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LENS AND MIRROR DESIGN VIA THE PRINCIPAL SURFACE\*

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MASTER

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## LENS AND MIRROR DESIGN VIA THE PRINCIPAL SURFACE\*

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### ABSTRACT

For many laser applications, it is desired to focus a collimated beam with a specified transformation of the intensity distribution. The transformation properties of a lens or mirror system can be specified in terms of the principal surface,  $r(\alpha)$ , which maps the height of the incident parallel ray onto a given angle at the focus. The intensity distribution at the focus is then given by the relation  $I(\alpha) = I(r)r(dr/d\alpha)/\sin\alpha$ . One aspheric surface in an optical system is sufficient to yield diffraction limited focusing. By means of two aspheric surfaces, diffraction limited performance with a specified principal surface can be achieved.

The problem of optical design is stated as follows: Given a principal surface  $r(\alpha)$ , and a maximum focal angle  $\alpha_m$ , find the pair of optical surfaces for which diffraction limited focusing is achieved. It is shown that specification of  $r(\alpha)$  and  $\alpha_m$  uniquely determines the lens design to within a scale factor, given the refractive index of the lens. It is further shown that one straightforward Runge-Kutta integration routine generates both surfaces for either a lens or a pair of mirror surfaces.

The complete family of aplanatic lenses will be described. Deviation from sphericity will be discussed, as will the possibility of realizing the specified lens designs. The family of lenses which map uniform incident intensity into uniform illumination about the focus will also be described. Extension of the method to off-axis aberrations will be considered.

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## INTRODUCTION

In the design of focusing optics for laser fusion experiments, special requirements arise which are significantly different from the considerations which govern the design of typical imaging optics. In this paper, we shall outline the special features of laser focusing optics which differentiate their design from other systems, and report a philosophy of lens design particularly suited to laser focusing optics. In particular, we shall develop the formalism for the design of lenses from the principal surface, which represents the mapping of ray height in the entrance pupil onto ray angle at the focal point. The equivalent formulation for reflecting optics will be given. Specific examples of families of lens surfaces will be presented.

In laser fusion experiments, one illuminates a spherical target as uniformly as possible, over its entire surface, keeping the light as near to normal incidence as possible.<sup>(1)</sup> The dual requirement of near-normal incidence and uniform illumination arises from the desirability of creating a uniform heating of the plasma over the entire target surface.<sup>(2)</sup> The light is generated as the output of a large, short-pulse laser, usually a Nd:glass laser, radiating at  $1.06\mu\text{m}$ . The beam profile is generally a function of ray height in the entrance pupil only, and is given by the operating constraints of the laser system.

In the design of a large, short-pulse glass laser, the crucial parameter is the total integral of the laser intensity along that part of the optical path that lies in glass, either laser glass or optical glass.<sup>(3)</sup> This parameter, the so-called "B-Integral", must be kept to a minimum, in order to prevent the growth of high spatial frequencies (ripples) on the beam, due to the nonlinear coupling of the intense

light to the optical medium. Thus in designing laser focusing optics, the total thickness of the lenses must be kept to a minimum. In addition, since optical surfaces are most vulnerable to damage, at high intensities, the number of surfaces must be kept to a minimum. These considerations militate against the use of a large number of elements in the focusing optics. In general, a number of elements no greater than two is desirable in any given focusing lens.

Laser fusion optics fall in the category of energy delivery systems, rather than imaging systems. The quality of the image, in the focal plane of the lens, is less important than the pattern of illumination generated on the target surface. In addition, due to the special features of laser illumination, there are fewer additional constraints on the lens design. For example, since the laser beam is generally well-collimated, and incident parallel to the axis of the system, off-axis aberrations can be neglected. Also, since the laser is monochromatic, chromatic aberration is of no concern. Thus the merit of a particular design is entirely specified by how well the illumination requirements are met.

Clearly, to effect a given transformation of ray height onto focal angle with a few elements requires aspheric surfaces. Recent advances in aspheric fabrication strongly support this approach. It is becoming increasingly feasible to fabricate steep and complicated aspheric surfaces at reasonable cost. It must be understood, however, that these are one-of-a-kind optical designs, of which only a few copies will be made. The initial costs of design, special tooling, and test setup must be distributed over these few copies. The large laboratories working in laser fusion must be prepared to bear this increased cost, as part of the price of developing a sophisticated and specialized industrial base to support their needs.

It is hoped that in the long run, the entire optical industry will benefit from the advances made and paid for under such specialized programs as laser fusion.

#### EXPLICIT METHODS OF LENS DESIGN

Since we are considering very specialized optical systems consisting only of a few elements, the customary interactive methods of lens design, such as are embodied in generally available lens design programs, are not the most efficient methods available for design. It is more efficient to design the lens surfaces explicitly in order to obtain exactly the desired transformation properties. We shall refer to this approach as the use of "explicit methods", instead of iterative adjustment of surface parameters to obtain an optimal approximation to the desired performance.

In geometrical optics, the objective of design is to determine a set of refractive or reflecting surfaces which map a set of rays, specified in terms of ray height and angle to the symmetry axis, on a plane in object space,  $Z_1$ , onto a set of rays on a plane in image space,  $Z_2$ , with prescribed ray height and angle to the axis. This transformation can be expressed as

$$(Z_1, R_1, \theta_1) \rightarrow (Z_2, R_2, \theta_2), \quad (1)$$

where  $R$  and  $\theta$  represent ray height and angle to the axis respectively. In this discussion, the system is assumed to be cylindrically symmetrical, and skew rays are not considered. As is pointed out in Luneburg's treatise, (5) if the mapping is defined in terms of ray position at the focus, then a system free of spherical aberration can be constructed by the specification of a single aspheric surface. This is carried out by ensuring that the optical path from every point on the object plane to the focal point is the same. We shall refer to this as the equal path condition. The

mapping can be represented in the form

$$(Z_1, R_1, \theta_1) \rightarrow (F, \alpha) \quad (2)$$

where  $F$  denotes the focal plane, and  $\alpha$  the angle between the ray and the axis at the focal point. In this case, the system is free of on-axis aberrations, but the intensity distribution about the focal point is completely determined.

A single aspheric surface can also be used to obtain a desired intensity distribution, but in this case, the system is afocal, i.e., all rays do not pass through a single focal point. (6) The prescribing mapping is given by

$$(Z_1, R_1, \theta_1) \rightarrow (Z_2, R_2) \quad (3)$$

but the ray angle at the image plane is completely determined. The intensity in the image plane is given by

$$I(R_2) = I(R_1) R_1 dR_1/R_2 dR_2 \quad (4)$$

where  $I(R_1)$  is the incident intensity distribution in the object plane. This method has been used to design laser illumination systems, and is discussed in a separate paper. (7)

In order to satisfy the equal path condition, and, simultaneously, to obtain the desired intensity mapping, the use of two aspheric surfaces in the optical system is required. Since the equal path condition is satisfied in this case, we can define the transformation in terms of ray angle at the focal point (where all ray heights are identically zero). We shall assume in the following discussion that the incident beam is parallel to the symmetry axis, although the method readily generalizes to converging or diverging incident light. Under this assumption, the desired mapping is entirely specified by the function  $R_1(\alpha)$ , where  $R_1$

is the ray height in the object plane, and  $\alpha$  is the focal angle. This function defines a surface in space, referred to as the principal surface, since it is tangent to the secondary principal plane at the axis. The principal surface is sometimes called the equivalent refracting surface, since it represents the intersection between the incident rays in the object space, and the focused rays in the image space. We shall see that a knowledge of the principal surface completely determines two aspheric surfaces, to within a scale factor, given either the focal angle of the marginal ray,  $\alpha_m$ , or the ratio of the back focal length to the lens thickness.

#### CALCULATION OF THE LENS SURFACES

In order to simplify the discussion, we shall consider a system of two aspheric refracting surfaces. There is no essential difficulty in introducing an arbitrary number of spherical surfaces in the system, but to do so renders the exposition less straightforward. For ease of fabrication, one would generally use two elements (four surfaces) with one aspheric surface on each element.

We consider the two aspheric surfaces as shown in Fig. 1. The principal surface is defined by the function  $R(\alpha)$ , where the ray height extends to a maximum value  $R_m$ , corresponding to a maximum value of the focal angle,  $\alpha_m$ . The marginal ray which is incident on the system at height  $R_m$ , intersects the principal surface at the point at which the two optical surfaces cross. By inspection, we see that the principal surface always passes through the intersection of the two lens surfaces.

Referring to Fig. 1, we want to integrate the equation for the displacement of the first surface from the lens vertex,  $Z$ , as a function

of the focal angle  $\alpha$ , where  $R(\alpha)$  is known. From Snell's law, we can write the slope of the first surface in terms of the angle of deflection of the ray entering the optical medium,  $\phi$ , as

$$\frac{dZ}{dR} = - \frac{n \sin \phi}{n \cos \phi - 1} \quad (5)$$

where the minus sign arises from the definition of  $Z$ . Here  $n$  is the refractive index of the optical medium. We define the distance  $P$  as the distance in object space from the first refracting surface to the principal surface, the distance  $Q$  as the distance in image space from the principal surface to the second refracting surface, and the distance  $z$  as the distance traveled through the optical medium by the actual ray. From the law of sines, we have

$$\frac{P}{\sin(\alpha - \phi)} = \frac{Q}{\sin \phi} = \frac{z}{\sin \alpha} \quad (6)$$

We want to express  $\phi$  entirely in terms of the angle  $\alpha$ , and the distance  $Z$ . Simple geometry yields the result

$$P = Z + (R_m / \tan \alpha_m) - (R / \tan \alpha) \quad , \quad (7)$$

while the equal path condition takes the form

$$Z + (R_m / \sin \alpha_m) - (R / \sin \alpha) = P(n \sin \alpha - \sin \phi) / \sin(\alpha - \phi) \quad . \quad (8)$$

Equating two expressions for  $P$  yields the result,

$$\frac{\sin(\alpha - \phi)}{n \sin \alpha - \sin \phi} = f(Z, \alpha) = \frac{Z + (R_m / \tan \alpha_m) - (R / \tan \alpha)}{Z + (R_m / \sin \alpha_m) - (R / \sin \alpha)} \quad (9)$$



Equation 9 can be written as

$$\sin \alpha \cos \phi + [f(Z, \alpha) - \cos \alpha] \sin \phi = n \sin \alpha f(Z, \alpha), \quad (10)$$

and solved for  $\phi$  in any of several ways (see Appendix I). Combining with Eq. (5), we can write

$$\frac{dz}{d\alpha} = - \left( \frac{dR}{d\alpha} \right) \frac{n \sin \phi}{n \cos \phi - 1} \quad (11)$$

which can be integrated using Runge-Kutta integration. Since the function  $f(z, \alpha)$  in Eq. (9) is indeterminate at the vertex, where  $R = R_m$  and  $\alpha = \alpha_m$ , l'Hopital's rule must be applied to start the integration. At each step, the function  $Z(\alpha)$  is determined. Since  $R(\alpha)$  is known, the first surface is thus specified. The coordinates of the second surface are then given as

$$R' = R - Y \sin \alpha \sin \phi / \sin (\alpha - \phi) \quad (12)$$

and

$$Z' = P \sin \alpha \cos \phi / \sin (\alpha - \phi) - Z \quad (13)$$

where  $Z'$  is measured from the vertex, as shown in Fig. 1.

If we examine the ray along the axis, we express the equal path condition as

$$Z + (R_m / \sin \alpha_m) = (n - 1)t + (Z + R_m / \tan \alpha_m) \quad (14)$$

where  $t$  is the axial thickness of the lens. Canceling the  $Z$  in Eq. (14), we obtain a general relation among lens thickness, index, and marginal ray parameters, namely,

$$(n-1)t = R_m \tan (\alpha_m/2). \quad (15)$$

In the case in which the back focal length,  $F$ , and lens thickness,  $t$ , are specified, we define  $Z$  as the distance from the focal plane to the intersection of the ray with the first surface. Then all of the previous formalism carries over intact, with the exception

that in Eq. 9 and 10, the function  $f(Z, \alpha)$  now takes the form

$$f(Z, \alpha) = (Z - R/\tan \alpha)/(Z + (n-1)t - R/\sin \alpha) \quad (16)$$

and that the coordinate of the second surface, measured from the focal plane, is given by

$$Z' = Z - P \sin \alpha \cos \phi / \sin(\alpha - \phi) \quad (17)$$

For reflecting surfaces, exactly the same considerations apply. Referring to Fig. 2, we now define the angle  $(-\phi)$  as the angle of deflection of the incident ray. The distance  $P$  is the distance in object space from the first reflecting surface to the principal surface, which the distance  $(-O)$  is the distance in image space from the second reflecting surface to the principal surface. Using these definitions, and taking  $n=1$ , we find that Eqs. 10-13 carry over without change. We note that the point corresponding to the lens vertex is again the point of intersection of the two reflecting surfaces. This point, which is, of course, never realized in a practical system, corresponds to the deflection point of a ray which is tangent to the first reflecting surface.

#### EXAMPLES OF THE APPLICATION OF THE METHOD

To illustrate the application of the method, we have computed the family of aplanatic lenses, for which the principal surface is given by  $R = \sin \alpha / \sin \alpha_m$ , and the family of lenses which map equal beam areas onto equal solid angles at the focus, for which the principal surface is given by  $R = \sin(\alpha/2) / \sin(\alpha_m/2)$ . This latter mapping is obtained by requiring that  $R dR = \sin \alpha d\alpha$ . In both cases, the ray height at the marginal ray is taken to be  $R_m = 1$ . In Fig. 3, we see lens profiles for aplanatics of increasing numerical aperture. As is expected, the asphericity of both surfaces increases dramatically with

increasing N.A. The same is seen to be true in Fig. 4, where the lens profiles for equal area mapping are shown for increasing N.A. All of these cases were computed for  $n = 1.5$ . The achievable value of N.A. is limited by total internal reflection in the lens. The designs were computed on a CDC 7600 computer, and each design, embodying 100 points across the lens (100 rays) took approximately 200msec.

#### SUMMARY

We have shown that for laser focusing optics, which consists in general of a few, aspheric lens surfaces, explicit design methods are advantageous. We have presented the general solution of the two-surface problem, which is the simplest arrangement which can simultaneously give a prescribed mapping of intensity from a beam onto the surface of a sphere while satisfying the equal path condition. Examples of the family of aplanatic lenses and lenses which map equal beam areas onto equal solid angles have been given.

The explicit design methods outlined in this paper have been applied to a number of laser focusing lens designs in the course of the Lawrence Livermore Laboratory Laser Fusion effort. The reader is referred to the 1974 Annual Report of the Laser Fusion program for further details.<sup>(3)</sup>

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APPENDIX I

An equation in the form of Eq. 10, namely

$$a \cos \phi + b \sin \phi = c \quad (A-1)$$

must be solved with some care, due to ambiguities in sign. It is convenient to introduce complex notation, and write

$$(a + ib) = r \exp(i\alpha) \quad (A-2)$$

Equation A-1 then takes the form

$$\cos(\phi - \alpha) = c/r \quad (A-3)$$

the solution of which can be written

$$\phi = \cos^{-1}(a/r) + \cos^{-1}(c/r) \quad (A-4)$$

with  $r = (a^2 + b^2)^{1/2}$ . The sine and cosine of  $\phi$  can easily be obtained in terms of the coefficients a, b, and c, by use of standard trigonometric identities.

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FIGURE CAPTIONS

- Fig. 1 Definition of ray parameters for refracting surfaces.
- Fig. 2 Definition of ray parameters for reflecting surfaces.
- Fig. 3 Lens profiles for aplanatic lenses of varying N.A. The dotted line indicates the principal surface.  $n = 1.5$ .
- Fig. 4 Lens profiles for equal area mapping. The dotted line indicates the principal surface.  $n = 1.5$ .

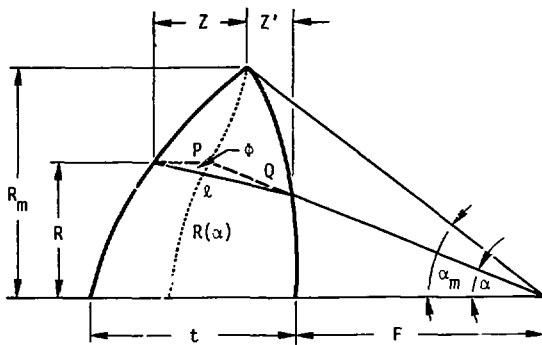


FIGURE 1

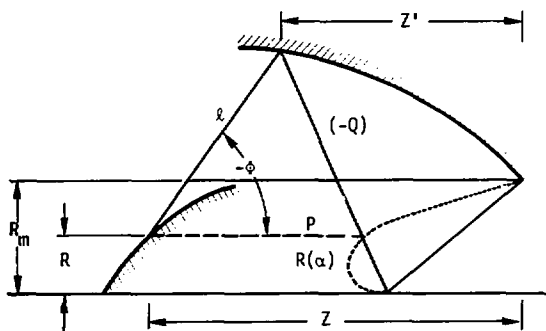
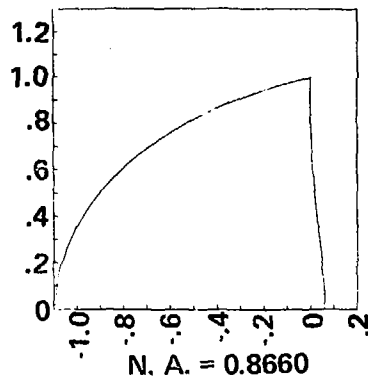
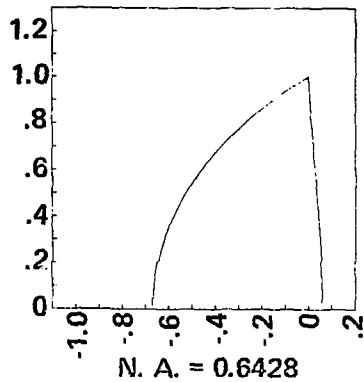
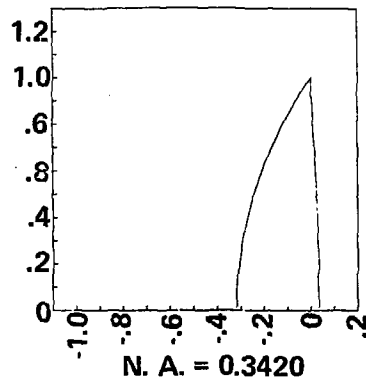
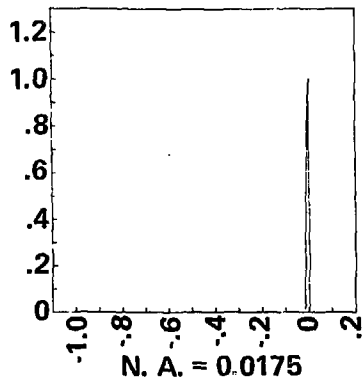


FIGURE 2





FIGURE 3



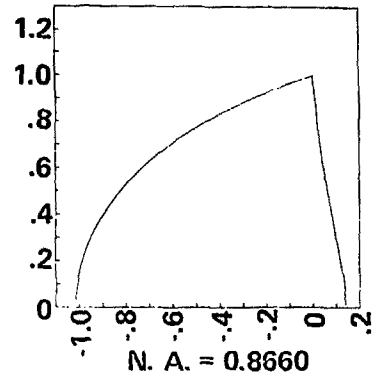
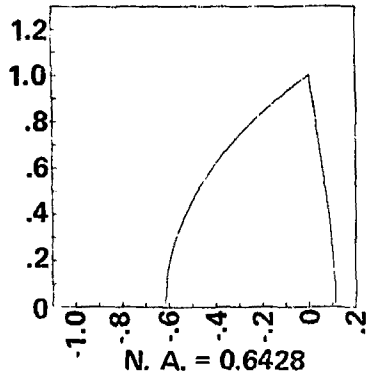
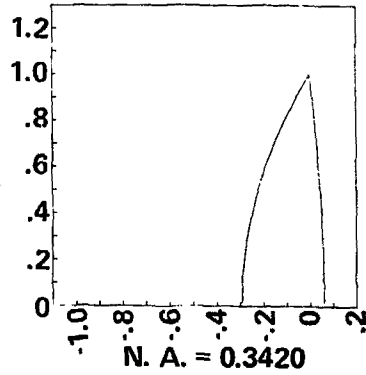
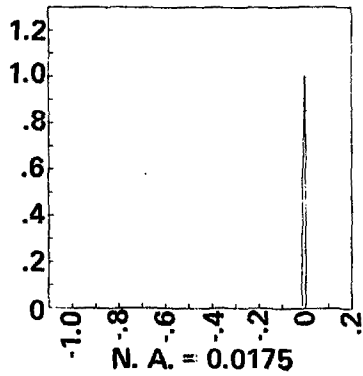


FIGURE 4