r=b} = 0$	(48-4) <sup>c</sup>
	$(u_1)_{x_1=0} = \left( u_r - \frac{t}{2} \frac{dw}{dr} \right)_{r=b}$	(48-1b) <sup>a,b</sup>		
Rotations	$(u_1')_{x_1=0} = \left( \frac{dw}{dr} \right)_{r=b}$	(48-2)		
Moments	$M_{r1} = -M_{ho} + \frac{1}{2} P_0 t$	(48-3)	$M_{r2} = 0$	(48-5) <sup>d</sup>
Shear along radius r			$Q = -\frac{dM_r}{dr} + \frac{M_t - M_r}{r} = \frac{W}{2\pi r}$	(48-6)

<sup>a</sup>Radial displacements.

<sup>b</sup>For an ASME-type calculation, Eq. (48-1a) is used.

<sup>c</sup>Axial displacements;  $(w)_{r=b} = 0$  is the reference point for all other axial displacements.

<sup>d</sup>Radial moment at outside edge of ring ( $r = a$ ).

Table 6. Matrix coefficients of the discontinuity equations<sup>a</sup> for a flange with a straight hub

Eq. No.	Coefficients of $C_n$						Loading parameters
	$C_5$	$C_6$	$C_7$	$C_8$	$C_9$	$C_{10}$	
(48-1a)	0	1.0	0	0	0	0	$bP^* + bC_{10} - U_{10}P$
(48-1b) <sup>b</sup>	$U_{14} - U_{13}$	$1 + U_{14} + U_{13}$	0	0	0	0	0
(48-2)	b	b	$-(2b \ln b + b)$	$-2b$		0	0
(48-3)	$2b^2 + 2b^3t/2$	$-2b_1^3t/2$	$-(2.6 \ln b + 3.3) \times (t/g_0)^3$	$-2.6(t/g_0)^3$	$(0.7/b^2)(t/g_0)^3$	0	0
(48-4)	0	0	$b^2 \ln b$	$b^2$	$\ln b$	1.0	0
(48-5)	0	0	$2.6 \ln a + 3.3$	2.6	$-0.7/a$	0	0
(48-6)	0	0	1.0	0	0	0	$\frac{-3(1-\nu^2)M}{2-1t^3(a-b)}$

<sup>a</sup>These equations are in the form  $[a]|C| + |B| = 0$ , where  $[A]$  is the coefficient matrix,  $|C|$  is the column matrix of unknown constants,  $|B|$  is the column matrix of loading parameters.

$$bU_3 = (b/E) \left[ \frac{(1+\nu)K^2 + (1-\nu)}{K^2 - 1} \right], \text{ where } K = a/b; U_{13} = \frac{2U_3 E g_0^3 b^3}{12(1-\nu^2)t}; U_{14} = tR/2.$$

## 5. BLIND FLANGES

Analysis Method

Blind flanges (or flat heads) are modeled as shown in Fig. 2. The general equations for a circular flat plate are:<sup>3</sup>

$$w = D_1 r^2 \ln r + D_2 r^2 + D_3 \ln r + D_4 + r^4 p / 64D, \quad (49)$$

$$\frac{dw}{dr} = D_1 (2r \ln r + r) + D_2 (2r) + D_3 / r + r^3 p / 16D, \quad (50)$$

$$\frac{d^2 w}{dr^2} = D_1 (2 \ln r + 3) + D_2 (2) - D_3 / r^2 + 3r^2 p / 16D, \quad (51)$$

and

$$\frac{d^3 w}{dr^3} = D_1 (2/r) + D_3 (2/r^3) + 3rp / 8D. \quad (52)$$

The radial and tangential moments  $M_r$  and  $M_t$  (see Fig. 2) are given by

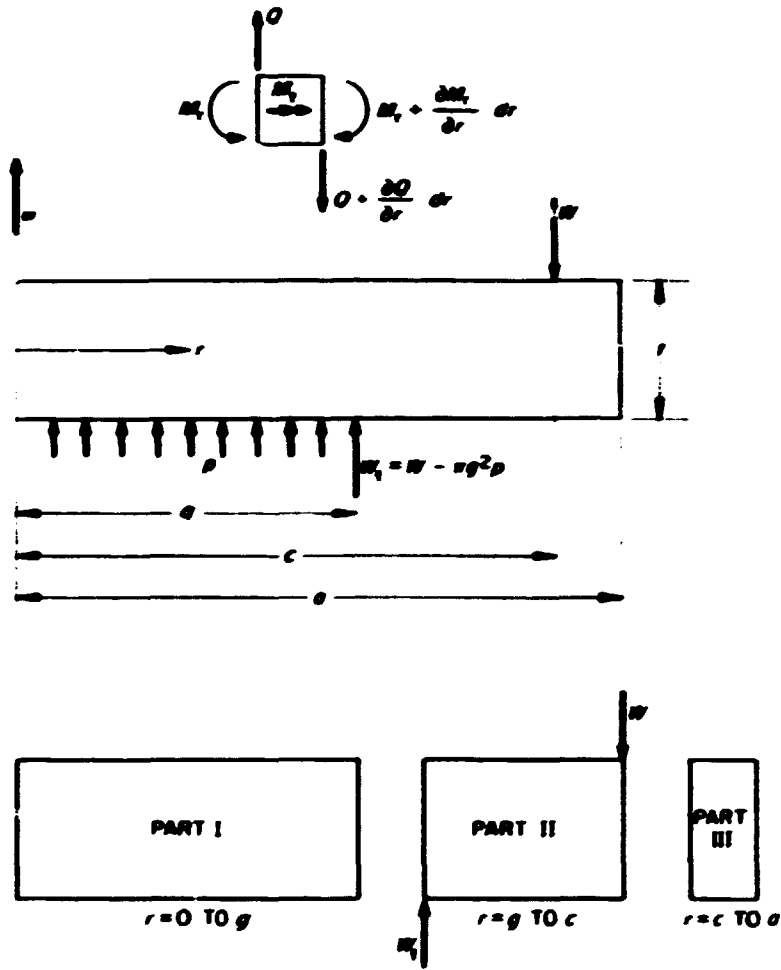
$$M_r = -D \left( \frac{d^2 w}{dr^2} + \frac{\nu}{r} \frac{dw}{dr} \right) \quad (53)$$

and

$$M_t = -D \left( \frac{1}{r} \frac{dw}{dr} + \frac{d^2 w}{dr^2} \right); \quad (54)$$

and the shear is given by

$$Q = -\frac{dM_r}{dr} + \frac{M_t - M_r}{r}. \quad (55)$$



CONSTANTS:

$$D_{11}, D_{12}, D_{13}, D_{14}$$

$$D_{21}, D_{22}, D_{23}, D_{24}$$

$$D_{31}, D_{32}, D_{33}, D_{34}$$

Fig. 2. Flat-plate analysis model of a blind flange or cover plate.

The moments and shears, in terms of the integration constants  $D_1$  through  $D_4$ , are:

$$M_r = -D_1 [2(1 + \nu) \ln r + (3 + \nu)] + D_2 [2(1 + \nu)] - D_3 [(1 - \nu)/r^2] - r^2 p / 16(3 + \nu), \quad (56)$$



$$M_t = -D\{D_1[2(1 + \nu) \ln r + (1 + 5\nu)] + D_2[2(1 + \nu)] + D_3[(1 - \nu)/r^2]\} - r^2 p / 16(1 + 5\nu), \quad (57)$$

and

$$Q = D \left( \frac{4D_1}{r} \right) + \frac{rp}{2}. \quad (58)$$

For analysis, the plate is divided into three parts as shown in Fig. 2. There are four integration constants for each segment. The boundary-condition equations used to evaluate these constants are shown in Table 7. These boundary conditions show that 3 of the 12 constants are zero. The set of simultaneous equations to be solved to establish the remaining 9 constants is shown in Table 8. Again, this table presents the elements of the matrix equation  $[A]|C| + |B| = 0$ .

Table 7. Boundary condition equations used for blind-flange analysis

Equation No.	Boundary condition
1	$2\pi r Q = \pi r^2 p$ for all of Part I. This gives $D_{11} = 0$ .
2	$(dw/dr)_I = 0$ at $r = 0$ . This gives $D_{13} = 0$ .
3	$(w)_I = 0$ at $r = g$
4	$(dw/dr)_I = (dw/dr)_{II}$ at $r = g$
5	$(Q)_{II} = (W/2\pi r) - (\pi g^2 p / 2\pi g)$ at $r = g$ . This gives $D_{21} = W/8\pi D - g^2 p / 8D$ . (For pressure loading, $W = \pi g^2 p$ ; hence $D_{21} = 0$ .)
6	$(w)_{II} = 0$ at $r = g$
7	$(M_r)_I = (M_r)_{II}$ at $r = g$
8	$(dw/dr)_I = (dw/dr)_{II}$ at $r = g$
9	$(Q)_{III} = 0$ . This gives $D_{31} = 0$ .
10	$(M_r)_{II} = (M_r)_{III}$ at $r = c$
11	$(M_r)_{III} = 0$ at $r = a$
12	$(w)_{II} = (w)_{III}$ at $r = c$

Table 8. Boundary equations<sup>a</sup> for a blind flange

No. <sup>b</sup>	Coefficients of $D_{ij}$									Loading parameter
	$D_{12}$	$D_{14}$	$D_{21}$	$D_{22}$	$D_{23}$	$D_{24}$	$D_{32}$	$D_{33}$	$D_{34}$	
3	$g^2$	1.0	0	0	0	0	0	0	0	$g^4 p / 64D$
4	$-2g$	0	$2g \ln g + g$	$2g$	$1/g$	0	0	0	0	$-g^3 p / 16D$
5	0	0	1.0	0	0	0	0	0	0	$-W / 8\pi D$
6	0	0	$g^2 \ln g$	$g^2$	$\ln g$	1.0	0	0	0	0
7	$-2.6$	0	$2.6 \ln g + 3.3$	$2.6$	$-0.7/g^2$	0	0	0	0	$-3.3g^2 p / 16D$
8	0	0	$2c \ln c + c$	$2c$	$1/c$	0	$-2c$	$-1/c$	0	0
10	0	0	$2.6 \ln c + 3.3$	$2.6$	$-0.7/c^2$	0	$-2.6$	$0.7/c^2$	0	0
11	0	0	0	0	0	0	$2.6$	$-0.7/c^2$	0	0
12	0	0	$c^2 \ln c$	$c^2$	$\ln c$	1.0	$-c^2$	$-\ln c$	$-1.0$	0

<sup>a</sup>These equations are in the form  $[A]|C| + |B| = 0$ , where  $[A]$  is the coefficient matrix,  $|C|$  is the column matrix of unknown constants, and  $|B|$  is the column matrix of loading parameters.

<sup>b</sup>Boundary condition number from Table 4.

### Stresses

After having established values for the integration constants, the stresses at any point in the blind flange can be readily obtained. Equations for stresses at the center of the flange and at  $r = g$  and  $r = c$  are given by

$$\sigma_t = \pm 6M_t/t^2 = \pm EtM_t/[2(1 - \nu^2)]D \quad (59a)$$

and

$$\sigma_r = \pm 6M_r/t^2 = \pm EtM_r/[2(1 - \nu^2)]D . \quad (59b)$$

At the center of the flange ( $r = 0$ ),

$$M_t = M_r = -D\{D_{12}[2(1 + \nu)]\} . \quad (60)$$

At the gasket ( $r = g$ ),

$$M_r = -D\{D_{12}[2(1 + \nu)] + g^2p(3 + \nu)/16D\} , \quad (61)$$

and

$$M_t = -D\{D_{12}[2(1 + \nu)] + g^2p(1 + 3\nu)/16D\} . \quad (62)$$

At the bolt circle ( $r = c$ ),

$$M_r = -D\{D_{32}[2(1 + \nu)] - D_{33}(1 - \nu)/c^2\} , \quad (63)$$

and

$$M_t = -D\{D_{32}[2(1 + \nu)] + D_{33}(1 - \nu)/c^2\} . \quad (64)$$

In all of the above, a positive moment produces a tensile stress on the back of the flange (positive  $w$  side of Fig. 2).

Displacements

In the third and sixth boundary conditions listed in Table 7, the axial displacement at the gasket has been arbitrarily set equal to zero. The relative displacement of the bolt circle to the gasket is therefore

$$w_c = D_{32}c^2 + D_{33} \ln c + D_{34} \quad (65)$$

## 6. THERMAL GRADIENTS

Two kinds of thermal gradients are included in the analysis: (1) a constant temperature in the pipe and hub that may be different from the assumed constant temperature in the ring and (2) a constant temperature in the bolts that may be different from the assumed constant temperature in the ring.

The significance of the bolt-to-ring thermal gradients is dependent upon the dimensional and material characteristics of the flanged joint and is covered later in Chapter 7.

The pipe/hub-to-ring temperature gradient is included in the analysis by an appropriate change in the "loading parameters" shown in Table 3. We define  $\Delta$  as the difference in temperature between the pipe/hub and the ring;  $\Delta$  is positive if the pipe/hub is hotter than the ring. The radial expansion of the tapered hub at its juncture with the ring is then:

$$u = \frac{b}{\sqrt{\psi_1}} (C_1 b_1' + C_2 b_2' + C_3 b_3' + C_4 b_4') + b \epsilon_f \Delta, \quad (66)$$

where  $b$  is the pipe radius;  $b_i'$  terms are the Bessel functions defined in Table 1 evaluated at  $x = h$ ,  $\eta = 2\gamma\rho^{1/2}/\alpha$ , as indicated in footnote *c* of Table 3; and  $\epsilon_f$  is the coefficient of thermal expansion of the flange material.

The effects of such a thermal gradient are taken into account by adding  $(\sqrt{\psi_1}/b)(b\epsilon_f\Delta)$  to the existing terms in the loading-parameter column in Table 3 [Eqs. (20-5a) and (20-5b)]. The analogous term is already included in Table 6.

## 7. CHANGE IN BOLT LOAD WITH PRESSURE, TEMPERATURE, AND EXTERNAL MOMENTS

A flanged joint is a statically indeterminate structure. Thus, in order to determine the residual bolt load in the joint, it is necessary to calculate the relative displacements of the parts when the joint is subjected to (1) initial bolt loading, (2) moment loading, (3) internal pressure, and (4) thermal gradients.

The object of the analysis is to determine the residual bolt load  $W_2$  in terms of (1) the loadings  $W_1$ ,  $p$ ,  $\Delta$ , and  $\Delta'$ ; (2) the component temperatures  $T_b$ ,  $T_g$ ,  $T_f$ , and  $T'_f$ ; (3) the flanged-joint dimensions; and (4) the material properties.

The basic analysis is given by Messtrom and Bergh,<sup>6</sup> and we follow their nomenclature, with additions as necessary. Reference 6 covers only the effect of initial bolt loading and part of the influence of internal pressure; the remaining influence from the internal pressure is discussed by Rodabaugh.<sup>7</sup> The extension of the analysis to cover thermal gradients is relatively simple and is covered below.

The nomenclature used in this development is:

- A = cross-sectional area of bolts or gasket
- B = inside diameter of ring
- C = bolt-circle diameter
- E = modulus of elasticity
- $g_0$  = wall thickness of pipe
- G = gasket centerline diameter
- $l$  = bolt length
- $p$  = internal pressure
- $p^*$  = equivalent pressure for external moment loading
- $q$  = elastic deformation coefficients
- $t$  = ring thickness
- T = final-state temperature (initial-state temperature is defined as zero)
- $v$  = gasket thickness
- W = bolt load

$\delta$  = relative axial displacement between the gasket centerline and the bolt circle

$\epsilon$  = coefficient of thermal expansion

$\Delta$  = temperature between hub/pipe and ring

The subscripts 0, 1, and 2 refer to the undeformed, initial deformed, and final deformed states, respectively; subscripts b, g, and f refer to the bolts, gasket, and flange, respectively. Quantities with a prime (') are for one of the flanges in a pair (e.g.,  $T_f'$  refers to the temperature of the right-hand flange in Fig. 3); quantities without a prime are for the other flange.

### Analysis

Figure 3 shows a schematic illustration of the general case of two dissimilar flanges and their mode of deformation. When the bolts are initially tightened to make up the joint, the resulting initial deformed bolt length is

$$l_1 = v_1 + t_1 + t_1' - \delta_1 - \delta_1' \quad (67)$$

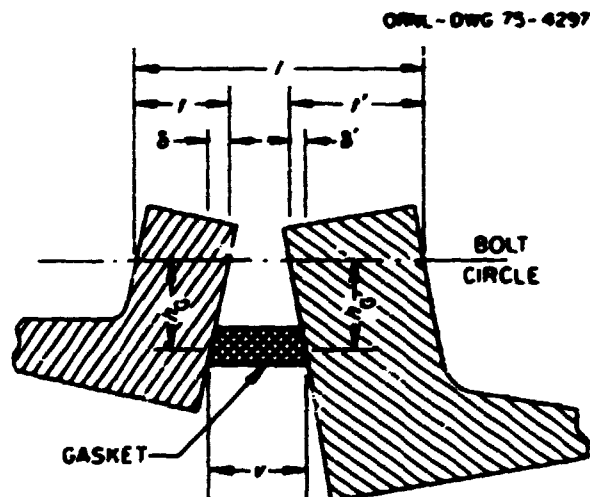


Fig. 3. General case of two dissimilar flanges and their mode of deformation.

After application of loadings, the bolt length becomes

$$i_2 = v_2 + t_2 + t_2' - \delta_2 - \delta_2' \quad (68)$$

The basic displacement relationship is thus

$$i_2 - i_1 = (v_2 - v_1) + (t_2 - t_1) + (t_2' - t_1') - (\delta_2 - \delta_1) - (\delta_2' - \delta_1') \quad (69)$$

We also use the following relationships:

$$i_2 = i_0 + T_b e_b i_0 + q_b W_2 \quad (a)$$

$$v_2 = v_0 + T_g e_g v_0 - q_g (W_2 - H_{D2} - H_{T2}) \quad (b)$$

$$t_2 = t_0 + T_f e_f t_0 \quad (c)$$

$$t_2' = t_0' + T_f' e_f' t_0' \quad (d)$$

$$\delta_2 = q_{f2} M h_G + q_p p h_G + q_t \Delta h_g \quad (e)$$

$$\delta_2' = q_{f2}' M' h_G + q_p' p h_G + q_t' \Delta' h_g \quad (f)$$

$$i_1 = i_0 + q_b W_1 \quad (g) \quad (70)$$

$$v_1 = v_0 - q_g W_1 \quad (h)$$

$$t_1 = t_0 \quad (i)$$

$$t_1' = t_0' \quad (j)$$

$$\delta_1 = q_{f1} M h_G \quad (k)$$

$$\delta_1' = q_{f1}' M' h_G \quad (l)$$



The elastic deformation coefficients  $q_{b1}$ ,  $q_{g1}$ ,  $q_{b2}$ , and  $q_{g2}$  in Eqs. (70a-d) are further defined as

$$q_{b1} = \frac{\lambda_0}{A_b E_b b_1}, \quad (71a)$$

$$q_{g1} = \frac{v_0}{A_g E_g g_1}, \quad (71b)$$

$$q_{b2} = \frac{\lambda_0}{A_b E_b b_2}, \quad (71c)$$

$$q_{g2} = \frac{v_0}{A_g E_g g_2}. \quad (71d)$$

In Eqs. (70a-d), the term  $q_{f1}$  is a rotation of the flange due to a unit moment load,  $q_p$  is a rotation of the flange due to a unit internal pressure, and  $q_t$  is a rotation of the flange due to a unit temperature gradient between the hub and the ring. The quantities  $q_{f1}$ ,  $q_p$ , and  $q_t$  are obtained from the functional expression

$$q(L) = \frac{-w_c(L) + w_g(L)}{h_G}, \quad (72)$$

where  $h_G = (C - G)/2$ ,  $C$  is the bolt-circle diameter, and  $G$  is the gasket-centerline diameter. Values for the displacements  $w_c(L)$  and  $w_g(L)$  are obtained from Eqs. (46) and (47) with the appropriate unit values for the load:  $\Delta$ ,  $P$ , and  $A$ .

For  $q_{f1}$ , the modulus of elasticity used is that for the initial condition. For  $q_p$  and  $q_t$ , the moduli used are those for the final condition. The term  $q_{f2}$  is obtained from  $q_{f1}$  and the ratio of the initial and final elastic moduli; thus:

$$q_{f2} = q_{f1} \frac{E_1}{E_2}.$$

The moments and loads are defined by Eqs. (73a-n). The nomenclature used in these equations is analogous to that used in the ASME Code.<sup>1</sup> The symbol  $W$  represents a load,  $h$  represents a lever arm, and  $H$  represents a moment. The term  $H_D$  is the hydrostatic end force (in pounds) on the area inside the flange,  $H_G$  is the gasket load in pounds,  $H_T$  is the difference between the total hydrostatic end force and the hydrostatic end force on the area inside the flange,  $h_D$  is the radial distance in inches from the bolt circle to the circle on which  $H_D$  acts (as prescribed in Table UA-50 of the Code),  $h_G$  is the radial distance in inches from the gasket-load reaction to the bolt circle, and  $h_T$  is the radial distance in inches from the bolt circle to the circle on which  $H_T$  acts (as prescribed in Table UA-50). Symbols,  $C$ ,  $B$ ,  $G$ ,  $g_0$ , and  $p$  are defined earlier in this chapter. Again, a subscript 1 refers to the initial deformed state, a subscript 2 refers to the final deformed state, and primed quantities refer to the mating flange.

$$h_D = (C - B - g_0)/2 , \quad (a)$$

$$h'_D = (C - B' - g'_0)/2 , \quad (b)$$

$$h_T = [C - (G + B)/2]/2 , \quad (c)$$

$$h'_T = [C - (G + B')/2]/2 , \quad (d)$$

$$h_G = (C - G)/2 , \quad (e)$$

$$H_{D2} = \frac{\pi}{4} B^2 p , \quad (f)$$

$$H'_{D2} = \frac{\pi}{4} (B')^2 p , \quad (g) \quad (73)$$

$$H_{T2} = \frac{\pi}{4} (G^2 - B^2) p , \quad (h)$$

$$H'_{T2} = \frac{\pi}{4} [G^2 - (B')^2] p , \quad (i)$$

$$H_{G2} = W_2 - H_{D2} - H_{T2} , \quad (j)$$

$$H'_{G2} = W_2 - H'_{D2} - H'_{T2} , \quad (k)$$

$$M_1 = W_1 h_G = H_{G1} h_G \quad (l)$$

$$M_2 = H_{D2} h_D + H_{T2} h_T + H_{G2} h_G \quad (m)$$

and

$$M_2' = H_{D2}' h_D' + H_{T2}' h_T' + H_{G2}' h_G \quad (n)$$

Substituting Eqs. (70a-l) into Eq. (69) gives

$$\begin{aligned} T_b \epsilon_b \ell_0 + q_{b2} M_2 - q_{b1} M_1 &= T_g \epsilon_g v_0 - q_{g2} (M_2 - H_{D2} - H_{T2}) \\ &+ q_{g1} M_1 + T_f \epsilon_f \ell_0 + T_f' \epsilon_f' \ell_0' - h_G (q_{f2} M_2 + q_p p + q_t \Delta - q_{f1} M_1) \\ &- h_G (q_{f2}' M_2' + q_p' p + q_t' \Delta' - q_{f1}' M_1') \quad (74) \end{aligned}$$

In order to eliminate  $M_1$  and  $M_2$  from Eq. (74), Eqs. (73l and m) are used; the sixth term on the right-hand side of Eq. (74) then becomes

$$-h_G \{ q_{f2} [H_{D2} h_D + H_{T2} h_T + (M_2 - H_{D2} - H_{T2}) h_G] + q_p p + q_t \Delta - q_{f1} M_1 h_G \} .$$

The last term in Eq. (74) is treated similarly. Collecting terms containing  $M_2$  on the left gives:

$$\begin{aligned} (q_{b2} + q_{g2} + h_G^2 q_{f2} + h_G^2 q_{f2}') M_2 &= (q_{b1} + q_{g1} + h_G^2 q_{f1} + h_G^2 q_{f1}') M_1 \\ &+ T_g \epsilon_g v_0 + T_f \epsilon_f \ell_0 + T_f' \epsilon_f' \ell_0' - T_b \epsilon_b \ell_0 + q_{g2} (H_{D2} + H_{T2}) \\ &- h_G q_{f2} [H_{D2} (h_D - h_G) + H_{T2} (h_T - h_G)] \\ &- h_G q_{f2}' [H_{D2}' (h_D' - h_G) + H_{T2}' (h_T' - h_G)] \\ &- h_G (q_p + q_p') p - h_G (q_t \Delta + q_t' \Delta') \quad (75) \end{aligned}$$

Defining

$$Q_1 = q_{b1} + q_{g1} + h_G^2 q_{f1} + h_G^2 q'_{f1}$$

and

$$Q_2 = q_{b2} + q_{g2} + h_G^2 q_{f2} + h_G^2 q'_{f2}$$

and using the given definitions of  $H_D$ ,  $H'_D$ ,  $H_T$ , and  $H'_T$ , Eq. (75) becomes

$$\begin{aligned} M_2 = & \frac{Q_1}{Q_2} M_1 + \frac{1}{Q_2} (T_g \epsilon_g v_0 + T_f \epsilon_f t_0 + T'_f \epsilon'_f t'_0 - T_b \epsilon_b t_0) \\ & + \frac{\pi h_G}{4Q_2} \left\{ \left[ \frac{q_{g2}}{h_G} - q_{f2} (h_T - h_G) - q'_{f2} (h_T - h_G) - q'_{f2} (h'_T - h'_G) \right] G^2 \right. \\ & \left. - [q_{f2} B^2 (h_D - h_T) + q'_{f2} (B')^2 (h'_D - h'_T)] \right\} p \\ & - \frac{h_G}{Q_2} (q_p + q'_p) p - \frac{h_G}{Q_2} (q_t \Delta + q'_t \Delta') \quad (76) \end{aligned}$$

In order to compute the flange stresses under the various loading conditions, it is necessary to compute the flange moment  $M_2$  or  $M'_2$ . From Eq. (75m) and the definitions in Eqs. (73a-k),

$$M_2 = \frac{\pi}{4} p [B^2 h_D + (G^2 - B^2) h_T - G^2 h_G] + M_2 h_G \quad (77a)$$

And similarly for the mating flange,

$$M'_2 = \frac{\pi}{4} p \left\{ (B')^2 h'_D + [G^2 - (B')^2] h'_T - G^2 h'_G \right\} + M'_2 h'_G \quad (77b)$$

The computer program was written to separately evaluate the various effects involved in bolt-load changes. The residual bolt load due to

temperature differences that produce differential axial strain is

$$W_{2a} = W_1 + \frac{1}{Q_1} (T_g \epsilon_g v_0 + T_f \epsilon_f t_0 + T_f' \epsilon_f' t_0' - T_b \epsilon_b t_0) \quad (78)$$

The residual bolt load, after internal pressure (acting in an axial direction) has transferred the bolt load on the gasket to a tensile load on the attached pipes due to a shift in lever arms, is given by:

$$W_{2b} = W_1 + \frac{\pi h_G}{4 Q_1} \left\{ \left[ \frac{q_{gl}}{h_G} - q_{f1} (h_T - h_G) - q_{f1}' (h_T' - h_G) \right] G^2 - [q_{f1} B^2 (h_D - h_T) + q_{f1}' (B')^2 (h_D' - h_T')] \right\} p \quad (79)$$

The total effect of internal pressure due to both the shift in the lever arms and the radial effect of pressure acting on the integral flange(s) and/or on the inside surface of a blind flange is given by:

$$W_{2c} = W_{2b} - \frac{h_G}{Q_1} (q_p + q_p') p \quad (80)$$

The residual bolt load due to a temperature difference between the hub and the ring is given by:

$$W_{2d} = W_1 - \frac{h_G}{Q_1} (q_t \Delta + q_t' \Delta') \quad (81)$$

A slight modification of the above is required for the case of a blind flange. If we designate the blind flange as that with the "primed" nomenclature, then all\* of Eqs. (70a-l) are valid except Eqs. (70f and l) for  $\delta_1'$  and  $\delta_2'$ .

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\* For  $v_2$  it should be noted that  $H_{D2} - H_{T2} = \pi G^2 p / 4$ ; hence, this equation is valid for blind flanges.

For blind flanges,  $N$  is used rather than  $M$  as the loading parameter because the relationship  $M = N(a - b)$  is not valid for the blind-flange analysis. For blind-flange analysis, Eq. (65) gives a value of  $w_c$ ; here  $-w_c$  is the equivalent of  $-w_c + w_g$  in Eq. (72) because  $w_g = 0$  in the blind-flange analysis. For blind flanges we define

$$q_f' = \frac{(-w_c)N}{h_G^2}, \quad (82)$$

where  $(-w_c)_N$  is the axial displacement per unit total bolt load  $N$ . The equation for  $N_2$  for a blind flanged joint is then:

$$\begin{aligned} N_2 = \frac{Q_1}{Q_2} N_1 + \frac{1}{Q_2} (T_g \epsilon_g v_0 + T_f \epsilon_f t_0 + T_f' \epsilon_f' t_0' - T_b \epsilon_b \lambda_0) \\ + \frac{\pi h_G}{4 Q_2} \left\{ \frac{q_{g2}}{h_G} - q_{f2} (h_T - h_G) G^2 - q_{f2} B^2 (h_D - h_T) \right\} p \\ - \frac{h_G}{Q_2} (q_p + q_p') p - \frac{h_G}{Q_2} q_t \Delta. \quad (83) \end{aligned}$$

In Eq. (83) the primed values refer to properties of the blind flange.

After the internal pressure has transferred the bolt load on the gasket to a tensile load on the attached pipe due to a shift in the lever arms, the residual bolt load for the case where a blind flange is used is

$$N_{2b} = N_1 + \frac{\pi h_G}{4 Q_1} \left\{ \frac{q_{g1}}{h_G} - q_{f1} (h_T - h_G) G^2 - q_{f1} B^2 (h_D - h_T) \right\} p. \quad (84)$$

It should be noted that  $q_t \Delta'$  does not exist for an integral flange mated to a blind flange.

The combined effect of all of the above is also obtained from the computer program by calculating  $N_2$  from Eqs. (76) and (83).

### External Moment Loading

Up to this point, all loads considered have been axisymmetric. For flanged joints in pipe lines, there is one other significant loading; that is, the bending moment imposed on the flanged joint by the attached pipe. To distinguish this from the local moments applied to the flange ring, the bending moment will be designated as an "external" moment. The external moment can be represented by a distributed axial edge force acting on the attached pipe:

$$F_M(\theta) = F_M \cos \theta , \quad (85)$$

where  $\theta$  = angle around the circumference ( $\theta = 0$  at the point of maximum tensile stress in the pipe due to the external moment). Since this report deals only with cases in which all contact occurs within the bolt-hole circle, a reasonably good first approximation for the effects of the external moment loading can be obtained by replacing the distributed axial force  $F_M(\theta)$  with the axisymmetric tensile force  $F_M = F_M(\max)$ . Then, since  $F_M$  is axisymmetric, there is some pressure  $p^*$  that will produce the same axial force in the pipe; or alternately, there is an equivalent pressure  $p^*$  that will produce an axial stress in the pipe which is equal to the maximum tensile stress  $S_b$  produced by an external moment. The relation between  $p^*$  and  $S_b$  is given by

$$p^* = 4S_b g_0 / D_o , \quad (86)$$

where  $S_b$  is the bending stress in the attached pipe due to the external moment. The change in bolt load  $W_{2b}$  is then obtained by replacing  $p$  with  $p + p^*$  in Eqs. (79) and (84). It should be noted that this equivalent pressure is included only in Eqs. (79) and (84) and not in Eq. (80).

## 8. COMPUTER PROGRAM

A Fortran computer program named FLANGE has been written to carry out the calculations according to the analyses described in this report. The program calculates appropriate loads, stresses, and displacements for the flanges, bolts, and gaskets when the flanged joint is subjected to internal pressure, moment, and/or thermal gradient loadings; thus, the program is much more general than that needed only to determine compliance with the ASME Boiler and Pressure Vessel Code. The program also has the advantage of internally computing the values of the Code variables  $F$ ,  $V$ , and  $f$  that must otherwise be extracted manually from the curves given in Code Figs. UA-51.2, UA-51.3, and UA-51.6. Loose hubbed flanges, which are covered by the Code, however, are not covered by the computer program.

The main function of this chapter is to describe the input and output for the various computational options available to the user. For more detailed information, the reader is urged to carefully study the examples given in Appendix A where a flanged joint, selected from API Standard 605 (Ref. 8), is analyzed. Several sample problems are worked, and the data input and program output are given for the various program options along with a discussion of the results. Flowcharts and listings of the program and its subroutines are given in Appendix B. In the following sections, the input data for option control and the input data and program output for Code compliance calculations and for more general calculations are discussed.

### Option Control Data Card

The first card of each data set, herein called the option control card, contains control information for execution of the various program options. It contains information specifying the type of flange being analyzed, the boundary condition placed on the displacement  $(u_r)_{x=h}$ , the stresses and other variables to be calculated, and the joint configuration and which flange (of the pair) is to be analyzed. These specifications are under control of the four variables ITYPE, IBOND,



ICODE, and MATE. The admissible values and their significance are as follows.

ITYPE (indicates the type of flange being analyzed)

- 1 for a tapered-hub flange
- 2 for a straight-hub flange
- 3 for a blind hub

IBOND (specifies the displacement  $u_r$  at  $x = h$ )

- 0 for  $(u_r)_{x=h} = 0$  to conform with the ASME Code basis
- 1 (see footnote)\*
- 2 for  $(u_r)_{x=h} \neq 0$  [see Eq. (20-6) of this report]

ICODE (controls the amount of output data)

- 0 for a wide variety of stresses, moments, and loads for specified moment, pressure, and  $\Delta T$
- 1 (see footnote)\*
- 2 for a select list for checking Code compliance in accordance with Section VIII, Div. 1 of the ASME Code

MATE (specifies the joint configuration and the flange to be analyzed)

- 1 for only one flange to be analyzed (This is the situation for ASME-Code related calculations.)
- 2 for two identical flanges mated together
- 3 for the first of two flanges that are not identical!, neither of which is a blind flange
- 4 for the second of two flanges that are not identical, neither of which is a blind flange
- 5 for a blind flange
- 6 for a flange that is mated with a blind flange.

The data card with the above information is followed by other data cards containing physical-property data, etc., for the particular flange being analyzed. Since the program can be used to analyze any number of flanges

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\* In the original conception of the program, IBOND and ICODE were envisioned as controlling additional calculations that were not implemented in the present version. As it is now written, the program does not distinguish between values of 0 or 1 nor between 2 and numbers greater than 2 for either IBOND or ICODE.

or flanged joints sequentially (as done in the examples of Appendix A), the data card set for each flange must start with an option-control data card.

Different types of flanges and different types of calculations have different input data requirements. These data and their formats are discussed in the following sections.

### Input for Code-Compliance Calculations

Since the ASME Code calculation procedures consider only one flange at a time, the input data requirements for the computer program are quite simple and straightforward. Input data are completely prescribed by the three data cards illustrated in Table 9. The nomenclature is the same as that used in the Code.

The first card is the option control card discussed in the previous sections. The first variable ITYPE may be equal to 1, 2, or 3, depending on the type of flange being analyzed. The next variable IBOND will always be 0, in which case the displacement  $u_r$  will be equal to zero at  $x = h$ , as specified by the Code. The third variable ICODE will always be 2 and will therefore cause the program to compute the stresses in accordance with Code paragraph UA-50 for straight or tapered-hub flanges or paragraph UG-34(c)(2) for blind flanges. The last variable MATE will always be 1 for Code-compliance calculations. This variable essentially controls the bolt-load-change calculations made by the program. Since the ASME Code does not consider bolt-load changes in determining compliance, when MATE = 1 these calculations are not performed.

The second card in the data set enters the physical dimensions of the flange being analyzed, as shown in Table 9. These dimensions are the outside and inside diameters of the flange ring A and B, the ring thickness  $t$ , the pipe-wall thickness  $g_0$ , the hub thickness at the hub-to-ring juncture  $g_1$ , the hub length  $h$ , the bolt-circle diameter  $C$ , and the internal pressure. All dimensions are expressed in inches; the pressure is in pounds per square inch.

Table 9. Input data for ASME bolt and flange stress calculation, using symbols defined in ASME Code, Section VIII, Division 1, Appendix 11

Option-Control Card (Read-in in FLANGE)

Column number	5	10	15	20
Variable	ITYPE <sup>d</sup>	IBOND	ICODE	MATE
Value	1, 2, or 3	0	2	1

Second Card (Read-in in TAPHUB, STIRUB, or BLIND)<sup>b,c</sup>

Column number	0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80
Quantity	Flange outer diameter A	Flange inner diameter B	Ring thickness t	Pipe-wall thickness $R_2$	Hub thickness $R_1$	Hub length h	Bolt-circle diameter c	Pressure p
Variable	XA	XB	TH	GO	G1	HL	C	PRESS

Third Card (Read-in in ASMEIN)<sup>d</sup>

Column number	0-10	11-20	21-30	31-40 <sup>f</sup>	41-50	51-60	61-70 <sup>e</sup>	72 <sup>g</sup>	73-80 <sup>h</sup>
Quantity	Gasket factor m	Minimum design seating stress y	Gasket outer diameter $G_o$	Gasket inner diameter $G_i$	Allowable bolt stress at design temperature $S_b$	Allowable bolt stress at atmospheric temperature $S_a$	Bolt cross-sectional area $A_b$	Option l	Basic gasket seat width $b_o$
Variable	XM	Y	GOUT	GIN	SB	SA	AB	INBO	BO

<sup>a</sup>When ITYPE = 2 for a ring flange,  $g_o$ , on the second card, should be a suitably small value, but not zero (e.g., 0.01).

<sup>b</sup>Subroutines TAPHUB and STIRUB call both ASMEIN and FLGDW; BLIND calls ASMEIN.

<sup>c</sup>For ITYPE = 2,  $g_o$  must be entered;  $g_i$  and h are not used. For ITYPE = 3, B,  $g_o$ ,  $g_i$ , and h are not used.

<sup>d</sup>If l (Column 72) is 0, the program computes b,  $b_o$ , and G for the particular case of  $b_o = N/2 = 1/2(G_o - G_i)/2$  as defined in Table UA-49.2 sketches (1a) and (1b) of the Code. Columns 73-80 may then be left blank. For other values of  $b_o$ , enter l = 2. In this case, the value of  $G_i$  is not used and thus columns 31-40 may be left blank.

<sup>e</sup>Column 71 is blank.

The third card inputs other physical data, including the gasket factor  $m$ , the minimum-design seating stress  $y$ , the outside diameter of the gasket  $G_o$ , the inside diameter of the gasket  $G_i$ , the allowable bolt stress at design temperature  $S_b$ , the allowable bolt stress at ambient temperature  $S_a$ , the total cross-sectional area of the bolts  $A_b$ , an option-selecting variable  $l$ , and the basic gasket-seating width  $b_o$ . The option variable  $l$  controls the calculation of  $b$  and  $G$ .

#### Output for Code-Compliance Calculations

For Code-compliance calculations, all of the output for each flange being analyzed is printed on a single page (e.g., see examples 1 and 2 of Appendix A). The program prints the input data followed by the effective gasket seating width  $b_o$  and the loads, bolt stresses, and moments identified under the headings shown in Table 10. For compliance with Code criteria, the value of SB1 must not exceed the allowable bolt stress at design temperature, and the value of SB2 must not exceed the allowable bolt stress at atmospheric\* temperature.

Immediately below, the program prints the flange stresses needed for comparison with the ASME Code criteria. For tapered-hub and straight-hub flanges (ITYPE = 1 or 2), the program prints five stresses under the two headings "ASME FLANGE STRESSES AT OPERATING MOMENT, MOP" and "ASME FLANGE STRESSES AT GASKET SEATING MOMENT." The stresses are identified as follows:

- $2/3(SH)$  = two-thirds of the longitudinal stress on the outside surface at the small end of the hub,
- ST = the tangential stress on the hub side of the ring,
- SR = the radial stress on the hub side of the ring,
- $(SH + ST)/2$  = the average of SH and ST, and
- $(SR + ST)/2$  = the average of SR and ST.

---

\* Although "ambient" would probably be a better term here, the word "atmospheric" is used as it is used in the Code.

Table 10. Output data identification, ICODE = 2,  
(ASME Code stresses)

ASME Code symbol <sup>a</sup>	Program symbol	Description <sup>a</sup>	
b <sub>o</sub>	BO	See ASME Code, Table UA-49.2. (This will be input data for I = 2.) <sup>b</sup>	
H	HM11	$\pi G^2 p / 4$	
	HM12	$2\pi b G m p$	
W <sub>m1</sub>	HM1	$\pi G^2 p / 4 + 2\pi b G m p$	
	SB1	Bolt stress, $W_{m1} / A_b$	
W <sub>m2</sub>	HM2	$\pi b G r$	
	SB2	Bolt stress, $W_{m2} / A_b$	
(c)	MOP	$H_G h_G + H_I h_I + H_D h_D$	
(d)	MGS	$[(A_m + A_b) S_a / 2] \times [(C - G) / 2]$	Except for ITYPE = 3 (Blind flanges)
	MGS1	$W_{m2} \times [(C - G) / 2]$	

<sup>a</sup>All symbols are defined in the ASME Boiler Code, Section VIII, Div. 1 (1971), Appendix II.

<sup>b</sup>See Footnote d of Table 9.

<sup>c</sup>MOP is the operating moment as defined by the ASME Code.

<sup>d</sup>MGS is the gasket seating moment as defined by the ASME Code.

For compliance with the Code Criteria, each of the above values printed under the first heading must not exceed the allowable stress for the flange material at the design temperature. The values printed under the second heading must not exceed the allowable stress for the flange material at atmospheric temperature.

For blind flanges (ITYPE = 3), the program prints the following five quantities under the heading "ASME CODE STRESSES FOR BLIND FLANGE":

SP = the stress due to pressure loading only,

SM1 = the stress due to the bolt load  $W_{m1}$  only, where  $W_{m1} =$   
 $\pi G^2 p / 4 + 2\pi b G m p,$

SOP = the stress at operating conditions,

SM2 = the stress due to the bolt load  $W_{m2}$ , where  $W_{m2} = \pi b D y$ , and

SGS = the stress at gasket-seating conditions.

For Code compliance, SOP must not exceed the allowable stress for the flange material at design temperature, and SGS must not exceed the allowable stress at atmospheric temperature.

### Input for General Purpose Calculations

When the computer program is used for general purpose calculations, (i.e., when it is used for calculating displacements and stresses other than those needed specifically for checking Code compliance), the user may select almost any combination of admissible values for the four variables ITYPE, I~~BN~~D, I~~CD~~E, and MATE coded in the option control data card. The only specific requirement is that the variable I~~CD~~E must be less than two for other than Code-compliance calculations. In this case the input data are structured somewhat differently than those described in the previous section.

When I~~CD~~E = 0 and MATE = 1, (i.e., only one flange is to be analyzed and the user does not wish to obtain bolt load changes), three data cards are needed as shown in Table 11. These are the option-control card (for which ITYPE may be 1, 2, or 3 and I~~BN~~D may be 0 or 2) and two physical-property data cards.

When I~~CD~~E = 0 and MATE = 2, 3, ... 6, the program will analyze a pair of flanges mated together and give bolt load changes. If MATE = 2, the program performs the calculations for a pair of identical flanges mated together. The input data requirements include the data cards shown in Table 11 plus the three cards shown in Table 12. These last three cards contain data on the physical properties of the bolts and gasket, supplemental data on the initial and final state of the flange, and other conditions. For this case, the six cards listed below complete the input data set when MATE = 2.

Table 11. Input data for the general purpose analysis of a single flange and partial data for paired flanges

Option-Control Card: [FORMAT (4I5) read-in in FLANGE]

Column number	5	10	15	20
Variable	ITYPE <sup>a,b</sup>	IBOND	ICODE	MATE <sup>c</sup>
Value	1, 2, or 3	0 to 2	0	1 or (2)

Second Card: [FORMAT (8E10.5); read-in in TAPIUB, STHUB, or BLIND]

Column number	0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80
Quantity	Flange outer diameter A	Flange inner diameter B	Ring thickness t	Pipe-wall thickness s <sub>0</sub>	Hub thickness s <sub>1</sub>	Hub length h	Bolt-circle diameter C	Pressure p
Variable	XA	XB <sup>b</sup>	TH	GO <sup>a,b</sup>	G1 <sup>a,L</sup>	HL <sup>a,b</sup>	C	PRESS

Third Card: [FORMAT (5E10.5); read-in in TAPHUB, STHUB, or BLIND]

Column number	0-10	11-20	21-30	31-40	41-50
Quantity	Moment applied to flange ring M	Coefficient of thermal expansion c <sub>f</sub>	Thermal gradient pipe or hub to ring Δ	Modulus of elasticity flange E	Gasket centerline diameter 2g
Variable	XMOA <sup>b</sup>	EF <sup>b</sup>	DELTA <sup>b</sup>	YM	G

<sup>a</sup>When ITYPE = 2, GO must be entered; G1 and HL are not used.

<sup>b</sup>When ITYPE = 3, XB, GO, G1, HL, EF, and DELTA are not used; the value for XMOA is the total bolt load W.

<sup>c</sup>When MATE = 2, additional data as described in Table 12 are also required.

Table 12. Last three input data cards for the general purpose analysis of paired flanges

Card No. 4 or 7:<sup>3</sup> [FORMAT (7E10.5); read-in in FLGDW]

Column number	0-10	11-20	21-30	31-40	41-50	51-60	61-70
Quantity	Nominal bolt diameter	Initial state; bolt modulus of elasticity $E_b$	Bolt coefficient of thermal expansion $\alpha_b$	Final state; bolt temperature $T_b$	Outside diameter of gasket	Inside diameter of gasket	Cross-sectional root area of all bolts
Variable	BSIZE <sup>b</sup>	YB	EB	TB	XG0 <sup>c</sup>	XG1 <sup>c</sup>	AB

Card No. 5 or 8:<sup>3</sup> [FORMAT (6E10.5); read-in in FLGDW]

Column number	0-10	11-20	21-30	31-40	41-50	51-60
Quantity	Gasket thickness $t_0$	Initial state; gasket modulus of elasticity $E_g$	Gasket coefficient of thermal expansion $\alpha_g$	Final state; gasket temperature $T_g$	A free bolt length variable	Equivalent pressure see Eq. (8b) of text $p^*$
Variable	VO	YG	EG	TI <sup>d</sup>	FACE <sup>b</sup>	PBE

Card No. 6 or 9:<sup>4</sup> [FORMAT (7E10.5); read-in in FLGDW]

Column number	0-10	11-20	21-30	31-40	41-50	51-60	61-70
Quantity	Initial bolt load $W_1$	Final state temperature of flange, side one $T_{f1}$	Final state temperature of flange, side two $T_{f2}$	Final state flange modulus of elasticity, side one $E_{f1}$	Final state flange modulus of elasticity, side two $E_{f2}$	Final state bolt modulus of elasticity $E_{b2}$	Final state gasket modulus of elasticity $E_g$
Variable	W1	TF <sup>d</sup>	TFP <sup>d</sup>	YF2	YFP2	YB2	YG2

<sup>3</sup>First card number applies when MATE = 2; second number applies when MATE = 3 and 4 or 5 and 6.

<sup>b</sup>The effective bolt load is calculated as  $t_0 = XLB = TI + THP + VO + BSIZE + FACE$ .

<sup>c</sup>Values for  $G_1$  and  $A_g$  are calculated using input variables XG0 and XG1.

<sup>d</sup>Initial-state temperatures are defined as zero.



<u>Card No.</u>	<u>Identification</u>
1	Option control card with MATE = 2
2 } 3 }	Data cards per Table 11
4 } 5 } 6 }	Data cards per Table 12

When ICODE = 0 and MATE = 3, the program performs the calculations for a pair of nonidentical flanges, neither of which, however, is blind (i.e., ITYPE = 1 or 2 ≠ 3 on the option-control card). Data for the first flange of the pair follows the option-control card. Data for the second flange in the pair will follow an option-control card with MATE = 4. The three cards described in Table 12 will then complete the data requirements. The complete input data set for analyzing a pair of nonidentical flanges (neither of which is blind) consists of the following nine cards.

<u>Card No.</u>	<u>Identification</u>
1	Option-control card, ITYPE ≠ 3, ICODE = 0, MATE = 3
2 } 3 }	Data cards per Table 11 for first flange of pair
4	Option-control card, ITYPE ≠ 3, ICODE = 0, MATE = 4
5 } 6 }	Data cards per Table 11 for second flange of pair
7 } 8 } 9 }	Data cards per Table 12

When ICODE = 0 and MATE = 5, the program performs the calculations for a flanged joint that is closed with a blind flange. For this option,

the blind flange is designated as the first flange and the mating flange is designated as the second with MATE = 6. As before, the input data set is completed by using the data cards described in Table 12. The complete input data set for this case consists of the following nine cards.

<u>Card No.</u>	<u>Identification</u>
1	Option-control card, ITYPE = 3, ICODE = 0, MATE = 5
2 } 3 }	Data cards per Table 11 for blind flange
4	Option-control card, ITYPE = 1 or 2, ICODE = 0, MATE = 6
5 } 6 }	Data cards per Table 11 for second flange
7 } 8 } 9 }	Data cards per Table 12

#### Output from General Purpose Calculations

The amount and format of the data printed out are determined predominantly by the number and types of flange being analyzed, which in turn are determined by the value of the option-control variable MATE. When MATE = 1, the output consists of one page of printout, which gives (1) the input data; (2) the three sets of stresses for moment loading only (the bolt load for blind flanges), pressure loading only, and temperature-gradient (hub to ring) loading only (except for blind flanges); and (3) the displacements produced by the calculated stresses. The symbols used on the printout are explained in Tables 13 and 14.

When MATE = 2, the output consists of three pages of printout. The first page gives (1) the input data and (2) the parameters involved in the bolt load-change calculations. The second page gives (1) the loadings, (2) the residual bolt loads, and (3) the initial and residual moments. The symbols used in the first and second page of printout are explained in Tables 15 and 16. The third page gives the stresses and

Table 13. Output data identification, stresses, displacements, and rotation

Theory Symbol	Program symbol	Description
$(\sigma_t)_o$	SLSO <sup>a</sup>	Stress, longitudinal, small end of hub, outside surface
$(\sigma_t)_i$	SLSI <sup>a</sup>	Stress longitudinal, small end of hub, inside surface
$(\sigma_c)_o$	SCSO <sup>a</sup>	Stress, circumferential, small end of hub, outside surface
$(\sigma_c)_i$	SCSI <sup>a</sup>	Stress, circumferential, small end of hub, inside surface
$(\sigma_t)_o$	SLLO	Stress, longitudinal, large end of hub, outside surface
$(\sigma_t)_i$	SLLI	Stress, longitudinal, large end of hub, inside surface
$(\sigma_c)_o$	SCLO	Stress, circumferential, large end of hub, outside surface
$(\sigma_c)_i$	SCLI	Stress, circumferential, large end of hub, inside surface
$(\sigma_t)_o$	STH	Stress, tangential, hub side of ring, at $r = b$
$(\sigma_t)_i$	STF	Stress, tangential, face side of ring, at $r = b$
$(\sigma_r)_o$	SRH	Stress, radial, hub side of ring, at $r = b$
$(\sigma_r)_i$	SRF	Stress, radial, face side of ring, at $r = b$
$\delta_g$	ZG	Axial displacement at $r = g$
$\delta_c$	ZC	Axial displacement at $r = c$
$q_r^h$	QFHG	$-\delta_c + \delta_g$
$y_o$	YO	Radial displacement, small end of hub
$y_i$	YI	Radial displacement, large end of hub
	THETA	Rotation of ring at $r = b$
		<u>For blind flanges<sup>b</sup></u>
$\sigma_r, \sigma_t, r = o$	SRTO	Stress, $r = o$ , radial and tangential
$\sigma_r, r = g$	SGR	Stress, $r = g$ radial
$\sigma_t, r = g$	SGT	Stress, $r = g$ , tangential
$\sigma_r, r = c$	SCR	Stress, $r = c$ , radial
$\sigma_t, r = c$	SCT	Stress, $r = c$ , tangential
$\sigma_t, r = a$	SAT	Stress, $r = a$ , tangential
$\delta_c$	ZC	Axial displacement at $r = c$ ( $\delta = 0$ at $r = g$ )

<sup>a</sup>For "Straight Hub Flange," these are at juncture of hub with ring.

<sup>b</sup>All stresses are for the side of the flange opposite the pressure-bearing side. Stresses on the pressurized side of the flange have reversed signs.

Table 14. Output data identification when MATE = 2, 3 and 4,  
or 5 and 6

Theory symbol	Program symbol	Description
$q_{f1}^{h_G}$	QFHG	Axial displacement from C to G, unit moment load
$q_{p1}^{h_G}$	QPHG	Axial displacement from C to G, unit pressure load
$q_{t1}^{h_G}$	QTHG <sup>a</sup>	Axial displacement from C to G, unit DELTA
2b	XB <sup>a,b</sup>	Inside diameter
$g_0$	GO <sup>a,b</sup>	Pipe wall thickness
t	TH	Ring thickness
$E_{f1}$	YM <sup>b</sup>	Modulus of elasticity of flange material, initial state
$E_{f2}$	YF2 <sup>c</sup>	Modulus of elasticity of flange material, final state
$\epsilon_f$	EF <sup>b</sup>	Coefficient of thermal expansion of flange material
( )'	( )P	The above nine symbols with a prime mark (') on the theory symbols are for the mating flange. The program symbol has the added final letter "p."

<sup>a</sup>For blind flanges, these values are not significant; an artificial value of -1.0000 is printed out.

<sup>b</sup>These values are input data for flange side one, input cards 2 and 3 (see Table 11). For MATE = 2, these values, along with calculated values of QFHG, QPHG, and QTHG, are used for side one and side two (i.e., an identical pair). If MATE = 3 or 5, the primed values are stored; the unprimed values are read in by input cards 5 and 6, and values of QFHGP, QPHGP, and QTHGP are calculated.

<sup>c</sup>Input from card 6 for MATE = 2, card 9 for MATE = 3 and 4 or 5 and 6 (see Table 11).

Table 15. Output data identification, MATE = 2, 3 and 4,  
or 5 and 6, bolts, gasket, and loadings data

Theory symbol	Program symbol	Description <sup>a</sup>
$l$	XLB	Effective bolt length
$A_b$	AB	Cross-sectional root area of all bolts
C	C	Bolt-circle diameter
$E_{b1}$	YB	Modulus of elasticity, bolts, initial state
$E_{b2}$	YB2	Modulus of elasticity, bolts, final state
$\epsilon_b$	EB	Coefficient of thermal expansion, bolts
$v_0$	VO	Gasket thickness
	XGO	Outside diameter of gasket
	XGI	Inside diameter of gasket
$E_{g1}$	YG	Modulus of elasticity of gasket, initial state
$E_{g2}$	YG2	Modulus of elasticity of gasket, final state
$\epsilon_g$	EG	Coefficient of thermal expansion, gaskets
$W_1$	W1	Initial total bolt load
$T_b$	TB	Temperature of bolts, final state
$T_{f2}$	TF	Temperature of flange ring, side one, final state
$T'_{f2}$	TFP	Temperature of flange ring, side two, final state
$T_g$	TG	Temperature of gasket, final state
$\Delta$	DELTA	Thermal gradient, pipe/hub to ring, side one
$\Delta'$	DELTA P	Thermal gradient, pipe/hub to ring, side two
P	PRESS	Internal pressure

<sup>a</sup>All values are input data, except XLB which is calculated by the equation:  $XLB = TH + THP + VO + BSIZE + FACE$ .

Table 16. Output data identification, MATE 2, 3 and 4, or 5 and 6, residual bolt loads and moments

Theory symbol	Program <sup>a</sup> symbol	Effect included
W <sub>2a</sub>	W2A	Relative change in temperature of bolts, gasket, flange (AXIAL THERMAL)
W <sub>2b</sub>	W2B	Change in moment arms (MOMENT SHIFT)
W <sub>2c</sub>	W2C	Total pressure
W <sub>2d</sub>	W2D	Thermal gradient, pipe/hub to ring (DELTA THERMAL)
W <sub>2</sub>	W2	All of the above, plus change in modulus of elasticity (COMBINED)

<sup>a</sup>The change in bolt load (e.g.,  $W1 - W2A$ ) and ratio of residual to initial bolt load (e.g.,  $W2A/W1$ ) are also printed out, along with the corresponding values of the initial moment ( $M1$ ) and residual moments,  $M2A$ , ...,  $M2P$ . The residual moment identifiers with final letter P (for prime) are for the first entered of a pair of nonidentical flanges. If the pair of flanges are identical, then  $M2B = M2BP$ , etc. The residual moment values are not significant for blind flanges,  $ITYPE = 3$ ; therefore, residual bolt loads are used for blind flanges.

displacements as for the case when  $MATE = 1$  plus the stresses and displacements for combined loading. The heading includes the value of the residual moments  $M2 = M2P$  used for the combined-loading calculations.

When  $MATE = 3$  and 4 or 5 and 6, the output consists of four pages of printout. The first two pages have the same format as for the case when  $MATE = 2$ , except input data for both of the (nonidentical) flanges are printed. The residual moments on the last line of page 2 apply to flange one; those on the preceding line apply to flange two. The last two pages of printout are for flange one and flange two, respectively, and are identical in format to the third page of the printout for the case when  $MATE = 2$ .

### Acknowledgment

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**APPENDIX A**  
**EXAMPLES OF APPLICATION OF COMPUTER PROGRAM FLANGE**



## APPENDIX A

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## INTRODUCTION

Several examples have been selected to illustrate the input/output data of the computer program FLANGE and the significance of the results. The flange selected for analysis is one included in API Standard 605.\* The particular size and rating selected was the 60-in., 300-lb tapered-hub flange. This particular flange represents a design in which the bolt stresses and flange stresses are close to the upper limits set in API-605.

Six examples are included:

1. A Code stress calculation is performed for a tapered-hub flange at its rated pressure of 720 psi at 100°F. The results show that this particular flange does indeed meet the criteria given in API-605 at 720 psi and 100°F.
2. A Code stress calculation is performed for a blind flange to match the 60-in., 300-lb API-605 tapered-hub flange. The thickness of the blind flange was selected so that its maximum stress was the allowable flange stress of 17,500 psi used in API-605.
3. A blind flange bolted to a tapered-hub flange under pressure loading only is analyzed.
  - (a) For an initial bolt stress equal to the API-605 allowable stress for the bolting material of 20,000 psi, the results indicate that the flanged joint will probably leak at its rated pressure of 720 psi at 100°F.
  - (b) For an initial bolt stress of 44,300 psi, the results indicate that the flanged joint will pass a hydrostatic test of 1.5 x 720 psi at ambient temperature.
4. A tapered-hub flange bolted to an identical tapered-hub flange with an initial bolt stress of 46,100 psi is analyzed.

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\* *Large-Diameter Carbon Steel Flanges (Size: 26 Inches to 30 Inches, Inclusive, Nominal Pressure Rating: 75, 150, and 300 lb), API Standard 605, 1st Ed., American Petroleum Inst., New York, 1967.*

- (a) For pressure loading only, the results indicate that the flanged joint will hold a hydrostatic test pressure of  $1.5 \times 720$  psi.
- (b) For pressure loading of 300 psi (API-605 rated pressure at 850°F) plus an external bending moment that produces an axial stress in the attached pipe of 7500 psi, the results indicate that the flanged joint is adequate to carry these loads.

### DETAILS OF THE FLANGE USED IN THE EXAMPLES

A sketch of the tapered-hub flange is shown in Fig. A.1. The dimensions are as specified in API-605. The inside diameter and dimensions B (and therefore  $g_0$  and  $g_1$ ) are not specified in API-605. For the purpose of checking ratings, the following equation given in API-605 was used to establish B:

$$B = D_o - 2t_p \quad (A.1)$$

where

$D_o$  = nominal outside diameter of pipe, in.;

$t_p = p_1 D_o / 2(0.875)S$  (but not less than 0.25), in.;

$p_1$  = rated pressure at 100°F, psi;

0.875 = assumed pipe-wall tolerance; and

$S = 20,000$  psi, the allowable stress at 100°F.

The definition of  $t_p$ , with  $D_o = 60$  in. and  $p_1 = 720$  psi, leads to  $t_p = g_0 = 1.2343$  in. Equation (A.1) gives  $B = 57.5314$  in. and  $g_1 = (X-B)/2 = 2.7030$  in.

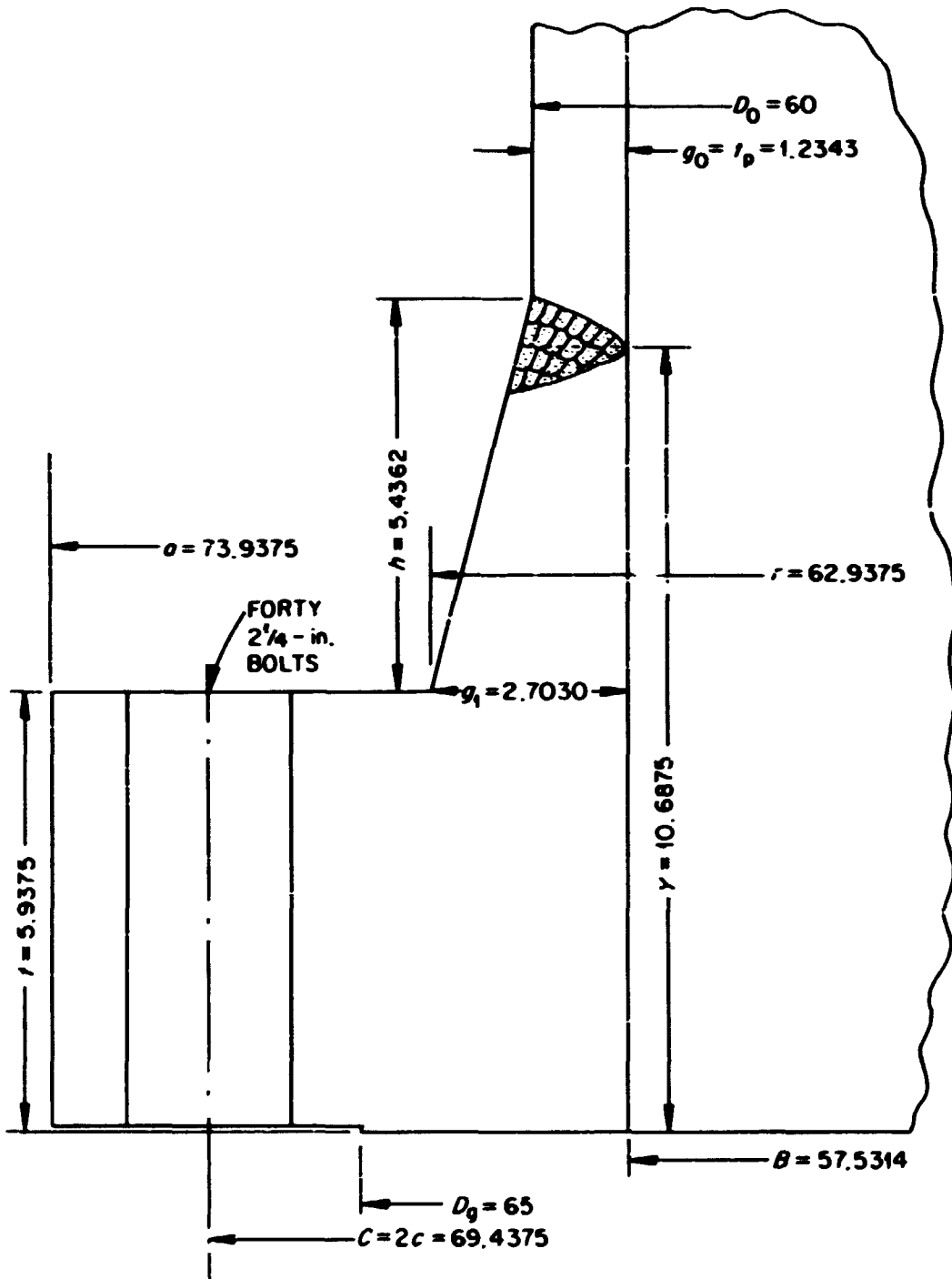
For the purpose of checking ratings, the hub length  $h$  was calculated by the equation given in API-605:

$$h = Y - t + 0.176g_0 + 0.469$$

Dimensions  $Y$  and  $t$  are shown in Fig. A.1. For this flange:

$$h = 10.6875 - 5.9375 + 0.176(1.2343) + 0.469 = 5.4362 \text{ in.}$$

The API-605 standard states that flange ratings were based on use of a 1/16-in.-thick, compressed-asbestos, flat ring-shaped gasket, with an inside diameter 1/4 in. larger than the outside diameter of the pipe and with an outside diameter equal to the raised-face diameter. For the 60-in., 300-lb flange, the gasket inside diameter is 60.25 in.; its



**DIMENSIONS IN INCHES**

Fig. A.1. Dimensions (in inches) of 60-in., 300-lb API-605 tapered-hub flange. The terms B, R, C,  $D_0$ , X, and A are diameters expressed in inches.

outside diameter is 65 in. According to the ASME Code, for a 1/16-in.-thick asbestos gasket,  $m = 2.75$ , and  $y = 3700$  psi.

The 60-in., 300-lb flange has forty 2-1/4-in.-diam. bolts. For an 8-pitch thread, the root area per bolt is 3.423 in.<sup>2</sup>, giving a total bolt root area of 136.92 in.<sup>2</sup>.

## ASME CODE CALCULATIONS, EXAMPLES 1 AND 2

The input data for examples 1 and 2 are shown in Table A.1. The source of all input for Cards 2 and 3 are contained in the previous section on flange details, except that the thickness of the blind flange was selected\* so that the controlling flange stress is 17,500 psi. Note that Card 2 is identical for examples 1 and 2 except for the value of  $t$ ; however,  $B$ ,  $g_0$ ,  $g_1$ , and  $h$  are not used for example 2 (blind flange), and any number (including zero) can be entered for these dimensions.

Example 1 is a Code stress calculation for the 60-in., 300-lb API-605 tapered-hub flange at its rated pressure of 720 psi at 100°F. The output data are shown in Table A.2. The value of  $SBI = 20,033$  psi is the controlling bolt stress, which essentially meets the API criterion value of a bolt stress not greater than 20,000 psi. The value of  $(SH + ST)/2 = 17,293$  psi under the heading "ASME FLANGE STRESSES AT OPERATING MOMENT, MOP" is the controlling flange stress and meets the API-605 criterion of a controlling flange stress not greater than 17,500 psi. The results, therefore, confirm that the 60-in., 300-lb API-605 tapered-hub flange meets the stated criteria.

The reader who is accustomed to using hand calculations for checking flange designs according to Code rules will note that the program input does not require either the factors  $T$ ,  $U$ ,  $Y$ ,  $Z$  from Code Fig. UA-51.1, or  $F$ ,  $V$ , and  $f$  from Code Figs. UA-51.2, UA-51.3, and UA-51.6, respectively. These factors are calculated by the computer program. In addition to simplifying the input, the program accurately calculates  $F$ ,  $V$ , and  $f$  values for any values of  $h/h_0$  and  $g_1/g_0$ , including those beyond the range of the Code figures.

Example 2 is a Code stress calculation for a blind flange to match the 60-in., 300-lb API-605 tapered-hub flange. The calculation method is that given in UG-34 [Eq. (2)], with  $C = 0.3$ . The output data are shown in Table A.3. The controlling flange stress is  $SOP = 17,500$  psi;

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\*API-605 does not give blind-flange thicknesses.

Table A.1. Input data for ASME Code stress calculations, examples 1 and 2

First card

Column number	5	10	15	20
Variable	ITYPE	IBOND	ICODE	MATE
Example 1	1	0	2	1
Example 2	3	0	2	1

Second card

Column number	0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80
Variable	A	B	t	$g_0$	$g_1$	h	C	P
Example 1	73.9375	57.5314	5.9375	1.2343	2.7030	5.4362	69.4375	720.
Example 2	73.9375	57.5314 <sup>a</sup>	7.9044	1.2343 <sup>a</sup>	2.7030 <sup>a</sup>	5.4362 <sup>a</sup>	69.4375	720.

Third card

Column number	0-10	11-20	21-30	31-40	41-50	51-60	61-70 <sup>b</sup>	72	73-80
Variable	m	y	$G_o$	$G_i$	$S_b$	$S_a$	$A_h$	I	$b_o$
Example 1	2.75	3700.	65.	60.25	20000.	20000.	136.92	0	0
Example 2	2.75	3700.	65.	60.25	20000.	20000.	136.92	0	0

<sup>a</sup>Not used in calculations for a blind flange.

<sup>b</sup>Column 71 is blank.



Table A.2. Output data for example 1, ASME Code analysis of a tapered-hub flange

FLANGE O.D.,A	FLANGE I.D.,B	FLANGE THICK.,T	PIPE WALL,G0	HUB AT BASE,G1	HUB LENGTH,H	BOLT CIRCLE,C	PRESSURE, P
73.93750	57.53140	5.93750	1.23430	2.70300	5.43620	69.43750	720.000
	H	T	GOUT	GIN	SB	SA	AB
	2.75000	3700.00000	65.00000	60.25000	20000.00000	20000.00000	136.92060
	BO	BN11	BN12	BN1	SB1	BN2	SB2
	1.1875D 00	2.3097D 06	4.3322D 05	2.7430D 06	2.0033D 04	4.0477D 05	2.9563D 03
	NOP	NGS	NGS1				
	1.1719D 07	7.5742D 06	1.1186D 06				
<b>ASME FLANGE STRESSES AT OPERATING MOMENT, NOP</b>							
(2/3)*SN= 1.5608D 04 ST = 1.1174D 04 SR = 8.4442D 03 (SN+ST)/2= 1.7293D 04 (SN+SR)/2= 1.5928D 04							
<b>ASME FLANGE STRESSES AT GASKET SEATING MOMENT, NGS</b>							
(2/3)*SN= 1.0097D 04 ST = 7.2216D 03 SR = 5.4576D 03 (SN+ST)/2= 1.1176D 04 (SN+SR)/2= 1.0294D 04							

Table A.3. Output data for example 2, ASME Code analysis of a blind flange

FLANGE O.D., A	FLANGE I.D., B	FLANGE THICK., T	PIPE WALL, GO	HUB AT BASE, G1	HUB LENGTH, H	BOLT CIRCLE, C	PRESSURE, P
73.93750	0.0	7.90440	0.0	0.0	0.0	69.43750	720.000
N 2.75000	Y 3700.00000	GOUT 65.00000	GIN 60.25000	SB 20000.00000	SA 20000.00000	AB 136.92000	
DO 1.1875D 00	WN11 2.3097D 06	WN12 4.3322D 05	WN1 2.7430D 06	SB1 2.0033D 04	WN2 4.0477D 05	SB2 2.9563D 03	
ASME CODE STRESSES FOR BLIND FLANGE							
SP 1.4121D 04	SW1 3.3792D 03	SOP 1.7500D 04	SW2 4.9865D 02	SAS 3.3763D 03			

the flange thickness of 7.9044 in. was selected to obtain this result. This example was included to illustrate that a blind flange may have to be considerably thicker than a mating flange in order for both to meet the Code stress limitations.

## BLIND-TO-TAPERED-HUB FLANGED JOINT, EXAMPLES 3(a) AND 3(b)

Input Data

The input data for examples 3(a) and 3(b) are shown in Table A.4. In addition to the basic purpose of illustrating input/output data for the program FLANGE, this pair of examples was selected to show how the program can be used to estimate required initial bolt stresses. In addition, example 3(2) shows how the general purpose option (ICODE # 2) gives stresses as obtained from Code calculations plus deformation data and additional stresses.

Examples 3(a) and (b) do not involve temperature gradients or temperatures other than ambient; hence, the modulus of elasticity is the same for the initial and final states. Values of temperatures for the flanges, bolts, and gaskets in the final state have been entered as zero. The initial-state reference temperature is zero; hence, a zero in the final state denotes a zero thermal gradient. However, the value of DELTA (the hub-to-ring thermal gradient) cannot be entered as zero without causing a divide-check error, so a value of 0.01 was used. A smaller value could be used (e.g., 0.001 or 0.0001), but the output data shows that DELTA = 0.01 is sufficiently small so that its influence is negligible. A coefficient of thermal expansion of  $6 \times 10^{-6}$  has been entered but is not significant in these examples.

The value of FACE, which is intended to permit use of a bolt length other than  $t_0 = TII + THP + VO + BSIZE$ , was entered as zero. The modulus of elasticity for both the flanges and the bolts was assumed to be  $3 \times 10^7$  psi. The modulus of elasticity for the 1/16-in.-thick asbestos gasket was assumed to be  $3 \times 10^6$  psi.

Some comments on the use of a modulus of elasticity of  $3 \times 10^6$  for a 1/16-in. asbestos gasket may be appropriate. The stress-strain relationship for such a gasket, which is confined between the two rigid flange faces, is highly nonlinear and both time and history dependent. Starting out with a new gasket, the first increment of bolt stress to produce a gasket stress of 1000 psi might decrease the gasket thickness

Table A.4. Input data for blind-to-tapered-hub flanged joint, examples<sup>a</sup> 3a and 3b

Card No.	Variables and numerical values								Read format
1	ITYPE	IBOND	ICODE	MATE					415
	3	0	0	5					
2	A	B	t	g <sub>0</sub>	g <sub>1</sub>	h	C	P	
	73.9375	57.5314	7.9044	1.2343	2.7030	5.4362	69.4375	720.	8E10.5 (1080.)
3	XMOA <sup>b</sup>	EF	DELTA <sup>c</sup>	YM	G				
	2.7430D+6 (6.0656D+6)	6. D-6	.01	3. D+7	62.625				5E10.5
4	ITYPE	IBOND	ICODE	MATE					
	1	0	0	6					415
5	A	B	t	g <sub>0</sub>	g <sub>1</sub>	h	C	P	
	73.9375	57.5314	5.9375	1.2343	2.7030	5.4362	69.4375	720.	8E10.5 (1080.)
6	XMOA	EF	DELTA <sup>c</sup>	YM	G				
	1.1719D+7 (2.0661D+7)	6. D-6	.01	3. D+7	62.625				5E10.5
7	BSIZE	YB	EB	TB	XG0	XG1	AB		
	2.25	3. D+7	6. D-6	0	65.	60.25	136.92		7E10.5
8	VO	YG	EG	TG	FACE	PBE			
	.0625	3. D+6	6. D-6	0	0	0			6E10.5
9	M1	TF	TFB	YF2	YFP2	YB2	YG2		
	2.7430D+6 (6.0656D+6)	0	0	3. D+7	3. D+7	3. D+7	3. D+6		7E10.5

<sup>a</sup>Values in parentheses are for example 3b.

<sup>b</sup>Initial bolt load is used here since ITYPE = 3; see footnote *b* to Table 11 in the text.

<sup>c</sup>Since DELTA cannot be entered as zero, 0.01 was used as a satisfactorily small value.

by 20%, so that the modulus would be  $1000/(0.2 \times 0.0625) = 8 \times 10^4$  psi. Crude observations indicate that, at a bolt stress that produces a gasket stress of 40,000 psi, the gasket thickness is about one-half of its original thickness, so that the average modulus up to this stress is  $40,000/0.03125 = 1.28 \times 10^6$  psi. These numbers are dependent upon the ratio of width to thickness of the gasket and the time under stress, particularly for low gasket stress. However, for the flanged-joint analysis, we are not interested in the gasket stress-strain characteristics when the bolt load is applied but rather in the gasket stress-strain characteristics when the gasket stress is decreased after the gasket has been under bolt load for several days or many months. No data on the "spring-back" of asbestos gaskets are available, but in most flanged joints using 1/16-in.-thick asbestos gaskets, the assumed modulus of elasticity of the gasket is not very significant provided it is not unrealistically low. This can be shown for example 3 by noting that the change in the bolt load depends upon the sum of the load-displacement characteristics of the bolts, the flanges, and the gasket. The displacements for a unit bolt load are -

$$\text{for bolts: } \frac{l_0}{A_b E_b} = \frac{16.15}{136.92 \times 3 \times 10^7} = 3.93 \times 10^{-9} ,$$

$$\text{for flanges: } 2 \times QFHG = 2(1.197 \times 10^{-9}) = 2.40 \times 10^{-9} ,$$

and

$$\text{for gasket: } \frac{V_0}{A_G E_G} = \frac{0.0625}{467.26 \times E_G} = \frac{1.34 \times 10^{-4}}{E_G} .$$

As  $E_G$  varies from  $10^5$  to  $10^7$ , the sum of these three displacements varies as follows:

$E_G$	$10^5$	$3 \times 10^5$	$10^6$	$3 \times 10^6$	$10^7$
Sum of displacements ( $\times 10^9$ in.)	7.67	6.78	6.46	6.37	6.34

From the above, it can be seen that changing the gasket modulus by two orders of magnitude changes the sum of the displacement by only 17%.

The initial bolt stress used in example 3(a) is 20,033 psi, giving an initial bolt load of  $W1 = S_b A_b = 20,033 \times 136.92 = 2.743 \times 10^6$  lb;  $W1$  is entered in place of  $XMOA$  on card 6 (see footnote *b* to Table 11 of text). The initial moment,  $XMOA$ , used in example 3(a) is  $1.1719 \times 10^7$  in.-lb. The initial bolt stress used in example 3(b) is 44,300 psi, giving an initial bolt load of  $W1 = 6.0656 \times 10^6$  lb. The initial moment,  $XMOA$ , used in example 3(b) is  $2.0661 \times 10^7$  in.-lb. The reasons for using these particular values of  $W1$  and  $XMOA$  are discussed in connection with the output data for these examples.

### Output Data

#### Residual Bolt Loads

The output data for example 3(a) are shown in Table A.5. The output starts with a printout of all input data on the first page (Table A.5a).<sup>\*</sup> The parameters involved in the bolt-load-change calculations are then printed, followed by residual bolt loads and moments, all on the second page (Table A.5b). The initial bolt load under "LOADINGS" is  $2.743 \times 10^6$  lb; the residual bolt load after application of the pressure of 720 psi is given following "COMBINED" as  $W2 = 1.0948 \times 10^6$  lb. The loss in bolt load is given by  $W1 - W2 = 1.6482 \times 10^6$  lb, and the ratio of residual to initial bolt load is given by  $W2/W1 = 0.39911$ . Calculated stresses for the blind flange and for the tapered-hub flange are printed on the third and fourth pages (Tables A.5c and A.5d, respectively). These are discussed later.

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<sup>\*</sup> For convenience in referring to specific pages of multipage tables, we have used alphabetic suffixes on table numbers. For example, the first page of Table A.5 is designated Table A.5a; the second page is Table A.5b, the third is Table A.5c, etc.

Table A.5a. Output data for example 3(a), blind flange bolted to a tapered-hub flange, with initial bolt stress = 20,033 psi\*

FLANGE O.D.,A	FLANGE I.D.,B	FLANGE THICK.,T	PIPE WALL,GO	HUB AT BASE,G1	HUB LENGTH,H	BOLT CIRCLE,C	PRESSURE,P				
73.93750	57.53180	7.90480	1.23430	2.70300	5.83620	69.41750	720.000				
BOLT LOAD	COEFF. OF THERMAL EXP.	DELTA	MOD. OF ELASTICITY	MOD. OF NEAR GASKET ELASTICITY	DIAMETER	ITYPE	IBCHD	ICODE	WATE		
2.7430	06	6.0000-06	1.0000-02	3.0000	07	6.2630	01	3	0	0	5
FLANGE O.D.,A	FLANGE I.D.,B	FLANGE THICK.,T	PIPE WALL,GO	HUB AT BASE,G1	HUB LENGTH,H	BOLT CIRCLE,C	PRESSURE,P				
73.93750	57.53180	7.93750	1.23430	2.70300	5.83620	69.43750	720.000				
MOMENT LOAD	COEFF. OF THERMAL EXP.	DELTA	MOD. OF ELASTICITY	MOD. OF NEAR GASKET ELASTICITY	DIAMETER	ITYPE	IBCHD	ICODE	WATE		
1.1720	07	6.0000-06	1.0000-02	3.0000	07	6.2630	01	1	0	0	6
BSIZE	YB	YD	YF	YG	YH	YI	YJ	YK	YL		
2.25000 00	3.00000 07	4.00000-06	0.0	6.50000 01	6.02500 01	1.36920 02					
VO	YB	YD	YF	YG	YH	YI	YJ	YK	YL		
6.25000-02	3.00000 07	4.00000-06	0.0	0.0	0.0						
W1	YB	YD	YF	YG	YH	YI	YJ	YK	YL		
2.74300 04	0.0	0.0	3.00000 07	3.00000 07	3.00000 07	3.00000 07	3.00000 07	3.00000 07	3.00000 06		

FLANGE JOINT BOLT LOAD CHANGE DUE TO APPLIED LOADS, BLIND TO INTERGR DATA

FLANGE JOINT SIDE ONE (PRINTED QUANTITIES)

QPHG= 9.89900-10 QPHG= 6.53500-06 QTHG= -1.00000 00 XB = -1.00000 00 GC= -1.00000 00 TH = 7.90480 00  
 YH = 3.00000 07 YF2 = 3.00000 07 EF = 6.00000-06

FLANGE JOINT SIDE TWO (UNPRINTED QUANTITIES)

QPHG= 1.19460-09 QPHG= 8.04220-06 QTHG= 9.54900-05 XB = 5.74310 01 GC= 1.23430 00 TH = 5.81750 00  
 YH = 3.00000 07 YF2 = 3.00000 07 EF = 6.00000-06

BOLTING

BOLT LENGTH= 1.41540 01 BOLT AREA= 1.36920 02 BOLT CIRCLE= 6.94380 01  
 YB = 3.00000 07 YD2 = 3.00000 07 YF = 6.00000-06

GASKET

VO = 6.25000-02 XGO = 6.50000 01 XGI = 6.02500 01  
 YG = 3.00000 06 YG2 = 3.00000 06 YH = 6.00000-06

\*For the convenience of the user, the first page of Table A.5 is designated Table A.5a, the second page is Table A.5b, the third is Table A.5c, etc. This convention is also used in the following tables.



Table A.5b (continued)

LOADINGS

INITIAL BOLT LOAD = 2.7430D 06 BOLT TEMP. = 0.0  
 GASKET TEMP. = 0.0 DELTA = 1.0000D-02 DELTA =  
 FLANGE ONE TEMP. = 0.0 FLANGE TWO TEMP. = 0.0  
 INITIAL BOLT LOADS AFTER INTERNAL-PRESSURE LOADS

AXIAL INTERNAL W2A = 2.7430D 06 MOMENT SMIP, W2B = 2.2294D 06

TOTAL PRESSURE, W3C = 1.0949E 06 DELTA INTERNAL W2B = 2.7429D 06

COMBINED W2 = 1.0949D 06

W1-W2A = 0.0 W1-W2B = 5.1359D 05 W1-W2C = 1.6481D 06 W1-W2D = 1.0333D 02 W1-W2E = 1.6482D 06

W2A/W1 = 1.0000D 00 W2B/W1 = 0.1276D-01 W2C/W1 = 3.9915D-01 W2D/W1 = 9.9996D-01 W2E/W1 = 3.9911D-01

INITIAL AND RESIDUAL MOMENTS AFTER INTERNAL PRESSURE LOADS.

W1 = 9.3433D 06 W2A = 9.3433D 06 W2B = 1.1646D 07 W2C = 7.7818D 06 W2D = 9.3433D 06 W2E = 7.7814D 06  
 W2BF = 4.2000D 07 W2CF = 3.9015D 07 W2DF = 3.9015D 07

Table A.5c (continued)

## BLIND FLANGE

## CALCULATIONS FOR POL: LOADING

SOFT= 4.0213D 03 SGR= 4.0213D 03 SGT= 4.0213D 03 SCR= -1.6197D 02 SCT= 2.5764D 03 SAT= 2.4140D 03  
 ZC= -2.6C7D-03

## CALCULATIONS FOR PRESSURE LOADING

SOFT= 1.3140D 04 SGR= -8.3015D 02 SGT= 5.0937D 03 SCR= -2.8472D 02 SCT= 4.5403D 03 SAT= 4.2555E 03  
 ZC= -4.7042D-03

## CALCULATIONS FOR COMBINED LOADING, M2 OR M2P FOR ITYPE=1 OR 2, W2 FOR ITYPE=3, = 1.0940D 06

SOFT= 1.4749D 04 SGR= 7.6681D 02 SGT= 6.6987D 03 SCR= -3.4921D 02 SCT= 5.5685D 03 SAT= 5.2193D 03  
 ZC= -5.7452D-03

Table A.5d (continued)

TAPERED HUB PLANGE

CALCULATIONS FOR MOMENT LOADING

SLSO= 2.3042D 04 SLST= -2.3042D 04 SCSS= 1.9763D 04 SCST= 5.9379D 03  
 SLLO= 2.3411D 04 SLLT= -2.3411D 04 SCLO= 7.0234D 03 SCLT= -7.0234D 03  
 STN= 1.1173D 04 STP= -1.8482D 04 SRN= 8.4441D 03 SRP= -6.6480D 03  
 ZG= -1.0421D-02 ZC= -2.4446D-02 QPNG= 1.4026E-02 Y0= 1.2322D-02 Y1= 1.0058D-18 THETA= -4.0579D-01

CALCULATIONS FOR PRESSURE LOADING

SLSO= 1.4194D 04 SLST= 2.5863D 03 SCSS= 1.4398D 04 SCST= 1.0915D 04  
 SLLO= 1.8645D 03 SLLT= 5.7979D 03 SCLO= 5.5935D 02 SCLT= 1.7394D 03  
 STN= 9.3311D 03 STP= -1.1062D 03 SRN= -2.2932D 03 SRP= 2.7038D 02  
 ZG= -4.5118D-03 ZC= -1.0302D-02 QPNG= 5.7904D-03 Y0= 9.7224D-03 Y1= 6.0715D-18 THETA= -1.8088D-01

CALCULATIONS FOR TEMPERATURE LOADING

SLSO= 1.2228D 00 SLST= -1.2228D 00 SCSS= 1.0649D-01 SCST= -6.2722D-01  
 SLLO= -1.3977E-01 SLLT= 1.3977D-01 SCLO= -1.8419E 00 SCLT= -1.7581D 00  
 STN= 1.1087D 00 STP= -6.1330D-01 SRN= -2.7247D-01 SRP= 1.5072D-01  
 ZG= -7.4476D-07 ZC= -1.7007D-06 QPNG= 9.5590D-07 Y0= -2.4965D-07 Y1= -1.7259D-06 THETA= -2.9860D-07

CALCULATIONS FOR COMBINED LOADING, W2 OR W2P FOR ITYPE=1 OR 2, W2 FOR ITYPE=3, = 7.7814D 06

SLSO= 3.3385D 04 SLST= -1.6603D 04 SCSS= 3.0857D 04 SCST= 1.5860D 04  
 SLLO= 2.1362D 04 SLLT= -1.3700D 04 SCLO= 6.4068D 03 SCLT= -4.1117D 03  
 STN= 1.8638D 04 STP= -1.6493D 04 SRN= 4.7191D 03 SRP= -5.2661D 03  
 ZG= -1.2191D-02 ZC= -3.0663D-02 QPNG= 1.7472D-02 Y0= 1.9984D-02 Y1= -1.7259D-06 THETA= -5.1886D-03

To avoid leakage,\* the residual bolt load must not be less than the critical value  $W_c$ , which may be obtained from simple equilibrium considerations; thus,

$$W_c = \frac{\pi}{4} G_0^2 p, \quad (\text{A.2})$$

where

$W_c$  = "critical" bolt load,

$G_0$  = outside diameter of gasket (65 in. in this example), and

$p$  = pressure (720 psi in this example).

In this example, the value of  $W_c$  is

$$W_c = \frac{\pi}{4} \times 65^2 \times 720 = 2.389 \times 10^6 \text{ lb.}$$

Because  $W_c$  is significantly greater than  $W_2 = 1.0948 \times 10^6$  lb, the result for example 3(a) indicate that the joint will leak at the rated pressure with the initial bolt stress of 20,033 psi. The results illustrate an aspect of ASME-designed flanges that is well known to many users; that is, the joints often cannot be made leaktight (especially in order to pass the hydrostatic test) by applying an initial bolt stress equal to the Code-allowable bolt stress.

The output data for example 3(b) are shown in Table A.6. Example 3(b) is the same as 3(a), except that the initial bolt stress has been increased from 20,033 psi to 44,300 psi ( $W_1$  input under XMOA increased to  $2.0661 \times 10^7$ ); the initial moment has been correspondingly increased; and the pressure has been increased from 720 psi to 1080 psi, the latter being the hydrostatic-test pressure of 1.5 times the cold rating pressure. It can be seen in Table A.6 (on the second page, Table A.6b) that the

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\* Leakage is defined as the gross type of leakage that occurs when the load on the gasket is reduced to zero. Slow, diffusion-type leakage may occur at lower pressures.

Table A.6a. Output data for example 3(b), blind flange bolted to a tapered-hub flange, with initial bolt stress = 44,300 psi

FLANGE O.D.,A	FLANGE I.D.,B	FLANGE THICK.,T	PIPE WALL,G0	HUB AT BASE,G1	HUB LENGTH,H	BOLT CIRCLE,C	PRESSURE, P	
73.93750	57.93140	7.90440	1.23430	2.70300	5.43620	69.43750	1080.000	
BOLT LOAD	COEFF. OF THERMAL EXP.	DELTA	MOD. OF ELASTICITY	NEAR GASKET DIAMETER	ITYPE	IBOND	ICODE	NATE
6.066D 06	6.000D-06	1.000D-02	3.000D 07	6.263D 01	3	0	0	5
FLANGE O.D.,A	FLANGE I.D.,B	FLANGE THICK.,T	PIPE WALL,G0	HUB AT BASE,G1	HUB LENGTH,H	BOLT CIRCLE,C	PRESSURE, P	
73.93750	57.93140	5.93750	1.23430	2.70300	5.43620	69.43750	1080.000	
MOMENT LOAD	COEFF. OF THERMAL EXP.	DELTA	MOD. OF ELASTICITY	NEAR GASKET DIAMETER	ITYPE	IBOND	ICODE	NATE
2.066D 07	6.000D-06	1.000D-02	3.000D 07	6.263D 01	1	0	0	6
<b>BSIR</b>	<b>YB</b>	<b>EB</b>	<b>TB</b>	<b>XG0</b>	<b>XG1</b>	<b>AB</b>		
2.2500D 00	3.0000D 07	6.0000D-06	0.0	6.5000D 01	6.0250D 01	1.3692D 02		
<b>VG</b>	<b>YG</b>	<b>EG</b>	<b>TG</b>	<b>FACE</b>	<b>PBE</b>			
6.2500D-02	3.0000D 06	6.0000D-06	0.0	0.0	0.0			
<b>W1</b>	<b>TP</b>	<b>TFP</b>	<b>YF2</b>	<b>YFP2</b>	<b>YB2</b>	<b>YG2</b>		
6.0656D 06	0.0	0.0	3.0000D 07	3.0000D 07	3.0000D 07	3.0000D 06		

FLANGE JOINT BOLT LOAD CHANGE DUE TO APPLIED LOADS, BLIND TO INTEGER PAIR

FLANGE JOINT SIDE ONE (PRINTED QUANTITIES)

QPNG= 9.4494D-10 QPHG= 6.5350D-06 QTHG= -1.0000D 00 XB = -1.0000D 00 GO= -1.0000D 00 TN = 7.9044D 00  
 YH = 3.0000D 07 YF2 = 3.0000D 07 YF = 6.0000D-06

FLANGE JOINT SIDE TWO (UNPRINTED QUANTITIES)

QPHG= 1.1968D-09 QPHG= 8.0422D-06 QTHG= 9.5590D-05 XB = 5.7531D 01 GO= 1.2343D 00 TN = 5.9375D 00  
 YH = 3.0000D 07 YF2 = 3.0000D 07 YF = 6.0000D-06

BOLTING

BOLT LENGTH= 1.6154D 01 BOLT AREA= 1.3692D 02 BOLT CIRCLE= 6.9438D 01  
 YB = 3.0000D 07 YB2 = 3.0000D 07 EB = 6.0000D-06

GASKET

VC = 6.2500D-02 XG0 = 6.5000D 01 XG1 = 6.0250D 01  
 YG = 3.0000D 06 YG2 = 3.0000D 06 EG = 6.0000D-06

Table A.6b (continued)

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LOADINGS

INITIAL BOLT LOAD= 6.0656D 06 BOLT TEMP.= 0.0 FLANGE ONE TEMP.= 0.0 FLANGE TWO TEMP.= 0.0  
 GASKET TEMP.= 0.0 DELTA= 1.0000D-02 DELTAP= 1.0000D-02 PRESSURE= 1.0000D 03

RESIDUAL BOLT LOADS AFTER THERMAL-PRESSURE LOADS

AXIAL THERMAL, W2A= 6.0656D 06 MOMENT SHIFT, W2E= 5.2952D 06

TOTAL PRESSURE, W2C= 3.5934D 06 DELTA THERMAL, W2D= 6.0655D 06

COMBINED, W2= 3.5933D 06

W1-W2A= 0.0 W1-W2E= 7.7038D 05 W1-W2C= 2.4722D 06 W1-W2D= 1.0333D 02 W1-W2= 2.4722D 06

W2A/W1= 1.0000D 00 W2E/W1= 8.7299D-01 W2C/W1= 5.9242D-01 W2D/W1= 9.9998D-01 W2/W1= 5.9240D-01

INITIAL AND RESIDUAL MOMENTS AFTER THERMAL PRESSURE LOADS.

M1= 2.0461D 07 M2A= 2.0661D 07 M2B= 2.4115D 07 M2C= 1.8319D 07 M2D= 2.0661D 07 M2= 1.8318D 07

M2BP= 7.0966D 07 M2CP= 6.5169D 07 M2P= 6.5168D 07

Table A.6c (continued)

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BLIND FLANGE

CALCULATIONS FOR BOLT LOADING

SOBT= 8.8924D 03    SGR= 8.8924D 03    SGT= 8.8924D 03    SCR= -3.5727D 02    SCT= 5.6971D 03    SAT= 5.3399D 03  
SC= -5.7620D-03

CALCULATIONS FOR PRESSURE LOADING

SOBT= 1.9716D 04    SGR= -1.2572D 03    SGT= 7.6405D 03    SCR= -4.2709D 02    SCT= 6.8104D 03    SAT= 6.3833D 03  
SC= -7.0578D-03

CALCULATIONS FOR COMBINED LOADING, W2 OR W2P FOR ITYPE=1 OR 2, W2 FOR ITYPE=3, = 3.5933D 06

SOBT= 2.4984D 04    SGR= 4.0107D 03    SGT= 1.2908D 04    SCR= -6.3873D 02    SCT= 1.0185D 04    SAT= 9.5467D 03  
SC= -1.0471D-02

Table A.6d (continued)

TAPERED HUB FLANGE									
CALCULATIONS FOR MOMENT LOADING									
SLSO=	4.0624D 04	SLSY=	-4.0624D 04	SCSO=	3.4843D 04	SCSY=	1.0469D 04		
SLLO=	4.1275D 04	SLLY=	-4.1275D 04	SCLO=	1.2382D 04	SCLY=	-1.2382D 04		
STN=	1.9699D 04	STP=	-3.2584D 04	SRN=	1.4887D 04	SRP=	-1.1721D 04		
SG=	-1.8372D-02	SC=	-4.3100D-02	QFNG=	2.6728D-02	YO=	2.1724D-02	YI=	2.1553D-19
								THETA=	-7.1542D-03
CALCULATIONS FOR PRESSURE LOADING									
SLSO=	2.1290D 04	SLSY=	3.8794D 03	SCSO=	2.1596D 04	SCSY=	1.6373D 04		
SLLO=	2.7967D 03	SLLY=	8.6918D 03	SCLO=	8.3902D 02	SCLY=	2.6090D 03		
STN=	1.3997D 04	STP=	-1.2503D 03	SRN=	-3.4397D 03	SRP=	4.0556D 02		
SG=	-6.7671D-03	SC=	-1.5053D-02	QFNG=	8.6856D-03	YO=	1.4584D-02	YI=	6.0715D-18
								THETA=	-2.7132D-03
CALCULATIONS FOR TEMPERATURE LOADING									
SLSO=	1.2428D 00	SLSY=	-1.2228D 00	SCSO=	1.0649D-01	SCSY=	-6.2722D-01		
SLLO=	-1.3977D-01	SLLY=	1.3977D-01	SCLO=	-1.8419D 00	SCLY=	-1.7581D 00		
STN=	1.1087D 00	STP=	-6.1330D-01	SRN=	-2.7247D-01	SRP=	1.5072D-01		
SG=	-7.4476D-07	SC=	-1.7007D-06	QFNG=	9.5590D-07	YO=	-2.4965D-07	YI=	-1.7259D-06
								THETA=	-2.9860D-07
CALCULATIONS FOR CORRUDED LOADING, W2 OR W2P FOR ITYPE=1 OR 2, W2 FOR ITYPE=3, = 1.8318D 07									
SLSO=	5.7309D 04	SLSY=	-3.2139D 04	SCSO=	5.2489D 04	SCSY=	2.5654D 04		
SLLO=	3.9392D 04	SLLY=	-2.7898D 04	SCLO=	1.1816D 04	SCLY=	-8.3712D 03		
STN=	3.1463D 04	STP=	-3.0541D 04	SRN=	9.7592D 03	SRP=	-9.9860D 03		
SG=	-2.3057D-02	SC=	-5.3667D-02	QFNG=	3.0610D-02	YO=	3.3844D-02	YI=	-1.7259D-06
								THETA=	-9.0565D-03



residual bolt load after application of a pressure of 1080 psi is  $W_2 = 3.5933 \times 10^6$  lb. The value of the critical bolt load to prevent gross leakage is

$$W_c = \frac{\pi}{4} \times 65^2 \times 1080 = 3.584 \times 10^6 \text{ lb} .$$

With an initial bolt stress of 44,300 psi, the residual bolt load is now greater than  $W_c$ . Accordingly, the results of example 3(b) indicate that an initial bolt stress of 44,300 psi is sufficient for the joint to pass a hydrostatic test to 1080 psi, albeit with no margin of safety. As the reader may have surmised, the initial bolt stress of 44,300 psi was preselected for example 3(b) to achieve this final result. It is pertinent to note that, because of the linear nature of the calculations, it is not necessary to iterate in order to find a value for the initial bolt stress that would make  $W_2 = W_c$ . Note that  $(W_1 - W_2) = 1.648 \times 10^6$  in example 3(a) and that  $(W_1 - W_2)$  varies linearly with pressure. To find the required value of  $W_1$  to make  $W_2 = W_c$  at an arbitrary pressure  $p$ , we need only solve the equation:

$$W_1 = \frac{\pi}{4} G_0^2 p + \frac{p}{720} (1.648 \times 10^6) . \quad (\text{A.3})$$

For  $p = 1080$ , Eq. (A.3) gives  $W_1 = 6.056 \times 10^6$ , and the corresponding initial bolt stress is  $W_1/A_b = 6.056 \times 10^6/136.92 = 44,228$  psi, which was rounded off to 44,300 psi for Example 3(b).

#### Blind Flange Stresses, Example 3(a)

Example 3(a) was run with an initial bolt stress of 20,033 psi to permit direct comparison of the blind-flange stresses with the stresses calculated in example 2, where the controlling bolt stress was  $S_{B1} = 20,033$  psi.

Stresses for the blind flange are shown in Table A.5c. The maximum stress due to initial bolt loading only is  $S_{ORT} = 4021.3$  psi. A comparable stress from the Code calculation (Table A.3), is  $S_{GS} = 3376.3$  psi.

This also represents a stress at the center of the blind flange due to bolt loading only. The maximum stress due to pressure loading only of the blind flange (Table A.5c) is SORT = 13,144 psi. A Comparable stress from the Code calculation (Table A.3) is SP = 14,121 psi.

The maximum stress due to combined bolt loading and pressure loading (Table A.5c) is SORT = 14,749 psi. Note that this combined stress is not the sum of the stress due to the initial bolt load and the stress due to pressure. Rather, the program recognizes that the pressure changes the bolt load – in this example, from  $2.743 \times 10^6$  lb down to  $1.0948 \times 10^5$  (Table A.5b). Stresses for combined loadings are related to stresses for initial bolt loading only and pressure only by the equation

$$\sigma_c = \sigma_b \cdot \frac{W2}{W1} + \sigma_p, \quad (A.4)$$

where  $\sigma_c$  = combined stress,  $\sigma_b$  = stress due to initial bolt load only, W2 = bolt load at pressure, W1 = initial bolt load, and  $\sigma_p$  = stress due to pressure only.

The Code equation for combined stresses [i.e.,  $S = (d/t)^2(0.3p + 1.78Wh_G)$  from paragraph UG-34 and Figs. UG-34 (j) and (k)] can be derived by assuming that the blind flange is a flat circular plate of outside diameter equal to the effective gasket diameter  $d$ . The metal outside the diameter  $d$  is ignored. The plate is simply supported along  $d$  and loaded by edge moment  $Wh_G$  and pressure  $p$ .  $Wh_G$  is either the operating moment or the gasket-seating moment, as obtained in Appendix II of the Code. The method used in this report is theoretically more accurate than that used in the Code, and the relatively good agreement between stresses in Table A.5c and those in Table A.3 is, in part, coincidental. Large differences can exist, particularly when there is a significant amount of flange material outside the gasket diameter  $d$ .

#### Tapered-Hub Flange Stresses, Example 3(a)

Example 3(a) was run with an initial moment of  $1.1719 \times 10^7$  in.-lb to permit direct comparison with the stresses given for example 1 in

Table A.2 under the heading "ASME FLANGE STRESSES AT OPERATING MOMENT, MOP." In example 1, the value for MOP was determined to be  $1.1719 \times 10^7$  in.-lb. To be consistent with the Code calculation in this example [3(a)], we chose  $IBOND = 0$ .

Calculated stresses for the tapered-hub flange are shown in Table A.5d. The Code method covers only moment loading. The stresses in Table A.5d for initial moment loading only are the same as those in Table A.2 for operating moment, MOP:

<u>Stress values from Table A.5d</u>	<u>Stress values from Table A.2</u>
SLLO = 23,411 psi	SH = 23,412 psi
STH = 11,173 psi	ST = 11,174 psi
SRH = 8,444 psi	SR = 8,444 psi

The Code method gives stresses at the small end of the hub if the Code factor  $f$  is greater than 1.0; otherwise, it gives stresses for the large end of the hub. The Code method calculates radial and tangential stresses on the hub side of the flange only. Usually these are higher than the corresponding stresses on the face side of the flange, but in this example,  $STH = 11,173$  psi is less than  $STF = -18,482$  psi in absolute magnitude. The Code method does not give circumferential stresses in the hub.

Stresses for pressure loading only, temperature loading only, and combined loadings are shown as the 2nd, 3rd, and 4th groups of stresses in Table A.5d. The small values under the heading "CALCULATIONS FOR TEMPERATURE LOADINGS" come from using  $DELTA = 0.01$ , since  $DELTA = 0$  is not a permissible input value.

Combined stresses are not the sum of the stresses due to the three individual loads. Rather, the program recognizes that pressure and temperature change the moment from  $M1 = 9.3433 \times 10^6$  in.-lb to  $M2 = 7.7814 \times 10^6$  in.-lb in this example\* (Table A.5b). The maximum stress

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\* It should be noted that  $M1$  is not the same as the input moment  $XMOA$ . The program will accept any value for calculating stresses but, for calculating bolt load changes, it assumes that the moment is equal to  $W(C-G)/2$ .

under combined loads (in this example, residual moment and pressure) is  $SLSO = 33,385$  psi. Under initial moment only, the maximum stress is  $SLLO = 23,411$  psi.

#### Blind and Tapered-Hub Flange Stresses, Example 3(b)

Stresses are shown in Table A.6c and A.6d for blind and tapered-hub flanges, respectively. It can be seen that maximum stresses are quite high for the realistic initial bolt stress of 44,300 psi needed to pass the hydrostatic test pressure of 1080 psi [i.e.,  $SORT = 24,984$  psi for the blind flange (Table A.6c) and  $SLSO = 57,309$  psi for the tapered-hub flange (Table A.6d)]. Comments on the significance of these high calculated stresses are included later in the discussion of examples 4a and 4b.

#### Displacements

Tables A.5 and A.6 include, along with stresses, the displacements  $ZC$  for the blind flange or  $ZG$ ,  $ZC$ ,  $QFHG$ ,  $Y0$ ,  $Y1$ , and  $THETA$  for the tapered-hub flange. One potential application for these displacements is discussed later in connection with examples 4(a) and 4(b).

## IDENTICAL PAIR OF TAPERED-HUB FLANGES, EXAMPLES 4(a) AND 4(b)

Input Data

The input data for Examples 4(a) and 4(b) are shown in Table A.7. The initial bolt stress of 46,100 psi and corresponding  $W_1 = 6.312 \times 10^6$  lb were selected by a preliminary calculation so that  $W_2$  would equal  $W_c$  at the hydrostatic-test pressure of 1080 psi. The value of  $W_1 = 6.312 \times 10^6$  lb leads to initial moment  $M_{MOA} = W_1(C-G)/2 = 2.1500 \times 10^7$  in.-lb. Example 4(a) is for hydrostatic test conditions at atmospheric temperature. Example 4(b) is for steady-state operating conditions at the rated pressure of 300 psi and corresponding API-605 temperature of 850°F.

The modulus of elasticity of the flange, bolt, and gasket materials was assumed to be  $2.25 \times 10^7$  psi at 800°F, as compared with  $3.0 \times 10^7$  at atmospheric temperature. It is assumed that at steady-state operating conditions there is an external bending moment such that the axial stress in the attached pipe is 7500 psi. This axial stress gives 617 psi as the input value for PBE for example 4(b), as shown below:

$$PBE = 4 S_b g_0 / D_0 = 4 \times 7500 \times 1.2343 / 60 = 617 \text{ psi} .$$

Output DataResidual Bolt Loads

The output data for example 4(a) are shown in Table A.8. The output data starts with a printout of all input data. The parameters involved in the bolt-load-change calculations are then printed, followed by residual bolt loads and moments (Table A.8b).

The residual bolt load is given by  $W_2 = 3.585 \times 10^6$  lb. The critical bolt load, derived from Eq. (A.2), is  $W_c = \pi G_0^2 p / 4 = 3.584 \times 10^6$  lb. Accordingly, the results of example 4(a) indicate that an initial bolt stress of 46,100 psi is sufficient for the joint to pass a hydrostatic test to 1080 psi, albeit with no margin of safety.

Table A.7. Input data for tapered-hub-to-tapered-hub flanged joint, examples<sup>a</sup> 4a and 4b

Card No.	Variables and numerical values								Read format
1	ITYPE	IBOND	ICODE	MATE					
	1	0	0	2					415
2	A	B	t	g <sub>0</sub>	g <sub>1</sub>	h	C	P	
	73.9375	57.5314	5.9375	1.2343	2.7030	5.4362	69.4375	1080. (300.)	8E10.5
3	XMOA	EF	DELTA <sup>b</sup>	YM	G				
	2.1500D+7	6. D-6	.01	3. D+7	62.625				5E10.5
4	BSIZE	YB	EB	TB	XG0	XG1	AB		
	2.25	3. D+7	6. D-6	0	65.	60.25	136.92		7E10.5
5	VO	YG	EG	TG	FACE	PBE			
	.0625	3. D+6	6. D-6	0	0	0			6E10.5
						(617.)			
6	W1	TF	TFP	YF2	YFP2	YB2	YG2		
	6.3120D+6	0	0	3. D+7 (2.25D+7)	3. D+7 (2.25D+7)	3. D+7 (2.25D+7)	3. D+6 (2.25D+6)		7E10.5

<sup>a</sup>Values in parentheses are for example 4b.

<sup>b</sup>Since DELTA cannot be entered as zero, 0.01 was used as a satisfactorily small value.

Table A.8a. Output data for example 4(a), identical pair of tapered-hub flanges, with initial bolt stress of 46,100 psi

FLANGE O.D.,A	FLANGE I.D.,B	FLANGE THICK.,T	PIPE WALL,GO	HUB AT BASE,G1	HUB LENGTH,H	BOLT CIRCLE,C	PRESSURE, P
73.93750	57.53140	5.93750	1.23430	2.70300	5.43620	69.43750	1080.000
MOMENT COEFF. OF THERMAL EXP.	DELTA MOD. OF ELASTICITY	HUB DIAMETER	GASKET ITYPE	IBOND	ICODE	MATE	
2.150D 07	6.000D-06	1.000D-02	3.000D 07	6.263D 01	1	0	0 2
B SIZE	YB	EB	TB	XGO	XG1	AB	
2.250D 00	3.000D 07	6.000D-06	0.0	6.500D 01	6.025D 01	1.3492D 02	
VO	YG	EG	TG	FACE	PFB		
6.250D-02	3.000D 06	6.000D-06	0.0	0.0	0.0		
H1	TF	TFP	YF2	YFP2	YR2	YG2	
6.312D 06	0.0	0.0	3.000D 07	3.000D 07	3.000D 07	3.000D 06	

FLANGE JOINT BOLT LOAD CHANGE DUE TO APPLIED LOADS, IDENTICAL PAIR

FLANGE JOINT SIDE ONE (PRINTED QUANTITIES)

QPNG= 1.1968D-09 QPNG= 8.0422D-06 QPNG= 9.5590D-05 XB = 5.7531D 01 GO= 1.2343D 00 TH = 5.9375D 00  
 YH = 3.000D 07 YF2 = 3.000D 07 EF = 6.000D-06

FLANGE JOINT SIDE TWO (UNPRINTED QUANTITIES)

QPNG= 1.1968D-09 QPNG= 8.0422D-06 QPNG= 9.5590D-05 XB = 5.7531D 01 GO= 1.2343D 00 TH = 5.9375D 00  
 YH = 3.000D 07 YF2 = 3.000D 07 EF = 6.000D-06

BOLTING

BOLT LENGTH= 1.4188D 01 BOLT AREA= 1.3492D 02 BOLT CIRCLE= 6.9438D 01  
 YB = 3.000D 07 YB2 = 3.000D 07 EB = 6.000D-06

GASKET

VO = 6.250D-02 XGO = 6.500D 01 XG1 = 6.025D 01  
 YG = 3.000D 06 YG2 = 3.000D 06 EG = 6.000D-06

Table A.8b (continued)

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LOADINGS

INITIAL BOLT LOAD= 6.3120D 06 BOLT TEMP.= 0.0 FLANGE ONE TEMP.= 0.0 FLANGE TWO TEMP.= 0.0  
 GASKET TEMP.= 0.0 DELTA= 1.0000D-02 DELTAP= 1.0000D-02 PRESSURE= 1.0000D 03

RESIDUAL BOLT LOADS AFTER THERMAL-PRESSURE LOADS

AXIAL THERMAL, W2A= 6.3120D 06 MOMENT SHIFT, W2B= 5.0760D 06  
 TOTAL PRESSURE, W2C= 2.5852E 06 DELTA THERMAL, W2D= 6.3118D 06  
 COMBINED, W2= 3.5850D 06

W1-W2A= 0.0 W1-W2B= 1.2360D 06 W1-W2C= 2.7268D 06 W1-W2D= 1.6408D 02 W1-W2= 2.7270D 06  
 W2A/W1= 1.0000D 00 W2B/W1= 8.0418D-01 W2C/W1= 5.6799D-01 W2D/W1= 9.9997D-01 W2/W1= 5.6796D-01

INITIAL AND RESIDUAL MOMENTS AFTER THERMAL PRESSURE LOADS.

M1= 2.1500D 07 M2A= 2.1500D 07 M2B= 2.3369D 07 M2C= 1.8291D 07 M2D= 2.1500D 07 M2= 1.8290D 07  
 M2BP= 2.3369E 07 M2CP= 1.8291D 07 M2P= 1.8290D 07



Table A.8c (continued)

TAPERED END FLANGE

CALCULATIONS FOR MOMENT LOADING

SLS0= 0.2270 04 SLSI= -0.2270 04 SC50= 3.6250 04 SC5I= 1.0000 04  
 SLL0= 0.2940 04 SLLI= -0.2940 04 SCL0= 1.2000 04 SCLI= -1.2000 04  
 ST0= 2.0000 04 STP= -3.3000 04 SRR= 1.5000 04 SRP= -1.2100 04  
 SC= -1.0100-02 SC= -0.0050-02 QP00= 3.5720-02 V0= 2.2000-02 V1= 1.6520-10 THERA= -7.0000-03

CALCULATIONS FOR PRESSURE LOADING

SLS0= 2.1200 04 SLSI= 3.0700 03 SC50= 2.1500 04 SC5I= 1.4370 04  
 SLL0= 2.7000 03 SLLI= 0.6000 03 SCL0= 0.3000 03 SCLI= 2.6000 03  
 ST0= 1.3000 04 STP= -1.6000 03 SRR= -2.0000 03 SRP= 0.0000 02  
 SC= -6.7070-03 SC= -1.5050-02 QP00= 0.6050-03 V0= 1.5000-02 V1= 6.0710-10 THERA= -2.7120-03

CALCULATIONS FOR TEMPERATURE LOADING

SLS0= 1.2200 00 SLSI= -1.2200 00 SC50= 1.0000-01 SC5I= -6.2720-01  
 SLL0= -1.3070-01 SLLI= 1.3070-01 SCL0= -1.0000 00 SCLI= -1.7000 00  
 ST0= 1.1000 00 STP= -6.1000-01 SRR= -2.7000-01 SRP= 1.5070-01  
 SC= -7.0000-07 SC= -1.7000-06 QP00= 9.5500-07 V0= -2.0000-07 V1= -1.7250-06 THERA= -2.9000-07

CALCULATIONS FOR COMBINED LOADING, H2 O2 R2P FOR T77P=1 OR 2, H2 FOR T77P=2, = 1.0000 07

SLS0= 5.7250 04 SLSI= -3.2000 04 SC50= 5.2000 04 SC5I= 2.5000 04  
 SLL0= 3.9330 04 SLLI= -2.7000 04 SCL0= 1.1750 04 SCLI= -0.3000 03  
 ST0= 3.1000 04 STP= -3.0000 04 SRR= 9.7000 03 SRP= -9.9000 03  
 SC= -2.3030-02 SC= -9.3000-02 QP00= 3.0500-02 V0= 3.3000-02 V1= -1.7250-06 THERA= -9.0000-03

The output data for example 4(b) are shown in Table A.9, which is identical in format to Table A.8 for example 4(a). The residual bolt load for example 4(b) is given by  $N_2 = 3.2718 \times 10^6$  lb. The pressure is lower in example 4(b) than in 4(a), but there is a modulus-of-elasticity decrease which, by itself, makes  $N_2 = N_1 \times 2.25 \times 10^7 / (3 \times 10^7)$  and makes the effect of the equivalent pressure correspond to the external moment PBE. We can check to see if the residual bolt load is sufficient to prevent leakage by an extension of the concept of the initial bolt load  $N_c$ , which was discussed in the previous section. We made the conservative assumption that the maximum tensile stress due to the external bending moment (which exists only at one point on the pipe circumference) acts around the complete circumference of the pipe. The value of  $N_c$ , the critical bolt load to prevent gross leakage, is then the sum of Eq. (A.2) and the axial load due to the bending moment; thus

$$N_c = \frac{\pi}{4} G_0^2 p + A_p S_b, \quad (\text{A.5})$$

where

$A_p = \pi(B + g_0) g_0$  = cross-sectional area of attached pipe, and

$S_b$  = axial stress in attached pipe due to an external moment.

For example 4(b), Eq. (A.5) gives:

$$\begin{aligned} N_c &= \left( \frac{\pi}{4} \times 65^2 \times 300 \right) + (\pi \times 58.7657 \times 1.2343 \times 7500) \\ &= 2.7045 \times 10^6 \text{ lb} . \end{aligned}$$

Because  $N_2 = 3.2718 \times 10^6$  lb is greater than  $N_c = 2.7045 \times 10^6$  lb, the results indicate that the flanged joint with an initial bolt stress of 46,100 psi can carry, at least for a short time at 850°F, an external moment giving both an axial bending stress of 7500 psi in the attached pipe of 1.2343-in. wall thickness and an internal pressure of 300 psi.

At 850°F, the carbon-steel flanges and bolts would be expected to undergo significant relaxation due to creep in the flanges and bolts,

Table A.9a. Output data for example 4(b), identical pair of tapered-hub flanges, steady-state operation at 300 psi and 850°F

FLANGE	FLANGE	PIPE	NOB AT	NOB	DELT	PRESSURE,
O.D.	I.D.	THICK.	CALL.	BASE	CIRCLE	P
73.83750	57.53140	5.93750	1.23030	2.70300	5.03030	60.83750
300.000						
MOMENT CORP. OF DELTA NOB. UP HEAD GASKET TYPE ICODE RATE						
THERMAL STP, PLASTICITY DIAMETER						
2.1500	07	6.0000-06	1.0000-02	3.0000	07	0.2030 01
						1 0 0 2
SIZE						
2.25000	00	3.0000	07	6.00000-04	0.0	70
						NOI
						6.02500 01
						1.20920 02
						NOI
						6.17000 02
						NOI
						2.25000 07
						2.25000 07
						2.25000 07
FLANGE JOINT DOLT LOAD CHANGE DUE TO APPLIED LOADS, IDENTICAL PAIR						
FLANGE JOINT SIDE ONE (PRINTED QUANTITIES)						
OPRC=	1.19000-04	OPRC=	0.00220-06	OPRC=	0.55900-05	IS = 5.75310 01
IS =	3.0000 07	IS =	2.25000 07	IS =	2.25000 07	IS = 6.00000-06
FLANGE JOINT SIDE TWO (UNPRINTED QUANTITIES)						
OPRC=	1.19000-09	OPRC=	0.00220-06	OPRC=	0.55900-05	IS = 1.75310 01
IS =	3.0000 07	IS =	2.25000 07	IS =	2.25000 07	IS = 6.00000-06
BOLTING						
BOLT LENGTH=	1.81000 01	BOLT AREA=	1.26020 02	BOLT CIRCLE=	6.00300 01	
IS =	3.0000 07	IS =	2.25000 07	IS =	6.00000-06	
GASKET						
VO =	6.25000-02	NO =	6.50000 01	NOI =	6.02500 01	
IS =	3.0000 06	IS =	2.25000 07	IS =	6.00000-06	

Table A.9b (continued)

LOADINGS

INITIAL BOLT LOAD= 4.3120D 06 BOLT TEMP.= 0.0 PLANGE ONE TEMP.= 0.0 PLANGE TWO TEMP.= 0.0  
 GASKET TEMP.= 0.0 DELTA= 1.0000D-07 DELTA1= 1.0000D-02 PRESSURE= 1.0000D 02

RESIDUAL BOLT LOADS AFTER THERMAL-PRESSURE LOADS

AXIAL THERMAL,W2A= 6.1120D 06 MOMENT SHIPT,W2B= 5.2621D 06

TOTAL PRESSURE,W2C= 6.8480E 06 DELTA THERMAL,W2D= 6.1110D 06

COMBINED,W2= 1.2710D 06

W1-W2A= 0.0 W1-W2B= 1.0495D 06 W1-W2C= 1.6616D 06 W1-W2D= 1.6400D 02 W1-W2= 1.0402D 06  
 W2A/W1= 1.0000D 00 W2B/W1= 0.1374D-01 W2C/W1= 7.6813D-01 W2D/W1= 9.9997D-01 W2/W1= 5.1935D-01

INITIAL AND RESIDUAL MOMENTS AFTER THERMAL PRESSURE LOADS.

M1= 2.1500D 07 M2A= 2.1500D 07 M2B= 1.9614D 07 M2C= 1.8201D 07 M2D= 2.1500D 07 M2= 1.2813D 07  
 M2BP= 1.9614E 07 M2CP= 1.8201D 07 M2P= 1.2813D 07

Table A.9c (continued)

TAPPED NUD FLANGE									
CALCULATIONS FOR MOMENT LOADING									
SLSO=	4.2271D 04	SLST=	-4.2271D 04	SCSO=	3.6258D 04	SCSI=	1.0894D 04		
SLLQ=	4.2951D 04	SLLT=	-4.2951D 04	SCLO=	1.2885D 04	SCLI=	-1.2885D 04		
STN=	2.0499D 04	STP=	-3.3907D 04	SRN=	1.9492D 04	SRP=	-1.2197D 04		
SG=	-1.9118D-02	SC=	-4.4850D-02	QPMG=	2.5732D-02	VO=	2.2406D-02	VI=	1.4524D-10
								THETA=	-7.4448D-01
CALCULATIONS FOR PRESSURE LOADING									
SLSO=	9.9140D 03	SLST=	1.0776D 03	SCSO=	9.9990D 03	SCSI=	4.5481D 03		
SLLQ=	7.7487D 02	SLLT=	2.4158D 03	SCLO=	2.3306D 02	SCLI=	7.2471D 02		
STN=	3.8880D 03	STP=	-4.5841D 02	SRN=	-9.5549D 02	SRP=	1.1266D 02		
SG=	-1.8798D-03	SC=	-4.2924D-03	QPMG=	2.4127D-03	VO=	4.0510D-03	VI=	8.4734D-10
								THETA=	-7.5365D-04
CALCULATIONS FOR TEMPERATURE LOADING									
SLSO=	1.2228D 00	SLST=	-1.2228D 00	SCSO=	1.0649D-01	SCSI=	-4.2722D-01		
SLLQ=	-1.3977D-01	SLLT=	1.3977D-01	SCLO=	-1.8419D 00	SCLI=	-1.7581D 00		
STN=	1.1087D 00	STP=	-6.1330D-01	SRN=	-2.7247D-01	SRP=	1.5072D-01		
SG=	-7.4474D-07	SC=	-1.7007D-04	QPMG=	9.5590D-07	VO=	-2.4965D-07	VI=	-1.7259D-04
								THETA=	-2.9840D-07
CALCULATIONS FOR COMBINED LOADING, U2 OR W2P FOR ITYPE=1 OR 2, U2 FOR ITYPE=3, = 1.2833D 07									
SLSO=	3.1147D 04	SLST=	-2.4156D 04	SCSO=	2.7641D 04	SCSI=	1.1050D 04		
SLLQ=	2.6411D 04	SLLT=	-2.3221D 04	SCLO=	7.9222D 03	SCLI=	-6.9680D 03		
STN=	1.6125D 04	STP=	-2.0498D 04	SRN=	8.2510D 03	SRP=	-7.1471D 03		
SG=	-1.3292D-02	SC=	-3.1064D-02	QPMG=	1.7772D-02	VO=	1.7544D-02	VI=	-1.7255D-06
								THETA=	-5.1976D-03

particularly with the high bolt stresses and flange stresses involved in example 4(b). For long-term service (many years) at 850°F, one might expect the flanges and/or bolts to creep so that a residual bolt stress of around 20,000 psi would exist, at which time  $M_2 = 2000 \cdot 156.92 = 2.7584 \cdot 10^5$  lb. Because this is larger than  $M_c = 2.7045 \cdot 10^5$  lb obtained from Eq. (A.5), indications are that the flanged joint could still carry the external moment and pressure, albeit with almost no margin of safety.

It should be noted that, if bolts relax in high-temperature service, then the bolt load does not return to its initial value upon returning to initial conditions. The permanent loss in bolt load would be  $M_2 - S_{br} A_b$ , where  $S_{br}$  = relaxed bolt stress, assumed here to be 20,000 psi. The permanent loss in bolt load, in this example, is  $3.2718 \cdot 10^5 - 20,000 \cdot 156.92 = 553,400$  lb. The load is theoretically not sufficient to pass a hydrotest of 1080 psi, but it is extremely unlikely such a hydrotest would be required for a system operating at 500 psi and 850°F.

### Flange Stresses

Tables A.8c and A.9c show the flange stresses for examples 4(a) and 4(b), respectively. The maximum calculated stress occurs in example 4(a) where SLSO = 57,253 psi for combined loadings. Note that this is not the sum of the stresses due to initial moment loading only plus pressure loading only (first two groups of stresses), but rather it is the stress due to the moment as changed by pressure,  $M_2 = M_2P = 1.829 \cdot 10^7$  in.-lb, plus the stress due to pressure only.

The question arises as to whether the flanges in the flanged joint are strong enough to pass the hydrostatic test. To pursue this question, it is appropriate to tabulate the tangential and radial stresses at initial and pressurized conditions:

Condition	STH	STF	SRH	SRF
Initial	20,499	-53,907	15,492	-12,197
Pressurized	31,436	-30,496	9,739	-9,970

It should be noted that the stresses are, in large part, bending stresses. Before large plastic deformations occur, these stresses must reach about  $1.5S_y$ , where  $S_y$  is the yield strength of the flange material. Further, high stresses in the hub will not lead to large plastic deformations if there is reserve strength in the flange ring as indicated by relatively low tangential and radial stresses. If the capability for calculating these stresses has been attained, the next logical step is to conduct an extensive study to develop suitable design criteria for stress limits in flanged joints. Until such a study is conducted, however, the following limits are suggested as appropriate for stresses under hydrostatic test conditions:

Stress	Limit
Longitudinal hub stresses	$< 1.5S_y$
Radial stress or tangential stress	$< S_y$
Averages of radial or tangential stress and longitudinal hub stress	$< S_y$

The above criterion makes the average of SLSO and STH under pressurized conditions [i.e.,  $1/2(5.7255 \times 10^4 + 3.1436 \times 10^4) = 44,344$  psi] the controlling stress and infers that the flanged joint is acceptable, provided the flange-material yield strength is not less than 44,344 psi.

### Displacements

In tightening the belts to 46,100 psi, the question arises as to whether the flanges will rotate so that contact occurs on the outer edge. Table A.8c shows values of THETA, the rotation of the ring at the mean radius of the pipe wall. An estimate\* of the displacement of the ring edge with respect to the gasket centerline can be obtained by

---

\*The deformation of the ring is not exactly linear across the ring, but in this example it is sufficiently close to linear.

multiplying  $\theta$  by  $(A-G)/2$ , the radial distance between the ring edge and gasket centerline. In example 4(a),  $A = 75.9575$ ,  $G = 62.625$ , and  $\theta = -9.0466 \times 10^{-7}$  under combined loading; the minus sign means that the rotation is such that clearance is reduced at the outer edge. The displacement of A with respect to C is  $9.0466 \times 10^{-7} \times (75.9575 + 62.625)/2 = 0.0512$  in. Because API-605 flanges have 1/16-in. raised faces, the outer edges of the flanges will not contact each other. The clearance will then be  $(0.0625 - 0.0512) \times 2 = 0.0056$  in. plus the thickness of the gasket.



## COMPUTER TIME

The six examples discussed in this appendix were run on Battelle's CDC 6400 computer and also on ORNL's IBM 360/91. The IBM FORTRAN source deck (converted to double precision for use on the IBM machine) has 1585 cards. The total length of the program is 80K bytes (10,240 actual words), and it needs no auxiliary storage devices except standard read and write units. The program requires 270K bytes for compilation and has a compilation time of 19.4 sec. The total execution time for the six examples was 1.15 sec.

**APPENDIX B**  
**FLOWCHARTS AND LISTING OF COMPUTER PROGRAM FLANGE**  
**AND ATTENDANT SUBROUTINES**

APPENDIX B

CONTENTS

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1. Flowcharts of Program FLANGE and Attendant Subroutines . . . . .	10
2. Listing of Program FLANGE and Attendant Subroutines . . . . .	11

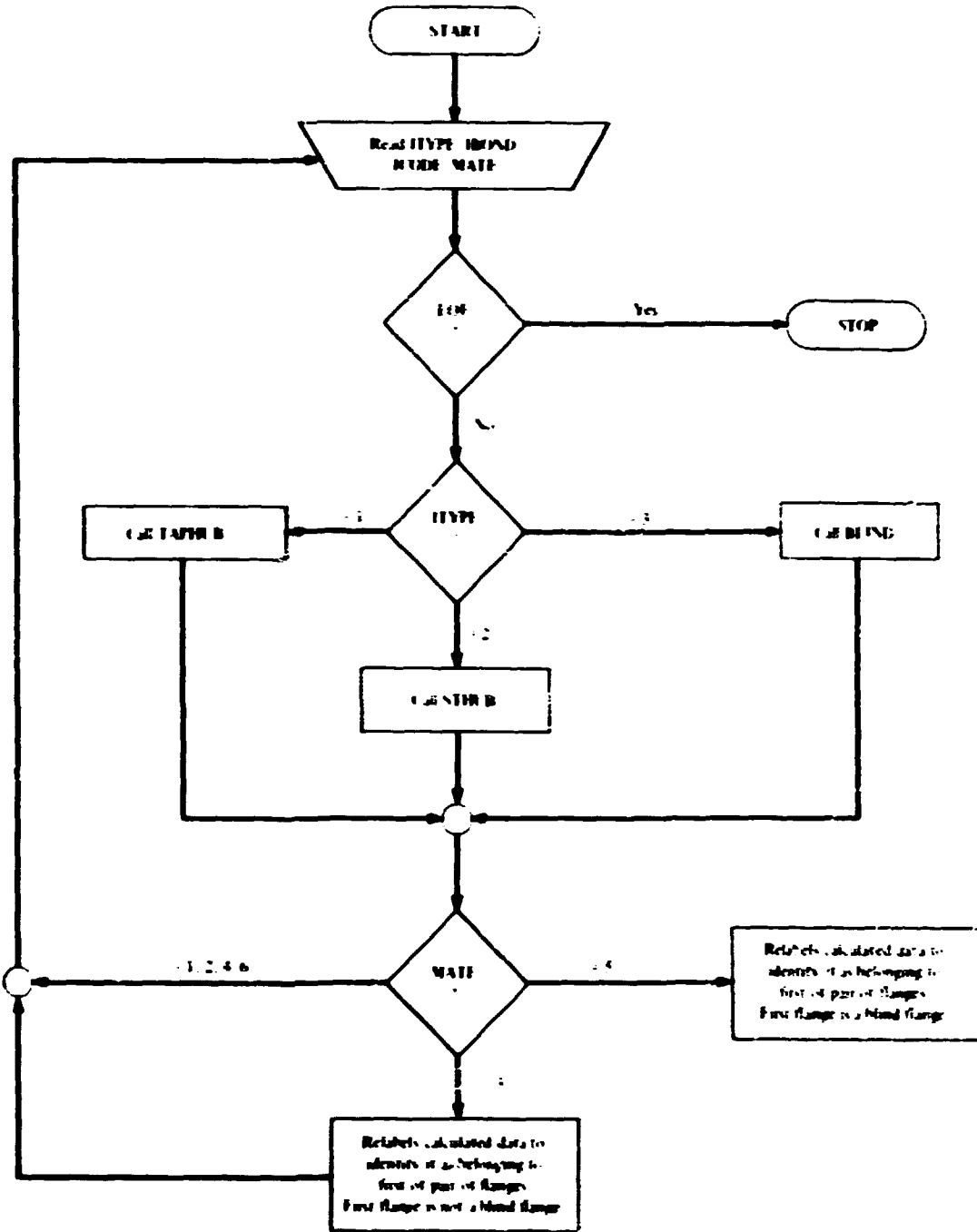


Fig. B.1. Program FLAGE.

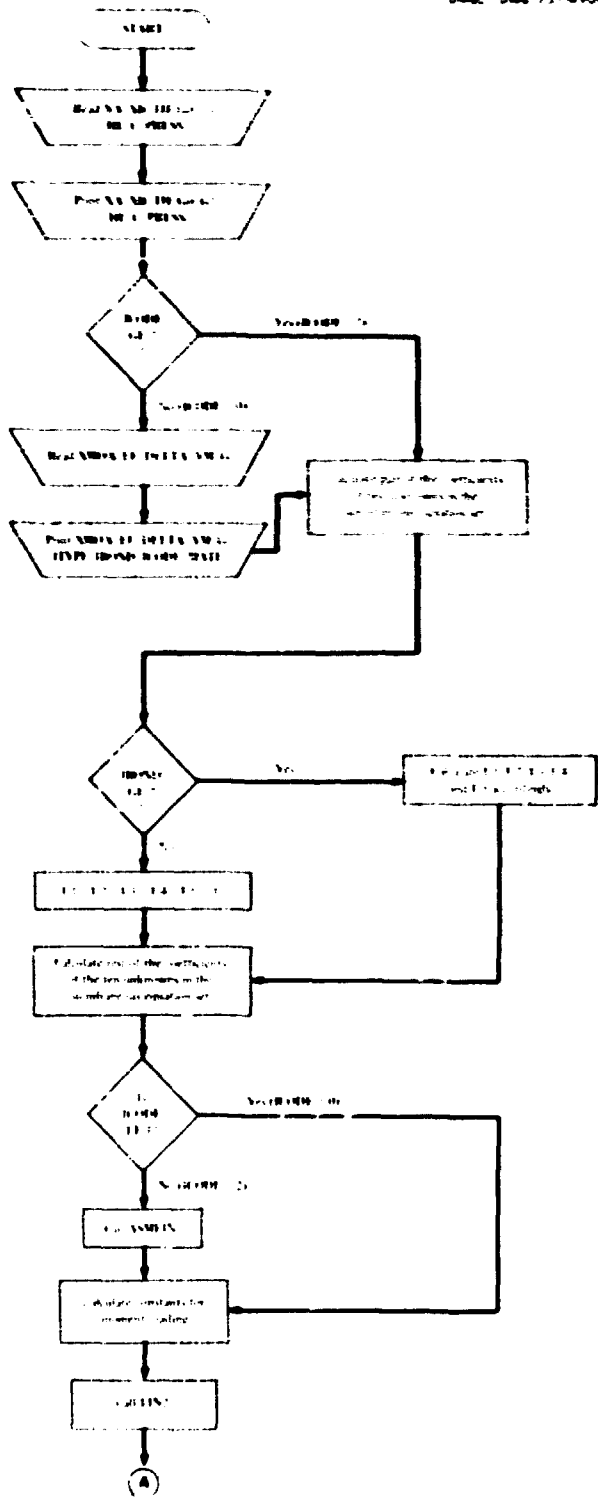


Fig. B.2. Subroutine TAPHUB (Part 1).

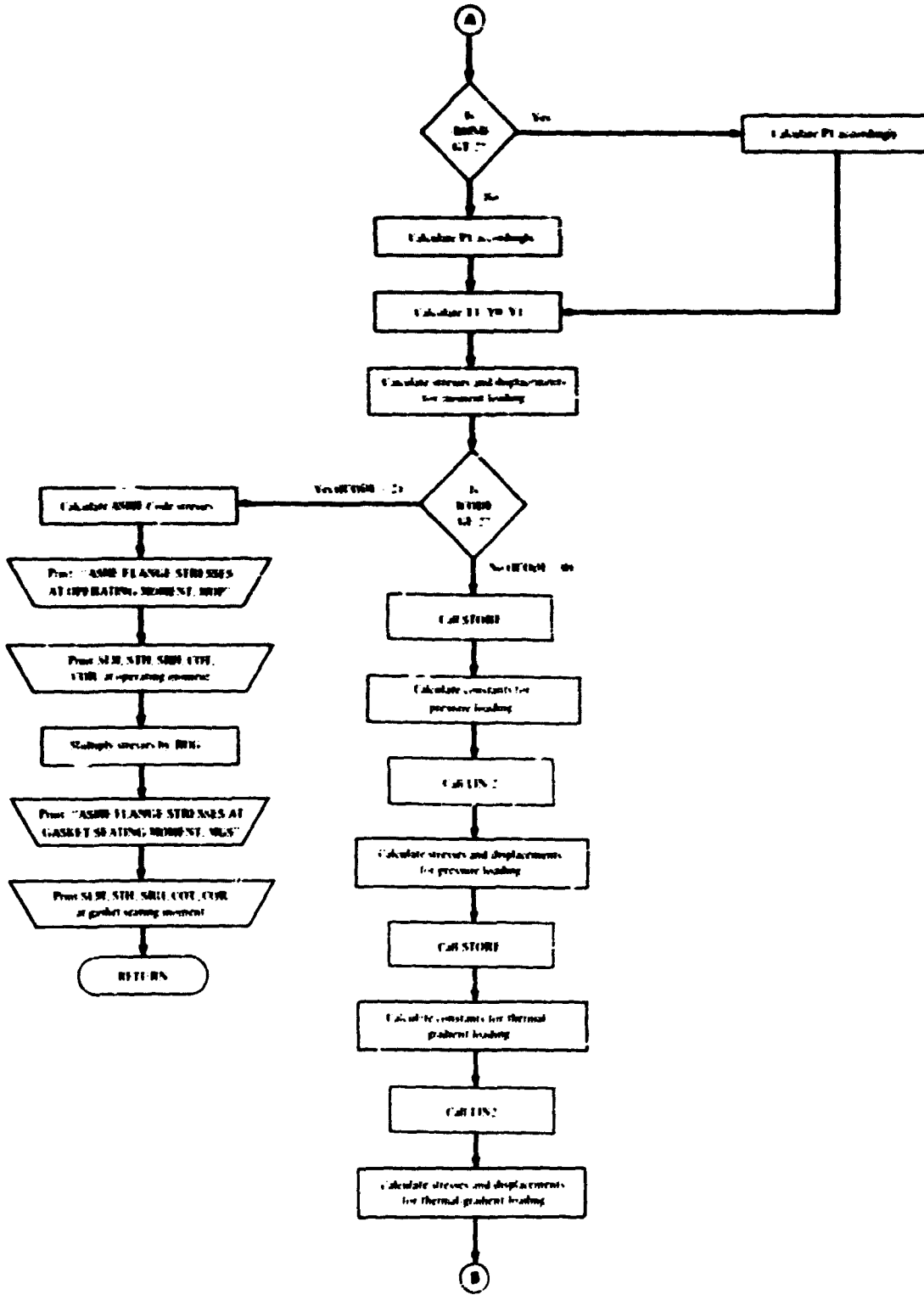


Fig. B.2. Subroutine TAPHUB (Part 2).

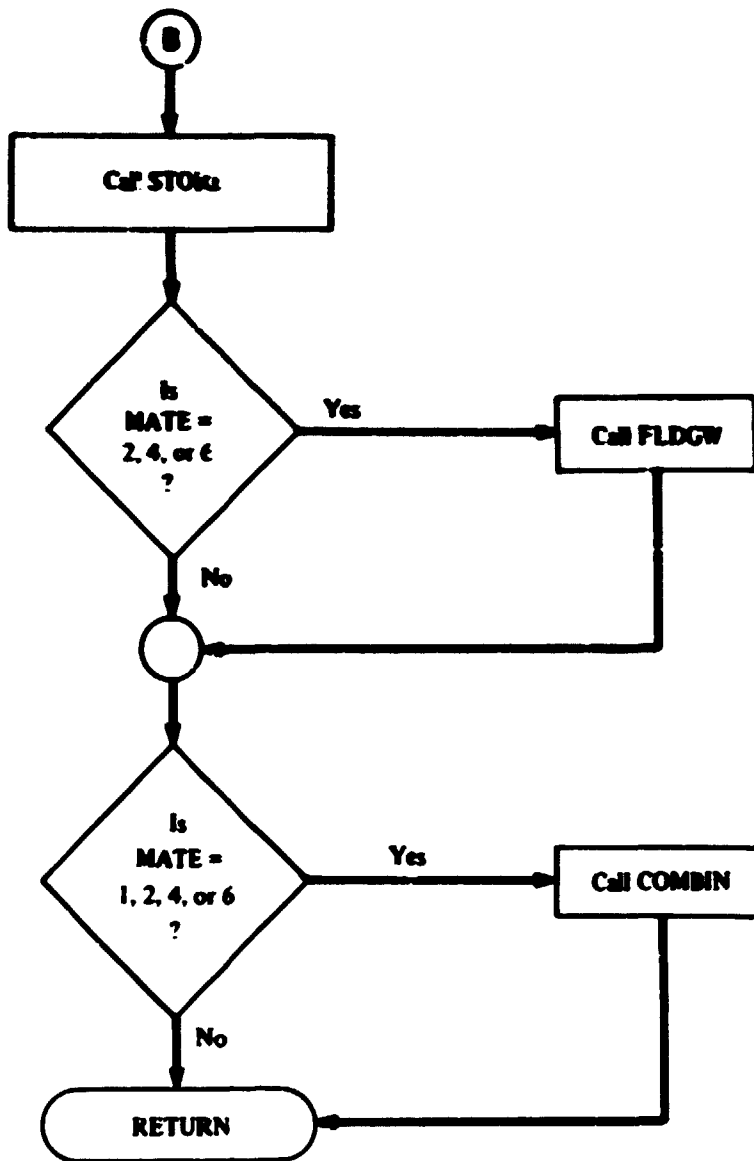


Fig. B.2. Subroutine TAPHUB (Part 3).

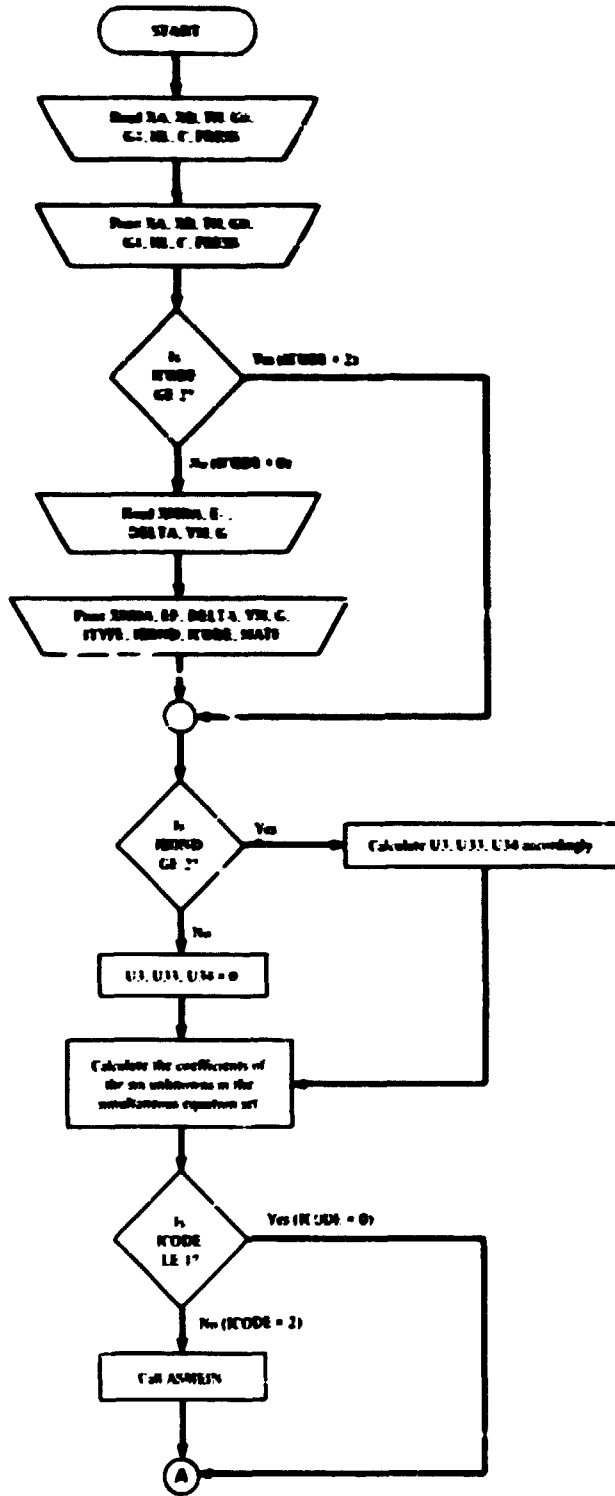


Fig. B.3. Subroutine STHUB (Part 1).





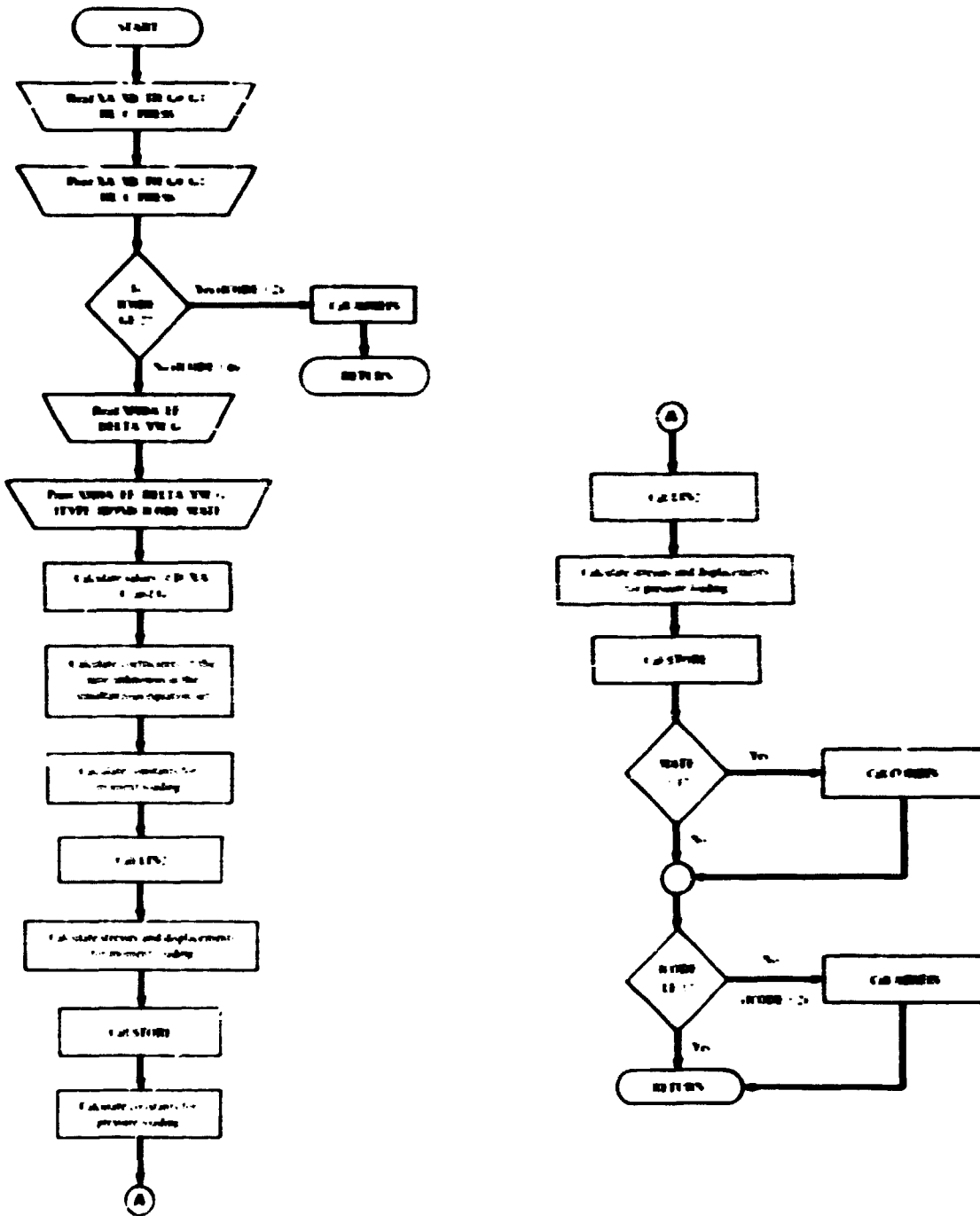


Fig. B.4. Subroutine BLIND.

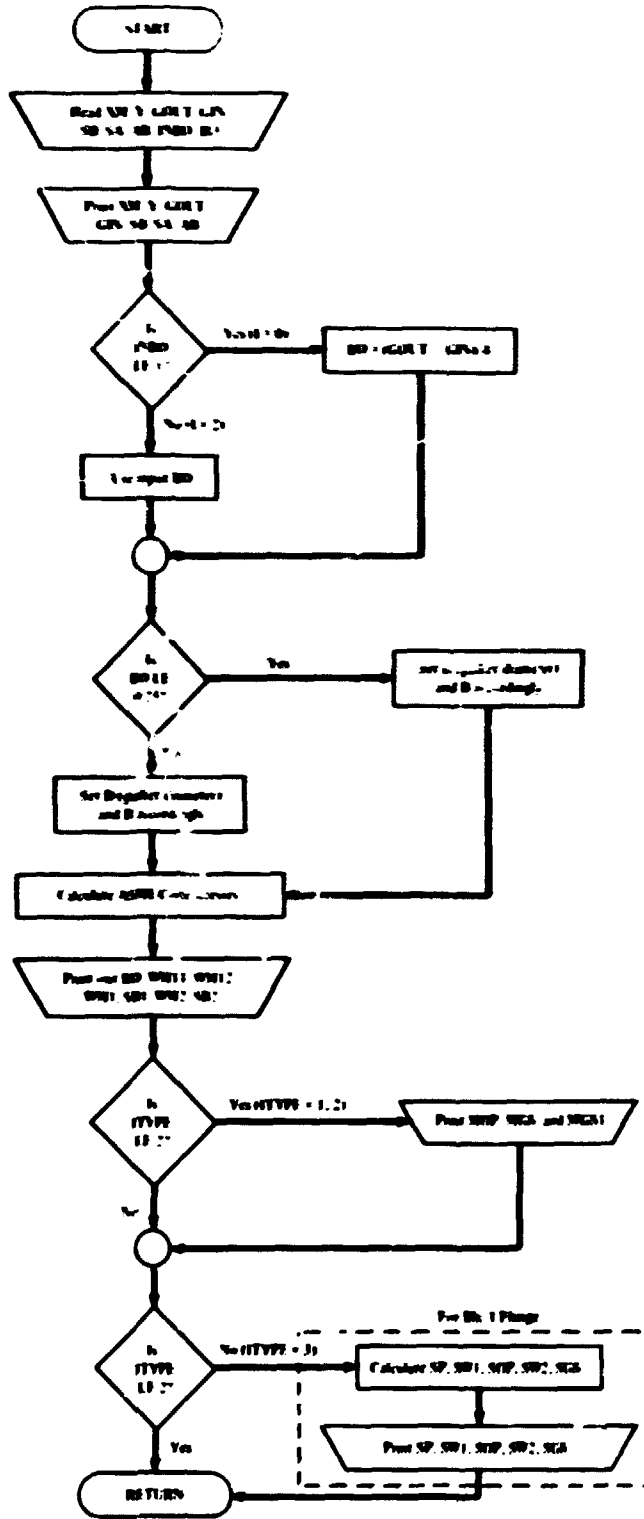


Fig. B.5. Subroutine ASMEIN.



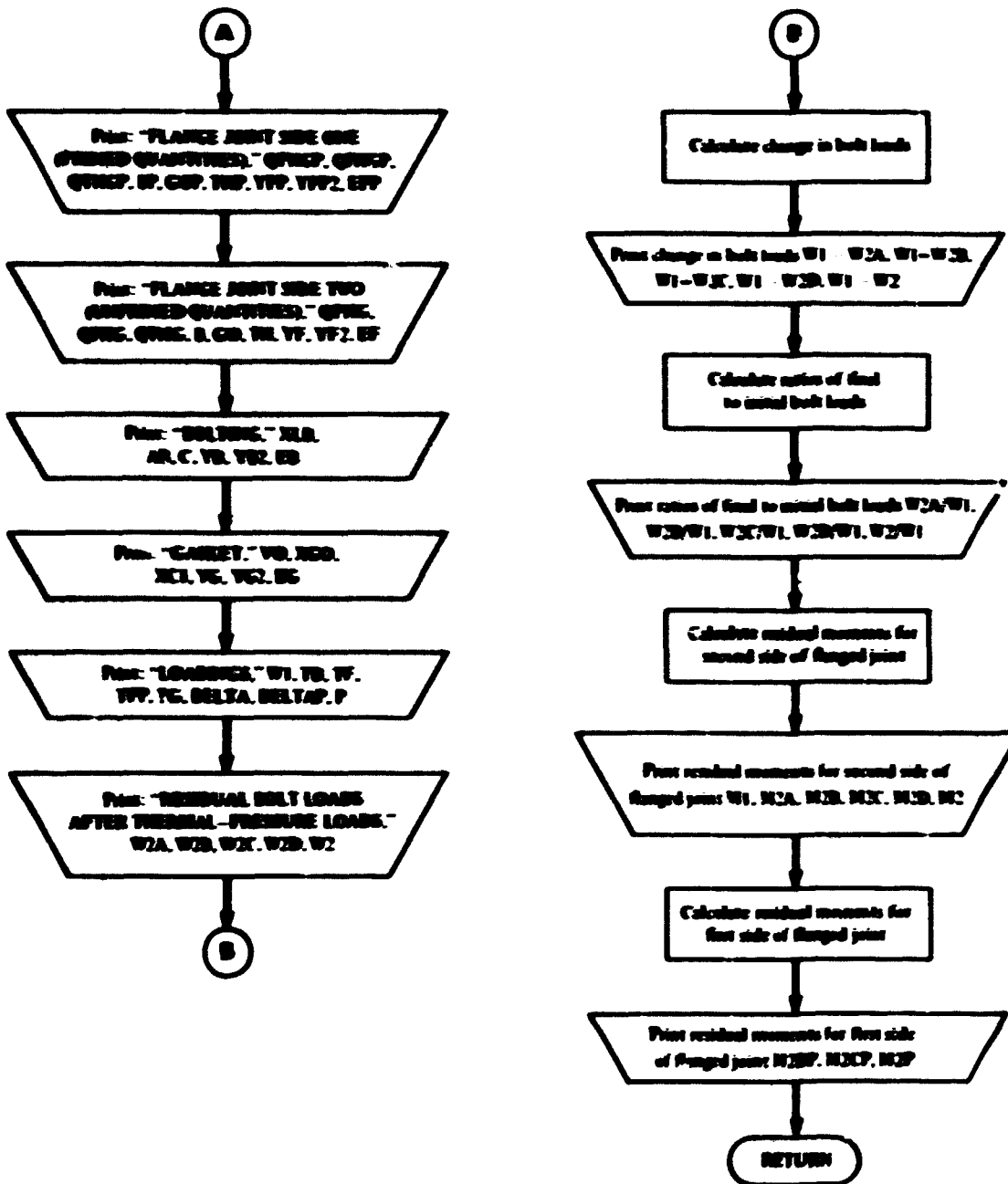


Fig. B.6. Subroutine FLGDW (Part 2).

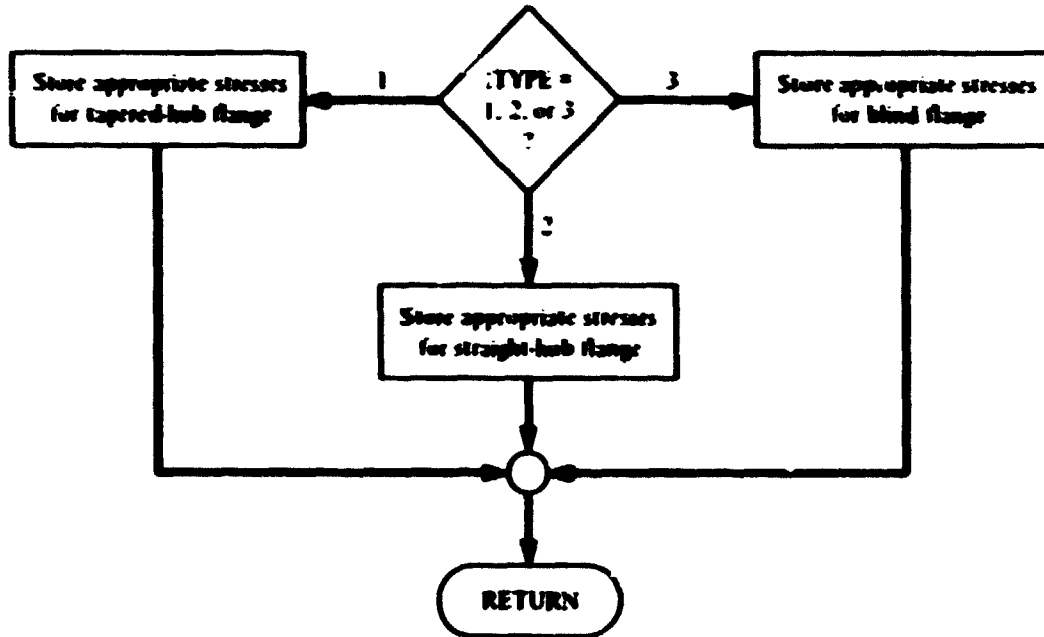


Fig. B.7. Subroutine STORE.

















```

A(9,5)=0
A(9,6)=0
A(9,7) = 2.00BLOC (HA) *J.)
A(9,8)=2.0
A(9,9)=-0.7/(HA-HB)
A(9,10)=0.
A(10,1)=0.
A(10,2)=0.
A(10,3)=0.
A(10,4)=0.
A(10,5)=0.
A(10,6)=0.
A(10,7)=1.0
A(10,8)=0.
A(10,9)=0.
A(10,10)=0.
C PRINT J, A(1), B(2), B(3), B(4), B(5), B(6), B(7), B(8), B(9), B(10)
DO 11 I=1,10
DO 12 J=1,10
A(I,J)=A(I,J)
12 CONTINUE
13 CONTINUE
C CALCULATIONS FOR MOMENT LOADING, TAPPED HUB
P=0.
PS=0.
DELT=0.
IF (ICOR=7) 10,14,15
10 IHO=IHOA
10 TO 16
15 CALL AINZIX
IHO=IOP
J=(COLI+ZIB)/2.
C 16 PRINT 50
16 CONTINUE
DO 17 I=1,10
B(I)=0.
17 CONTINUE
B(10)=- (2.7) / (B.20) * YH * YH * 3. * (A-HB) * IHO
CALL LINZ (A, 10, 10, 0., 0.1, 10, LTEMP, IERR, DEFT, NPT, LPA, LPA)
B17=(-I*DEIX-2.*DEIX)
B18=(-I*DEIX-2.*DEIX)
B19=(-I*DEIX-2.*DEIX)
B20=(-I*DEIX-2.*DEIX)
P1=(-YH*G100.) * IHO * IHO * 2. / (07.36 * PH1 * 100.5 * HL * 100.) * (B17 * B(1) + B18 * B(2) + B19 * B(3) + B20 * B(4))
B9=0. * IHO * IHO * 0. * DEIX - IHO * DEIX
B10=0. * IHO * IHO * 0. * DEIX - IHO * DEIX
B11=0. * IHO * IHO * 0. * DEIX - IHO * DEIX
B12=0. * IHO * IHO * 0. * DEIX - IHO * DEIX
A1=(1./ (0. * PH1 * 100.5 * HL * 100.)) * (B5 * B(1) + B11 * B(2) + B12 * B(3)) * 2. * ALPT
HA=2 * PS / ((1. * A1 * HA) * 3)
T1 = B(7) * (2. * IHO * LOG (HA) * IHO) * 2. * 0.5 * (8) * IHO * B(9) / IHO
P1A=P1/A1
COF=(YH * G100 * IHO * IHO * 3.) / (IHO * 2.7 * 100.5 * G100 * 1.)
P=P1A / COF
T1A=T1/A1
COV=(IHO * 2.7 * 100.5 * G100 * 1.) / (IHO * G100 * 1.)
V=P1A / COV
-----C-17-75
C IF (IHO=7) 10,14,15
C 10 CONTINUE
-----C-17-75
19 IHO=0
HA=1
20 SLB=1.016 * YH * B(5)
IF (IHO=4) 21,21,42
21 P1=(-YH * G100.) * IHO * IHO * 2. / (07.36 * PH1 * 100.5 * HL * 100.) * (B17 * B(1) + B18 * B(2) + B19 * B(3) + B20 * B(4))
10 TO 21
22 P1=(-YH * G100.) * IHO * IHO * 2. / (07.36 * PH1 * 100.5 * HL * 100.) * (B17 * B(1) + B18 * B(2) + B19 * B(3) + B20 * B(4))

```







	SUBROUTINE SUBR9	STN	2
C	THIS CALCULATION IS FOR ITYPE = 2, STRAIGHT ROD FLANGES	STN	0
	INFLUENT FLOW (F-D, G-Z)	STN	11-20-75
	DIMENSION A(10,10), B(10), ITEMP(10), LP(100), LFC(10), AN(10,10)	STN	6
	DIMENSION SC(4,10), SC(10)	STN	6A
	COMMON ITYPE, ICOD, ICOD2, DATE, IA, IB, IC, C, PRESS, RGS, RCP, G1, G2, TH, TN, STN	STN	0
	IE, OFE (M), AL, DEL IE, AL, XNO, OFRIF, JNGP, CNGP, bi, cur, CRP, IFF, EFP, STM	STN	10
	DELTA, OUT, G2, CG	STN	12
	J, JCS, SISC, SCS, JCS, SLAG, SILL, SCLA, SCL, STH	STN	0A
	STF, SFF, SIF, ZC, ZC, FNG, YC, Y1, Y2, TETA, SOFT, SCL, SC2, SC3, SC4, S4C	STN	0B
	S, H2, W1, S3, X1, Z1, X2, X2E	STN	0C
	DATA /1000./, /2/1000./, /LTEMP/1000./, /LP/1000./, /LFC/1000./, /AN/10000./		
C			
	1 READ J2, IA, IE, TH, GC, G1, HL, C, PRESS	STN	10
	PRINT J2	STN	10
	PRINT J2, IA, IE, TH, GC, G1, HL, C, PRESS	STN	10
	J=1.	STN	20
	YY=1.	STN	22
	IF (ICOD2.GE.2) GO TO 2	STN	20
	READ J5, ICOD, OF, TETA, TH, C	STN	26
	PRINT J5	STN	20
	AL=LP	STN	30
	PRINT J7, IVAL, EP, DELTA, TH, C, ITYPE, ICOD2, ICOD, DATE	STN	32
	2 KA=IA/2.	STN	30
	KB=IE/2.	STN	30
	KH=AL/KB	STN	30
	KB2=KB*KB	STN	40
	I=6/2.	STN	02
	C=C/2.	STN	00
	SFF2 = .73*(C.25/BS)*Y (KB*G)	STN	06A
	IF (ICOD2-1) J=0.	STN	00
	3 J3=0.	STN	50
	TJ3=0.	STN	52
	U3=0.	STN	50
	GO TO 5	STN	56
	4 U3=(IB/YH)*((1.3)*KB2+.7)/(KB2-1.)	STN	50
	TJ3=2.*U3*YH*(GC+TETA)**3/(TH*10.92)	STN	60
	J3=2*U3*EIE/2.	STN	62
	5 K71=U3*-U3	STN	60
	T72=1.+TJ3+*TJ3	STN	66
	PS=(.05*KB/(YH*GC)) *PRESS	STN	60
	A(1,1)=K71	STN	70
	A(1,2)=K72	STN	72
	A(1,3)=0.	STN	70
	A(1,4)=C.	STN	76
	A(1,5)=C.	STN	70
	A(1,6)=C.	STN	00
	A(2,1)=EETA	STN	02
	A(2,2)=EETA	STN	00
	A(2,3) = -(2.0*KB*DLOG(KB)*IB)	STN	06A
	A(2,4)=-2.*Tb	STN	00
	A(2,5)=-1./IB	STN	90
	A(2,6)=C.	STN	92
	A(3,1)=2.*DZC*KB**2*(1.+DZTA*TH/2.)	STN	90
	A(3,2)=-2.*DZTA**3*TH/2.	STN	96
	A(3,3) = -(2.0*DZC*(KB)*1.3)*(TH/GC)**3	STN	90A
	A(3,4)=-2.*6*(TH/GC)**3	STN	100
	A(3,5) = (.7/(KB*IB))* (TH/GC)**3	STN	102
	A(3,6)=0.	STN	100
	A(4,1)=0.	STN	106
	A(4,2)=C	STN	100
	A(4,3) = IB*DLOG(KB)	STN	110A
	A(4,4)=K3*Tb	STN	112
	A(4,5) = TLOG(Kd)	STN	110A
	A(4,6)=1.	STN	110
	A(5,1)=C.	STN	110
	A(5,2)=0.	STN	120
	A(5,3) = 2.4*DLOG(KA)*3.3	STN	122A
	A(5,4)=2.0	STN	120







```

A(3,3)=1.
A(3,4)=0.
A(3,5)=3.
A(3,6)=0.
A(3,7)=0.
A(3,8)=0.
A(3,9)=0.
A(4,1)=0.
A(4,2)=0.
A(4,3)=EXP(1/LOG(6))
A(4,4)=EXP(6)
A(4,5)=DLOG(6)
A(4,6)=1.
A(4,7)=0.
A(4,8)=0.
A(4,9)=6.
A(5,1)=-2.6
A(5,2)=0.
A(5,3)=2.6*LOG(6)*3.3
A(5,4)=2.6
A(5,5)=-.7/(6*6)
A(5,6)=0.
A(5,7)=0.
A(5,8)=0.
A(5,9)=0.
A(6,1)=0.
A(6,2)=0.
A(6,3)=2.0*EXP(1/LOG(6))*C
A(6,4)=2.0C
A(6,5)=1./C
A(6,6)=0.
A(6,7)=-2.0C
A(6,8)=-1./C
A(7,4)=2.6
A(7,5)=-.7/(6*6)
A(7,6)=0.
A(7,7)=-2.6
A(7,8)=0.
A(7,9)=0.
A(8,1)=0.
A(8,2)=0.
A(8,3)=0.
A(8,4)=0.
A(8,5)=0.
A(8,6)=0.
A(8,7)=2.6
A(8,8)=-.7/(6*6)
A(8,9)=0.
A(9,1)=0.
A(9,2)=0.
A(9,3)=COS(1/LOG(6))
A(9,4)=C**C
A(9,5)=DLOG(6)
A(9,6)=1.
A(9,7)=-C**C
A(9,8)=-DLOG(6)
A(9,9)=-1.
DO J=1,9
DO I=1,9
A(I,J)=A(I,J)
I=0
J=0
I=J

```

C CALCULATION FOR SHEAR LOADING, PLATE FLANGES  
 YA = 1  
 Y0 =  
 Y1 = JA

```

0LI 02
3LI 04
0LI 06
0LI 08
0LI 10
0LI 12
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3LI 16
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0LI 216
0LI 218
0LI 220

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JF10=QF8 (2)/IRES1	FLG 78
JF10=QF8 (3)/IRES1	FLG 80
J=I**2.	FLG 82
YF=YY	FLG 84
ZF=ZZ	FLG 86
VAF=IHI*2+VC*FACI*BSIZE	FLG 88
1 P=IRES1	FLG 90
JF1=XL0/(A0*YF)	FLG 92
JF2=XL1/(A0*YF2)	FLG 94
J=(I50*YF)/Z.	FLG 96
AG=(HG0-K11)*1.57*G0:	FLG 98
QG1=V0/(AG*Y0)	FLG 100
QG2=V0/(AG*Y02)	FLG 102
H0=(C-G)/A.	FLG 104
JF1=QF8/H0	FLG 106
JF2=QF10*(YF/YF2)	FLG 108
IF (NATE-5) 5, 6, 8	FLG 110
4 JFP1=QF8/(HG*HG)	FLG 112
GO TO 6	FLG 114
5 JFP1=JF8/P/HG	FLG 116
6 JF2=QF10*(YF/YF2)	FLG 118
J1=Q1*Q1+HG*HG*(P+1+YF-1)	FLG 120
J2=Q5*Q5+Q2*HG*HG*(QF2+YF2)	FLG 122
H1=(C-(G*E)/Z.)/Z.	FLG 124
H2P=(C-(G*E)/Z.)/Z.	FLG 126
H3=(C-P-20)/Z.	FLG 128
H0P=(C-E-20P)/Z.	FLG 130
COFAL=.7854*HG/C1	FLG 132
42A=01*(1./J1)*(-10**20*XL0**20*EG*VC*YF*YF**20*YF2**20*YF2)	FLG 134
IF (NATE-5) 8, 9, 7	FLG 136
7 42B=01*COFAL*((G1/HG-JF1*(H1-HG)-JF2*(H2P-HG))*G0-(JF1*YF**20*(H1-42))**2	FLG 138
GO TO 9	FLG 140
4 42C=41*COFAL*((G1/HG-JF1*(H1-HG)-JF2*(H2P-HG))*G0-(JF1*YF**20*(H1-42))**2	FLG 142
H1)*JF1**20*P**20*(H1-H2P))**(P*P*E)	FLG 144
4 42D=02E-(CP*HG/HG*(C1*HG/HG)*P*HG/A1	FLG 146
IF (NATE-5) 11, 11, 10	FLG 148
10 42D=01-(JF1*HG/HG)*I(ELT*HG/C1	FLG 150
GO TO 12	FLG 152
11 42D=01-(JF1*HG*DELTA*(C1*HG*DELTA*P)/C1	FLG 154
12 IF (NATE-5) 14, 14, 13	FLG 156
13 42E=(Q1/Q2)*01*(1./J2)*(-YB*YB*XL0**20*EG*VC*YF*YF**20*YF2**20*YF2*YF2*YF2*YF2)*	FLG 158
1/2*(J2)*COFAL*((G2/HG-JF2*(H1-HG)-JF2*(H2P-HG))*G0-(JF2*YF**20*(H1-H2P))*	FLG 160
2)*YF/YF2)**(Q1*Q1/HG)*YF/YF2)**(JF1*YF**20*(H1-42))**2-(JF1*HG/HG)*YF/YF2	FLG 162
*DELTA*P	FLG 164
GO TO 15	FLG 166
14 42E=(Q1/Q2)*01*(1./J2)*(-YB*YB*XL0**20*EG*VC*YF*YF**20*YF2**20*YF2*YF2*YF2*YF2)*	FLG 168
1/2*(J2)*COFAL*((G2/HG-JF2*(H1-HG)-JF2*(H2P-HG))*G0-(JF2*YF**20*(H1-H2P))*	FLG 170
2)*YF/YF2)**(Q1*Q1/HG)*YF/YF2)**(JF1*YF**20*(H1-42))**2-(JF1*HG/HG)*YF/YF2	FLG 172
*DELTA*P*(YF/YF2))**(HG/C2-(JF1*HG*DELTA*(YF/YF2)*G2*HG*DELTA*P*(YF/YF2)))/	FLG 174
YF2	FLG 176
15 GO TO (0, 16, 20, 17, 20, 19), NA 22	FLG 178
16 PRINT 24	FLG 180
GO TO 19	FLG 182
17 PRINT 29	FLG 184
GO TO 19	FLG 186
18 PRINT 30	FLG 188
19 PRINT 31	FLG 190
PRINT 32, QF8, YF, H0, Q1, Q2, Q3, Q4, Q5, YF1, YF2, EY	FLG 192
PRINT 33	FLG 194
PRINT 32, QF8, YF, Q1, Q2, E, G0, H, YF, YF2, EY	FLG 196
PRINT 34	FLG 198
PRINT 35, XL0, AP, C, YB, YF2, EB	FLG 200
PRINT 36	FLG 202
PRINT 37, V0, H0, Q1, Y0, Y02, ZG	FLG 204
PRINT 38	FLG 206
PRINT 34	FLG 208
PRINT 39, 0 1, 10, 5 P, 7 P, 10, DELTA, DELTA*P, P	FLG 210
PRINT 40	FLG 212
PRINT 41, 02A, 02B, 02C, 02D, 02	FLG 214
PRINT 41-02A	FLG 216

000-01-021	PLG	216
000-01-020	PLG	218
000-01-020	PLG	220
000-01-021	PLG	222
000-01-021	PLG	224
000-01-021	PLG	226
000-01-021	PLG	228
000-01-021	PLG	230
000-01-021	PLG	232
000-01-021	PLG	234
000-01-021	PLG	236
000-01-021	PLG	238
000-01-021	PLG	240
000-01-021	PLG	242
000-01-021	PLG	244
000-01-021	PLG	246
000-01-021	PLG	248
000-01-021	PLG	250
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000-01-021	PLG	256
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000-01-021	PLG	260
000-01-021	PLG	262
000-01-021	PLG	264
000-01-021	PLG	266
000-01-021	PLG	268
000-01-021	PLG	270
000-01-021	PLG	272
000-01-021	PLG	274
000-01-021	PLG	276
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000-01-021	PLG	380
000-01-021	PLG	382
000-01-021	PLG	384
000-01-021	PLG	386
000-01-021	PLG	388
000-01-021	PLG	390
000-01-021	PLG	392
000-01-021	PLG	394
000-01-021	PLG	396
000-01-021	PLG	398
000-01-021	PLG	400

2 CONTINUED, (M=12.0)	FLS	348
02 POSNAT (1/78) 01-12=12.0, 02 01-12=12.0, 02 01-12=12.0, 02	FLS	350
1 01-12=12.0, 01 01-12=12.0	FLS	352
03 POSNAT (1/78) 02/01=12.0, 03 02/01=12.0, 03 02/01=12.0, 03	FLS	354
02/01=12.0, 02 02/01=12.0	FLS	356
04 POSNAT (1/78) 03/01=12.0, 04 03/01=12.0, 04 03/01=12.0, 04	FLS	358
03/01=12.0, 03 03/01=12.0	FLS	360
05 POSNAT (1/78) 04/01=12.0, 05 04/01=12.0, 05 04/01=12.0, 05	FLS	362
04/01=12.0	FLS	364
06 POSNAT (1/78)	FLS	366
END	FLS	368

```

SUBROUTINE LINE(A,B,3N,EPS,B,N,N2,LINP,LEPA,DET,DELTA,DELTA2,
1      LDC)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(N,N),B(3N,N)
      DIMENSION LTRN(1),LEP(1),LDC(1)
C
C SUBROUTINE LINE
C EICK 0006A
C
C SUBROUTINE CALLED - EICK
C
C THIS ROUTINE SOLVES THE MATRIX EQUATION AX+B=C OVERLAPPING A WITH THE
C SOLUTION MATRIX X. A MUST BE SQUARE AND NON-SINGULAR. A MUST
C HAVE THE SAME NUMBER OF ROWS AS B. THE DETERMINANT OF A IS
C COMPUTED. BOTH A AND B ARE EXTRACTED.
C
C THIS ROUTINE IS RECOMMENDED FOR THE SOLUTION OF SIMULTANEOUS LINEAR
C EQUATIONS.
C
C THE METHOD CONSISTS OF GAUSSIAN ELIMINATION FOLLOWED BY BACK
C SUBSTITUTIONS. THIS IS MORE EFFICIENT THAN SOLUTION BY MATRIX
C INVERSION REGARDLESS OF THE NUMBER OF COLUMNS IN B. BOTH ROWS AND
C COLUMNS ARE SEARCHED FOR MAXIMAL PIVOTS. INTERCHANGING OF ROWS OR
C COLUMNS OF A IS AVOIDED. CHAPTER 1 OF E.L. STIEGLER, INSTRUCTIONS TO
C NUMERICAL MATHEMATICS, ACADEMIC PRESS, N.Y., 1963, SHOULD BE HELPFUL IN
C FOLLOWING THE CODE.
C
C THE CALLING PROGRAM MUST SET A,B,3N,EPS,B,N,N2,LINP TO-
C
C   A-THE COEFFICIENT MATRIX
C
C   B-THE ORDER OF A
C
C   3N-THE NUMBER OF WORDS OF STORAGE PROVIDED FOR EACH COLUMN OF
C     A IN THE CALLING PROGRAM
C
C   EPS-A NON-NEGATIVE NUMBER WHICH EACH PIVOT IN THE ELIMINATION
C     PROCESS IS REQUIRED TO EXCEED IN ABSOLUTE VALUE (CUSTOMARILY
C     1E-04)
C
C   B-THE CONSTANT TERM MATRIX
C
C   N-THE NUMBER OF COLUMNS OF B
C
C   ? IN THE CALLING PROGRAM
C
C   3N-THE NUMBER OF WORDS OF STORAGE PROVIDED FOR EACH COLUMN OF
C
C   LINP-A BLOCK OF AT LEAST N WORDS OF TEMPORARY INTERNAL STORAGE

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C
C IN ADDITION TO JUDGING b WITH THE SELECTOR STATE A, THE ROUTINE
C WILL TEST, DET, S, T, U, V, W, X, Y, AND Z FOR
C
C      IZAP= 1 IF NO COLUMNS OF b ARE FOUND, THE ELIMINATION PROCESS
C           BEING HALTED BECAUSE THE CURRENT PIVOT FAILS TO EXCEED
C           EPS IN MAGNITUDE
C
C           2 IF ALL COLUMNS OF b ARE FOUND, NO trouble signs detected
C
C DET-PLUS OF ROWS THE PRODUCT OF THE CURRENT AND ALL PREVIOUS
C PIVOTS
C
C EPV-THE NUMBER OF THE CURRENT PIVOT (FIRST, SECOND, ETC.)
C
C PIV-THE CURRENT PIVOT
C
C LPS-THE FIRST LPS POSITIONS LIST THE FIRST AND SECOND IN ORDER
C OF USE, 2 VECTORS OF LENGTH 3
C
C LPC-THE FIRST LPS POSITIONS LIST THE FIRST COLUMN INDICES IN
C ORDER OF USE, 2 VECTORS OF LENGTH 3
C
C IF THE ELIMINATION PROCESS IS HALTED PREMATURELY (EPS NEGATIVE), THEN
C THE DATA BEING b, V, W, X, Y, Z, AND LPS, MAY BE HELPFUL IN DIAGNOSING THE UNDERLYING
C CAUSE OF THE TROUBLE. IF THE PROCESS GOES TO COMPLETION THE OUTPUT,
C LPS SHOULD BE THE SUPPLEMENT OF b, PIV WILL BE THE STR PIVOT, AND LPS
C AND LPC LIST ALL PIVOT POSITIONS.
C
C DO INITIALIZATIONS
C
C   1 IZAP=0
C     DET=1.
C     DO 2 I=1,3
C       LPS(I)=1
C       2 LPC(I)=1
C
C BEGIN ELIMINATION PROCESS
C
C   DO 10 I=1,3
C     PIV=0
C
C   SELECT PIVOT
C
C     PIV=0.
C     DO 5 K=SP,3
C       I=LPS(K)
C       DO 5 L=SP,3
C         J=LPC(L)
C         IF( ABS(A(I,J))-ABS(PIV) ) 0,J,J
C       5 KPIV=K
C         LPIV=L
C         IPIV=I
C         JPIV=J
C         PIV=A(I,J)
C     4 CONTINUE
C
C UPDATE DET, W, X, Y, Z AND PIVOT FOR THE COLUMN LISTS
C
C     DET=DET*PIV
C     IZAP=IZAP+(PIV)
C     LPS(W)=LPS+(KPIV)
C     LPS(X)=LPS+(LPIV)
C     IZAP=IZAP+(IPIV)
C     LPS(Y)=LPS+(IPIV)
C     LPS(Z)=LPS+(JPIV)
C
C EXIT IF PIVOT TOO SMALL
C
C   IF( EPS-ABS(PIV) ) 0,7,7

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7 IER= 2
  IFTER
C
C RECIPIENT POSITION OF A AND B (ELEMENTS IS PRESENT IN PREVIOUS FROM
C COLUMNS OF A AND B)
C
  8 IF (IP=3) 9, 11, 5
  9 IER=IP+1
  10 10 I=IBP, 5
  11 J=LPC(I)
  12 A(IPIV,J)=-A(IPIV,J)/PIV
  13 DO 12 J=1, 2
  14 B(IPIV,J)=-B(IPIV,J)/PIV
C
C SUBSTITUTION SCHEM OF A AND B (ELEMENTS IS PRESENT IN PREVIOUS
C COLUMNS OF COLUMNS ARE SKIPPED)
C
  15 IF (IP=5) 13, 16, 13
  16 DO 17 I=IBP, 5
  17 I=LPC(I)
  18 IER=I(L, JPIV)
  19 IF (IPIV) 16, 17, 16
  20 DO 15 I=IBP, 5
  21 J=LPC(I)
  22 A(I, J)=A(I, J)+A(IPIV, J)*TERP
  23 DO 16 J=1, 2
  24 B(I, J)=B(I, J)+B(IPIV, J)*TERP
  25 CONTINUE
  26 CONTINUE
C
C END SUBSTITUTION SCHEM
C
C DO BACK SUBSTITUTIONS
C
  27 DO 23 J=1, N
  28 DO 21 I=2, 5
  29 I=I-1
  30 I=LPC(I)
  31 DO 21 I=2, N
  32 LL=I-1
  33 II=LPC(LL)
  34 JJ=LPC(LL)
  35 B(I, J)=B(I, J)+B(LL, J)*A(I, JJ)
  36 CONTINUE
C
C UNSCRAMBLE ROWS OF SOLUTION MATRIX AND ADJUST SIGN OF DETERMINANT
C
  37 DO 26 I=1, 5
  38 I=LPC(I)
  39 LTERP(I)=LPC(I)
  40 DO 26 I=1, 5
  41 K=LTERP(I)
  42 IF (I-K) 26, 29, 26
  43 DET=-DET
  44 DO 27 J=1, N
  45 TERP=B(I, J)
  46 B(I, J)=B(K, J)
  47 B(K, J)=TERP
  48 LTERP(I)=LTERP(K)
  49 LTERP(K)=I
  50 GO TO 25
  51 CONTINUE
  52 IER=
  53 END

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0006119  
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SUBROUTINE COORDS
  IMPLICIT REAL*8 (A-H,O-Z)
  DIMENSION S(6,6), SC(10)
  DATA SC/1990.0/
  COMMON ITR, ITR2, ICODE, DATE, IS, IS2, IPRIS, RCS, IOP, I1, I2, I3, I4, I5, I6, I7, I8, I9, I10
  ITR = ITR(0), ITR2 = ITR2(0), ICODE = ICODE(0), DATE = DATE(0), IS = IS(0), IS2 = IS2(0), IPRIS = IPRIS(0), RCS = RCS(0), IOP = IOP(0), I1 = I1(0), I2 = I2(0), I3 = I3(0), I4 = I4(0), I5 = I5(0), I6 = I6(0), I7 = I7(0), I8 = I8(0), I9 = I9(0), I10 = I10(0)
  I = ITR, IS2 = IS2, IPRIS = IPRIS, RCS = RCS, IOP = IOP, I1 = I1, I2 = I2, I3 = I3, I4 = I4, I5 = I5, I6 = I6, I7 = I7, I8 = I8, I9 = I9, I10 = I10
  O = ITR, IS, IS2, IPRIS, RCS, IOP, I1, I2, I3, I4, I5, I6, I7, I8, I9, I10

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C

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  IC = 0
  IF(MATE-LL-2) GOTO 1
  DO 10 ( 1, 2 ), IC
  1 IC = IC + 1
  IF(IC-6) GOTO 99
  IF(MATE-SI-1) PRINT 50
  ITR = 10
  DO 10 ITR = 1, J
  GO TO( 5, 6, 7 ), ITR
  5 PRINT 53
  GO TO 6
  6 PRINT 54
  GO TO 6
  7 PRINT 55
  8 DO 20( 12, 13 ), IC
  12 PRINT 60, (S (M+1, I), I=1, 50)
  GO TO 9
  13 PRINT 60, (S (M+1, I), I=1, 50)
  9 CONTINUE
  IF(MATE-LQ-1) GC IC 99
  DO 9 I=1, 50
  GO TO( 10, 11 ), IC
  10 SC(I) = S(I, 1)*R22/R11 + S(3, I)
  GO TO 9
  11 SC(I) = S(I, 1)*R22/R11 + S(5, I) + S(6, I)
  9 CONTINUE
  GO TO( 40, 41 ), IC
  40 PRINT 56, ITR2
  GO TO 42
  41 PRINT 56, ITR
  42 PRINT 60, (SC(I), I=1, 50)
  IF(MATE-LQ-2) GC IC 99
  IF(IT-LQ-IT2) GO TO 1
  IF(MATE-LQ-0) GC IC 1
  IF(MATE-LQ-4) GO TO 99
  2 IC = IC + 1
  IF(IC-6) GOTO 99
  IF(MATE-SI-1) PRINT 50
  ITR = 10
  DO 10 ITR = 1, J
  GO TO( 15, 16, 17 ), ITR
  15 PRINT 53
  GO TO 16
  16 PRINT 54
  GO TO 16
  17 PRINT 55
  18 DO 20( 22, 23 ), IC
  22 PRINT 61, (S (M+1, I), I=1, 50)
  GO TO 19
  23 PRINT 61, (S (M+1, I), I=1, 50)
  19 CONTINUE
  IF(MATE-LQ-1) GC IC 99
  DO 19 I=1, 50
  GO TO( 20, 21 ), IC
  20 SC(I) = S(I, 1)*R22/R11 + S(3, I)
  GO TO 19
  21 SC(I) = S(I, 1)*R22/R11 + S(5, I) + S(6, I)
  19 CONTINUE
  GO TO( 43, 44 ), IC

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GO TO ( 4, 4, 4, 4, 4 ), NBR
S 24 = 24 * J
GO TO ( 1, 2, 3 ), ICRF
1 S(74, 1) = SLSC
  S(74, 2) = SLEZ
  S(74, 3) = SCDB
  S(74, 4) = SCZC
  S(74, 5) = SLEB
  S(74, 6) = SLEI
  S(74, 7) = SCLD
  S(74, 8) = SCZL
  S(74, 9) = SIF
  S(74, 10) = SIF
  S(74, 11) = SIF
  S(74, 12) = SIF
  S(74, 13) = SIF
  S(74, 14) = SIF
  S(74, 15) = SIF
  S(74, 16) = SIF
  S(74, 17) = SIF
  S(74, 18) = SIF
  GO TO 50
2 S(74, 1) = SLSC
  S(74, 2) = SLEI
  S(74, 3) = SCSC
  S(74, 4) = SCSE
  S(74, 5) = SIF
  S(74, 6) = SIF
  S(74, 7) = SIF
  S(74, 8) = SIF
  S(74, 9) = SIF
  S(74, 10) = SIF
  S(74, 11) = SIF
  S(74, 12) = SIF
  S(74, 13) = SIF
  GO TO 50
3 S(74, 1) = SCPC
  S(74, 2) = SGB
  S(74, 3) = SGT
  S(74, 4) = SCB
  S(74, 5) = SGT
  S(74, 6) = SAT
  S(74, 7) = SC
50 RETURN
END

```

S700E  
S720E