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SUBJECT: Calculated Reactivity Changes Produced by the Oscillation of an Epithermal Neutron Absorber in the Bulk Shielding Reactor

TO: Distribution

FROM: R. Perez-Belles

ABSTRACT

Based on a simplified model of the Bulk Shielding Reactor, a lower limit is established for the amount of resonance absorption which can be measured by the pile oscillator technique.

## DISCLAIMER

## INTRODUCTION

In this report we attempt to ascertain the optimum working conditions of a pile oscillator and to compare this method with the danger coefficient technique from the point of view of minimum amount of resonance absorption (from a fission product, for example) which can be detected. This aspect of the problem is of importance because the available amount of isolated fission product may be very small in some cases. The model used in the present calculations uses one group of thermal neutrons corrected for resonance absorption. Changes in slowing down power produced by the sample and scattering effects are not accounted for.

The influence of sample self-absorption as well as spatial dependence of the epithermal flux will be phenomenologically included by using an effective resonance integral.

The space-time behavior of the absorption properties of the sample will be described by means of a $\delta$-function.[1] The reactor kinetic equations will be obtained by using a single mode expansion of the flux, which is very accurate for low frequencies of oscillation.[1,2] The theoretical results will be applied to a bare reactor model equivalent to the fully reflected parallelepiped BSR-I by using suitable reflector savings[3] and the same homogeneous core composition.

## SYMBOLS

$A$ = Amplitude of the mechanical oscillation.

$C_j$ = Concentration of the jth group of delayed neutrons precursors.

$D$ = Diffusion coefficient.

$\ell^*$ = Prompt neutron generation time.

$N_{FP}$ = Number of atoms/cm$^3$ of fission products.

$p_o, p$ = Resonance escape probability for the initial and perturbed reactor, respectively.

$\underline{r}'(t)$ = Instantaneous position of the sample.

$x_o, y_o, z_o$ = Average position of the center of the sample.

$v$ = Speed of thermal neutrons.

$V_{FP}, V_R$ = Volume of the sample and reactor, respectively.

$\beta_j$ = Fractional yield of the jth delayed neutron group.

$\phi$ = Prompt neutron flux.

$\gamma$ = Ratio of nonescape probability of delayed-to-prompt neutrons.

$\nu$ = Neutron multiplicity. (Also frequency = $\frac{\omega}{2\pi}$ sec$^{-1}$)

$\omega$ = Frequency (rad/sec).

$\Sigma_f$ = Macroscopic fission cross section.

$\sigma(E)$ = Epithermal absorption cross section of fission products.

$X, Y, Z$ = Dimensions of reactor, including reflector savings.

$I_{eff}$ = Effective resonance integral.

$\xi$ = Average logarithmic energy change by collision.

$\Sigma_s$ = Macroscopic scattering cross section.

## CALCULATIONS

1. For a sample which is oscillated with a frequency $\omega$ and amplitude A in a region of zero thermal flux, the change in reactivity due to epithermal absorptions is given by (see App. A)

1.1
$$\rho = \rho_o \left[1 - \cos\left(\frac{2\pi}{Z} Z_o\right)\left\{J_o(\eta) - 2J_2(\eta)\cos 2\omega t\right\} + 2\sin\left(\frac{2\pi}{Z} Z_o\right)\left\{J_1(\eta)\cos\omega t - J_3(\eta)\cos 3\omega t\right\}\right]$$

where

1.2
$$\rho_o = \frac{8\, N_{FP}\, V_{FP}}{V_R}\left(\frac{I_{eff}}{\xi\, \Sigma_s}\right)$$

$$\eta = \text{modulation index} = 2\pi\,\frac{A}{Z}$$

the reactor being initially critical.

The condition of zero thermal flux can be approximated by isolating a region of the reactor with a cadmium box.[4]

The harmonic content of the reactivity (Eq. 1.1) can be partially eliminated by locating the sample at

$$Z_o = Z/4.$$

In this way all the even harmonics vanish. By equating the modulation index to the fourth root of $J_3$ the value of $J_1$ is made to fall near to its first (larger) maximum. Therefore the third harmonic disappears while the fundamental component is enhanced. The rest of the odd harmonics are weighted by Bessel's functions of higher order and can be neglected.

For the sake of further discussion we will assume that the reactor reactivity input is given by

1.3
$$\Delta\rho = \Delta\rho_o \cos\omega t$$

1.4
$$\Delta\rho_o = 2\rho_o\, J_1(\eta)$$

where $\rho_o$ is given by Eq. 1.2.

2. The lower limit to the amount of fission product resonance absorption which can be detected by using a pile oscillator depends on the achievable signal-to-noise ratio which in turn depends on the reactor transfer function and the band width of the electronic equipment used in the measurement.

L. Dresner[5] has shown that the maximum signal-to-noise ratio R is given by

2.1 $$R_{max} = \Delta \rho_o \; | G(\nu) | \sqrt{\frac{N_c}{\Delta \nu}}$$

where

$\Delta \rho_o$ = reactivity worth of the sample,

$|G(\nu)|$ = reactor transfer function,

$\Delta \nu$ = Band width of the signal selection device,

$N_c$ = Maximum counting rate of the detecting device.

Introducing Eq. 1.2 and Eq. 1.4 for $\Delta \rho_o$ in Eq. 2.1 we get

2.2 $$(V_{FP} N_{FP} I_{eff}) = S \geq \left(\frac{V_R}{16 J_1(\eta)}\right) \left(\xi \Sigma_s\right) \frac{\sqrt{\frac{\Delta \nu}{N_c}}}{\left[G(\nu)\right]} R_{max}$$

where S is the resonance absorption of the sample in $cm^2$.

Inspection of Eq. 2.2 shows the convenience of working at low frequencies provided the reactor transfer functions $|G(\nu)|$ increase very fast in this frequency range, a limit being set by a reasonable duration of the oscillator runs. The parameter $R_{max}$ is related to the design of whatever electronic equipment is being used to analyze the output of the reactor-ionization chamber system. By using the parameters of Table I we get

2.3 $$S \geq 1.94 \; 10^{-2} \; R_{max} \; cm^2$$

Therefore assuming a signal to noise ratio of 2 we get $S = 3.88 \; 10^{-2} \; cm^2$.

Amounts of 50 mg of $Sm_2O_3$ ($I_{eff}$ for Sm = 1750 barns) can be measured by the danger coefficient method.[4] This is equivalent to S = 2.9 $10^{-1}$ $cm^2$ of resonance absorption as compared with the above figure of 3.88 $10^{-2}$ $cm^2$.

TABLE I

Magnitudes used in computing Eq. 2.2

| BSR-I (from Ref. 3) | Mechanical Oscillator | Reactor Transfer Functions and Detector |
|---|---|---|
| $P_o$ = .930 | $\frac{2\pi A}{Z}$ = 1.64 = fourth root of $J_3$ | |
| $\xi \Sigma_s$ = .803 $cm^{-1}$ | | |
| X = 28.83 cm | A = .261Z = 20.02 cm | $\nu = \Delta\nu$ = 4 $10^{-2}$ $sec^{-1}$ |
| Y = 79.5 cm | $J_1 (\frac{2\pi A}{Z})$ = .57 | G($\nu$) = 160 |
| Z = 77.2 cm | | $N_c$ = $10^6$ $sec^{-1}$ |
| $V_R$ = 1.77 $10^5$ $cm^3$ | | |

RECOMMENDATIONS and CONCLUSIONS

The oscillator should work at low frequencies because the reactor transfer function is independent of the neutron lifetime, and the higher flux modes are eliminated. The signal-to-noise ratio is also optimum at low frequencies.[5]

Previous experimentation in locating the detector in order to minimize the scattering effect should be performed using samples of pure scatterers. The sample should be located around Z/4 in the vertical axis (Z is the axial reactor length) to minimize the harmonic content of the reactivity signal.

A carefully designed integrator followed by a null system or analog computer analyzer should be used for analyzing the reactor output. Studies have

to be performed of the influence of sample size on self-absorption and also of the effect of slowing down inside the sample. It is concluded that by the use of reasonably well designed analyzing equipment, capable of working with an input signal-to-noise ratio of two, the oscillator technique can be made roughly one order of magnitude better than the danger coefficient method.

## APPENDIX A

### Derivation of the reactivity for an oscillating sample

Using one group of thermal neutrons corrected for resonance absorption and fast leakage, and neglecting scattering as well as the slowing down by the sample, the reactor equations are:

$$\text{A.1} \quad \frac{1}{v}\frac{\partial \phi(r,t)}{\partial t} = (D\nabla^2 - \Sigma_a)\,\phi(r,t) + p\,P_1(1-\beta)\,\nu\Sigma_f\,\phi(r,t) + P_2\sum_j \lambda_j\,C_j(r,t)$$

$$\text{A.2} \quad \frac{\partial C_j(r,t)}{\partial t} = \beta_j(\nu\Sigma_f)\,\phi(r,t) - \lambda_j\,C_j(r,t)$$

where

$$\text{A.3} \quad p = p_o \exp-\left[\frac{\int_{E_{th}}^{E_o}\sigma(E)\,\phi(E)dE}{\xi\,\Sigma_s}\,N_{FP}\,V_{FP}\,\delta(r-r')\right] \simeq p_o\left[1 - \frac{I_{eff}}{\xi\Sigma_s}\,N_{FP}\,V_{FP}\,\delta(\underline{r}-\underline{r}')\right]$$

A.4 $p_o$ = unperturbed reactor resonance escape probability

$$\text{A.5} \quad I_{eff} = \int_{E_{th}}^{E_o}\sigma(E)\,\phi(E)dE.$$

A.6 $P_1$ = fast nonleakage probability for prompt neutrons = $(1 + \tau_p B^2)^{-1}$

A.7 $P_2$ = nonleakage probability for delayed neutrons = $(1 + \tau_d B^2)^{-1}$

$\tau_p, \tau_d$ = age to thermal for prompt and delayed neutrons, respectively

$B^2$ = buckling of the equivalent bare reactor.

The adjoint flux equations for the reactor which has been brought to critical with the cadmium box inserted are:

A.8 $$(D\nabla^2 - \Sigma_a)\ \phi_o^+(r) + P_1\ p_o(1-\beta)\ \nu\Sigma_f\ \phi_o^+(r) + \sum_j \beta_j\ \nu\Sigma_f\ P_2\ \phi_o^+(r) = 0.$$

Multiplying Eq. A.1 by $\phi_o^+(r)$ and Eq. A.8 by $\phi(r,t)$, integrating over the system, subtracting and applying Green's Theorem and the pertinent boundary conditions, we get

A.9 $$\frac{\partial}{\partial t}\int \underline{dr}\ \frac{\phi_o^+(r)\ \phi(r,t)}{v} = -\int \underline{dr}\ \phi_o^+(r)\ P_1(1-\beta)\ (\nu\Sigma_f)\ p_o\left(\frac{I_{ff}\ N_{FP}\ V_{FP}}{\xi\ \Sigma_s}\ \delta(\underline{r}-\underline{r}')\right)$$
$$-\sum_j \beta_j \int \underline{dr}\ \phi_o^+(r)\ P_2(\nu\Sigma_f)\ \phi(r,t)$$
$$+\sum_j \lambda_j \int \underline{dr}\ \phi_o^+(r)\ P_2\ C_j(r,t).$$

Multiplying Eq. A.2 by $P_2\ \phi_o^+(r)$ and integrating over the system we get

A.10 $$\frac{\partial}{\partial t}\int \underline{dr}\ P_2\ \phi_o^+(r)\ C_j(r,t) = \beta_j \int \underline{dr}\ P_2\ \phi_o^+(r)\nu\Sigma_f\ \phi(r,t) - \lambda_j \int \underline{dr}\ P_2\ \phi_o^+(C_j(r,t).$$

Using the definitions

A.11 $$n(t) = \text{weighted neutron population} = \frac{1}{v}\int \underline{dr}\ \phi_o^+(r)\ \phi(r,t)$$

A.12 $$C_j(t) = \int \underline{dr}\ P_2\ \phi_o^+(r)\ C_j(r,t)$$

A.13 $$\rho = \text{prompt reactivity} = \frac{\text{weighted change of rate of prompt neutron production}}{\text{weighted total rate of neutron production}} =$$
$$= -\frac{\int \underline{dr}\ \phi_o^+(r)\ P_1(1-\beta)\ \nu\Sigma_f\ p_o\left(\frac{I_{eff}\ N_{FP}\ V_{FP}}{\xi\ \Sigma_s}\ \delta(\underline{r}-\underline{r}')\right)\phi(r,t)}{\int \underline{dr}\ \phi_o^+(r)\ \nu\Sigma_f\left[p_o(1-\beta)\ P_1 + \beta\ P_2\right]\phi(r,t)}$$

A.14 $$\gamma = \text{delayed neutron effectiveness} = \frac{\int \phi_o^+(r)\ P_2\ \nu\Sigma_f\ \phi(r,t)}{\int \underline{dr}\ \phi_o^+(r)\ \nu\Sigma_f\left[p_o(1-\beta)\ P_1 + \beta\ P_2\right]\phi(r,t)}$$

A.15 $\ell^*$ = prompt neutron generation time = $\frac{\text{weighted neutron population}}{\text{weighted rate of production}}$

in equations A.9 and A.10 we get

A.16 $$\frac{dn(t)}{dt} = \frac{\rho - \gamma\beta}{\ell^*}\, n(t) + \sum_j \lambda_j C_j(t)$$

A.17 $$\frac{d C_j(t)}{dt} = \frac{\gamma \beta_j}{\ell^*}\, n(t) - \lambda_j C_j(t)$$

which are the usual macroscopic kinetic equations.

Now a first order approximation to the reactivity input is computed by setting $\phi(r,t) = \phi_o(r)$ in Eq. A.13. Also we neglect $\beta = 6.4 \times 10^{-3}$ versus unity and the product $\beta P_2 \sim 5.5 \times 10^{-3}$ versus $p_o P_2 \sim 0.675$, and we get

A.18 $$\rho(t) = -\left(\frac{8 N_{FP} V_{FP}}{V_R}\right)\left(\frac{I_{eff}}{\xi \Sigma_s}\right) \int dx \int dy \int dz \, \sin^2\left(\frac{\pi}{X} x\right) \sin^2\left(\frac{\pi}{Y} y\right) \sin^2\left(\frac{\pi}{Z} z\right) \delta(\underline{r}-\underline{r}_o)$$

where

$$\delta(\underline{r} - \underline{r}_o) = \delta(x - x_o)\, \delta(y - y_o)\, \delta(z - z_o + A \cos \omega t)$$

where $x_o$, $y_o$, $z_o$ is the position of the center of the sample which is being oscillated along the z-axis with the amplitude A and frequency $\omega$.

Use of the properties of the $\delta$ function and the conditions

$$x_o = X/2$$

$$y_o = Y/2$$

yields

A.19 $$\rho(t) = -\left(\frac{8 N_{FP} V_{FP}}{V_U}\right)\left(\frac{I_{eff}}{\xi \Sigma_s}\right) \sin^2\left(\frac{\pi}{Z}(z_o + A \cos \omega t)\right).$$

After using straightforward trigonometric transformations and the relations[6]

$$\cos (x \cos y) = J_o(x) - 2 J_2(x) \cos 2y + 2 J_4(x) \cos 4y \ldots$$

$$\sin (x \cos y) = 2 J_1(x) \cos y - 2 J_3 \cos 3y + 2 J_5 \cos 5y \ldots$$

we get

$$A.20 \quad \rho(t) = -\left(\frac{8\, N_{FP}\, V_{FP}}{V_R}\right)\left(\frac{I_{eff}}{\xi\, \Sigma_s}\right)\left[1 - \cos\left(\frac{2\pi}{Z}\, z_o\right)\left\{J_o\left(\frac{2\pi A}{Z}\right) - J_2\left(\frac{2\pi A}{Z}\right)\cos 2\omega t + \ldots\right\} + \right.$$

$$\left. + 2 \sin\left(\frac{2\pi}{Z}\, z_o\right)\left\{J_1\left(\frac{2\pi A}{Z}\right)\cos \omega t - 2\, J_3\left(\frac{2\pi A}{Z}\right)\cos 3\, \omega t + \ldots\right\}\right].$$

## APPENDIX B

### Reactor Transfer Function

The solution of the reactor kinetic equations for a small sinusoidal perturbation

B.1 $$\rho = \Delta \rho_o \cos \omega t$$

is well known. It involves the linearization of the set of equations A.14, A.15 and solution of the linearized equations by straightforward application of the Laplace transform technique. The result is

B.2 $$\frac{1}{n_o}\left(\frac{n}{\Delta \rho_o}\right) = \frac{1}{i\omega\left[\ell^* + \sum_j \frac{\gamma \beta_j}{\lambda_j + i\omega}\right]}$$

which is called the Reactor Transfer Function. Its amplitude is given by

B.3 $$\frac{1}{n_o}\ \frac{n}{\Delta \rho_o} = \frac{1}{\omega\left[\omega^2\left[\sum_j \frac{\gamma \beta_j}{\lambda_j^2 + \omega^2}\right]^2 + \left[\ell^* + \sum_j \frac{\gamma \beta_j \lambda_j}{\lambda_j^2 + \omega^2}\right]^2\right]^{1/2}}$$

where

$\ell^*$ = neutron generation time,

$n$ = amplitude of the reactor output,

$n_o$ = initial reactor flux level,

$\Delta \rho_o$ = amplitude of the reactivity input.

For low frequencies the condition

B.4 $$\ell^* << \sum_j \frac{\gamma \beta_j}{\lambda_j}$$

is satisfied, and Eq. B.3 becomes

B.5 $$\frac{1}{n_o}\left(\frac{n}{\Delta \rho_o}\right) = \frac{1}{\omega \gamma \left[\omega^2\left[\sum_j \frac{\beta_j}{\lambda_j^2}\right]^2 + \left[\sum_j \frac{\beta_j}{\lambda_j}\right]^2\right]^{1/2}}$$

A plot of Eq. B.5 versus frequency for a set of different values of $\ell^*$ shows that the simplified result is very accurate for low frequencies[7], and this result presents the advantage of being independent of the neutron lifetime.

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