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DETECTION OF INTERNALLY DEPOSITED ACTINIDES

PART II: STATISTICAL TECHNIQUES AND RISK ANALYSIS

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Abstract

Since a considerable number of workers at Oak Ridge National Laboratory work with compounds of the transuranic elements, computer techniques have been developed to evaluate phoswich spectra in order to determine lung burdens following accidental inhalation of ^{239}Pu, ^{241}Am, ^{244}Cm or other isotopes. Two unfolding methods which have been found useful in the analysis of such cases are presented and discussed. These techniques have been used successfully to detect low levels of ^{239}Pu, ^{241}Am, ^{244}Cm, ^{233}U, ^{90}Sr, and ^{153}Gd in contaminated workers; but because of the current importance of ^{239}Pu, emphasis is placed on detection of that isotope in the presence of ^{241}Am and natural human background. In the health physics tradition of emphasizing benefit vs. risk, we also analyze uncertainties inherent in external counting of the actinides from the viewpoint of statistical risk analysis and derive decision criteria which are useful in determining whether various radioactive species have, in fact, been detected. These criteria are somewhat different from those encountered using traditional counting statistics and derive from the realization that some errors will always be made in scanning large numbers of radiation workers. The optimum decision strategy for the determination of lung burden is, therefore, one which minimizes the long-term risk of error. The usefulness of this approach to whole body counting will be discussed and analyzed.

Introduction

A topic of considerable concern to the health physics profession is the development of sensitive methods to detect low levels of actinides in the lung. In the case of inhalation of the insoluble oxides of these elements, it is common practice at many installations to determine the lung burden by external counting with either a proportional counter or a phoswich detector. As part of our program at Oak Ridge National Laboratory to develop new techniques for analysis of phoswich spectra, a general computer unfolding method was presented recently for determining lung burdens of ^{239}Pu in the presence of ^{241}Am, human background, and other interfering isotopes.(1) In Figure 1 typical phoswich spectra for ^{239}Pu, ^{241}Am and human background are presented. The net count in each channel of the human spectrum is assumed to be composed of a linear combination of counts due to the various isotopes under consideration. The fraction α_i of the i^{th} reference standard contained in the human spectrum ($i = 1, \ldots 3$ corresponding to ^{239}Pu, ^{241}Am and human background, respectively) is obtained by minimizing the weighted sum of the squares of the residuals with respect to each α_i, thereby leading to a matrix equation

$$A\alpha = X \qquad (1)$$

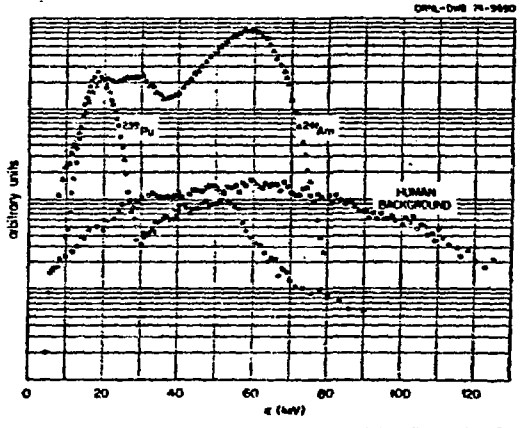

Fig. 1 Phoswich calibration spectra.

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to invert for the respective α_i. The elements of the n x n symmetric matrix A are

$$A_{ij} = \sum_K \varphi_K^i \varphi_K^j W_K \tag{2}$$

and those of α and X are, respectively, α_i and

$$X_i = \sum_K C_K \varphi_K^i W_K \tag{3}$$

In Eq. (2) - (3) C_K is the net count in channel K of the human spectrum, W_K is an arbitrary weighting factor, and φ_K^i is the count in channel K due to Q_i nanocuries of the i^{th} standard. The summations are taken over all energies of interest, typically 10-128 keV. The product

$$q_i = \left(\alpha_i \pm Z_\gamma \sigma_{ii}\right) Q_i \tag{4}$$

gives the activity of the i^{th} isotope in the human spectrum along with the associated 100 γ % confidence interval. In Eq. (4) σ_{ii} is the ii element of the covariance matrix associated with the unfolding and Z_γ is a statistical factor corresponding to different levels of confidence.

It has been apparent to health physicists for some time that there is an inherent spectral decomposition in phoswich spectra containing ^{239}Pu and ^{241}Am since spectra of these isotopes cover mainly the regions 10-24 keV and 10-75 keV, respectively, while human background covers the entire spectral range, 10-128 keV. If, in some energy band, $\epsilon_1 \leq \epsilon \leq \epsilon_2$, the phoswich spectrum has only one component, then Eqs. (1) - (4) reduce to

$$q = \frac{Q_i \sum_K W_K C_K \varphi_K}{\sum_K W_K \varphi_K^2} \tag{5}$$

for a least squares estimate of the activity q of that component in the human spectrum. Using the fact that counts in the region 75-128 keV are entirely human background, a least squares estimate of this contribution can be made from Eq. (5), where φ is a human background spectrum. φ may be either the subject's own uncontaminated spectrum, if that is available, or a suitable averaged spectrum taken from uncontaminated workers of the same weight-to-height ratio. After using Eq. (5) to determine the amount of ^{40}K in the spectrum, that amount is subtracted, leaving the ^{241}Am and ^{239}Pu contribution. The ^{241}Am chest burden can be determined by again using Eq. (5), where φ is now the appropriate library standard, in this case ^{241}Am, and summing over the channels corresponding to the energy range 51-75 keV. Subtracting the ^{241}Am contribution from the ^{239}Pu spectral region (10-24 keV) and again using Eq. (3) yields a good estimate of the ^{239}Pu lung burden.

Statistical Techniques and Risk Analysis

The preceeding discussion has presented both a direct matrix inversion method (Eq. 1) and a method of successive stripping (Eq. 5) for the determination of lung burdens of ^{239}Pu and ^{241}Am. Normally, both algorithms are used to provide independent evaluations of the lung burden. Determination of the amount of ^{40}K present in the phoswich spectrum is particularly important in the context of Pu detection, since small errors in this quantity may lead to large uncertainties in the calculated actinide lung burden. At ORNL we have observed that the ratio

$$\frac{^{40}\text{K scatter in 12-24 keV region}}{\text{integral of 1.46 MeV } ^{40}\text{K photopeak}} = \frac{r_{\text{phoswich}}}{r_{9x9}} \tag{6}$$

is reasonably constant in uncontaminated individuals. The 1.46 MeV gamma photon from chest ^{40}K is routinely monitored by placing a 9" x 9" NaI crystal opposite the phoswich assembly. In Fig. 2 the raw data are presented along with a linear least squares fit of the form

$$r_{\text{phoswich}} = A + B\, r_{9x9} \tag{7}$$

where A was found to be statistically zero ($A \sim 19 \pm 34$) within the precision of the experiment and B was determined to be 0.042 ± 0.003. Use of the dual detector technique is particularly helpful since it gives another evaluation of the ^{40}K

scatter in the subject's phoswich spectrum. This additional information is important and may be used in conjunction with the previous unfolding algorithms for improved determination of lung burden in routine personnel monitoring and in accident situations.

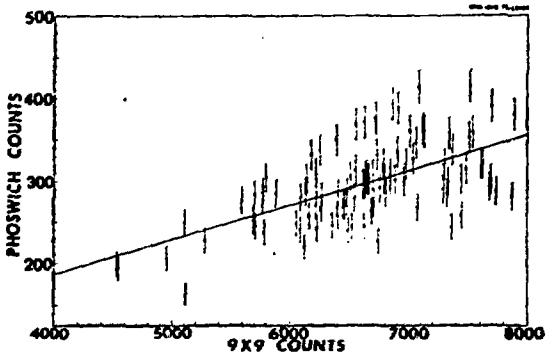

Fig. 2 Comparison between ^{40}K scatter in 12-24 keV region of phoswich and 9 x 9 NaI determination of chest ^{40}K.

In all computer unfolding methods, after examining the net counting rate in a specific region of the spectrum (12-24 keV for detection of the 17 keV X rays from ^{238}, ^{239}Pu and ^{244}Cm), it is necessary to decide whether or not various species have, in fact, been detected. Hypothesis testing such as this involves two kinds of errors: (1) deciding that the contaminant is present when it is not (probability = α, known as error of the first kind) and (2) failing to decide that contamination is present when it is (probability = β, known as error of the second kind). From a health physics viewpoint it is necessary for installation management to decide which error is more critical, and any development of decision criteria must reflect this fact.

Since all ORNL workers at risk are routinely screened for various transuranic elements at the Whole Body Counter, it is useful to formulate a decision strategy which gives the analyst an objective criterion for deciding whether or not actinide activity is present. Various criteria have, in fact, been developed over the years based on the application of traditional counting statistics.[2, 3] Since many workers are analyzed at the Whole Body Counter each year, it is evident that some errors in the determination of lung burden will always be made. Therefore, in the health physics tradition of emphasizing benefit vs. risk, we assert that it is best for those analysts who must make a large number of decisions under similar circumstances to employ decision criteria which minimize the average total error.

In all actinide detection problems involving external counting, a residual counting rate r is observed in the L X-ray band after room background and the appropriate amount of human background (^{40}K) have been subtracted, and one desires to determine in some objective manner whether this rate belongs to a statistical distribution centered about zero or about a rate X_q corresponding to q nCi of the actinide under consideration. In order to make such a decision, two statistical hypotheses, H_0 (no activity present) and H_1 (q nCi of actinide present) are formulated and a simple dichotomy of the range of values is determined so that H_0 is chosen when $r < X_0(q)$ while H_1 is chosen when $r > X_0(q)$. These two distributions are shown schematically in Fig. 3. $X_0(q)$ is dependent upon the level of actinide activity q, the relative costs of errors of the first and second kinds, and the decision criteria used. In this note concise expressions for $X_0(q)$ will be presented using the Bayes and Neyman-Pearson criteria. From Fig. 3 it is evident that, occasionally, a wrong decision will be made regardless of the value of $X_0(q)$. The utility of this method lies in the fact that an error profile can be developed which is dependent both on the decision criterion used and on the range of q examined.

For simplicity it is convenient to assume that the net counting rate r is normally distributed with distribution function $f_K(r)$ (K = 0, 1) under the hypotheses H_0 and H_1, respectively. Using $X_0(q)$ as a quantity to be evaluated under the Bayes and Neyman-Pearson criteria, the probability of deciding that contamination is present when it actually is absent is

$$\alpha = \int_{X_0(q)}^{\infty} f_0(X)\, dX = \frac{1}{2}\ \mathrm{erfc}\ \left(\frac{X_0(q)}{\sqrt{2}\ \sigma}\right) \qquad (8)$$

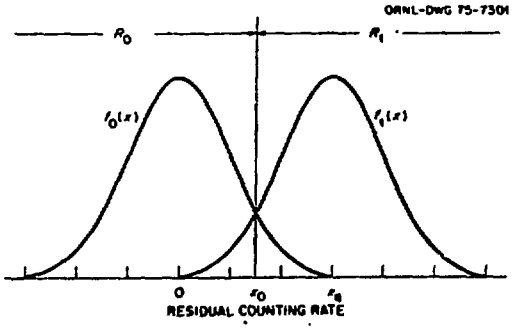

Fig. 3 Probability density functions under hypotheses H_0 and H_1.

where the complementary error function is defined by $erfc(X) = 2/\sqrt{\pi} \int_X^\infty e^{-t^2} dt$ and σ is the standard deviation of the net counting rate. Likewise, the probability of missing a level q is

$$\beta = \int_{-\infty}^{X_0(q)} f_1(X)dX = \frac{1}{2}\left[erf\left(\frac{X_0(q)-X_q}{\sqrt{2}\,\sigma}\right)+1\right]. \tag{9}$$

where the error function is defined by $erf(X) = 1 - erfc(X)$ and X_q is the counting rate in the L X-ray band corresponding to q nCi of actinide. In order to evaluate $X_0(q)$, it is useful to observe that the long-term probability of error P_e is given by

$$P_e = \xi\,\alpha + (1-\xi)\,\beta \tag{10}$$

where ξ is the prior probability of observing an uncontaminated subject (probability of obtaining H_0). Minimizing P_e with respect to $X_0(q)$ gives

$$\frac{f_1[X_0(q)]}{f_0[X_0(q)]} = \frac{\xi}{(1-\xi)} = \Lambda_0 \tag{11}$$

as the minimum risk statistic to test at the completion of each count. Given the observed rate r, we compute the likelihood ratio $\Lambda(r) = f_1(r)/f_0(r)$ and choose H_1 if $\Lambda(r) > \Lambda_0$ and H_0 if $\Lambda(r) < \Lambda_0$. This decision philosophy is known as the Bayes solution to the problem of detection of actinides in the lung. From Eq. (4) the decision level $X_0(q)$ corresponding to q nCi in the lung is found to be

$$X_0(q) = \frac{X_q}{2} + \frac{\sigma^2}{X_q} \ln \Lambda_0 \tag{12}$$

For some values of q, r will be less than $X_0(q)$ and hypothesis H_0 will be chosen while for other values of q, r will be greater than $X_0(q)$ and, consequently, hypothesis H_1 will be chosen. An error profile can, thus, be generated since values of α and β can be calculated from Eqs. (8) and (9) for each q. In accident situations the most pertinent parameter is the maximum organ burden that could be present, given the net counting rate r. Under the Bayesian decision criterion this would be the value of q such that $r = X_0(q)$. Solving for X_q in Eq. (12), therefore, gives

$$X_q = r + \sqrt{r^2 - 2\sigma^2 \ln \Lambda_0} \tag{13}$$

as the counting rate corresponding to q_{max}. Knowledge of X_q, the subject's chest wall thickness, and the oppropriate counter calibration factors, therefore, leads to determination of the maximum lung burden possible.

In many situations involving chest counting, the population at risk contains only a few individuals who would have significant probability of obtaining > 10 nCi of actinide in the lung region. In these situations the principal factor in the error expression P_e is the fraction of the time α in which H_1 is incorrectly chosen and an error of the first kind is made.

Such a false alarm may involve personnel apprehension and costly action taken by the laboratory in vain. Under such circumstances it is appropriate for the health physicist to determine the value of the probability α that he can afford and to seek a decision strategy that attains this value and at the same time yields the minimum possible probability β of making an error of the second kind. This type of decision strategy is said to fulfill the Neyman-Pearson criterion and corresponds to maximizing the probability of detecting actinide activity in the lung for a given false alarm probability.

Using the Neyman-Pearson strategy, X_0 is chosen from the predetermined value of α by

$$X_0 = \sqrt{2}\ \sigma\ \mathrm{erfc}^{-1}(2\alpha) \tag{14}$$

Once α is chosen, the error profile on β can be evaluated under different assumptions of q as follows

$$\beta(X_q) = \frac{1}{2}\left[\mathrm{erf}\left(\frac{X_0 - X_q}{\sqrt{2}\ \sigma}\right) + 1\right] \tag{15}$$

In Appendix I a brief generalization of the Bayes criterion will be presented for the case where the risks associated with errors of the first and second kinds are unequal, and in Appendix II practical calculations involving both criteria will be given to emphasize the utility of these decision strategies in applied health physics.

Appendix I

It is possible to generalize the Bayes criterion by observing that errors of the first and second kinds may involve different risks, both for the analyst and for the worker. The long-term probability of error P_e now becomes the long-term risk function P_R given by

$$P_R(X_0) = C_0\,\xi\,\alpha + C_1\,(1-\xi)\,\beta \tag{16}$$

where ξ is the prior probability of examining an uncontaminated worker (probability of obtaining hypothesis H_0), α and β are errors of the first and second kinds, respectively, and C_i, $i=0, 1$ are risks attendant to α and β, respectively. The function $C_1(q)/C_0(q)$ gives the relative risk of errors of the second kind as compared to those of the first kind. From a radiobiological viewpoint it might be argued that errors of the second kind are more costly since there may be health problems for the worker and legal problems for the employing installation if amounts of Pu significantly in excess of several lung burdens are missed. On the other hand a significant number of false alarms tend to promote employee apprehension and distrust and may involve costly action taken by the employing installation. From a pragmatic viewpoint it is necessary to reach a compromise corresponding to maximizing detection probability while minimizing the number of false alarms. Clearly, errors of the first and second kinds may involve different risks, but it is difficult at this time to assess the relative magnitudes of these quantities. Use of the relative risk functions $C_i(q)$ simply allows inclusion of this fact in the development of decision criteria. Minimization of the long-term risk function, therefore, gives

$$X_0(q) = \frac{X_q}{2} + \frac{\sigma^2}{X_q}\ln\frac{\xi\,C_0}{(1-\xi)\,C_1} \tag{17}$$

as the decision level for the Bayes criterion.

Appendix II

Consider a hypothetical lung counting example where the following parameters have been obtained from a 40-minute count:

room background (narrow window) rate = $r_b = 2$ cpm for two 5" diameter phoswich detectors

The 9" x 9" NaI rate obtained by placing the crystal opposite the phoswich assembly and under the subject's back was 7000 counts in 40 minutes, so that $r_{9x9} = 175$ cpm, and the estimate of ^{40}K scatter in the 12-24 keV region is, therefore, $r_{phoswich} = 0.042\,(r_{9x9}) = 7.35 \pm 0.52$ cpm. The gross counting rate in the 12-24 keV region was found to be 10.35 cpm and, using the above data, the net rate and its standard deviation σ is $r_{net} = 1.0 \pm 0.75$ cpm. Furthermore, in the population of workers under consideration, 80% were found to be uncontaminated on previous counts, so that $\xi = 0.8$ and

$5/1-5 = \Lambda_0 = 4$. The decision level derived using the Bayes criterion is then given by

$$X_0 = \frac{X_q}{2} + \frac{\sigma^2}{X_q} \ln \Lambda_0 = \frac{X_q}{2} + \frac{0.78}{X_q} \tag{18}$$

Close examination of $X_0(X_q)$ shows that X_0 possesses a minimum at some point X_q^{min} and, for valid use of the Bayes criterion, analysis must be restricted to those values of X_q such that $X_q > X_q^{min}$. For $r_{net} < X_0^{min}$, hypothesis H_0 will always be chosen so that X_q^{min} may be considered to be the minimum detectable activity under the Bayes criterion. Solving $\partial X_0 / \partial X_q = 0$ for X_q^{min} gives

$$X_0^{min} = X_q^{min} = \sigma \sqrt{2 \ln \Lambda_0} = 1.67\sigma \tag{19}$$

Using the values previously given $X_0^{min} = X_q^{min} = 1.25$ cpm. Since $r_{net} < X_0^{min}$, in this case the analyst must choose hypothesis H_0 and report that this subject is uncontaminated using the Bayes decision criterion. The pertinent error here is β, the fraction of times where one will fail to decide that activity is present when it actually is. The probability of missing a level of q nCi corresponding to the counting rate X_q is

$$\beta = \int_{-\infty}^{X_0} f_1(X)\, dX = \frac{1}{2} \left[1 + \text{erf} \left(\frac{X_0 - X_q}{\sqrt{2}\,\sigma} \right) \right] \tag{20}$$

At $X_0^{min} = X_q$, $\beta = 0.5$ while for $X_q = 2$ cpm, $\beta = 0.21$. Suppose, for example, that the appropriate calibration factor for the subject is 5 nCi/cpm. Then, if the net rate had corresponded to ~ 6 nCi, 50% of the time hypothesis H_0 would have been chosen erroneously. Furthermore, if r_{net} had corresponded to 10 nCi in the lung, then only 21% of the time would the wrong hypothesis have been chosen.

Suppose that the net counting rate in the 12-24 keV region had been 2 ± 0.75 cpm. Clearly, $r_{net} > X_0^{min}$ so that hypothesis H_1 is chosen. For reporting purposes, we must assume that some internal contamination is present and proceed accordingly. The maximum lung burden that could correspond to r_{net} is given by

$$X_q^{max} = r_{net} + \sqrt{r_{net}^2 - 2\sigma^2 \ln \Lambda_0} \tag{21}$$

and equals 3.57 cpm or 17.8 nCi.

Turning from the Bayes criterion to the Neyman-Pearson criterion, in this strategy, X_0 is chosen from a predetermined value of α as follows:

$$X_0 = \sqrt{2}\,\sigma\, \text{erfc}^{-1}(2\alpha) = \sqrt{2}\,\sigma\, \text{erf}(1-2\alpha) \tag{22}$$

Setting $\alpha = 0.05$ gives $X_0 = 1.23$ cpm. For the first case $r_{net} = 1$ cpm and H_0 would be chosen since $r_{net} < X_0$. In the second case $r_{net} = 2$ cpm $> X_0$ which necessitates choice of hypothesis H_1.

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