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SIMPLIFIED ANALYTICAL MODEL FOR SIMULATION OF BOILING WATER REACTORS

by

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INTRODUCTION

During the course of physics work undertaken for the research and development project associated with the Consumers Big Rock Point Reactor, a need was seen for a relatively simple and inexpensive calculational method to determine core reactivity and power distributions. A need for this type of analytical tool is not unique to the Consumers project, and in fact, exists during the design and operation of any nuclear plant. However, three factors made scoping calculations of this type of particular importance to the Consumers project:

1. The core will be operated in a number of off-standard conditions of pressure, flow, subcooling, and power. A very detailed examination of each operating state would be prohibitive in terms of cost and time required.

2. One goal of the development program is to study alternate methods for scheduling refueling and control rod movement in order to optimize power shapes throughout the core's life. Calculations of this type are very expensive unless simplified techniques are employed.

3. The Consumers reactor has an on-line digital computer which evaluates all important plant operating characteristics on the basis of input plant observables. An analytical model is required to generate power distributions from the observables.

To assist in accomplishing the above tasks, the digital computer program FLARE was developed. Before specifying the FLARE equations it was decided that as a minimum the code should include:

1. a physically correct representation of the coupled effects of flow, power, and voids,

2. a calculation for the effects of fuel burnup,

3. an independent representation of each control rod,

4. a limitation of size, permitting calculation in-core in a 16K-bit fast memory.

It was felt that the first three items represent the two most-significant factors affecting reactivity and power distributions and that the last item was essential in achieving a fast, inexpensive calculational tool.

Because of the independent representation of the control rods a three dimensional calculational scheme was employed. This three dimensional geometry when combined with the computer size limitations restricted the calculations to a relatively coarse mesh. For example, the 84 fuel element Consumers core might be represented by 672 mesh points or nodes (8 axial and one horizontal in each fuel element). Despite the severe approximations inherent in this model, relatively good agreement has been obtained in comparisons with both experimental data and more detailed calculational results.

DESCRIPTION OF CALCULATIONS MADE BY FLARE

A. The Basic Calculation

The basic calculation made by FLARE is the relative power density at each node in the reactor as well as the effective core multiplication constant. The following core parameters are required as input:

1. Total core power
2. Total core flow
3. Core inlet enthalpy
4. Position of each control blade
5. Description of nuclear characteristics of each fuel element
6. Exposure distribution in each fuel element

In addition to power distribution the moderator density is calculated at each node. Iterations are made between power and void distributions until a consistent solution is obtained.

B. Total Power Search

An option is provided by which the core power can be adjusted automatically until core multiplication equals some pre-set value (e.g., 1.0). This permits calculation of the expected power response to a given control rod motion, for example.

C. Fuel Burnup Calculation

After a power distribution is established the fuel exposure distribution in the core is incremented at each node by the value proportional to the local power density. A new power distribution and core multiplication are then calculated. If a change in the control system is required this must be specifically indicated in the input. Because of the one group nature of the model, fuel burnup changes are assumed to affect only the infinite multiplication constant of the fuel.

D. Refueling Option

If a specific multi-batch refueling schedule is described in the input it is possible to calculate the equilibrium end-of-life power and exposure distribution which yield a specified final core multiplication. In this option it is assumed that the core power distribution is held constant during each fuel cycle.

E. Xenon Transient Analysis

In one version of the code, the fuel depletion equations are replaced by xenon and iodine equations. With this option it is possible to follow the effects of spatially dependent xenon transients in three dimensional geometry. A specific application of this option would be the following of the slow transients subsequent to a shift in control pattern.

DESCRIPTION OF ANALYTICAL MODEL

The equations solved by the code are based on a one group theory utilizing a transport kernel for the probability that neutrons born in one node are ultimately absorbed in one of the adjacent nodes. If A_i is the neutron absorption rate at node i and S_i is the neutron production rate at node i,

$$S_i = \frac{k_{\infty i}}{\lambda} A_i \qquad (1)$$

in which λ is the usually defined effective multiplication constant. If, in addition, ω_{ij} is defined as the probability that a neutron born at node i is absorbed at node j, the absorption rate at i can be written

$$A_i = \sum_j{}' S_j \omega_{ji} + S_i \omega_{ii} \qquad (2)$$

in which the prime indicates a sum over the six nearest nodes. Combining equations (1) and (2) one obtains

$$S_i = \frac{k_{\infty i} \sum' S_j \omega_{ji}}{\lambda - k_{\infty i} \omega_{ii}} \qquad (3)$$

Equation (3) is the basic nodal equation solved at all interior mesh points. In the calculation, all mesh points are assigned to fuel regions and reflector effects are treated by an albedo at all nodes adjacent to the reflector.

The eigenvalue λ is recalculated after each source iteration on the basis of a calculated neutron balance summed over the entire core.

The infinite multiplication constant ($k_{\infty i}$) is specified through constants in the input as a function of

- moderator density
- power density (xenon and Doppler)
- presence of adjacent control rod
- fuel exposure

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The transport kernel, ω_{ij}, is assumed to be a function of the migration area of neutrons $(M^2)_i$ and the inter-node spacing $(\Delta x)_{ij}$ only.

The equation from which ω_{ij} is calculated is,

$$\omega_{ij} = g \frac{M_i^2}{(\Delta x)_{ij}^2} + (1-g) \frac{\sqrt{M_i^2}}{2(\Delta x)_{ij}} \qquad (4)$$

in which g is an arbitrary input parameter.

There is no rigorous justification for equation (4). The most straightforward justification for this equation is that it is simple and gives good results. However, the first term of equation (4) is that value of ω_{ij} which reduces equation (3) to an equation very similar to the diffusion equation. The second term of equation (4) is the leading term in the expression for a diffusion theory kernel assuming a slab source and a slab receiver. Variations in the mixing parameter g can be utilized to provide improved agreement to data if such data exist.

The Doppler reactivity is assumed to be a linear function of power density and voids. If greater precision were required it would be possible to increase the complexity of this fit. The equilibrium xenon reactivity is assumed to be dependent on power density alone and to be independent of such parameters as voids and exposure. Thus

$$\left(\frac{\Delta k}{k}\right)_{\text{Doppler}} = a_1 - (b_1 + c_1 U)P$$

$$\left(\frac{\Delta k}{k}\right)_{\text{xenon}} = \frac{a_2 P}{b_2 + c_2 P}$$

in which P is power density
U is relative moderator density

Given the channel inlet flow and enthalpy, the calculation of coolant quality at each node is straightforward. It is assumed that the coolant quality is the average of the inlet and exit value for each node. Thus

$$X_z = \frac{\sum_1^{z-1} P_i + \frac{P_z}{2}}{W h_{fg}} - \frac{\Delta h_s}{h_{fg}}$$

in which W is mass flow in lb/hr
P_i is nodal power in BTU/hr

The void-quality relationship is input as a fit to existing experimental data. It is possible to include the effects of subcooled voids by extending this fit into negative qualities.

The complex effects of fuel depletion are calculated externally to the FLARE code and these results are merely fitted to yield the dependence of k_∞ on average void content, average control presence, and exposure. A combination of a linear and an exponential term has been found to give a good representation of the exposure dependence for a wide variety of fuel types including those with significant burnable poison content.

Figure 1 is a flow chart for the FLARE calculation. For most of the iterative calculational loops it is possible to establish both a convergence criterion and a maximum number of iterations. Because a calculation can involve as many as four levels of iteration (source, voids, total power, and exposure) it is important that the number of inner iterations be kept to a minimum.

A typical running time for this code on a Philco Transac S-2000 is 2 - 3 minutes for adequate convergence of a single operating state of a 6 foot core with half-core symmetry.

CALCULATIONAL RESULTS AND COMPARISONS WITH OPERATING DATA

The FLARE code has been utilized for the solution of a wide class of reactor problems. One difficulty associated with a simplified model of this type is the determination of the accuracy and reliability of the solution. To date, no comparisons with more exact three dimensional calculations have been made. A number of comparisons with two dimensional problems and with three dimensional reactor operating data have been made.

Operating data from the 60 MWth Kahl boiling water reactor have been compared with FLARE calculations simulating the same reactor. Figure 2 shows a plan view of the Kahl core. It consists of fuel with two different enrichments and two types of channels. The channels near the core center contain significant amounts of boron as a burnable poison. This created an initial slight rise in core reactivity with burnup. Figure 3 shows the calculated core multiplication constant for actual reactor operating states during the first 2-1/2 years operation of the reactor. There appears to be no consistent drift in the calculated versus observed reactivity. It is interesting to note that these calculated states include power level changes from two-thirds to full power, significantly different control rod movement patterns, and a replacement of one-half of the steel channels with Zircaloy channels. Despite these rather severe core changes the calculated reactivity appears to be constant to within ±0.5 percent. This excellent agreement is probably more properly attributed to the calculation of k_∞ as a function of exposure by the General Electric burnup code than to the FLARE code.

Three times during this operating period the reactor was shut down and each fuel element was gamma scanned to give a measure of the core power distribution. Figure 4 shows a comparison of calculated and measured relative fuel element power levels. Figures 5-6 show comparisons of calculated and measured axial power shapes from the first gamma scan of the essentially unburned Kahl core. Figures 7-8 show similar comparisons for the same core after an average core exposure of approximately 2000 MWD/T. It can be seen that the calculation follows the major trends in power distribution, but consistently tends to underestimate the magnitude of the peaks. Although the Kahl reactor was shut down this spring to install an internal superheat loop, the calculations were extended to predict what would have been the performance of the first core up to the first refueling operation. Figure 9 shows the calculated fuel bundle exposures at the time that 25 percent of the core would be scheduled for refueling. The average exposure of the fuel to be discharged is calculated to be 10,600 MWD/T.

Similar calculations have been made and compared to gamma scans at both the Big Rock Point reactor and the Humboldt Bay reactor. Although neither of these cores have sufficient fuel exposure to test the reliability of the burnup effects, both reactors gave comparisons similar to the Kahl results for essentially clean cores.

Rod oscillator tests have been planned for the Big Rock Point reactor as a means of measuring the system's dynamic characteristics. As a guide to the expected core response, FLARE has been utilized to estimate the reactor response to both the high frequency and low frequency limits of the rod oscillation test. In the low frequency limit it is assumed that voids, temperature, and power distributions have all come into equilibrium, but xenon is still at its previous value. The following values were calculated for a six-inch displacement of the control rod which can be oscillated.

Control Rod Response
Rated Power, Rated Flow
Low Frequency Response -- 0.48 MW/inch
High Frequency Response -- 6.1 MW/inch

The relatively strong power coefficient results in a significantly reduced response in the low frequency regime; relative to the high frequency value.

Utilizing FLARE, a test calculation has been made of the xenon transient characteristics of a large BWR. In this idealized model the reactor was assumed to be 12 feet high by 12 feet wide and infinite in length. A control rod swap was made between central rods and edge rods, and the power magnitude and distribution were calculated as a function of time. Figure 10 shows the assumed core and control rod geometry. The core power coefficients were intentionally made somewhat less-negative than that which would exist in a typical BWR. Figure 11 shows the calculated local power density as a function of time for a point near the center of the core. The initial transient, which would appear to have a 25-30 hour period, can be seen to be almost completely damped. The code appears to be very useful in investigating transients of this type.

SUMMARY AND CONCLUSIONS

The limited experience we have had using this very simplified core representation indicates that it can be a very useful tool in following and understanding the characteristics of operating reactors. Because of its simple logic it is amenable to making adjustments in the equations to bring the results more nearly into agreement with observed data. In this sense, the method is not considered to be an exact, first principle solution, but more nearly a semi-empirical solution which can take advantage of observed results and use this to make improvements on future predictions. Very little effort has been applied to optimizing the numerical techniques in the code and it would be expected that improvements could be made to speed up the solution. From the results we have obtained, it would appear that the code has sufficient applicability to justify future improvements. One improvement which has been planned but not implemented is to automatically adjust control rod settings to maintain criticality throughout a core lifetime. An additional later modification would permit calculation by some algorithm the optimum control pattern as a function of time.

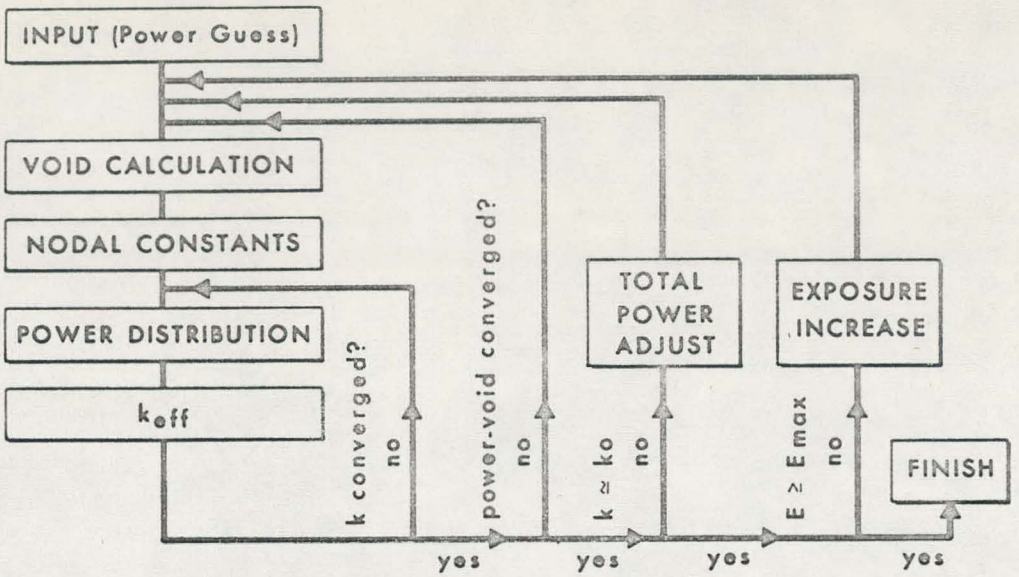

FIGURE 1 FLARE FLOW CHART

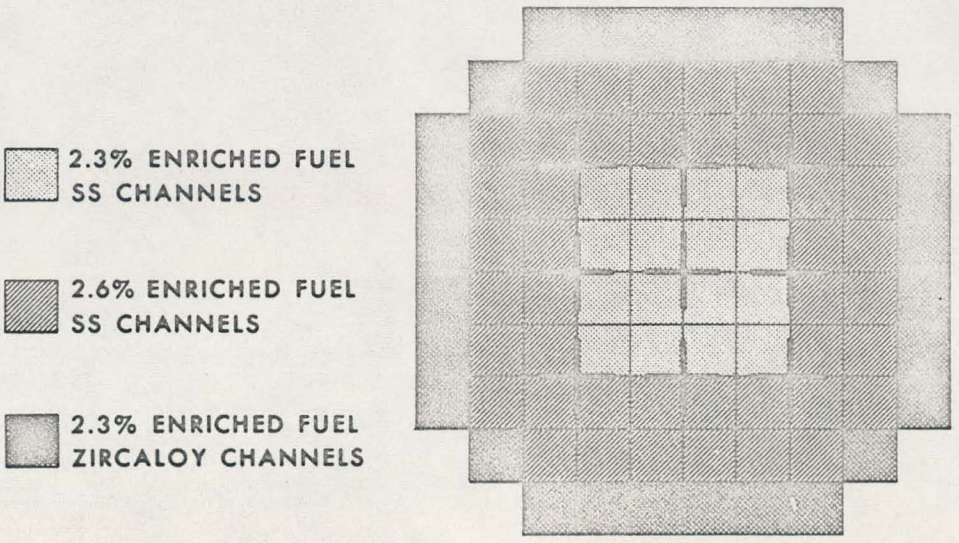

FIGURE 2 KAHL CORE GEOMETRY

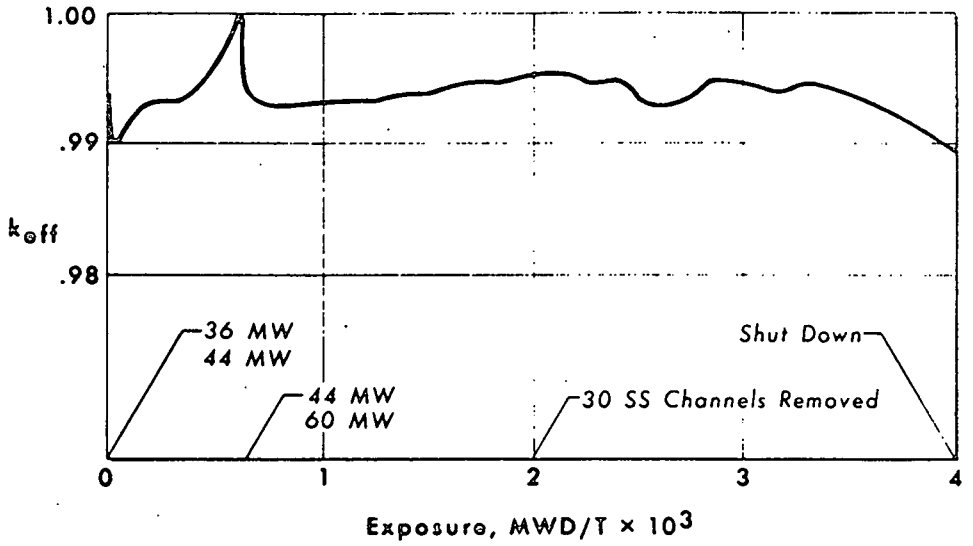

FIGURE 3 CALCULATED k_{eff} VS. EXPOSURE

TEST JUNE 25, 1961

.697 .69	.856 .89	1.025 .90	1.195 1.21	1.042 1.06
	1.047 1.16			
		1.185 1.17		
			.813 .77	

TEST JULY, 1962

.958 .941	1.049 1.043	1.121 1.066	1.140 1.098	.918 .943
	1.159 1.205			
		1.154 1.193		
			.732 .737	

TEST DEC 14, 1961

.918 .960	.976 960	1.069 .993	1.158 1.122	.970 .975
	1.119 1.163			
		1.171 1.159		
			.765 .772	

BUNDLE POWERS $\frac{\text{FLARE}}{\text{GAMMA SCAN}}$

FIGURE 4 KAHL RELATIVE BUNDLE POWER

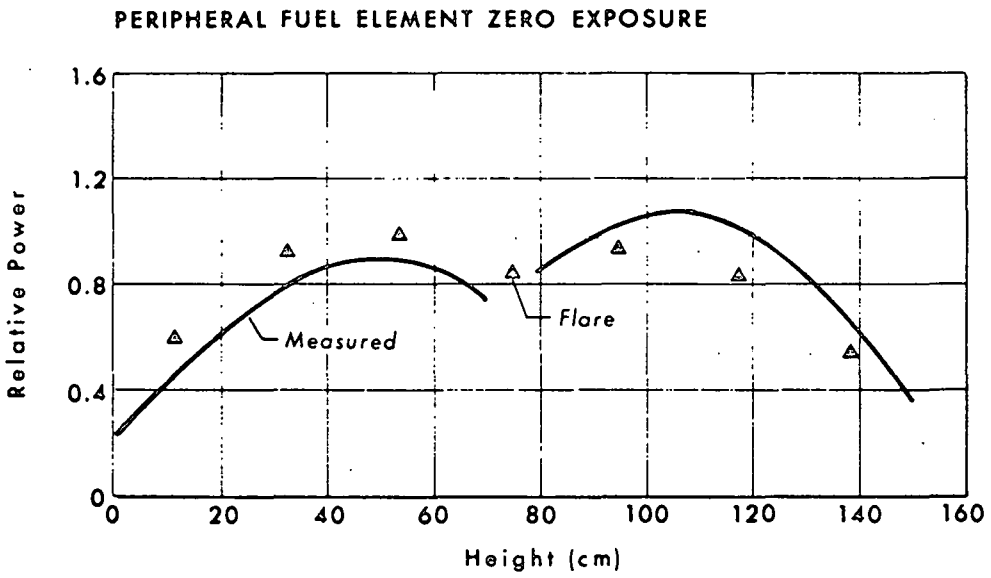

FIGURE 5 AXIAL POWER SHAPE

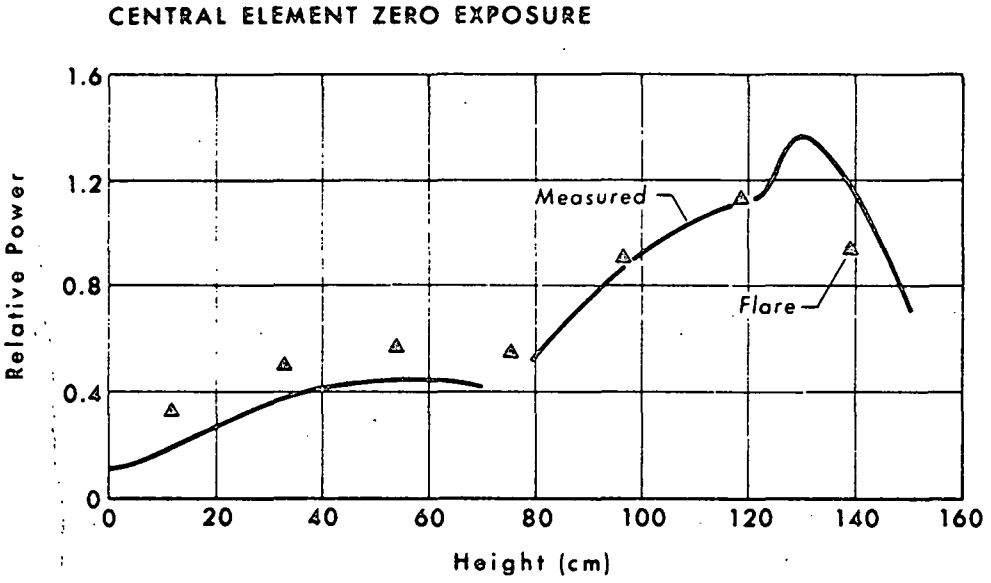

FIGURE 6 AXIAL POWER SHAPE

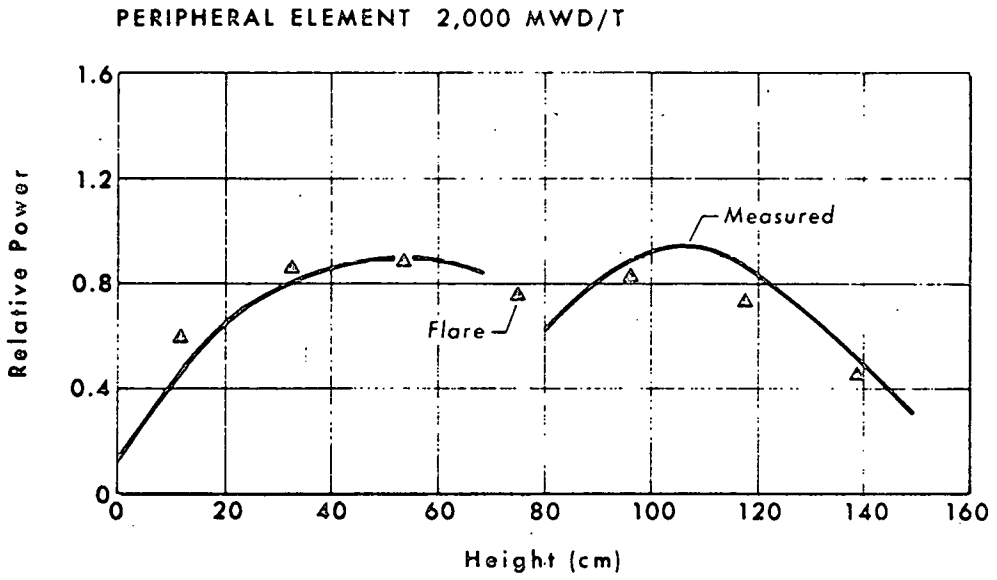

FIGURE 7 AXIAL POWER SHAPE

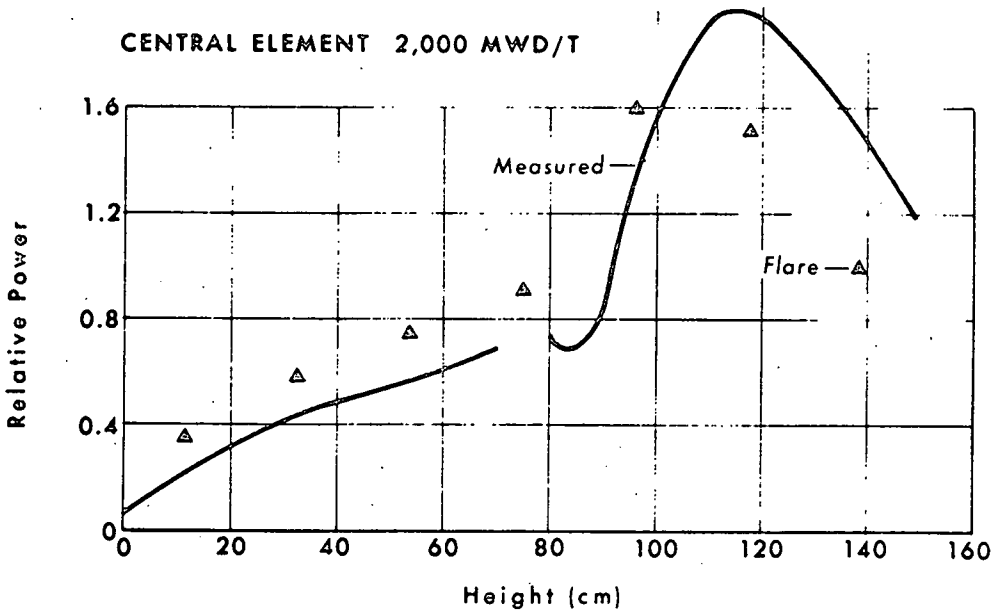

FIGURE 8 . AXIAL POWER SHAPE

FUEL ELEMENT EXPOSURES, INITIAL REACTIVITY
END OF LIFE

7.54	9.86	10.57	10.58	10.66	10.69	10.71	10.72	10.10	7.72
6.96	9.18	9.55	9.76	10.62	10.67	9.93	9.77	9.35	7.07
5.50	7.89	8.79	9.57	10.61	10.66	9.75	9.02	8.17	5.67
	5.77	7.90	9.21	9.92	10.00	9.31	8.15	6.02	
		5.51	6.99	7.59	7.65	7.04	5.65		

CORE AVG. EXPOSURE **8736 MWD/T**
FIRST DISCHARGE **10,600 MWD/T**

FIGURE 9 KAHL PROJECTED FUEL EXPOSURE

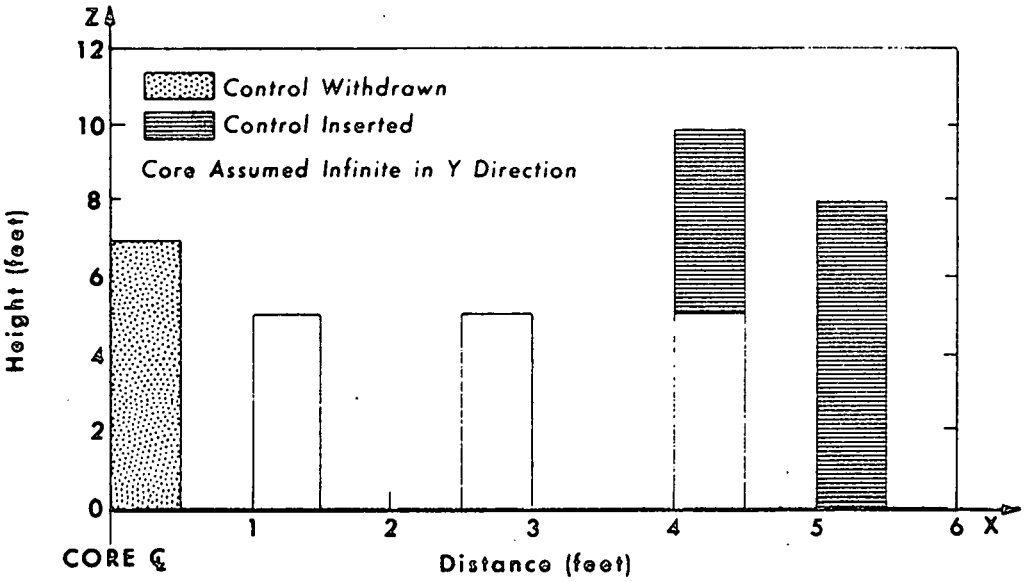

FIGURE 10 CORE GEOMETRY – XENON TRANSIENT

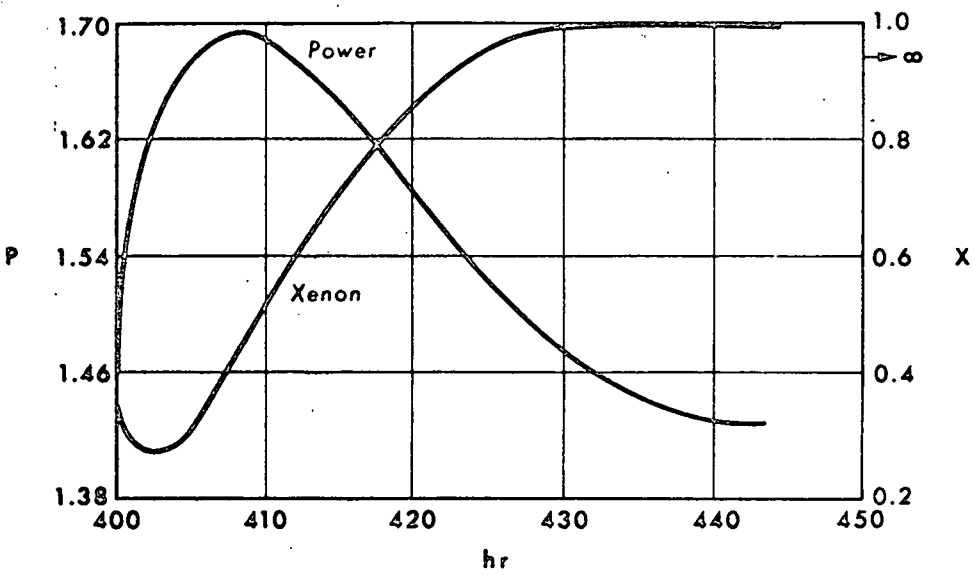

FIGURE 11 XENON TRANSIENT RESULTS