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Excitation of Alfvén Waves by High-Energy Ions in a Tokamak†

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ABSTRACT

It is shown that shear Alfvén waves can be destabilized by resonance with high-energy "beam" ions near the magnetic axis in a tokamak, if the beam is radially non-uniform.

In a plasma heated by high-energy neutral-beam injection[1,2] the non-equilibrium plasma stability properties must be examined carefully. Previous work has indicated that the distributions that are likely to arise from injection should be stable to velocity-space instabilities in a uniform medium.[3] In particular, for isotropic injection the beam slowing-down distribution is monotonically decreasing in energy, and thus stable to all such modes. More detailed study indicates that, even with parallel injection, distributions unstable to uniform-medium modes are unlikely to arise. Hence, it becomes of interest to look at the modes associated with non-uniform beams and plasmas. In this letter, we examine the possibility of beam excitation of the shear Alfvén waves. In projected experiments, beam velocities lie just below the Alfvén speed v_A, and hence might be in the proper range for resonant excitation.

As is well known, the dispersion relation for shear Alfvén waves is given by $\omega = k_{\parallel} v_A \equiv \omega_A$. In tokamak geometry, complications arise in describing the radial eigenmodes since $k_{\parallel}$ is a function of radius. Thus, for a mode like $\exp(-im\theta + in\zeta)$, where θ is the poloidal angle and ζ the toroidal angle, we have $k_{\parallel}(r) = (nq - m)/qR$ where $q(r) = rB_{\zeta}/RB_{\theta}$; for positive m and n, $k_{\parallel}(r)$ is an increasing function of r when the current density is centrally peaked. In the MHD approximation, the eigenmodes for the fluid displacement $\xi(r)$ are singular at a radius where $\omega = \omega_A(r)$. To resolve this singularity, it is necessary to include finite gyroradius effects outside the MHD approximation. We will see that these effects also introduce damping of the waves by means of electron dissipation, especially collisions of magnetically trapped electrons. We will also see that the high-energy beam ions can interact resonantly with the waves by means of their ∇B drifts; for non-uniform beams this interaction is destabilizing whenever $\omega < \omega_{*b}$, where ω_{*b} is the diamagnetic frequency of the beam.

It is clearly necessary to adopt a microscopic picture to evaluate these effects. Where $W = mv^2/2 + e\phi$ is the particle energy, and $\mu = v_{\perp}^2/2B$ the invariant magnetic moment, the guiding center drift equation for $f(W, \mu, \underset{\sim}{x})$ is

$$\partial f/\partial t + v_{\parallel}\underset{\sim}{n} \cdot \underset{\sim}{\nabla} f + \underset{\sim}{\nabla} \cdot [(\underset{\sim}{v}_E + \underset{\sim}{v}_D + \underset{\sim}{v}_p) f] + (dW/dt)\, \partial f/\partial W = 0 \qquad (1)$$

where $\underset{\sim}{n} = \underset{\sim}{B}/B$. To an adequate approximation, we have

$$\underset{\sim}{v}_E = \frac{\underset{\sim}{E} \times \underset{\sim}{B}}{B^2} \quad ; \qquad \underset{\sim}{v}_D = \frac{m(\mu B + v_{\parallel}^2)}{eB^2} \; \underset{\sim}{n} \times \underset{\sim}{\nabla} B$$

$$\underset{\sim}{v}_p = \frac{m}{eB^2}\left(1 + \frac{3}{4}\rho^2 \nabla_{\perp}^2\right)\frac{\partial \underset{\sim}{E}}{\partial t} \; ; \qquad \frac{dW}{dt} = e\left(\frac{\partial \phi}{\partial t} - v_{\parallel}\frac{\partial A_{\parallel}}{\partial t}\right) \tag{2}$$

with $\rho = mv_{\perp}/eB$, the gyroradius. Here, $\underset{\sim}{v}_p$ is the polarization drift including finite gyroradius effects. For our purposes, it is more convenient to transform to variables $\epsilon = v^2/2$, μ, $\underset{\sim}{x}$, in which case by straightforward manipulation we obtain

$$\begin{aligned} &\partial f/\partial t + v_{\parallel}\underset{\sim}{n}\cdot\underset{\sim}{\nabla} f + (\underset{\sim}{v}_E + \underset{\sim}{v}_D)\cdot\underset{\sim}{\nabla} f + \underset{\sim}{\nabla}\cdot(\underset{\sim}{v}_p f) \\ &\quad + (e/m)(v_{\parallel}E_{\parallel} + \underset{\sim}{v}_D\cdot\underset{\sim}{E})\,\partial f/\partial\epsilon = Cf \end{aligned} \tag{3}$$

where we have also introduced a collision operator C. For shear Alfvén waves in a low-β plasma, we can represent the perturbation electric field by

$$\underset{\sim}{E}_{\perp} = -\underset{\sim}{\nabla}_{\perp}\phi \; ; \qquad E_{\parallel} = -\nabla_{\parallel}\phi - \partial A_{\parallel}/\partial t \tag{4}$$

with $\phi, A_{\parallel} \sim \exp(-i\omega t - im\theta + in\zeta)$. In this limit, the magnitude B and $\underset{\sim}{v}_D$ are unaffected by the perturbation. We linearize Eq. (3) about an axisymmetric equilibrium. If we integrate the linearized version of Eq. (3) over all velocities using $d^3v = 2\pi B\, d\mu\, d\epsilon/v_{\parallel}$, multiply by the charge e, and sum over all species making use of the quasi neutrality condition, we obtain a moment equation for $j_{\parallel}$:

$$\underset{\sim}{n}_o\cdot\underset{\sim}{\nabla} j_{\parallel 1} + \underset{\sim}{n}_1\cdot\underset{\sim}{\nabla} j_{\parallel o} + \sum e\int d^3v\,\underset{\sim}{v}_D\cdot\underset{\sim}{\nabla} f_1 = -\frac{i\omega n_i m_i}{B^2}\left(1 + \frac{3}{4}\rho_i^2\nabla_{\perp}^2\right)\nabla_{\perp}^2\phi \, , \tag{5}$$

where $\rho_i = (m_i T_i)^{1/2}/eB$. In obtaining Eq. (5), we have made use of

$$\sum e \int \underset{\sim}{v}_E \cdot \underset{\sim}{\nabla} f_o d^3v + \sum \left(e^2/m\right) \int \underset{\sim}{v}_D \cdot \underset{\sim}{E}\ \partial f_o/\partial\epsilon\, d^3 v = 0,$$

which is a consequence of the quasi neutrality of the equilibrium.[4] Ultimately, we will combine Eq. (5) with the Maxwell equation $\nabla^2 A_{\|} = -4\pi j_{\|1}$ to yield one relation between ϕ and $A_{\|}$. This moment equation procedure make use of the quasi neutrality condition to high order; accordingly relatively small effects, such as the beam contribution to the third term on the left and the fourth derivative term, must be retained in Eq. (5). We must proceed to linearize Eq. (3) to obtain the f_1', for use both in Eq. (5) and in the lowest-order quasineutrality condition. We must include a perturbation in the unit vector $\underset{\sim}{n}$, arising from the magnetic perturbation given by

$$-i\omega \underset{\sim}{B}_1 = \underset{\sim}{\nabla} \times (\underset{\sim}{v}_E \times \underset{\sim}{B}) - \underset{\sim}{\nabla} \times (E_{\|}\underset{\sim}{n}) = (\underset{\sim}{B} \cdot \underset{\sim}{\nabla})\underset{\sim}{v}_E + \underset{\sim}{n} \times \underset{\sim}{\nabla} E_{\|} \tag{6}$$

where we have made use of the fact that the magnitude B of the unperturbed field is approximately uniform. The linearized version of Eq. (3) can then be put in the form

$$\begin{aligned}(-i\omega + v_{\|}\underset{\sim}{n} \cdot \underset{\sim}{\nabla} + \underset{\sim}{v}_D \cdot \underset{\sim}{\nabla} - C)&(f_1 - \underset{\sim}{v}_E \cdot \nabla f_o / i\omega) \\ &= -\frac{i\omega m f_o}{eB^2} \nabla_\perp^2 \phi - \frac{e}{m}\left(v_{\|} E_{\|} - \underset{\sim}{v}_D \cdot \underset{\sim}{\nabla}\phi\right) \frac{\partial f_o}{\partial \epsilon} \\ &+ \frac{1}{i\omega}\left[\frac{v_{\|}}{B}\, \underset{\sim}{n} \times \underset{\sim}{\nabla} E_{\|} \cdot \underset{\sim}{\nabla} f_o - \underset{\sim}{v}_D \cdot \underset{\sim}{\nabla}\left(\underset{\sim}{v}_E \cdot \underset{\sim}{\nabla} f_o\right)\right].\end{aligned} \tag{7}$$

Here, we have omitted the fourth derivative term and have made use of the fact that the collision operator acting on $\underset{\sim}{v}_E \cdot \underset{\sim}{\nabla} f_o$ vanishes, at least in the absence of temperature gradients. The procedure from this point is to solve Eq. (7)

for the distribution functions of the various species. These are substituted into the lowest-order quasi neutrality condition to yield a relation between ϕ and $E_{\parallel}$ (or $A_{\parallel}$).

In the case of the electrons, we have $\omega \ll k_{\parallel} v_{\parallel}$. Thus, the terms in $E_{\parallel}$ dominate on the right in Eq. (7), and to lowest order we have

$$v_{\parallel} \underset{\sim}{n} \cdot \underset{\sim}{\nabla} \left(f_1 + \frac{\underset{\sim}{v}_E \cdot \underset{\sim}{\nabla} f_o}{i\omega} \right) = - \frac{e}{m} v_{\parallel} E_{\parallel} \frac{\partial f_o}{\partial \epsilon} + \frac{v_{\parallel}}{i\omega B} \underset{\sim}{n} \times \underset{\sim}{\nabla} E_{\parallel} \cdot \underset{\sim}{\nabla} f_o \quad . \tag{8}$$

The second term on the right in Eq. (8) can be written as $-\omega_{*e}/\omega$ times the first term on the right, and may thus be neglected since $\omega \gg \omega_{*c}$ for the Alfvén waves of interest. The solution becomes

$$f_1 + \frac{\underset{\sim}{v}_E \cdot \underset{\sim}{\nabla} f_o}{i\omega} = g_e = - \frac{e}{m} \frac{\partial f_o}{\partial \epsilon} \int^{\ell} E_{\parallel} d\ell + \bar{g}_e (\mu, \epsilon) \quad . \tag{9}$$

Proceeding to next order in $\omega / k_{\parallel} v_{\parallel}$, we must include the terms in $\underset{\sim}{v}_D$ on the right in Eq. (7). Again the term in $\underset{\sim}{\nabla} f_o$ can be written as $-\omega_{*e}/\omega$ times the term in $\partial f_o / \partial \epsilon$, and may be neglected. In this order we obtain a solubility condition which determines $\bar{g}_e(\mu, \epsilon)$, namely

$$\int \frac{d\ell}{v_{\parallel}} \left(-i\omega + \underset{\sim}{v}_D \cdot \underset{\sim}{\nabla} - C \right) g_e = \frac{e}{m} \frac{\partial f_o}{\partial \epsilon} \oint \frac{d\ell}{v_{\parallel}} \underset{\sim}{v}_D \cdot \underset{\sim}{\nabla} \phi \quad . \tag{10}$$

For untrapped particles all inhomogeneous terms average to zero ($k_{\parallel} \neq 0$), and we conclude that $\bar{g}_e = 0$. For trapped particles, writing $Cg = -\nu_e^{eff} g$ and assuming $\omega \gg \underset{\sim}{v}_D \cdot \underset{\sim}{\nabla}$, we obtain

$$\bar{g}_e^T = \frac{e}{m} \frac{\partial f_o}{\partial \epsilon} \frac{\omega}{\omega + i \nu_e^{eff}} \left\langle \int^{\ell} E_{\parallel} d\ell \right\rangle \quad , \tag{11}$$

where $\langle A \rangle \equiv \left(\int A d\ell / v_{\parallel} \right) / \int d\ell / v_{\parallel}$. This procedure, of course, assumes trapped-electron collisions to be the dominant electron dissipation mechanism; electron Landau damping is small since $v_A \ll v_{Te}$.

In the case of background ions, we have $\omega \gg k_{\parallel} v_{\parallel}$ and $\omega \gg \nu_i$, and the solution of Eq. (7) is

$$f_1 + \frac{\underset{\sim}{v}_E \cdot \nabla \underset{\sim}{f}_o}{i\omega} = g_i = \frac{mf_o}{eB^2} \nabla_{\perp}^2 \phi + \frac{ie}{m\omega} \underset{\sim}{v}_D \cdot \underset{\sim}{\nabla} \phi \frac{\partial f_o}{\partial \epsilon} . \tag{12}$$

The last term in Eq. (12) is smaller than the second term on the left by a factor r/R and moreover, being proportional to $\cos\theta$, tends to average out; accordingly, we will omit it.[5]

Finally, we turn to the perturbed distribution function for the high-energy beam ions. For these ions, collisions may be neglected. Moreover, we are interested primarily in the possibility of mode excitation by beam resonances, where the operator on the left of Eq. (7) effectively vanishes; accordingly, we will keep only these terms. For simplicity we will take the beam distribution also to be Maxwellian, with $T_b \gg T_{c,i}$. For a Fourier mode like $\exp(in\zeta - im\theta)$ the right-hand side of Eq. (7) may be simplified, and the equation written

$$(-i\omega + v_{\parallel} \underset{\sim}{n} \cdot \underset{\sim}{\nabla} + \underset{\sim}{v}_D \cdot \underset{\sim}{\nabla}) g_b = (ef_o/T_b)(1 - \omega_{*b}/\omega)(v_{\parallel} E_{\parallel} - \underset{\sim}{v}_D \cdot \underset{\sim}{\nabla} \phi) , \tag{13}$$

where $\omega_{*b} = -(mT_b/eBr) d\ln n_b/dr$. In tokamak geometry, the particle drift has (r,θ) components given by $\underset{\sim}{v}_D = [m_b(2\epsilon - \mu B)/eBR] (\sin\theta, \cos\theta)$. Also, $E_{\parallel}$ is small and resonance is only possible for $\omega \sim k_{\perp} v_D \sim k_{\parallel} v_A \gg k_{\parallel} v_b$, so we will neglect the $E_{\parallel}$ term on the right of Eq. (13).

Our philosophy in evaluating the beam resonance contributions is as follows. When we treat the eigenmodes in a sheared field, we will see that the beam resonance is only important in the region $\omega \geq k_{\parallel} v_A \gg k_{\parallel} v_b$. The angular dependence of $\underset{\sim}{v}_D$ gives rise to terms like $\exp[in\zeta - i(m \pm 1)\theta]$. Beam resonances can then occur where $\omega = (nq - m + 1)v_{\parallel}/qR$ for positive m and n. The usual case of interest will be where $k_{\parallel}(r)$ is an increasing function of r, since then there will be relatively undamped Alfvén waves occurring between $r = 0$ and $r = r_o$ where $\omega = k_{\parallel}(r_o)v_A$. Where v_b now denotes the maximum beam velocity (i.e., the injection speed), the condition for resonance to occur will be $v_b/v_A \geq [nq(r_o) - m]/[nq(r_o) - m + 1]$. Noting that $q(r_o) > q(0)$ and $nq(0) > m$ (or else there would be strong electron damping at the radius where $k_{\parallel} = 0$), this condition on v_b/v_A is not too restrictive especially for $n = 1$, and we will assume in what follows that it is satisfied. In this approximation, the resonant beam term from Eq. (13) is

$$g_b = \frac{\pi i f_o m_b (2\epsilon - \mu B)}{2BRT_b} \left(1 - \frac{\omega_{*b}}{\omega}\right) \delta\left(\omega - \frac{(nq - m + 1)v_{\parallel}}{qR}\right) \cdot e^{i\theta} \left(\frac{m\phi}{r} + \frac{\partial\phi}{\partial r}\right) . \quad (14)$$

Next we apply the calculated distribution functions to the macroscopic equations. We first substitute the electron and background ion distributions given in Eqs. (9), (11), and (12) into the lowest-order quasineutrality condition, the beam contribution being negligible to this order. In Eq. (9), we write $\int^{\ell} E_{\parallel} d\ell = E_{\parallel}/ik_{\parallel}$. In the case of interest, we will typically have $(nq - m)\theta_{max} \lesssim 1$, where θ_{max} is the turning point of a typical trapped particle. Accordingly, as a rough average, we may write $\langle \int^{\ell} E_{\parallel} d\ell \rangle = E_{\parallel}/ik_{\parallel}$ in Eq. (11) provided we introduce a factor $(r/R)^{1/2}$ to take into account the number of trapped particles. Quasi neutrality then gives[6]

$$E_{\parallel} = -ik_{\parallel}[1+(r/R)^{1/2}\omega/(\omega+i\nu_e^{eff})]\rho_{ie}^2 \nabla_{\perp}^2\phi \tag{15}$$

where $\rho_{ie} = (m_i T_e)^{1/2}/eB$, and $E_{\parallel} = -ik_{\parallel}\phi + i\omega A_{\parallel}$.

Finally, we combine Eq. (5) and $\nabla^2 A_{\parallel} = -4\pi j_{\parallel 1}$. The beam contributes to the third term on the left in Eq. (5); we obtain

$$\begin{aligned} e\int d^3v\, \underset{\sim}{v}_D \cdot \underset{\sim}{\nabla} g_b &= -\frac{\pi}{T_b}\left(\frac{m}{2BR}\right)^2\left(1-\frac{\omega_{*b}}{\omega}\right)\nabla_{\perp}^2\phi \cdot \int f_o(2\epsilon-\mu B)^2\delta\left(\omega-\frac{(nq-m+1)v_{\parallel}}{qR}\right)d^3v \\ &\simeq -\frac{n_b T_b}{\omega B^2 R^2}\left(1-\frac{\omega_{*b}}{\omega}\right)\nabla_{\perp}^2\phi \ . \end{aligned} \tag{16}$$

Using Eq. (6), and noting that $\partial j_{\parallel o}/dr = -(B/4\pi m r^2)(\partial/\partial r)(r^3\partial k_{\parallel}/\partial r)$, the first two terms in Eq.(5) may be combined and written as $(-i/4\pi r^2)\{(\partial/\partial r) \times [k_{\parallel}^2 r^3(\partial/\partial r)(A_{\parallel}/rk_{\parallel})] - (m^2-1)k_{\parallel}A_{\parallel}\}$. We are now in a position to write down Eq. (5) as a relation between $A_{\parallel}$ and ϕ, and to substitute for $A_{\parallel}$ in terms of ϕ using Eq. (15). Changing from ϕ to a fluid displacement variable $\xi = -m\phi/rB$, we obtain

$$\begin{aligned} &\omega^2\rho_i^2\left(\frac{7}{4}-i\delta\right)\frac{\partial^4\xi}{\partial r^4} + \frac{1}{r^3}\frac{\partial}{\partial r}\, r^3\left[\omega^2(1+i\eta)-\omega_A^2\right]\frac{\partial\xi}{\partial r} \\ &\qquad - \frac{m^2-1}{r^2}\left[\omega^2(1+i\eta)-\omega_A^2\right]\xi = 0 \ , \end{aligned} \tag{17}$$

where $\omega_A(r) = k_{\parallel}(r)v_A$, $\delta = (r/R)^{1/2}\omega\nu_e^{eff}/(\omega^2+\nu_e^{eff\,2})$, and $\eta = (n_b T_b/n_i m_i \omega^2 R^2)(1-\omega_{*b}/\omega)$. In obtaining Eq. (17), we have kept only the term $\partial^4\xi/\partial r^4$ out of $\nabla^4\xi$, and have put $\omega^2 = \omega_A^2$ and $T_e = T_i$ in the coefficient of this term.[7]

A complete solution of Eq. (17) is evidently difficult. We observe that away from the singular layer around $\omega = \omega_A(r_o)$, the solution will consist of slowly verying solutions ξ_s and fast varying (evanescent or oscillatory) solutions ξ_f. We are interested in the case illustrated in Fig. 1, where the fast varying solution is evanescent for $r > r_o$ and oscillatory within $0 < r < r_o$. Considering the solution only near the singular layer, we may neglect the last term in Eq. (17), and integrate once to obtain an equation for $y = \xi'$, namely $\rho_i^2 (7/4 - i\delta) y'' + (1 + i\eta - \omega_A^2/\omega^2) y = \text{const}$. The solutions that decay for $r > r_o$ involve fast varying oscillatory solutions for $0 < r < r_o$. The condition at $r = 0$ in the present slab-like approximation will be $\xi = \xi'' = 0$ (physically, $\xi_r = B_r = 0$). The condition $\xi'' = 0$ will apply to the fast-varying oscillatory part of the solution; considering only this part, we have a WKB condition

$$(7/4 - i\delta)^{-1/2} \int_o^{r_o} (1 + i\eta - \omega_A^2/\omega^2)^{1/2} \, dr = n\pi\rho_i \; . \tag{18}$$

Marginal stability will occur when the imaginary part of Eq. (18) is satisfied for ω real. We write $\omega_A^2 = \omega_A^2(0) + r^2 \partial(\omega_A^2)/\partial r^2$. The marginal stability condition is then $(2\delta r_o^2/7)\,\partial \ln(\omega_A^2)/\partial r^2 + \eta = 0$. Evidently, for $\omega < \omega_{*b}$ there must always be unstable modes for small enough r_o. Writing $\partial \ln(\omega_A^2)/\partial r^2 =$ $[2m/(nq - m)]\,\partial \ln q/\partial r^2$, and $n_b(r) = (1 - r^2/2r_b^2)\, n_b(0)$, the condition $\omega < \omega_{*b}$ becomes

$$\frac{v_b}{v_A} \frac{\rho_b R}{r_b^2} \; \frac{mq(r_o)}{nq(r_o) - m} > 1 \; , \tag{19}$$

where $\rho_b = (m_b T_b)^{1/2}/eB$. Parenthetically, we note that if $\partial f_b/\partial v_{\|}^2 > 0$ then the condition $\omega < \omega_{*b}$ is no longer required for instability.

In the limit $\omega \ll \omega_{*b}$ the condition for instability becomes (in the case $\omega \gg \nu_e^{eff}$)

$$\frac{r_o^2}{r_b^2} < \frac{\beta_b v_b \rho_b}{\nu_e r_b^4} \left(\frac{r}{R}\right)^{1/2} \left(\frac{\partial \ln q}{\partial r^2}\right)^{-1} \frac{q(r_o)^2}{nq(r_o) - m} \tag{20}$$

where $\beta_b = 8\pi n_b T_b/B^2$. In applying Eqs. (19) and (20), we note that $nq(r_o) - m$ may be small, although a singular surface where $nq(r) = m$ must not occur. Thus, the worst situation would be where $nq(0) = m$ in which case $nq(r_o) - m = mr_o^2 \, \partial \ln q/\partial r^2$; this case is assumed in the discussion below.

As typical parameters of a large tokamak with intense high-energy injection,[2] we take $B = 50$ kG, $n = 10^{14}$ cm^{-3}, $T = 5$ keV, $\nu_e = 10^4$ sec^{-1}, $v_b = 3.10^8$ cm/sec (for 100 keV injection), $v_A = 10^9$ cm/sec, $\rho_b = 0.5$ cm, $R = 300$ cm, $r_b = 50$ cm, and $(\partial \ln q/\partial r^2)^{-1} = 10^4$ cm^2. The instability condition (19) then requires $r_o < 15$ cm, and for $\beta_b = 0.01$ the condition (20) requires $r_o < 30$ cm. Thus, the beam may be unstable, and anomalously flattened in radial profile, over the innermost 15 cm. For narrower beams (smaller r_b) the region of instability would be somewhat larger. In the case of α-particles in a reactor, we might have $B = 50$ kG, $n = 10^{14}$ cm^{-3}, $T = 15$ keV, $\nu_e = 2.10^3$ sec^{-1}, $v_b = 10^9$ cm/sec (for 3.5 MeV), $v_A = 10^9$ cm/sec, $\rho_b = 3$ cm, $R = 600$ cm, $r_b = 100$ cm, and $(\partial \ln q/\partial r^2)^{-1} = 4.10^4$ cm^2. In this case the instability condition (19) requires $r_o < 80$ cm, and for $\beta_\alpha = 0.01$ the condition (20) is then also satisfied; this represents a fairly severe condition.

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[4] Finite Larmor radius corrections to the expression given in Eq. (2) for $\mathbf{v}_E$ have the effect of multiplying the right-hand side of Eq. (5) by the factor $1 - \omega_{*i}/\omega$. These corrections are unimportant in the present calculation since our modes have $\omega \gg \omega_{*i}$; if required, the corrections can be obtained by introducing the usual factor $J_o^2(k_\perp \rho)$ multiplying $\mathbf{v}_E$.

[5] Again, finite Larmor radius corrections to $\mathbf{v}_E$ would result in a factor $1 - \omega_{*i}/\omega$ multiplying the first term on the right.

[6] Replacing ν_e^{eff} by $\nu_e^{\mathrm{eff}}(2T_e/m_e v^2)^{3/2}$ and computing the indicated velocity integrals, or using a simple Fokker-Planck model, agrees to within

a factor 2 with the simple form given here, if we use $\nu_e^{eff} = (R/r)\nu_e$ with $\nu_e = 2\pi n e^4 \ln\Lambda / m_e^{1/2} (2T_e)^{3/2}$. (W. Tang private communication).

[7] In the more general case where the correction factors $1 - \omega_{*i,e}/\omega$ are retained, Eq. (17) is modified as follows: the dispersion function in the square brackets becomes $[\omega(\omega - \omega_{*i}) + i\omega^2\eta - \omega_A^2]$, and the coefficient of $\partial^4\xi/\partial r^4$ becomes $[3\omega(\omega - \omega_{*i})/4 + k_\parallel^2 v_A^2 \tau(1 - i\delta)(\omega - \omega_{*i})/(\omega - \omega_{*e})]\rho_i^2$ with $\tau = T_e/T_i$.

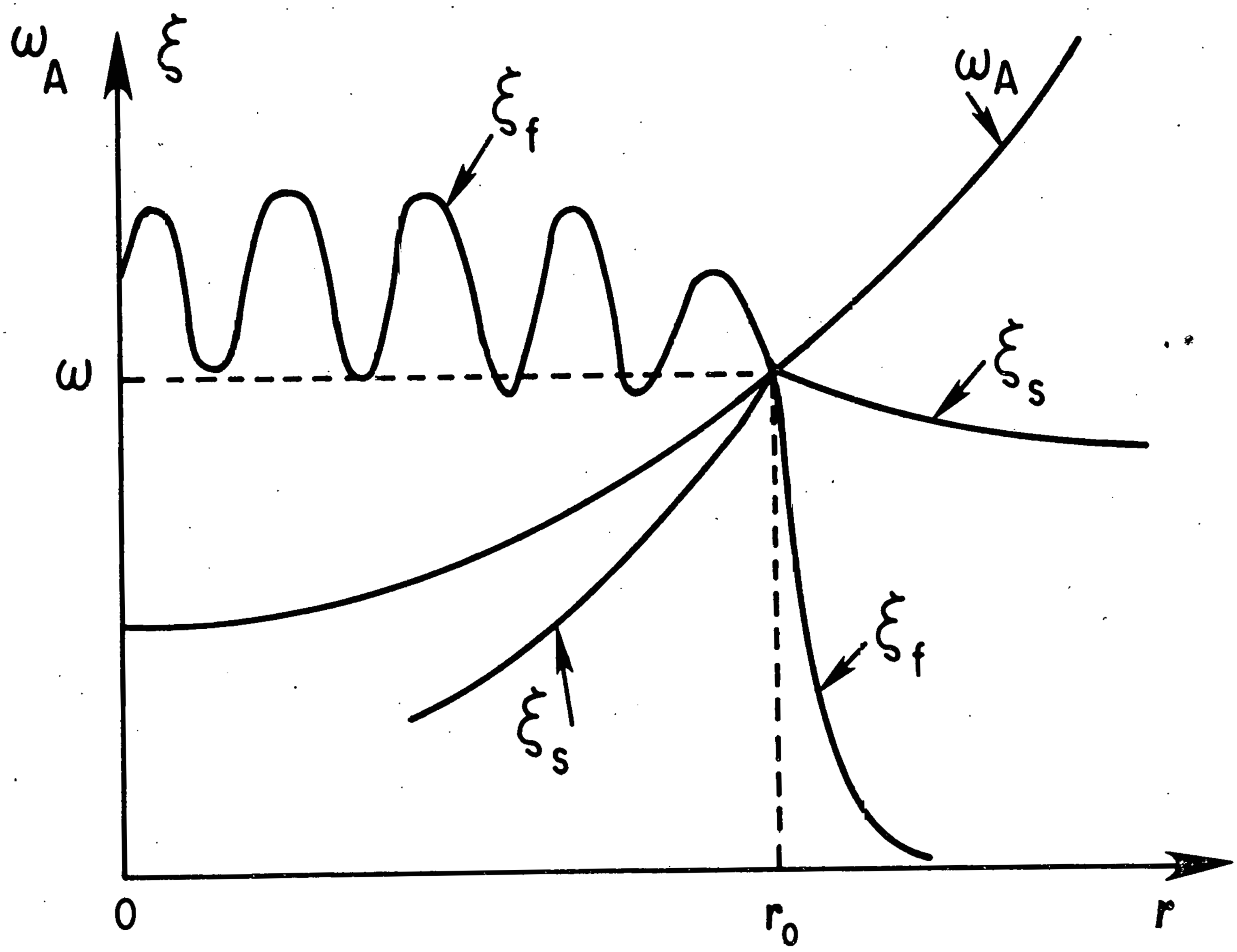

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Fig. 1. Illustration of shear Alfvén waves eigenmodes in a tokamak; $\omega_A(r) \equiv k_{\parallel}(r)\, v_A$; ξ_s are slowly varying MHD-like solutions; ξ_f are fast varying evanescent or oscillatory solutions.

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