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SUBJECT: Estimate of the Fast Effect in the Lid Tank Source Plate

TO: Distribution

FROM: Lawrence Dresner

Abstract

An estimate has been made of the fast effect in the Lid Tank source plate. The number of fast fissions per thermal fission is 0.019.

DISCLAIMER

Introduction

The Lid Tank source plate is a uranium disc 0.060 in. thick and 28 in. in diameter, composed of 20.8% U^{235} and 79.2% U^{238}. It is used as a fission neutron source in the Lid Tank Shielding Facility by placing it in a partially collimated thermal beam emerging from a hole in the shield of the ORNL graphite reactor. In obtaining the source strength of the plate three methods have been used, viz.: (i) measurements of the fission heat production in the plate, (ii) measurements of the fast-neutron production in the plate, and (iii) measurements of the thermal-neutron absorption in the plate. In the first two of these methods fissions caused by fast neutrons originating in the plate are included in the source strength; in the third method they are not. The purpose of this note is to estimate this fast effect and so estimate the difference expected between calibrations (i) and (ii), on one hand, and (iii) on the other.

Theory[1]

The number, ϵ, of fast fissions occurring per thermal fission in the source plate is given by

$$\epsilon = \nu_t P_c^{(1)} \frac{\sigma_f}{\sigma_{tr}} + \nu_t P_c^{(1)} \cdot \frac{\nu\sigma_f + \sigma_e}{\sigma_{tr}} \cdot P_c \cdot \frac{\sigma_f}{\sigma_{tr}} + \nu_t P_c^{(1)} \frac{\nu\sigma_f + \sigma_e}{\sigma_{tr}} \cdot P_c \frac{\nu\sigma_f + \sigma_e}{\sigma_{tr}} \cdot P_c \frac{\sigma_f}{\sigma_{tr}} + \dots \tag{1a}$$

$$= \frac{\nu_t P_c^{(1)} \frac{\sigma_f}{\sigma_{tr}}}{1 - P_c \frac{\nu\sigma_f + \sigma_e}{\sigma_{tr}}} \tag{1b}$$

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1. A. M. Weinberg and E. P. Wigner, "Physical Theory of Neutron Chain Reactors," University of Chicago Press, 1958, Ch. XX.

Here $P_c^{(1)}$ is the collision probability of fast neutrons whose spatial distribution is proportional to the absorption density of thermal neutrons in the plate. ν_t is the multiplicity of fast neutrons from thermal fission, ν is the corresponding quantity for fission neutrons, and σ_{tr}, σ_f, and σ_e are appropriately averaged transport, fission, and elastic scattering cross sections. σ_e is defined as the difference of σ_{tr} and the summed cross section for all absorptive or inelastic processes. Finally, P_c is the collision probability of fast neutrons which have made at least one collision in the plate.

$\nu_t P_c^{(1)}$ represents the average number of fission neutrons produced by a thermal fission which collide in the source plate; σ_f/σ_{tr} gives the fraction of these which initiate further fission. Originating from the first collision of these neutrons are $(\nu\sigma_f + \sigma_e)/\sigma_{tr}$ secondaries; of these $P_c(\sigma_f/\sigma_{tr})$ cause fission, etc. This reasoning leads to the geometric series given in Eq. (1a). Summing the series gives Eq. (1b). The numerator of Eq. (1b) is the "first generation" fast effect while the denominator represents the summed effect of the second and higher generations.

The use of the transport cross section in Eq. (1) rather than the total cross section is recommended by Weinberg and Wigner[1] in order to take into account the anisotropy of scattering of fast neutrons in uranium. In the example at hand only the first collision will produce any significant contribution to ϵ. At first sight it seems that the angular distribution of fast neutron scattering can only produce effects in the second and higher generations, and hence a first collision calculation should properly be carried out with the total cross section. This, however, is not the case; for, imagine the differential scattering cross section arbitrarily assumed to be the sum of an isotropic part and a straight-ahead part. Let these parts be so normalized that they sum to the correct total scattering cross section, σ_s, and also imply the correct mean cosine, $\bar{\mu}$, of the scattering angle. In this case, even if only the first collision is important, only the isotropic part of the scattering should be employed. The isotropic part of the scattering is easily shown to be of magnitude $\sigma_s(1 - \bar{\mu})$.*

In a sufficiently small finite lump the ratio P_c/σ becomes independent of σ and the difference between use of the transport and the total cross section disappears. In an infinite slab, on the other hand, P_c has a logarithmic singularity, so that P_c/σ always has some dependence on σ. As we shall see subsequently, however, this dependence is not strong and the difference between the use of the transport and the total cross sections causes only a 10% change in ϵ. In what follows we shall use the transport cross section in calculating ϵ as in Eq. (1).

The fact that inelastic scattering slows neutrons below the fission threshold of U^{238} but to energies at which there is always some U^{235} fission possible is specifically ignored. Indeed, all inelastic scattering is presumed to remove neutrons from further consideration. This approximation can only affect the contributions of the second and higher generations, which are of the order of 5% as we shall presently see. Hence, our simple treatment is justified.

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*The author wishes to thank Dr. R. L. Macklin for pointing out this argument.

The effect of the spatial distribution of thermal fissions should be small. To show this let us regard the source plate as an infinite slab. According to Case, Placzek, and deHoffmann[2] the collision probability $P_c^{(1)}$ should be independent of the source strength spatial distribution in the slab if the latter is sufficiently thin. To test this $P_c^{(1)}$ has been calculated in the appendix for a slab of thickness and for an exponentially varying source such as might arise from the absorption of normally incident thermal neutrons. A numerical comparison of $P_c^{(1)}$ for an exponential source and for a uniform source has been calculated using the following data: a = 1.0 thermal mean free path and Σ, the ratio of fast transport to thermal absorption cross section, = 0.03. (For a U density of 18.9, thermal absorption cross sections of 694 b for U^{235} and 2.7b for U^{238} (Ref. 3) and a fast transport cross section of 4.4b,[1] a = 1.067 and Σ = 0.03.) For the exponential source $P_c^{(1)}$ = 0.06649, while for the uniform source P_c = 0.06659, a difference of about 0.15%. Thus the expected independence of source distribution is verified with quite high accuracy.

It is furthermore the case that an increase or decrease in σ_{tr} by a factor of 3/2 from 4.4b[1] causes $(P_c^{(1)}/\sigma_{tr})$ to vary by only ±9%. (N.B.: σ_t = 6.8b = 1.55 x 4.4b.) The use of the transport cross section applicable to U^{238} (4.4b) for the entire source plate is valid for two reasons: (i) the U^{238} contributes 4/5 of the transport cross section since the microscopic cross sections of U^{235} and U^{238} are comparable, and (ii) according

2. K. M. Case, G. Placzek, and F. deHoffmann, "Introduction to the Theory of Neutron Diffusion," U. S. Government Printing Office, June 1953.
3. BNL-325, 2nd Ed., July 1, 1958.

to the optical model of Feshbach, Porter, and Weisskopf[4] the average over resonance of the total, absorption and differential scattering cross sections should vary little from nucleus to nucleus. Comparison of the total cross sections of U^{238} and U^{235} (Ref. 3) shows this rule to be well verified. Hence, we expect uncertainties in σ_{tr} to produce a quite small error. Moreover the relative constancy of $P_c^{(1)}/\sigma_{tr}$ makes possible to average the numerator at least over a fission spectrum by simply averaging σ_f.

According to Weinberg and Wigner σ_f for U^{238} averaged over a fission spectrum is 0.3b; the corresponding quantity for U^{235} estimated from (3) is 1.25b. These results, together with $\nu_t = 2.47$, and the results of the last paragraph give a "first generation" fast effect of 1.86%. Choosing $\sigma_e = 1.5b$[1] and $\nu = 2.75$[5] and P_c for the uniform source (which will be even better for the second and higher generations than for the first) one calculates for the denominator of Eq. (1b) the value 0.9567. ϵ then equals 1.94% with a relative accuracy of the order of 10%.

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4. H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev. 96, 448 (1954).
5. W. D. Allen and R. L. Henkel, Progress in Nuclear Energy, Series I, Vol. 2, Physics and Math., Pergamon Press, N. Y., 1958.

Appendix 1

$$1 - P_c^{(1)} = (1 - e^{-a})^{-1} \int_0^a e^{-x} \frac{1}{2} \left\{ E_2(\Sigma x) + E_2\left(\Sigma(a - x)\right)\right\} dx \qquad (1.1)$$

since e^{-x} is the source density for a unit normally incident current, of which, however, a fraction e^{-a} is transmitted. The unit of length is the thermal absorption mean free path, a is the slab thickness, and Σ the ratio of fast transport to thermal absorption cross section. The manner of evaluating the integrals is to employ the integral definition of the E_n-functions,

$$E_n(x) = \int_1^{\infty} \frac{du}{u^n} e^{-ux}, \qquad (2.1)$$

and interchange the order of integration. Thus, e.g.,

$$I_1 = \int_0^a e^{-x} E_2(\Sigma x) dx = \int_1^{\infty} \frac{du}{u^2(1 + \Sigma u)} \left[1 - e^{-a(1+\Sigma u)}\right] \qquad (3.1a)$$

$$I_2 = \int_0^a e^{-x} E_2\left(\Sigma(a - x)\right) dx = e^{-a} \int_1^{\infty} \frac{du}{u^2(1 - \Sigma u)} \left[e^{a(1-\Sigma u)} - 1\right] \qquad (3.1b)$$

Now the denominators of the right hand sides of Eqs. (3.1a) and (3.1b) are expanded in partial fractions. The integration of Eq. (3.1a) is straightforward and yields

$$I_1 = 1 - \Sigma \ln\left(\frac{\Sigma + 1}{\Sigma}\right) - e^{-a}E_2(\Sigma a) + \Sigma e^{-a}E_1(\Sigma a) - \Sigma E_1\left((\Sigma + 1)a\right) \tag{4.1}$$

Because of the singularity in the denominator of the right hand side of Eq. (3.1b) some care must be taken with the partial fraction expansion. I_2 can be written

$$I_2 = \int_1^\infty du \left\{ e^{-\Sigma a u}\left(\frac{1}{u^2} + \frac{\Sigma}{u}\right) - \frac{e^{-a}}{u^2} \right\} +$$

$$\Sigma^2 e^{-a} \int_1^\infty du \left\{ \frac{e^{(1-\Sigma u)a} - 1}{1 - \Sigma u} - \frac{1}{\Sigma u} \right\} \tag{5.1}$$

The first integral on the right hand side can be calculated straight forwardly; the second can be integrated by setting $y = (1 - \Sigma u)a$ in the first term and $y = \Sigma u$ in the second. The result is

$$I_2 = E_2(\Sigma a) + \Sigma E_1(\Sigma a) - e^{-a} - \Sigma e^{-a}E_1(\Sigma a) + \Sigma e^{-a} \int_{(\Sigma-1)a}^{\Sigma a} \frac{1 - e^{-y}}{y}\, dy \tag{6.1}$$

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