# SMALL-SCALE DENSITY FLUCTUATIONS <br> IN THE ADIABATIC TOROIDAL COMPRESSOR 

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Printed in the United States of America.
Available from
National Technical Information Service
U. S. Department of Commerce

5285 Port Royal Road
Springfield, Virginia 22151
Price: Printed Copy \$ * ; Microfiche \$1.45
*Pages
1-50
51-150
151-325
326-500
501-1000

NTIS
Selling Price
$\$ 4.00$
5.45
7.60
10.60
13.60

## SMALL-SCALE DENSITY FLUCTUATIONS IN THE

ADIABATIC TOROIDAL COMPRESSOR

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A new class of density fluctuations has been observed in the ATC tokamak by using spectral analysis of scattered microwaves. The observed frequency spectrum is consistent with that of drift waves with amplitudes that are maximum in the wavelength range $0.5-1.0 \mathrm{~cm}$ where finite ion Larmor radius effects are important for plasma stability. The total density fluctuation is $\tilde{\mathrm{n}}_{\mathrm{e}} \gtrsim 5 \times 10^{-3} \overline{\mathrm{n}}_{\mathrm{e}}$. We estimate that these fluctuations could account for a large fraction of the electron energy losses of the ATC discharge. $\qquad$
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One of the most intriguing and worrying phenomena in tokamaks is the large transport of the electron heat. It is universally acknowledged that this anomalous loss is caused by microinstabilities ${ }^{\perp}, \mathcal{L}$ but any direct evidence of the existence of these phenomena in tokamaks was missing up to now. In this letter we report the first experimental observation of smallscale density fluctuations in a tokamak.

To detect the presence of small-scale density fluctuations in the 'ATC discharge-we used the scattering of microwaves. The
output of a 70 GHz oscillator, with a power of 20 watts, was launched into the plasma. as a wave in the ordinary mode. An array of antennae was installed inside the vacuum vessel so that it was possible to make measurements at six scattering angles between $11^{\circ}$ and $64^{\circ}$. The geometry was such that the vector $\vec{k}=\vec{k}_{i}-\vec{k}_{s}\left(\vec{k}_{i}\right.$ and $\vec{k}_{s}$ being the wave vectors of the incident and the scattered waves respectively) was along the poloidal direction of the tokamak configuration. The scattering region 'was midway between the center and the edge of the plasma minor cross-section.

A homodyne detection system was used in which the received wave was mixed with a larger reference wave directly from the microwave oscillator. The signal from a crystal detector was sampled ( 2000 times with a maximum sampling frequency of 5 MHz ), stored in an electronic waveform recorder and analyzed with a spectrum analyżer. The frequency spectral resolution was primarily limited by the length of the sampling window (typically $0.4-1.0 \mathrm{msec})$.

The scattering cross section, which is defined as the fraction of the incident power which is scattered at the frequency $\omega_{0}+\omega$ ( $\omega_{0}=$ frequency of the incident wave) per unit volume, solid angle and frequency, is

$$
\begin{equation*}
\sigma=r_{0}^{2} s(\vec{k}, \omega) \tag{1}
\end{equation*}
$$

where $r_{o}$ is the classical electron radius and $S(\vec{k}, \omega)$ is the power spectrum of electron density fluctuations, given by ${ }^{3}$

$$
\begin{equation*}
S(\vec{k}, \omega)=\lim _{\substack{\mathrm{V} \rightarrow \infty \\ \mathrm{~T} \rightarrow \infty}} \frac{2}{V T}\left|n_{e}(\vec{k}, \omega)\right|^{2}, \tag{2}
\end{equation*}
$$

with

$$
\begin{equation*}
n_{e}(\vec{k}, \omega)=\int_{T} d t \int_{V} d \vec{r} e^{-i(\omega t+\vec{k} \cdot \vec{r})} n_{e}(\vec{r}, t) \tag{3}
\end{equation*}
$$

and $\vec{k}=\vec{k}_{i}-\vec{k}_{s}$.
The use of Eq. (1) implies that the linear dimensions of the scattering volume are much larger than the wavelength of density fluctuations. For tokamaks we expect that $k_{\|} \equiv|\vec{k} \cdot \vec{B}| /|\vec{B}| \simeq 1 / q R$, where $\vec{B}$ is the magnetic field; $G$ is the safety factor and $R$ is the major radius. For such long wavelengths the second condition is not satisfied but we may consider $n_{e}(\vec{r}, t)$ to be constant along the magnetic field lines and replace Eq. (1) by

$$
\begin{equation*}
\sigma=r_{0}^{2} L_{\|} S\left(\vec{k}_{\perp}, \omega\right), \tag{4}
\end{equation*}
$$

where $L_{\|}$is the average dimension of the scattering volume along the magnetic field and $S\left(k_{\perp}, \omega\right)$ is the spectral density obtained by replacing $\vec{k}$ by $\vec{k}_{\perp}=\vec{k}-(\vec{k} \cdot \vec{B}) \vec{B} /|\vec{B}|^{2}$ and $\vec{r}$ by $\vec{r}_{\perp}=\vec{r}-(\vec{r} \cdot \vec{B}) \vec{B} /|\vec{B}|^{2}$ in Eqs. (2) and (3).

The scattering volume, which was determined by the radiation patterns of the launching and the receiving antennae, was a decreasing function of the scattering angle $\theta_{S}$. Its average dimension along the minor radius varied from $\sim 10 \mathrm{~cm}$ for $\theta_{s}=11^{\circ}$ to $\sim 5 \mathrm{~cm}$ for $\theta_{S}=64^{\circ}$. Therefore the values of $S\left(\vec{k}_{\perp}, \omega\right)$ obtained from Eq. (4) were averaged over large portions of the plasma minor cross section.

The frequency spectra of the homodyne signals measured at four scattering angles are shown in Fig. 1. They were produced by density fluctuations with wavelengths $\left(\lambda_{\perp}=2 \pi /\left|\vec{k}_{\perp}\right|=\right.$ $\left.\pi /\left|\vec{k}_{i}\right| \sin \left(\theta_{s} / 2\right)\right)$ in the range $0.4-2.3 \mathrm{~cm}$. These results were obtained in a typical uncompressed ATC discharge ${ }^{4}$ where (with standard notation) $a=16 \mathrm{~cm}, \mathrm{R}=80 \mathrm{~cm}, \mathrm{~B}_{\mathrm{t}}=20 \mathrm{kG}$, $\bar{n}_{e}=2 \times 10^{13} \mathrm{~cm}^{-3}, \overline{\mathrm{~T}}_{\mathrm{e}}=400 \mathrm{eV}, \overline{\mathrm{T}}_{\mathrm{i}}=150 \mathrm{eV}, \mathrm{I}_{\mathrm{p}}=70 \mathrm{kA}$, $q=4.5, Z_{e f f}=5$. The gross electron energy confinement time was 5 msec .

The range of observed frequencies is that of drift waves. For the central region of the scattering volume we calculate a drift frequency of 160 kHz for $\lambda_{\perp} \simeq 1 \mathrm{~cm}$. Both the finite angular aperture of the antennae and the average over large scattering volumes contributed to the spectral broadening we observed. A considerable increase of the spectrum width occurs as the scattering angle is increased from $11^{\circ}$ to $40^{\circ}$. At larger angles no remarkable change of the spectrum shape was detected.

The total mean square density fluctuation is given by

$$
\begin{equation*}
\left.\left.\langle | \tilde{n}_{e}\right|^{2}\right\rangle=\frac{1}{(2 \pi)^{3}} \cdot \int S\left(\vec{k}_{\perp}, \omega\right) d \vec{k}_{\perp} d \omega \tag{5}
\end{equation*}
$$

where the region of integration is the entire $\left(k_{\perp}, \omega\right)$ space. The function $\left.\left.\langle | n_{0}\left(\vec{k}_{\perp}\right)\right|^{2}\right\rangle=\left(1 / 4 \pi^{3}\right) \int S\left(\vec{k}_{\perp}, \omega\right) d \omega$, which was obtained by integrating the spectral density over the measured frequency range, is shown in Fig. 2 for six values of $\vec{k}_{\perp}=\vec{k}_{i}-\vec{k}_{s}$. It reaches a maximum for values of $k_{\perp}=\left|\vec{k}_{\perp}\right|$ such that $k_{\perp} p_{i} \simeq I$
$\left(\rho_{i}=c\left(m_{i} T_{e}\right)^{1 / 2} / e B\right)$, indicating that finite ion Larmor radius effects might play a role in the observed fluctuations.

We measured only the scattering produced by waves with their wave vector along the poloidal magnetic field. For drift waves in tokamaks one expects a strong localization of perturbations around magnetic surfaces due to the shear of magnetic lines. Consequently the spectrum of the observed turbulence should be finite also in the region where $|\vec{k} \cdot \nabla p| /|\vec{\nabla} p|>1 \mathrm{~cm}^{-1}$ ( $\vec{\nabla} p=$ pressure gradient). By integrating the function $\left.\left.\langle | \tilde{n}_{e}\left(k_{\perp}\right)\right|^{2}\right\rangle$, shown in Fig. 2, over the variable $k_{\perp}=2\left|\vec{k}_{i}\right| \sin \left(\theta_{s} / 2\right)$ we can therefore obtain a lower limit. to the mean square density fluctuation induced by the observed microinstabilities. From the data of Fig. 2 we get $\left.\left(\left.\langle | \dot{\tilde{n}}_{e}\right|^{2}\right\rangle\right)^{1 / 2} \geqslant 5 \times 10^{-3} \bar{n}_{e}$.

For the plasma parameters in the central part of the scattering region we estimate $\omega_{b e} \simeq 1.7 \times 10^{6} \mathrm{rad} \mathrm{sec}{ }^{-1}$, for the bounce frequency of trapped electrons, and $\nu_{\text {eff }} \simeq 5.5 \times 10^{6}$ sec $^{-1}$, for their effective collision frequency. The regime of operation of ATC is therefore not far from the banana regime and we anticipate the appearance of dissipative trapped-electron modes. ${ }^{5}$ Nevertheless on the basis of present results we cannot rule out the existence of other types of drift instabilities.

To estimate the effects of the observed density fluctuations on the electron energy transport we shall assume that they were produced by low-frequency electrostatic waves for which we can take $\left(\tilde{n}_{e} / \bar{n}_{e}\right) \simeq\left(e \phi / T_{e}\right), \phi$ being the wave electrostatic potential. The effect of an electrostatic instability on the transport of a low- $\beta$ toroidal configuration is produced by the random walk
of particles across the magnetic surfaces under the influence of collisions and $\vec{E} \times \vec{B}$ drifts. The two frequencies, $\nu_{1}=(e \phi / T)^{3 / 2} \mathrm{k}_{\mathrm{I}} \mathrm{v}_{\mathrm{th}}$ and $\nu_{2}=\mathrm{k}_{11} \mathrm{v}_{\mathrm{th}}$ (where $\mathrm{v}_{\mathrm{th}}=(\mathrm{T} / \mathrm{m})^{1 / 2}$ ) define three possible regimes.

When the collision frequency $\nu<\nu_{1}$, those particles with a velocity component $\mathrm{v}_{\mathrm{II}}$ in the direction of the magnetic field smaller than $v_{t h}(e \phi / T)^{1 / 2}$ are trapped in the field of an electrostatic wave. The resultant electron heat transport coefficient in plane geometry has been estimated by Pogutse, ${ }^{6}$
$X_{e}^{(l)} \simeq\left(e \phi / T_{e}\right)^{1 / 2}\left(k_{\perp} / k_{11}\right)^{2} \nu_{e} \rho_{e}{ }^{2}$, where $\rho_{e}$ is the electron Larmor radius.

For $\nu>v_{2}$ electrostatic trapping does not play any role. In this regime the electron heat conductivity has been estimated by Yoshikawa, ${ }^{7} \chi_{e}^{(3)} \simeq\left(e \phi / T_{e}\right)^{2}\left(k_{\perp} / k_{\|}\right)^{2} \nu_{e} \rho_{e}^{2}$.

From the measured amplitude of fluctuations we infer that present tokamaks operate in an intermediate regime where $\nu_{1}<\nu_{e}<\nu_{2}$. In this regime trapping of particles still plays a role in the electron heat transport but its effect is less than in the first regime $\left(\nu<\nu_{1}\right)$. We estimate the heat conductivity to be $x_{e}^{(2)} \simeq x_{e}^{(1)}\left(\nu_{1} / \nu_{e}\right)$ where the ratio $\nu_{1} / \nu_{e}$ takes in account the reduction of the number of trapped particles caused by collisions. With $k_{\|} \simeq 1 / q R$, from $\tau \simeq a^{2} / 4 \chi_{e}^{(2)}$ we get that the observed electron energy confinement of a typical uncompressed ATC discharge ${ }^{4}$ could be determined by electrostatic fluctuations with $\lambda_{1} \simeq 1 \mathrm{~cm}$ and $\mathrm{e} \phi / \mathrm{T}_{\mathrm{e}} \simeq 10^{-2}$. Therefore we conclude that the density fluctuations we have observed are responsible for a large part of the electron energy losses of ATC.

In conclusion we have observed a new class of small-scale density fluctuations in the ATC tokamak. Their frequency spectrum is consistent with that of drift waves and their amplitudes are maximum in the range of wavelengths where finite ion Larmor effects play a role in the plasma stability. The amplitude of these fluctuations are sufficient to explain a large fraction of the electron energy losses of ATC.

ACKNOWLEDGMENT
I am grateful to H. P. Furth and R. A. Ellis for their encouragement and support and to F. W. Perkins and T. H. Stix for useful discussions. I should also like to express my particular appreciation to R. Shoemaker and his crew of technicians for their help in the operation of the ATC device. This work was supported by the U.S. Energy Research and Development Administration Contract $\mathrm{E}(11-1)-3073$.

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Fig. 1. Spectra of microwaves scattered by density fluctuations with wavelengths of: $\lambda\left(11^{\circ}\right)=2.3 \mathrm{~cm}, \lambda\left(26^{\circ}\right)=1.0 \mathrm{~cm}$, $\lambda\left(40^{\circ}\right)=0.6 \mathrm{~cm}, \lambda\left(64^{\circ}\right)=0.4 \mathrm{~cm} . \quad \theta$ is the scattering angle; the ordinate is an arbitrary unit and is proportional to the electric field of scattered microwaves.


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Fig. 2. Amplitude of density fluctuations as a function of $k=2\left|\vec{k}_{i}\right| \sin \left(\theta_{s} / 2\right) ;<n_{e}$ is the average electron density.

