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ANALYTIC SOLUTIONS FOR CONSTANT TENSION COIL SHAPES

BY

S. L. GRALNICK AND F. H. TENNEY

PLASMA PHYSICS LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

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S. L. Gralnick and F. H. Tenney
Plasma Physics Laboratory
Princeton University
Princeton, New Jersey 08540

ABSTRACT

An analytical solution of the differential equation describing the shape of a flexible filamentary conductor (incapable of supporting bending stresses) in a toroidal magnetic field has been obtained - previously only numerical solutions were available. The solution derives from a series expansion of modified Bessel functions of integer order. The characteristics of toroidal field magnets for proposed tokamak devices are obtainable by term by term integration of the solution series. General expressions are given for the following coil characteristics: the conductor turn length, the solenoid inductance, the area enclosed by the coil and the coil support dimensions. For several particular cases of interest these coil characteristics are obtained as closed form analytical formulae.

A new type of coil, called a compound-constant-tension coil, is proposed. It is formed by selecting and matching (point and slope) segments chosen from two or more members of the one parameter family of solution curves found for the shape equation. These coils may be supported by tension members at the intersections of the solution curves or by a compression ring support and provide a unique and highly attractive solution to the toroidal field coil centering force support problem of tokamak designs.

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I. Introduction

The shape assumed by a flexible filamentary conductor, (incapable of supporting bending stresses), in a toroidal magnetic field, ($B_T \propto 1/R$), has been of considerable interest to the designers of toroidal field magnets for future CTR experiments and eventual power reactors. A coil designed to closely approximate this ideal shape, that is described by the differential equation(1)

$$kr = \left\{1 + \left(\frac{dz}{dr}\right)^2\right\}^{3/2} \Bigg/ \frac{d^2Z}{dr^2} \qquad (1)$$

(r,θ,z cylindrical coordinates are used and k is a constant), would experience minimum stress levels for a given magnetic field production. As the design of existing, copper, toroidal field magnets has already necessitated the use of high design stresses to reach desired field levels, it is evident that the realization of high field, large bore, superconducting magnets is likely to depend on our ability to employ methods of reducing the excessive mechanical stress fields. Although actual magnets are not filamentary, and this, combined with the discrete distribution of coils about the torus produces non-toroidal components of magnetic field, the shapes described by equation (1) are an excellent approximation to the shape of actual reduced stress coils. Consequently, most of the proposed experimental and power reactor designs employ toroidal field coils of the familiar D shape, sometimes referred to as the Princeton-D,[2] that can be generated from equation (1).

The earliest analysis of the problem of generating a constant tension coil shape that we have found is that of Leites,[3] who determined the desired shape by a graphical construction. More recently, this problem has been addressed by File, Mills and Sheffield,[1] who have employed a numerical quadrature of equation (1), and by Shafranov[4] who has also presented a numerical solution but who derived equation (1) by posing the question, "What coil shape produces an extremum of the inductance of the toroidal solenoid?", and solving the resulting variational problem.

We have found an analytic solution of the equation which we present here. This solution is derived from an expansion in a series of Bessel functions. Term by term integration of this uniformly convergent series yields a one parameter family of integral curves of equation (1). Solution curves for two choices of the parameter k are displayed in figure 1.

Quite remarkably much of the information required by the designer is obtainable in closed form as exact analytical formulae. Specifically, for the Princeton D shape the following information is available: the turn length, the support cylinder height, the cross sectional area of the coil and its inductance.

The D shape is suitable for a coil design in which the centering force is taken by a support cylinder in the center of the machine. Space in this area is limited however, and the need to place ohmic heating windings in this same interior region also complicates matters. Alternative designs can be developed from the variety of coil shapes which can be generated from the family of solution curves of equation (1). These shapes comprise three generic families: the D-type coil formed by utilizing the larger outer lobe of the solution curve and closing the coil on the inside with a vertical conductor element, the C-type coil formed by utilizing the smaller lobe of the curve and closing the coil on the outside of the torus and, finally, a coil formed by combining elements chosen from two different members of the family of solution curves (different values of k). As we will demonstrate, such a coil can be in simple tension at every point. We will call coils of this type compound coils. Members of this family possess the novel design feature of being supportable by two tension members attached at the intersection points of the elements of the coil, thereby providing a unique and highly attractive solution to the toroidal field coil centering force support problem.

II. Solution of the Constant Tension Toroidal Solenoid Shape Equation

The solution of equation (1) is a straight forward matter, and accordingly we will proceed through it with some rapidity, leaving much of the algebraic detail to the reader. The physical lengths of this problem will all be normalized to the geometric mean radius of the outer and inner extremes of the solution curve,

$$r_o = \sqrt{r_1 \, r_2} \quad ; \qquad (2)$$

We define $x = r/r_o$ and $y = z/r_o$. The first integral of equation (1) may be obtained directly by simple integration yielding,

$$\frac{dy}{dx} = \frac{1/k \, \ell n \, x}{\sqrt{1 - \frac{1}{k^2} \, \ell n^2 \, x}} \; . \qquad (3)$$

We note that

$$x = e^{k\sin\theta} . \qquad (4)$$

and substituting θ in favor of x in equation (3) gives the integral in the form,

$$y = k \int_{o}^{\theta} \sin\theta' \, e^{k\sin\theta'} \, d\theta' . \qquad (5)$$

Using the definition of the arc length along the curve,

$$ds^2 = dz^2 + dr^2 , \qquad (6)$$

we can find the length of arc between the point at which $\theta = o$ ($x=1$, $y=o$) and any other point on the curve, namely

$$s = r_o k \, Sg(\theta) \int_{o}^{\theta} e^{k\sin\theta'} \, d\theta' , \qquad (7)$$

where $Sg(\theta)$ is the algebraic sign of θ. We will need to know s later on when we want to evaluate the turn length of a toroidal field coil.

We now turn our attention to the integral appearing in equation (7),

$$J = \int_0^{\theta} e^{k\sin\theta'} \, d\theta' \quad . \tag{8}$$

It turns out that all of the required information is obtainable from J and its derivatives with respect to k. Differentiating,

$$\frac{\partial J}{\partial k} = \int_0^{\theta} \sin\theta' \, e^{k\sin\theta'} \, d\theta' \quad , \tag{9}$$

and equation (5) becomes

$$y(k,\theta) = k \, \frac{\partial J}{\partial k} \quad . \tag{10}$$

The area element of a coil formed from the larger lobe of the solution curve is

$$dA = 2r_o^{\,2}(y_2 - y)dx \tag{11}$$

where $y_2 = y(k,\pi/2)$. Using θ in favor of x and integrating between a lower limit x_{min}, chosen as the interior radius of the coil, and x_2, the area is

$$A = 2r_o^{\,2}\left\{y_2\,\frac{\Delta}{r_o} - \int_{\theta=\theta_{min}}^{\pi/2}\int_{o}^{\theta} k^2\sin\theta' e^{k\sin\theta'}\, e^{k\sin\theta}\cos\theta\, d\theta\, d\theta'\right\}, \tag{12}$$

in which $\Delta = r_2 - R_1$, the bore width of the coil, and $\theta_{min} = \theta(x_{min})$. (n.b. $\theta_{max} = \pi - \theta_{min}$ for D-type coils.) Now,

$$\int_{\theta=\theta_{min}}^{\pi/2}\int_{\theta'=o}^{\theta} F(\theta,\theta')d\theta' d\theta = \int_{\theta'=o}^{\theta_{min}}\int_{\theta=\theta_{min}}^{\theta'} F(\theta,\theta')d\theta d\theta' + \int_{\theta'=o}^{\pi/2}\int_{\theta=\theta'}^{\pi/2} F(\theta,\theta')d\theta d\theta', \tag{13}$$

which fact facilitates our reducing equation (12) to

$$A = 2r_o^{\,2}k\left\{\frac{\Delta}{r_o}\frac{\partial}{\partial k}J(k,\pi/2) + e^{k\sin\theta_{min}}\frac{\partial}{\partial k}J(k,\theta_{min}) + \frac{\partial}{\partial(2k)}J(2k,\pi/2) - \frac{\partial}{\partial(2k)}J(2k,\theta_{min}) - e^{k}\frac{\partial}{\partial k}J(k,\theta/2)\right\}. \tag{14}$$

When the smaller lobe of the solution curve is used, the analogous formula is,

$$A = 2r_o^2 k \left\{ \left(\frac{\Delta}{r_o} + e^{-k}\right) \frac{\partial}{\partial k} J(k,-\pi/2) - \left(\frac{\Delta}{r_o} + e^{-k}\right) \frac{\partial}{\partial k} J(k,\theta_{max}) + \frac{\partial}{\partial(2k)} J(2k,\theta_{max}) - \frac{\partial}{\partial(2k)} J(2k,-\pi/2) \right\} , \qquad (15)$$

in which the maximum coil radius r_{max} is defined by,

$$r_{max} = R_1 + \Delta \quad , \qquad (16)$$

and $\theta_{max} = \theta(r_{max})$.

For D-type coils, the magnetic energy inductively stored in the field is,

$$U_B = \frac{B_o^2 r_o^3}{2} \int (y_2 - y) \frac{dx}{x} \quad , \tag{17}$$

the integration being performed between the minimum and maximum coil radii. Recalling that in a toroidal solenoid

$$B = \frac{B_o r_o}{r} = \frac{2I}{r} , \quad (\text{e.m.u.}), \tag{18}$$

and replacing U_B by $\frac{1}{2}LI^2$, we find that,

$$L = 4r_o k^2 \left\{ \sin \theta_{min} \left[\frac{\partial}{\partial k} J(k,\theta_{min}) - \frac{\partial}{\partial k} J(k,\pi/2) \right] - \left[\frac{\partial^2}{\partial k^2} J(k,\theta_{min}) - \frac{\partial^2}{\partial k^2} J(k,\pi/2) \right] \right\} , \tag{19}$$

while for C-type coils,

$$L = 4r_o k^2 \left\{ \sin \theta_{max} \left[\frac{\partial}{\partial k} J(k,-\pi/2) - \frac{\partial}{\partial k} J(k,\theta_{max}) \right] - \left[\frac{\partial^2}{\partial k^2} J(k,-\pi/2) - \frac{\partial^2}{\partial k^2} J(k,\theta_{max}) \right] \right\} . \tag{20}$$

The locked door which now stands between us and the desired information, (y, S, A, and L), is the integral $J(k,\theta)$ - the key that opens it is the recognition that

$$e^{k\sin\theta} = \sum_{n=-\infty}^{\infty} e^{in\theta} J_n(-ik) \qquad (21)$$

where the J_n are Bessel functions of integer order.(5) This series converges absolutely and uniformly, as does the series that results from its term by term integration on θ. The rest then is algebraic detail which we will not present here, several different and equivalent series representations of $J(k,\theta)$ being obtained; a convient series is,

$$J(k,\theta) = I_o(k)\theta + \sum_{1}^{\infty} \frac{i}{n}[e^{-in\theta} - 1]\,[\,1 + e^{in(\theta+\pi)}]\, e^{in\pi/2}\, I_n(k) \; ; \qquad (22)$$

in terms of the modified Bessel function,(6) I_n, of integer order.

In the next two sections of this paper we will focus our attention on D-type and C-type coils and demonstrate that when $\theta_{max} = \frac{3}{2}\pi$ (Princeton D) or when $\theta_{max} = o$ or π that closed form equations are found giving the desired data.

III. Formulae for D-type Coils

The earliest recent applications of constant tension coils were the use of the Princeton D shape in the tokamak reference design studies(7,8); more recently several proposed experimental devices have also used coils of this shape. This coil is generated by selecting $\theta_{max} = +3\pi/2$. Values of J and its first two derivatives will be needed at $\theta = -\pi/2$ and $\theta = \pi/2$. At the former value of θ,

$$J(k,-\pi/2) = -I_o(k)\frac{\pi}{2} - 2\sum_{n=1}^{\infty}\frac{i^{2n}}{(2n-1)} I_{(2n-1)}(k) \quad , \tag{23}$$

while at the latter,

$$J(k,\pi/2) = I_o(k)\frac{\pi}{2} - 2\sum_{n=1}^{\infty}\frac{i^{2n}}{(2n-1)} I_{(2n-1)}(k) \quad . \tag{24}$$

Using equation (7) provides the length of the curved portion of the D,

$$\ell = 2\pi r_o k\ I_o(k) \quad , \tag{25}$$

while the height of the support or straight section is obtained by differentiating equations (23) and (24) using the well-known formulae(9)

$$I_o{}'(z) = I_1(z) \quad , \tag{26}$$

$$I_n{}'(z) = \frac{I_{n-1}(z) + I_{n+1}(z)}{2} \quad , \tag{27}$$

and using equation (10) to evaluate y,

$$h = 2\pi r_o\ k\ I_1(k) \quad . \tag{28}$$

Summing equations (25) and (28) the total conductor turn length is,

$$\ell_{turn} = 2\pi r_o k\ [I_o(k) + I_1(k)] \quad . \tag{29}$$

The area and inductance are found in much the same manner, substituting $-\pi/2$ for θ_{min} in equations (14) and (19) and differentiating equation (23) and (24) as required. The results are as follows:

$$A = 2\pi r_o^2 k \left[I_1(2k) - e^{-k} I_1(k) \right] \quad , \tag{30}$$

$$L = 2\pi r_o k^2 \left[I_o(k) + 2I_1(k) + I_2(k) \right] \quad . \tag{31}$$

Another D-type coil for which closed form formulae result is obtained by setting $\theta_{max} = \pi$. The coil is formed by connecting the points of horizontal tangency of the larger lobe of the solution curve with a straight vertical element. The formulas for the turn length, area, inductance and support height of this coil will depend on the values of $J(k,\pi/2)$ (equation 24) and $J(k,\pi)$; the latter is readily shown to be,

$$J(k,\pi) = \pi \left[I_o(k) + L_o(k) \right] \quad , \tag{32}$$

where L_n is a modified Struve function.[10] Using this formula and others described previously, we find that;

$$\ell = \pi r_o k \, [I_o(k) + L_o(k)] \quad , \tag{33}$$

$$h = \pi r_o k \, [\, I_1(k) + L_1(k) + 2/\pi \,] \quad , \tag{34}$$

$$\ell_{turn} = \pi r_o k \, [\, I_o(k) + I_1(k) + L_o(k) + L_1(k) + 2/\pi \,] \quad . \tag{35}$$

The area and inductance of this coil are given by,

$$A = \pi r_o{}^2 k \, \{ I_1(2k) + L_1(2k) - [\, I_1(k) + L_1(k)] \} \quad , \tag{36}$$

and

$$L = 2\pi r_o k^2 \, \{ I_o(k) + L_o(k) - \frac{1}{k} [\, I_1(k) + L_1(k)] \} \quad . \tag{37}$$

IV. Formulas for C-type Coils

The family of coils obtained from the smaller lobe of the solution curve will be called C-type coils. They are formed with a vertical closure on the exterior side of the torus, in contrast to the D coils which are closed on the inside. The C-coil offers some advantages over the D-coil in that it is supported at the exterior side of the torus where space is more readily available. Also, the coil is closer to circular than the D coils; consequently, when the extra vertical height of the D coil is not essential, it gives a design solution having a shorter turn length and lower inductance, thereby reducing material requirements and the amount of energy needed to charge the solenoid. For the particular case in which $\theta_{max} = o$ closed form formulas are available as follows:

$$\ell = \pi r_o k \; [I_o(k) - L_o(k)] \tag{38}$$

$$h = \pi r_o k \left\{ \frac{2}{\pi} - [I_1(k) - L_1(k)] \right\} \tag{39}$$

$$\ell_{turn} = \pi r_o k \left\{ \frac{2}{\pi} + [I_o(k) - L_o(k)] - [I_1(k) - L_1(k)] \right\} \tag{40}$$

$$A = \pi r_o^{\,2} k \; \{I_1(2k) - L_1(2k) - [I_1(k) - L_1(k)]\} \qquad (41)$$

$$L = 2\pi r_o k^2 \; \{[I_o(k) - L_o(k)] - \frac{1}{k}\,[I_1(k) - L_1(k)]\} \;. \qquad (42)$$

One should bear in mind the significance of r_1 and r_2, the extreme extents of the solution curve when comparing formulae for C-type and D-type coils. Each coil is determined by specifying its minimum radius R_1, its width Δ and the maximum tangent angle θ_{max}. For C-type coils the resulting value of k is,

$$k = \ln(1 + \Delta/R_1) \;/\; (1 + \sin\theta_{max}) \quad . \qquad (43)$$

r_2 may than be found from,

$$r_2 = r_1\, e^{2k} \quad . \qquad (44)$$

For D-type coils on the other hand R_1 and r_1 are not equal. k is found to be,

$$k = \ln(1 + \Delta/R_1) \;/\; (1 - \sin\theta_{max}) \quad , \qquad (45)$$

and in this case,

$$r_1 = r_2\, e^{-2k} \quad . \tag{46}$$

It is interesting to note that a $\theta_{max} = 0$ C-type coil and a $\theta_{max} = 3/2\pi$ Princeton D coil which have the same R_1 and Δ possess k values which are in the relationship $k_C/k_D = 2$; while the $\theta_{max} = \pi$ D-type coil having the same R_1 and Δ has the same k value as the $\theta_{max} = 0$ C-type coil.

V. Compound Coils

Smooth coil shapes can be formed by nestling D and C portions of solution curves as indicated in figure (3). Such compound coil shapes are generalizations of the C and D coils discussed above. In figure 2 the coil bebcb can be thought of as a D-type coil, beb, with its straight segment replaced by a segment of a C-type coil, bcb. The coil adaca can be thought of as a C-type coil aca with its straight segment replaced by the D-type coil ada. Finally, the curve fgf suggests the hypothetical possibility of producing a non-planar, zero centering force configuration.

We observe that the C and D shaped segments correspond to different values of k in equation (1). This jump in k implies: either a difference in tension for the C and D segments, or that either the C or D segment carries a mechanical loading in addition to its electromagnetic loading, or a combination of these two. The necessity of a tension jump or mechanical loading comes easily from consideration of the force balance in a small segment of flexible filament as shown in figure 3. The segment is to be considered as part of a toroidal solenoid and therefore the magnetic field is taken to exist only on one side of the segment.* The segment is under constant tension and subject to a mechanical pressure, p(r), as well as the electromagnetic forces arising from carrying a current, I, in the toroidal magnetic field, B. If $p(r) = p_o r_o / r$ the force balance leads to the following expression for the radius of curvature of the filament,

$$\rho = \left[\frac{2T}{I\ B_o\ r_o + 2p_o\ r_o}\right] r \equiv k\ r \quad , \tag{47}$$

which is equivalent to equation 1. (For the C and D coils described in the above sections, the straight segments have an infinite radius of curvature. This circumstance can be described by equation (47) by taking $p_o \equiv \frac{1}{2} I\ B_o$.) For the compound coil, different values of k in each segment can be achieved in a variety of ways. For example if T is to be constant, p_o must be different in each segment. Physically, the proper p_o will arise from the reaction of a rigid

*The authors express their gratitude to P. Bonanos for reminding them of this aspect of the magnetic field and the consequent reduction in calculated coil tension.

support shaped to conform to the C or D segment of the compound coil with which it is in contact. If $p_o = 0$ everywhere, then T must differ in each segment of the compound coil since I, B_o and r_o are common to both segments. Physically this jump in tension can be provided by a tensile support mechanism at the juncture of the C and D segments. The tension supports allow the compound coil to hang from a surrounding structure such as a wall or external compression ring, yielding a novel solution to the toroidal field coil centering force problem.

For the compound coil the value of ℓ, A, and L are the sum of the corresponding quantities pertaining to the C and D type segments of the coil. For the particular compound coil composed of C and D segments that join with zero slope the above mentioned quantities can be expressed by the closed form formulae displayed above in Sections III and IV. These formulae require knowledge of k for both segments. The compound coil is determined by the specification of three quantities such as R_1, Δ and the slope of the coil at the point where the C and D segments join. To find the resulting values of k requires the solution of a transcendental relation arising from the requirement that the height of the C-segment must match the height of the D-segment.

VI. Concluding Remarks

We have found an analytical solution to the constant tension solenoid problem, (equation 1), in terms of an infinite series of Bessel functions. Using this solution we have been able to demonstrate that closed form formulas, (eqs. 25, 28-31), exist which provide desired design information for the Princeton D coil. Similar formulas are also obtained for two other coils, the $\theta_{max} \doteq \pi$ D-type coil (eqs. 33-37) and the $\theta_{max} = 0$ C-type coil, (eqs. 38-42).

The suggestion of using the C-type coil is not new to this writing. Our contribution to C-coils is the ability to make rapid calculations for design and estimating purposes. This is in fact true for both C and D type coils of arbitrary parameters. The series for $J(k,\theta)$ converges rapidly, and numerical evaluation of J and its k derivatives provides rapid information on the properties of any coil or coil segment formed from a solution curve of equation (1).

By joining two such segments we have produced a compound coil, presented here, we believe, for the first time. Great flexibility is available in the choice of shape and the support design of these coils. Figure 4 shows several different compound coil shapes and support schemes. The tension jump implied by the differing values of k for the two segments can be taken by a tension mechanism, effectively hanging the coils on structure outside of the torus and its congested central core. The particular solution composed of the $\theta_{max} = \pi$ D-type segment and $\theta_{max} = -\pi$ C-type segment; allows the possibility of using a plane, ring truss, support structure which can simultaneously be part of the toroidal torque frame. Alternatively by mounting one segment of the coil on a preformed support pad it is possible to generate a mechanical loading which exactly compensates the unbalanced tension. This form of support suggests the possibility of using structure already present in reactor designs, (blanket and shield), to support the toroidal field coils, possibly realizing a more economical design.

Finally, we would like to point out that compound coils may have more than two segments. A multiple segment coil having tension support fixtures or preshaped support blocks would allow one to tailor the shape of the coil to the particular needs of the experiment or reactor being designed without sacrificing the reduction of mechanical stresses and associated strain energy offered by constant tension coils.

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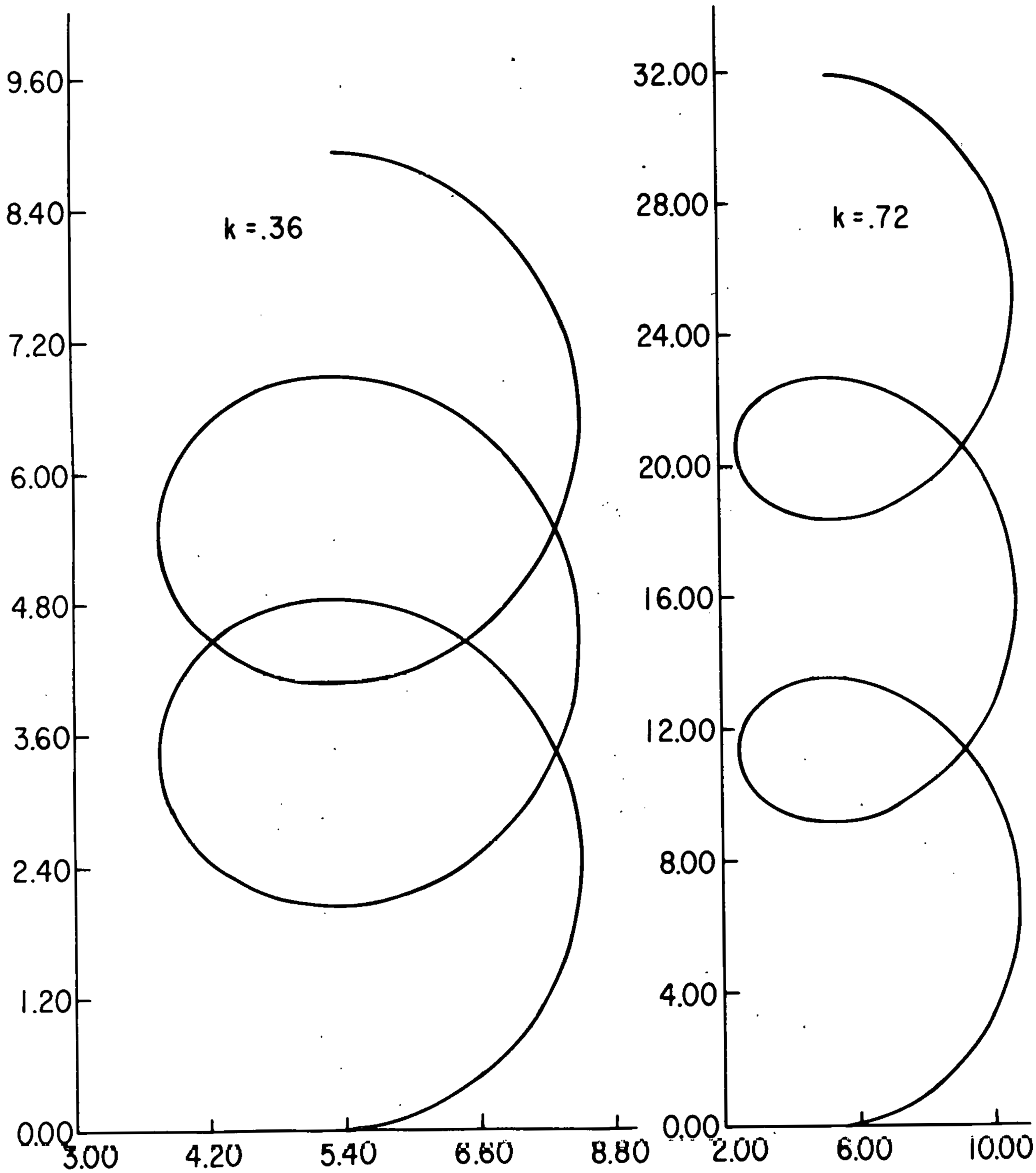

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Fig. 1. Solution curves of equation 1 for different values of k; $Z(r=r_o)=0$, $Z'(r=r_o)=0$; $r_o=5.27$, (n.b., scales differ).

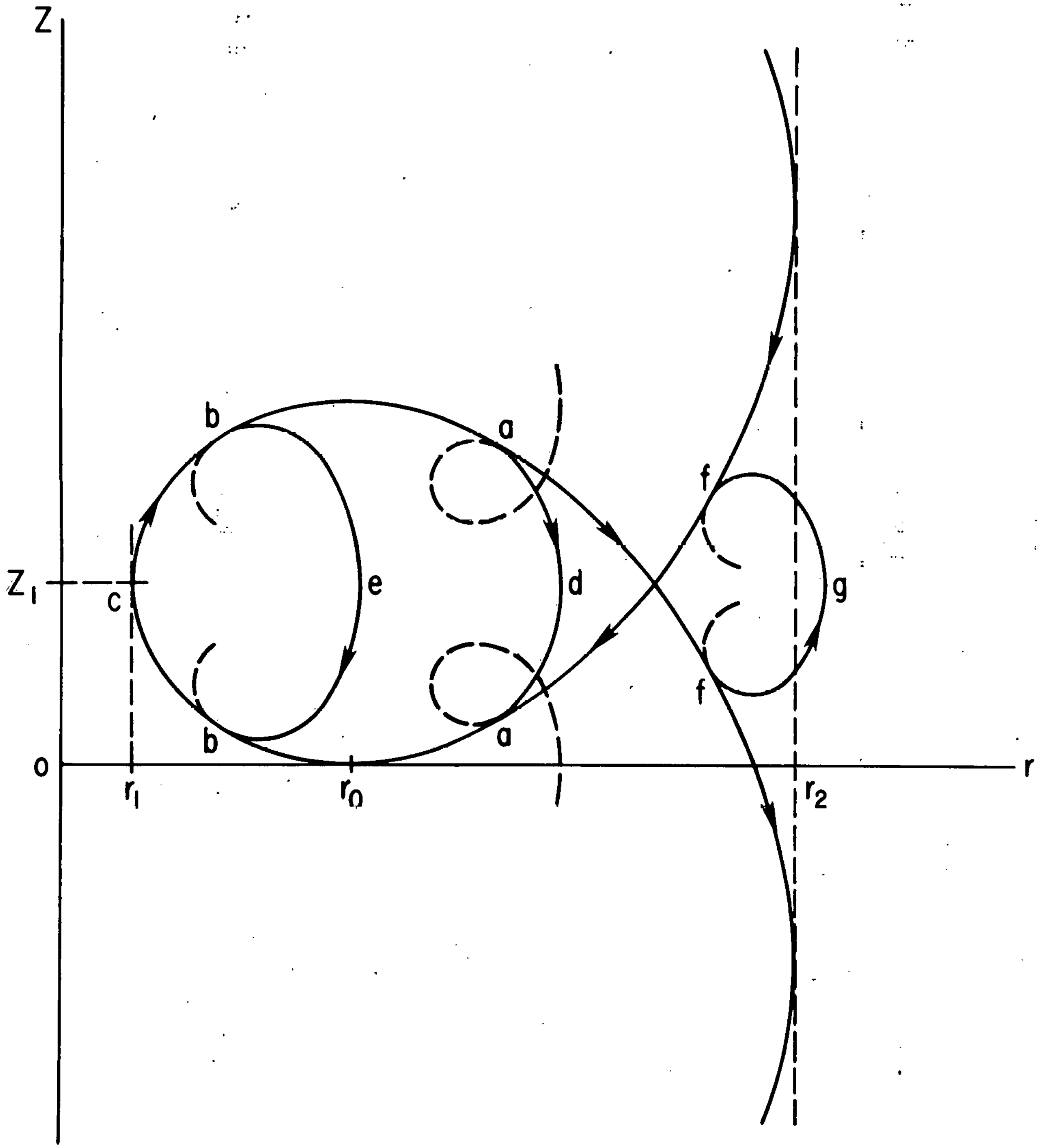

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Fig. 2. Compound-Coil shapes generated by combining segments of different solution curves of equation (1) - different k values.

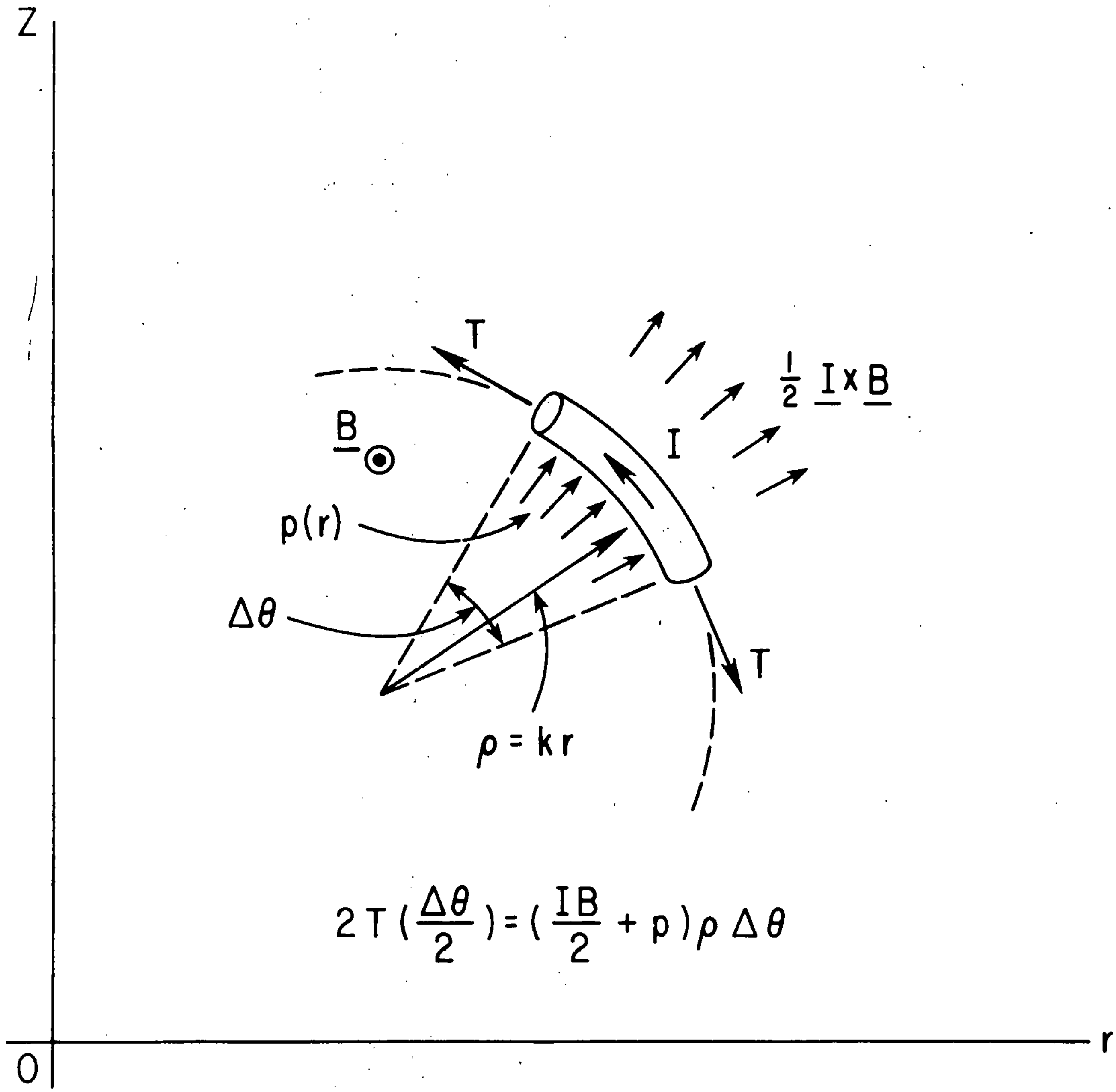

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Fig. 3. Force balance on an elemental conductor filament carrying a distributed pressure p(r) in addition to the electromagnetic loading $\frac{1}{2}\underline{I}\times\underline{B}$.

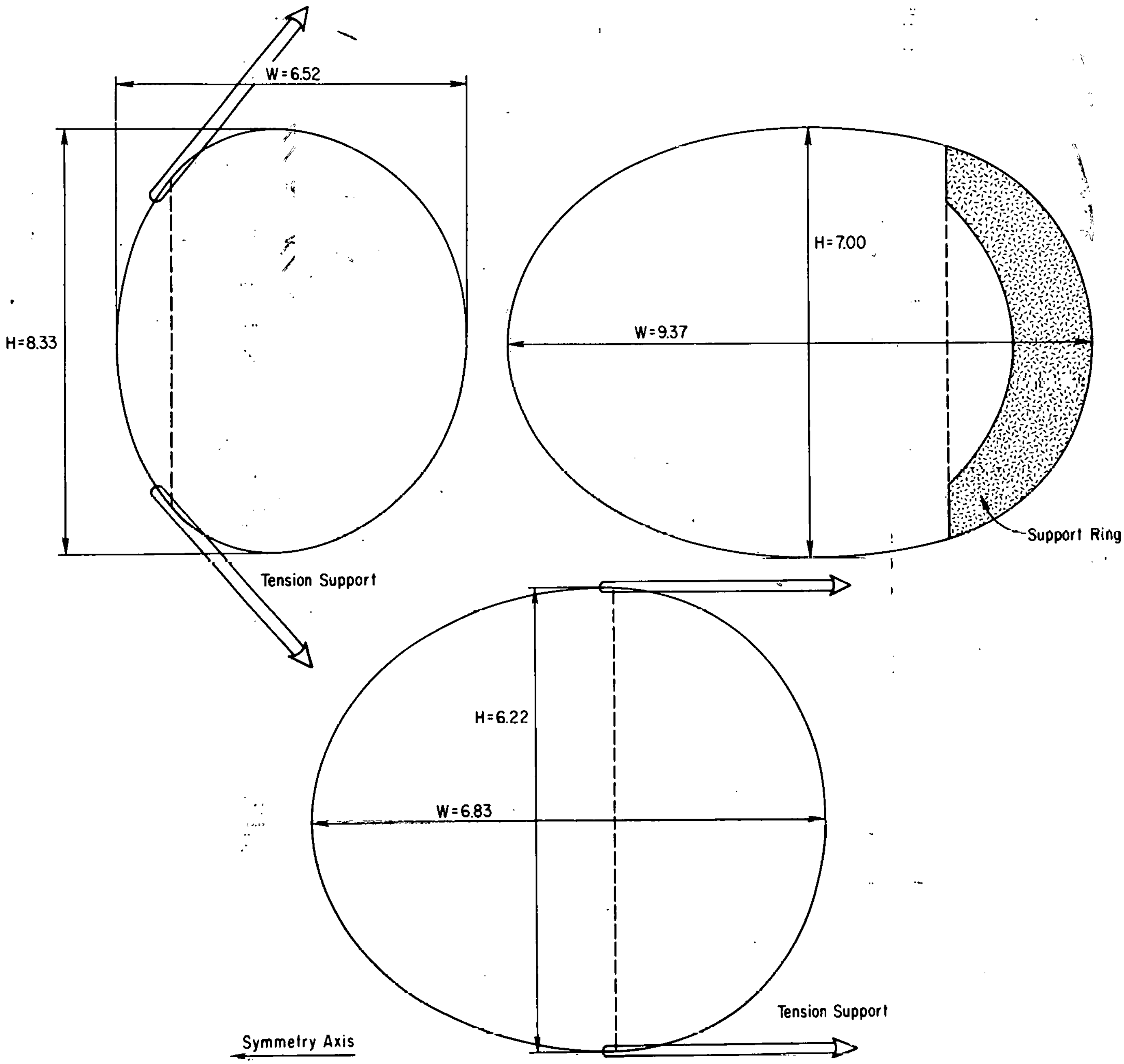

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Fig. 4. Several compound coils, illustrating the wide variety of shapes obtainable and two potential means of supporting the coil, tension attachments or a compression support ring.