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# ***Guide to Calculating Transport Efficiency of Aerosols in Occupational Air Sampling Systems***

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## INTRODUCTION

In nuclear facilities, radioactive aerosols can be a source of internal dose to workers. These particles are produced through inadvertent release of radioactive material from containment barriers such as piping, hoods, bags and glove boxes or produced through operations and maintenance of items and systems containing radioactive material. Resuspension of contamination is a recurring source of airborne radioactivity once contamination is established in a facility.

Air sampling is a key method of detecting changes in radiological conditions of a work place. It can help to evaluate the effectiveness of engineering and administrative controls, to identify the need for bioassay of workers and to demonstrate compliance with applicable regulations such as radiological posting and access control (Hadlock 2010, USNRC 1993).

Occupational air sampling at its simplest form involves drawing a measured flow of air through a filter directly mounted in the breathing zone. In this case, transport of aerosols is not an issue. However, local sampling is not always practical. For example, sampling lines can provide for remote monitoring of air from potentially hazardous areas (Whicker 1999). Air sampling systems are of various lengths and complexity based on the need to protect the sampler from the environment or for convenient connection to a vacuum header (Thames 2000, Hogue 1994).

With the use of sampling lines, inevitably comes the loss of some particles, especially of larger particles as they fail to be aspirated into the system or deposit somewhere along the way (Dorrian 1995, Maiello 2011, ANSI 1999). Elements of a sampling system typically include the entrance to the line (considered the “probe” even if this element is only the abrupt end of a pipe), tubing and bends. More complicated lines can include splitters,

contractions and expansions. Sampling lines may end with a direct connection to an air sample filter or end in a housing with a filter holder.

The potential for inefficiencies in the lines should not be ignored (Hinds 1999, Maiello 2011, Baron 2011). A common method of estimating loss of particles in sample lines is by the Deposition Code (McFarland 2002), an earlier version of which was recommended by the US Nuclear Regulatory Commission (NRC 1992). More recently, there has been increased awareness of the need to check the validity of software, verify it operates correctly and verify that it is used properly (USDOE 2011, ASME 2008). A key method for achieving these objectives is to perform hand calculations.

This report will present hand calculations for transport efficiency based on aspiration efficiency and particle deposition losses. Because the hand calculations become long and tedious, especially for lognormal distributions of aerosols, an R script (R 2011) will be provided for each element examined. Calculations are provided for the most common elements in a remote air sampling system, including a thin-walled probe in ambient air, straight tubing, bends and a sample housing.

One popular alternative approach would be to put such calculations in a spreadsheet, a thorough version of which is shared by Paul Baron via the Aerocalc spreadsheet (Baron 2012). To provide greater transparency and to avoid common spreadsheet vulnerabilities to errors (Burns 2012), this report uses R.

The particle size is based on the concept of activity median aerodynamic diameter (AMAD). The AMAD is a particle size in an aerosol where fifty percent of the activity in the aerosol is associated with particles of aerodynamic diameter greater than the AMAD.

This concept allows for the simplification of transport efficiency calculations where all particles are treated as spheres with the density of water ( $1 \text{ g}\cdot\text{cm}^{-3}$ ). In reality, particle densities depend on the actual material involved. Particle geometries can be very complicated. Dynamic shape factors are provided by Hinds (Hinds 1999). Some example factors are: 1.00 for a sphere, 1.08 for a cube, 1.68 for a long cylinder (10 times as long as it is wide), 1.05 to 1.11 for bituminous coal, 1.57 for sand and 1.88 for talc.

Revision 1 is made to correct an error in the original version of this report. The particle distributions are based on activity weighting of particles rather than based on the number of particles of each size. Therefore, the mass correction made in the original version is removed from the text and the calculations. Results affected by the change are updated.

## **MATERIALS AND METHODS**

The theoretical basis of this work is provided by references noted for each element and is not itself the subject of this paper. The evaluation of a sample system will consist of a composite of models for each element of the system.

### **Sample System Elements**

For practical reasons, this report will be limited to the following elements that are considered to be the most prevalent at the Savannah River Site:

1. Probes:
  - a. Thin-walled sample probes in calm air oriented up, down or horizontal.
  - b. Anisokinetic probes with adjustable angles to the sampled air velocity. This is for sampling in laminar flow ducts or stacks. The hand calculation does not calculate whether the stack or duct flow is laminar.
2. Bends (elbows): Typically we use 90 degree bends and sometimes 45 degree bends. The curvature ratio (bend radius/tube radius) is a required element.
3. Straight Tubing (at angle to the horizontal)
4. Sample filter housing: We do not have a reliable model, but estimate a best case efficiency.

### Overview

Overall transport system efficiency is calculated by taking the product of all the individual element efficiency. Individual element efficiency is a ratio of the concentration at the outlet of an element to that of the inlet of that element (ANSI 1999).

$$\eta_j = \frac{c_{e,j}}{c_{i,j}}$$

### Equation 1: General Transport Efficiency

where  $\eta$  is transport efficiency  
j is the element identifier  
e is the exit plane  
i is the inlet plane

For monodispersed particles, this is simply the product of the transport efficiencies of each element. The order of analysis makes no difference:

$$\eta_{overall} = \prod_i \eta_i$$

### Equation 2: Overall Transport Efficiency

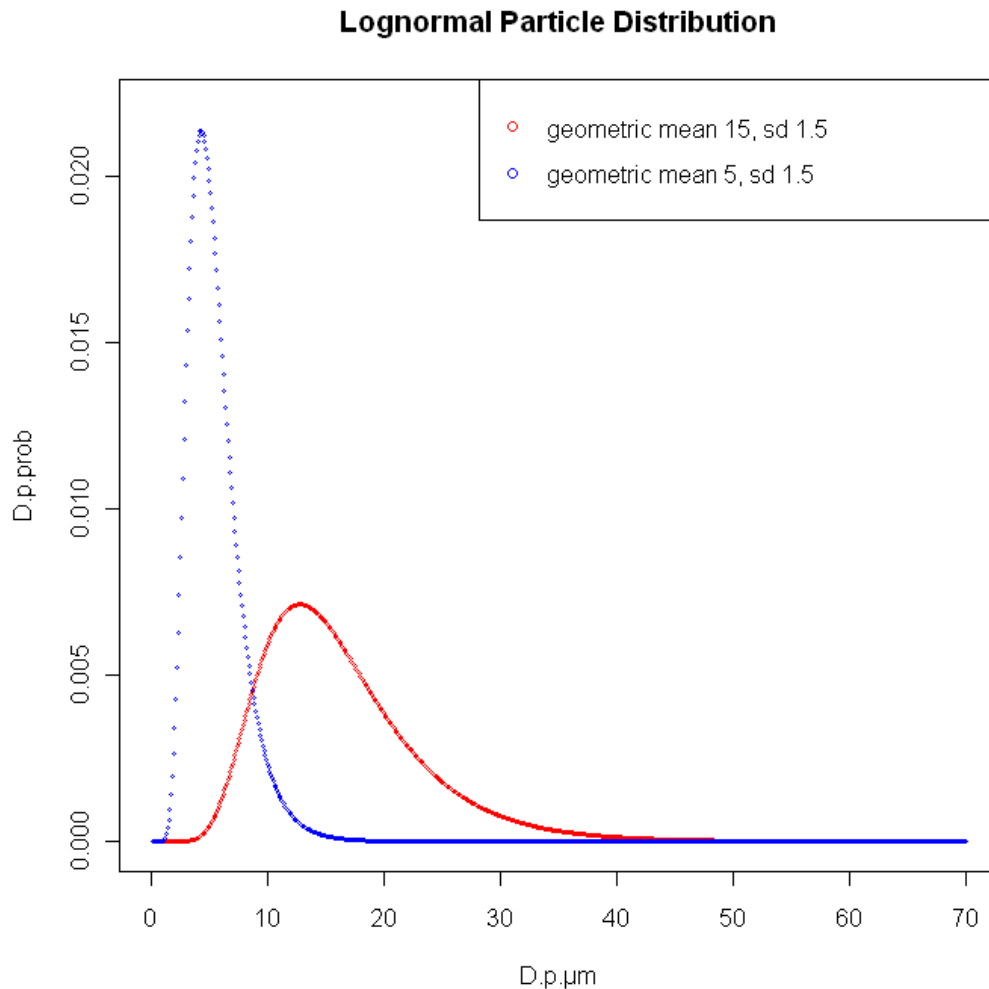
where  $\eta$  is transport efficiency

Equation 2 is meant to include the aspiration efficiency (shown below as A) as described in Brockman 2011. Note that the aspiration efficiency can be greater than 1. In this case, the probe can collect more of a particular particle size than is represented in the ambient air, resulting in a net gain for that particle size.



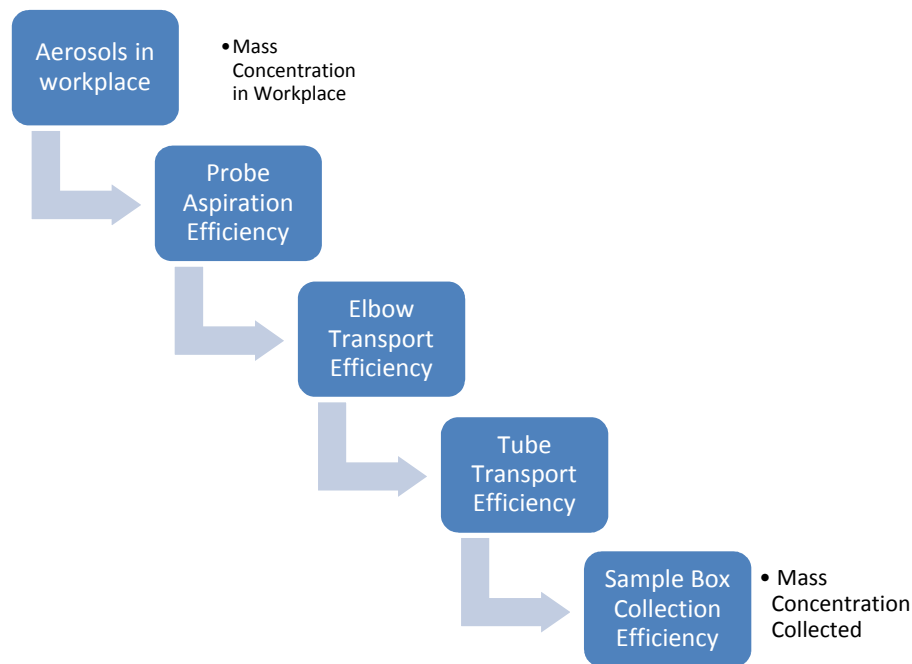
Dorrian (Dorrian 1995) reported on the results of aerosol activity size distributions in radiological facilities and found support for a 5  $\mu\text{m}$  lognormal distribution with a standard deviation of 2.5. Another often-used benchmark distribution is monodispersed (single size) 10  $\mu\text{m}$  (ANSI 1999). SRS has identified three size distributions for evaluation, 1  $\mu\text{m}$  and 10  $\mu\text{m}$  monodispersed and lognormal with a geometric mean of 5  $\mu\text{m}$  and standard deviation of 2.5 (Hadlock 2010).

The starting point for this latter distribution (geometric mean 5, standard deviation 2.5) along with a second distribution with geometric mean of 15 is shown in Figure 1. A distribution was generated starting at 0.1  $\mu\text{m}$ , extending to 70  $\mu\text{m}$  with 0.1  $\mu\text{m}$  increments. This produces a total of 700 particle sizes. A detailed R script is provided in the Attachment 1. The probability density parameter, D.p.prob is shown on the vertical axis.



**Figure 1: Lognormal distribution of particles, aerodynamic equivalent diameters**

When the aerosol being evaluated is distributed over a variety of particle sizes, then the approach will be to start with an activity-weighted distribution of the particles and to track the changes to a data set in the form of a histogram. This set of particles is evaluated at each sequential element. The overall efficiency in this case will be evaluated based on the total activity transported successfully through the system.



**Figure 2: Distribution of Aerosols by Size Tracked through System (example)**

### Limitations on Modeling

Modeling the transport efficiency is a valuable practice to help appreciate that system response can vary and can help to identify weaknesses in system design. It should be cautioned, however, that aerosol size distributions are rarely well known and may be more complex than the simple distributions presented here.

The uncertainty in the methods presented here is bound to be at best within 20% and probably much worse. Testing of the models has been limited by the difficulty and cost of developing the physical situations required. The models are specifically limited to only apply to clean, smooth transport systems. ANSI 13.1 (ANSI 1999, McFarland 2000) was revised to stipulate annual cleaning because of this limitation. Thus, the application of these calculations to unclean transport systems may be of little value.

Other effects that are not considered in these calculations are summarized by von der Weiden (von der Weiden 2009):

- Electrostatic deposition – Use only electrically grounded, conductive material. This requirement is not met by some sampling systems (Thames 2000), so these systems will have worse transport efficiency than calculated. Even the reference model (Metz 1993) is made of PVC, which is not conductive, so the experimental data in that

transport system may show less penetration than would be found in a comparable system made of conductive material.

- Diffusiophoresis – If significant temperature and concentration gradients exist in sample lines, then the system will have worse transport efficiency than calculated.
- Interception – This effect should be negligible for systems at SRS because it only becomes important when the particle size is not much smaller than the tubing dimension.
- Coagulation – Smaller particles may come together and form larger particles but it takes extraordinarily large concentrations or long residency time in the sample lines for this to be significant.
- Re-entrainment of deposited particles – This can be a significant problem in sample lines that do not have regular cleaning. As noted above, this is not the only problem with dirty transport systems, since deposition along the lines of dirty systems will also be higher.

### **A Note on Units**

Units in these calculations have to be uniform to avoid confusion. The R scripts are written on the basis of the cgs (centimeter gram second) unit system. For convenience, some parameters are entered in more convenient units, such as micrometers ( $\mu\text{m}$ ). However, these parameters are converted into the cgs units.

### **System and Environmental Parameters**

Transport efficiency depends on environmental factors and sample system parameters. For example, air density affects the degree to which aerosol particles interact with air molecules. Air density is affected by temperature and pressure. So, temperature and pressure are included in the calculation and intermediate terms in calculating transport efficiency, such as the settling velocity of a particle are calculated. (Baron 2011, Hinds 1999, Sajo 2011)

System parameters affecting transport efficiency that are common for a whole sampling system often include tube diameter and flow rate. Associated with these terms is the degree of turbulence in the system. The Reynolds number is used as a measure of this turbulence and plays a role in selection of models.

The Stokes number is used repeatedly in the transport efficiency calculations below. The Stokes number is the ratio of the stopping distance to a characteristic dimension. The selection of the characteristic dimension depends on the application. Adjustments are made to the Stokes number to account for aerodynamic drag on the particle by the Cunningham slip correction factor.

Constants and default system parameters are provided in Table 1. Other system parameters that are used by many of the individual element calculations provided in Table 2. Table 3 provides many of the intermediate calculated values that are used in the efficiency calculations.

**Table 1: System Parameters – Constants and Defaults**

Parameter	Value	Units	Comment	term in R script <sup>i</sup>
air density at NTP	1.20484e-3	g·cm <sup>-3</sup>	NTP = Normal Temperature and Pressure	density.air.ntp
air pressure at NTP	1013250.1	barye	= 1 atmosphere	P.ntp
air viscosity at NTP	1.8203e-04	Poise		viscosity.air.ref
Boltzmann constant	1.38e-16	g·cm <sup>2</sup> ·molecule <sup>-1</sup> ·K <sup>-1</sup> ·s <sup>-2</sup>		Boltzmann
air molecular mean free path (NTP)	6.65e-06	cm		mfp.air
acceleration due to gravity	980.7	cm·s <sup>-2</sup>		g=980.7
Reference Temperature (NTP)	293.15	K		T.ntp
Default System Temperature	298.15	K		T.C=25 T=T.C+273.15
Tube Diameter (example value)	2.54	cm	1” diameter	D.tube
Particle Density	1	g·cm <sup>-3</sup>	Standard particle density (Ref. e.g. Baron 2001 equation 3-1)	density.particle

**Table 2: System Parameters – Variable**

Parameter	Example Value	Units	Comment	term in R
Flow Rate	20	l·min <sup>-1</sup>	l=liters, converted to cm <sup>3</sup> ·s <sup>-1</sup>	Q.lpm
Particle Size	10	µm	converted to cm In the case of lognormal distribution, a set of values is produced based on entered mean and standard deviation.	D.µm=10 D.p=D.µm*1e-4

<sup>i</sup>The R scripts use the assignment character string, “<-“ rather than “=”.

**Table 3: Common Calculated Terms**

Parameter	Units	Comment	term in R	References
air speed in sample stream	cm·s <sup>-1</sup>	Based on flow rate and tube diameter	$U.o=4*Q/(\pi*(D.tube)^2)$	NA
Sutherland constant	K	Used in Air Viscosity Adjustment	Sutherland=110.4	Kulkarni 2011 Table 2-1
air viscosity	Poise	Adjusted to T and P	$viscosity.air=viscosity.air.ref*(T.ntp+Sutherl and)/(T+Sutherland)*(T/T.ntp)^{1.5}$	Kulkarni 2011 equation 2-8
air density	g·cm <sup>-3</sup>	Adjusted to T and P	$density.air.correction=(P/P.ntp)*(T.ntp/T)$ $density.air=density.air.ntp*density.air.correct ion$	Clapeyron 1834
mean free path	cm	Adjusted to T and P Units of T are K, Units of P are barye	$mfp=mfp.air*(1010000/P)*(T/293)* (1+110/293)/(1+110/T)$	Kulkarni 2011 equation 2-10
Knudsen Number	none	Used to develop the Cunningham Slip Coefficient	$K.n=2*mfp/D.p$	Kulkarni 2011 equation 2-11
Cunningham Slip Correction Factor	none	Needed for the slip effect in calculating diffusion, settling velocity and the Stokes number.	$alpha.C=1.142$ $beta.C=0.558$ $gamma.C=0.999$ $C.c=1+K.n*(alpha.C+beta.C* exp(-gamma.C/K.n))$	Kulkarni 2011 equation 2-15

Parameter	Units	Comment	term in R	References
Stokes Number	none	Ratio of the stopping distance to a characteristic dimension. The dimension depends on the application. Stk is used for most applications and Stk2 for probes.	$\text{Stk} = \frac{\text{density.particle} \cdot D_p^2 \cdot U_o \cdot C_c}{9 \cdot \text{viscosity.air} \cdot D_{\text{tube}}}$ $\text{Stk2} = \text{Stk} / 2$	Glissmeyer 2010 equation 15.6, Su 2004 equation 6
Reynolds Number (gas)	none	The ratio of the inertial force of the gas to the friction force of the gas moving over the surface.	$\text{Re} = \frac{\text{density.air} \cdot U_o \cdot D_{\text{tube}}}{\text{viscosity.air}}$	Kulkarni 2011 equation 2-4
Reynolds Number (particle)	none	The ratio of the inertial force of the gas to the friction force of the gas moving around a particle.	$\text{Re.p} = 6.5 \cdot U_o \cdot D_p / \text{viscosity.air}$	Kulkarni 2011 equation 2-5
particle settling velocity	cm·s <sup>-1</sup>	Used in probe aspiration efficiency	$v_{\text{settling}} = U_o \cdot R,$ where $R = \frac{D_p^2 \cdot g}{(18 \cdot \text{viscosity.air} \cdot U_o) \cdot q}$	Su 2004 (7)
particle relaxation time	s	Used in gravitational settling	$t_{\text{relax}} = \frac{\text{density.particle} \cdot D_p^2 \cdot C_c}{(18 \cdot \text{viscosity.air})}$	Kulkarni 2011 equation 2-37
curvature ratio	none	Used in bend transmission	$\text{ratio.curvature} = \delta = R_b / R_t$ where $R_b$ is the bend radius and $R_t$ is the pipe radius <sup>ii</sup>	Zhang 2012

<sup>ii</sup> Others, e.g. ANSI 1999, use the ratio of the bend radius to the pipe diameter.

### **Probe Aspiration Efficiency**

The transport efficiency model starts with the aspiration efficiency into the system. The first element is called a probe even if, as is usually the case with occupational air sampling, the probe is a simple cut off pipe.

Until this method is developed, however, the probe models that have been used at SRS have been those included in the code Deposition (McFarland 2002) (the 2001a version and earlier versions). The probe models in this code were developed for stack modeling, where a probe is aligned with the exhaust stream and there is considerable velocity of the particles in the stack stream. The only appropriate application of this code for occupational air sampling is for the case where the occupational sample has been designed into an exhaust ventilation duct. The code can then be used if a match to the codes' models can be made. The code's models include unshrouded isokinetic, unshrouded anisokinetic, shrouded, and seven models of commercial probes manufactured by ThermoAndersen.

### ***Thin-Walled Probe in Calm Air***

The most common air sampling system at SRS is one that begins with an open pipe in a relatively calm air space. For the purposes of this guide, this open pipe is considered a thin-walled probe. The aspiration efficiency of a thin-walled probe in calm air has been modeled by Su and Vincent (Su 2004).

The model for the thin-walled probe is based on a basic physical model for the entry of particles into a point sink in calm air. This model is modified based on empirical studies to account for the effects of particle inertia, gravitational settling and sampler orientation.

Su's general equation for aspiration efficiency for calm air:

$$A_{calm-air} = 1 - 0.8(4 \cdot Stk2 \cdot R^{3/2}) + 0.08 \left( 4 \cdot Stk2 \cdot R^{3/2} \right)^2 \\ - \alpha(\theta)[0.5(R^{1/2}) - R (B^2 - 1)] - \beta(\theta)[\{(0.12 \cdot R^{-0.4}(e^{-p} - e^{-q})\} \\ - R^{3/2}(B^{1/2} - 1)]$$

where R = ratio of the particle settling velocity to the mean air velocity across the inlet

$$p = 2.2 \cdot R^{1.3} \cdot Stk2$$

$$q = 75 \cdot R^{1.7} \cdot Stk2$$

$\alpha$  and  $\beta$  and B are chosen according to the following:

Thin-walled probe facing downwards:  $\alpha = 1$ ;  $\beta = 0$ ; B = 1

Thin-walled probe facing upwards:  $\alpha = 0$ ;  $\beta = 1$ ; B = 1

Horizontal thin-walled probe:  $\alpha = 0.8$ ;  $\beta = 0.2$ ; B = 1

All other terms are in Table 3. Stk2 is found under Stokes Number.

**Equation 3: Su's general equation for aspiration efficiency**

### ***Anisokinetic probe***

The anisokinetic probe is modeled based on a study of laminar flow (Vincent 1986). The model is used in the Deposition code (McFarland 2000) but without the terms for various angles<sup>iii</sup>.

$$A = 1 + \left[ 1 - \frac{1}{1 + k \cdot Stk \cdot (\cos(\theta) + 4 \cdot (R \cdot \sin(\theta))^{0.5})} \right] \cdot (R \cdot \cos(\theta) - 1)$$

where

R is the ratio of airstream velocity to velocity at the entrance plane of the probe.

(Note that this R is different from the ratio used for Thin-Walled Probes in Calm Air)

$\theta$  is the angle of the probe to the air velocity in the duct or stack

k is identified by Vincent 1986 as an empirically derived variable of approximately 2.1, but Vincent's Stokes number is the Stk2 identified in Table 2; the Deposition Code uses Stk and k=1.05.

Stk is the Stokes number from Table 3.

#### **Equation 4: Vincent's anisokinetic probe aspiration efficiency**

### **Bend Transmission Efficiency**

The model for bend transmission efficiency is from Zhang 2012. This model was selected because it appears to be more accurate based on the comparisons in the article and because it benefits from the most recent research. The calculation is:

$$\eta = \exp(-0.528 \cdot \theta \cdot Stk^{2\frac{1}{\delta}} \cdot \delta^{0.5})$$

where  $\delta$  is the curvature ratio in Table 3.

Stk is the Stokes number from Table 3

$\theta$  is the angle of the bend in radians

#### **Equation 5: Bend Transport Efficiency**

In the R script, an alternative bend efficiency is provided for comparison. Figure 3 shows that the selected model is close to the model in the Deposition code and significantly higher than the simple Pui 1987 model that is referenced in ANSI 1999 and is a reference model in Metz 1993.

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<sup>iii</sup> The Deposition code applies this formula only for probes at angles of 0° with respect to the free air stream velocity, such that the cosine terms equal 1 and the sine terms equal 0.



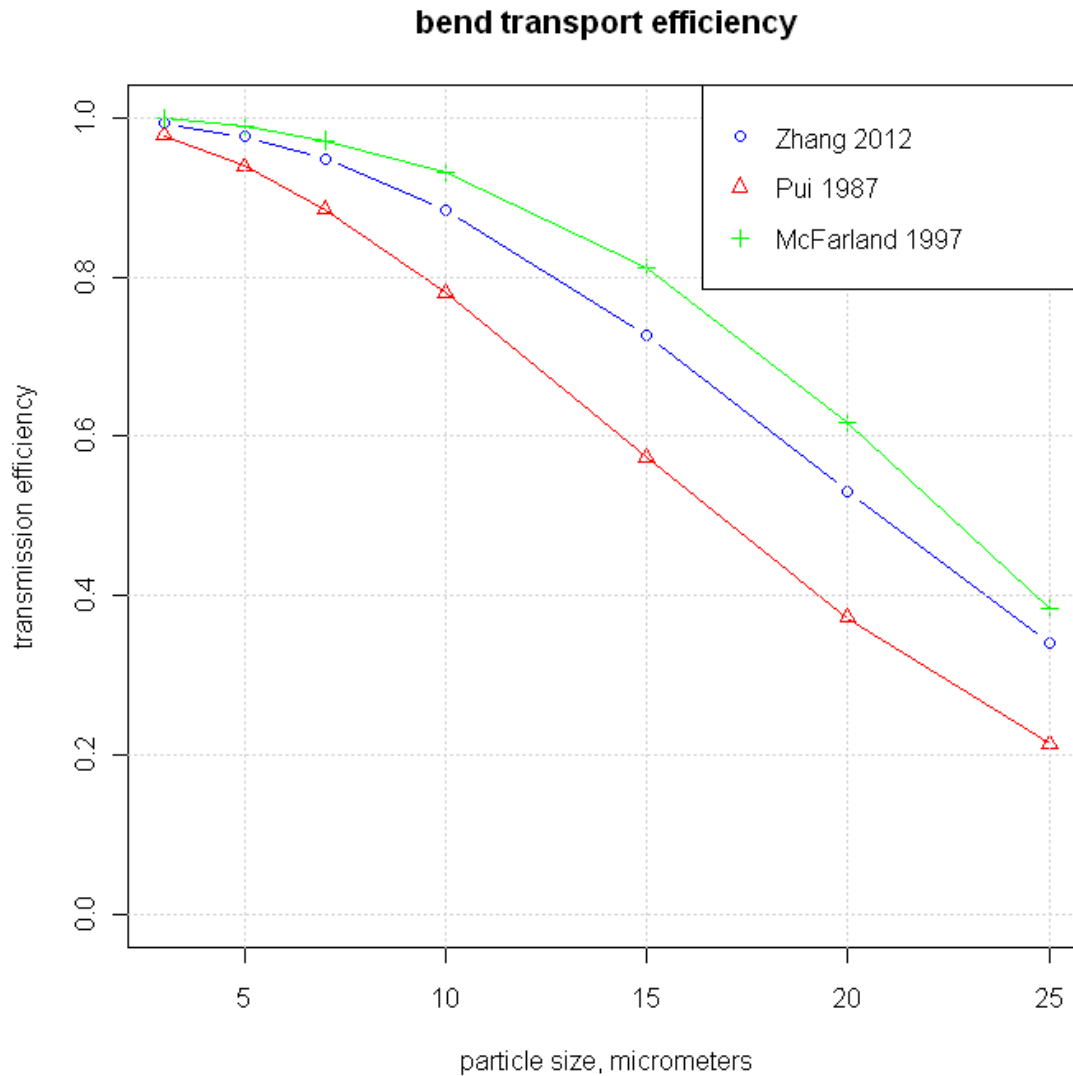


Figure 3: Comparison of Bend Transport Efficiency Models

### Tubing Transport Efficiency

A model for transport efficiency in tubing addresses two mechanisms for particle deposition: turbulent inertial deposition and gravitational losses. The gravitational losses require both a laminar and turbulent model. A third mechanism, diffusion, is included in a commented-out section of the R script but is only applicable to particle sizes less than 100 nm (0.1  $\mu\text{m}$ ).

The Turbulent Inertial Deposition Transport Efficiency is from Brockman 2011 Equation 6-61.

$$\eta_{turb.inert} = \exp\left(-\frac{\pi \cdot D \cdot p \cdot L \cdot V_t}{Q}\right)$$

where

D.p is the particle size,

L is tubing length

Q is the flow rate

V<sub>t</sub> is the experimentally determined turbulent deposition velocity per Equation 7.

**Equation 6: Turbulent Inertial Deposition Transport Efficiency**

$$V_t = \frac{\left(6 \times 10^{-4} \cdot \left(0.0395 \cdot Stk \cdot Re^{3/4}\right)^2 + 2 \times 10^{-8} \cdot Re\right) U \cdot o}{5.03 \cdot Re^{1/8}}$$

where

Re is the Reynold's Number in Table 3.

Stk is the Stokes number from Table 3

U is the particle speed in the sample stream.

**Equation 7: Inertial Deposition Velocity**

The laminar gravitational settling transport efficiency is from (von der Weiden 2009, Heyder 1977, and Brockman 2011 Equations 6-61 through 6-64):

$$\eta_{\text{settling.laminar}} = 1 - \frac{2}{\pi} \cdot (2 \cdot k \cdot \sqrt{1 - k^{2/3}} - k^{1/3} \cdot \sqrt{1 - k^{2/3}} + \arcsin(k^{1/3}))$$

where k is provided in Equation 9

#### Equation 8: Laminar Gravitational Settling Transport Efficiency

$$k = \frac{3}{4} \cdot Z \cdot \cos \theta$$

where

$\theta$  is the angle with respect to the horizontal

Z is the gravitational deposition parameter defined below

#### Equation 9: K factors for Laminar Flow Gravitational Settling

$$Z = \frac{L \cdot v.\text{settling}}{D.\text{tube} \cdot U}$$

where

L is the tubing length

v.settling is the particle (terminal) settling velocity in Table 3

D.tube is the tube diameter

U is the air velocity in the tube

#### Equation 10: Gravitational Deposition Parameter

The Turbulent Inertial Deposition Transport Efficiency is from von der Weiden (von der Weiden 2009)

$$\eta_{\text{settling.turb}} = \exp\left(-\frac{D.\text{tube} \cdot L \cdot v.\text{settling} \cdot \cos(\theta)}{Q}\right)$$

where

L is the tubing length

v.settling is the particle (terminal) settling velocity in Table 3

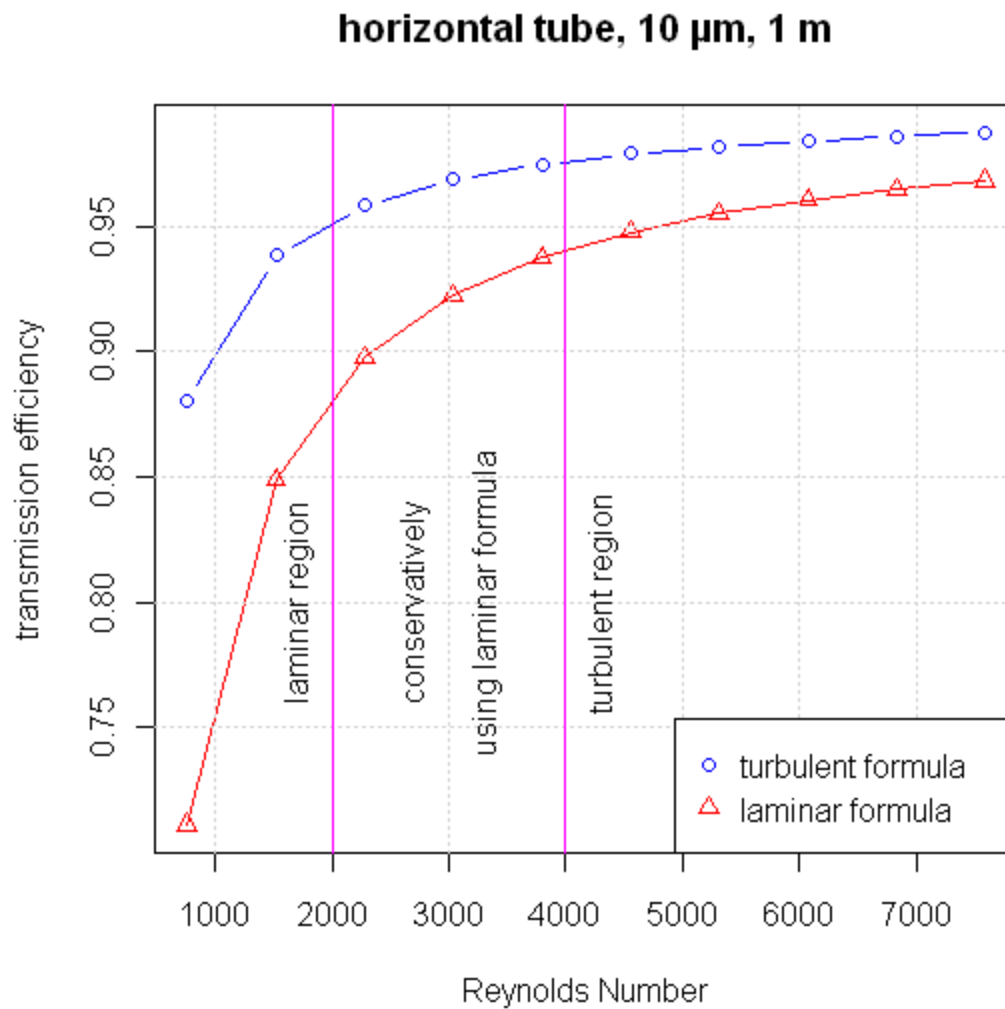
D.tube is the tube diameter

$\theta$  is the angle with respect to the horizontal

Q is the flow rate

#### Equation 11: Turbulent Gravitational Settling Transport Efficiency

The combined tubing transport efficiency is the product of additive inverses of the turbulent inertial and gravitational losses (Efficiency = [1-loss fraction, turbulence]x[1-loss fraction, gravitational]). The laminar gravitational settling equation is applicable below Reynolds' number of 2000 whereas the turbulent gravitational settling equation is applicable above a Reynolds' number of 4000. In between 2000 and 4000, the laminar formula is conservatively applied. The effect of applying this to a horizontal tube 1 m long, 2.54 cm diameter is demonstrated in Figure 4. The flow rates producing the Reynolds Numbers in this figure range from 14 l·m<sup>-1</sup> to 141 l·m<sup>-1</sup>.



**Figure 4: Tube Efficiency (Gravitational Settling) - Laminar vs. Turbulent Regions**

### Sample Filter Housing

In this section the sample filter housing is examined for additional potential losses. In the best scenario, a sample filter is aligned with the previous element, most likely a tube, and there is no place for a particle to deposit, except on the filter. While no filter is 100% efficient, filter efficiency is a separate and relatively minor issue compared to transport efficiency for particle sizes in the range of interest (ANSI 1999).

As an example, a cross-section of an air sampler in use at SRS is shown in Figure 5 with some sample particle flowpaths. Deposition in such a sampler can be considered negligible.

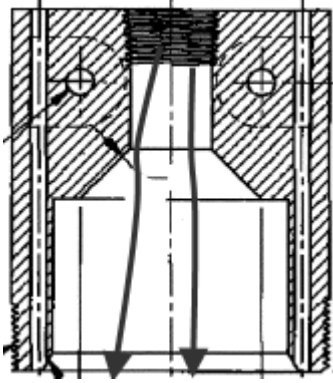
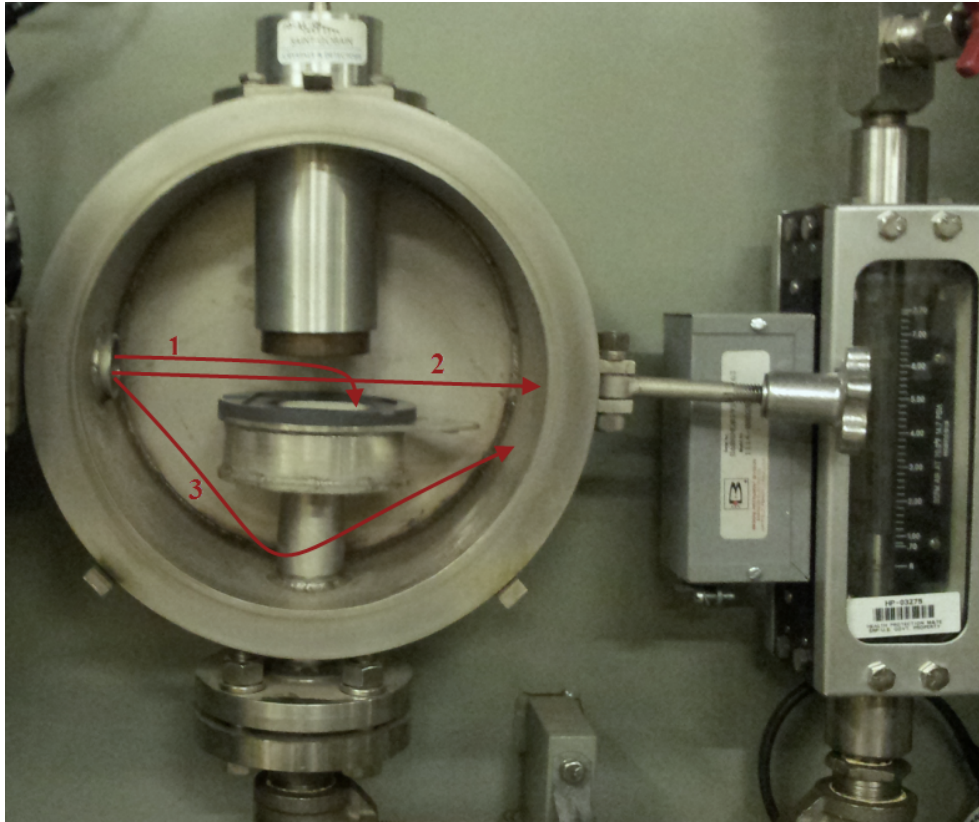


Figure 5: SRS Gaudin Sampler

On the other hand, some samplers are not so ideal. In Figure 6, the particles must make a 90° turn to land on the filter. Turbulence is possible and can be predicted with use of the Reynolds number in effect at the entrance to the sampler. Three example particle lines are shown with 1) making the turn 2) following the inertial line and 3) following a possible turbulence line.



**Figure 6: Example Air Sampler/Continuous Air Monitor shown with door open**

Neither an accurate model nor experimental data for deposition in this sampler are available, but a reasonable estimate of a best case efficiency can be made. Three different models from previous elements in this section were considered:

1. The anisoaxial anisokinetic probe model, where the entry velocity into the box is substituted for the sample stream velocity in a stack.
2. The calm air probe model.
3. The bend model.

All three models are expected to underestimate the losses, but the bend model predicted a higher loss rate. Therefore, the bend model is recommended as a “best case” estimate.

#### Activity-Weighted Averaging for Lognormal Distributions

In monodispersed populations, only a single particle size is analyzed and only one efficiency is calculated. With lognormal populations, a range of efficiencies are calculated. To properly weight them, the following equation is used:

$$\eta_{average} = \frac{\sum(\eta_i * D.p. prob_i)}{\sum(D.p. prob_i)}$$

**Equation 12: Weighting Efficiencies in a Lognormal Distribution**

### **Model Validation Methods**

The models will be checked in three ways:

1. Aspiration efficiency and transmission efficiency for each element are computed at various parametric settings. These results are compared for internal consistency and compared to the reference models. Common parameters such as Reynolds number are checked as these parameters are varied.
2. An example air sampling setup is modeled. This shows that the method produces results. This is not exactly validation except for the vague notion of plausibility in the results.
3. Models in published reports are run and the results compared with the published results.



## RESULTS AND DISCUSSION

### Model Validation Results

#### *Thin Walled-Probe in Calm Air*

This model will be used for checking common parameter changes (pressure and temperature).

This model was run for a series of particle sizes at normal temperature and pressure and at 2 cfm (56.6 lpm). This is the baseline model.

Temp, C    Pressure, Torr    Reynolds No.  
25                    760                    3037

orientation Flow Rate, lpm  
h                    56.6

#	Particle Size, $\mu\text{m}$	Stokes Number	Aspiration Eff.
1	1	0.000255	0.9984
2	3	0.00209	0.9954
3	5	0.005691	0.9925
4	7	0.011058	0.9898
5	10	0.022421	0.9859
6	15	0.050192	0.9798
7	20	0.089002	0.9742
8	25	0.138851	0.9688

Comments: The Stokes number increases as expected with particle size. The aspiration efficiency decreases as expected with particle size.

The baseline model was repeated for 5 cfm (141.5 lpm):

Temp, C	Pressure, Torr	Reynolds No.
25	760	7594

> report3

Orientation	tube diameter, cm	Flow Rate, lpm
h	2.54	141.5

> report4

#	Particle Size, $\mu\text{m}$	Stokes Number	Aspiration Eff.
1	1	0.000638	0.999
2	3	0.005224	0.997
3	5	0.014227	0.995
4	7	0.027646	0.9929
5	10	0.056053	0.9899
6	15	0.125479	0.9848
7	20	0.222504	0.9796
8	25	0.347128	0.9742

Comments: The aspiration efficiency increases somewhat with the larger flow rate especially on the larger particles. This is reasonable since the horizontal probe has to overcome the force of gravity especially on the larger particles.

The baseline model was repeated for 30 C:

Temp, C	Pressure, Torr	Reynolds No.
30	760	2949

> report3

Orientation	tube diameter, cm	Flow Rate, lpm
h	2.54	56.6

> report4

#	Particle Size, $\mu\text{m}$	Stokes Number	Aspiration Eff.
1	1	0.000253	0.9984
2	3	0.002065	0.9954
3	5	0.005622	0.9926
4	7	0.010923	0.9898
5	10	0.022144	0.9859
6	15	0.049565	0.9799
7	20	0.087886	0.9744
8	25	0.137106	0.9689

Comments: The slightly higher temperature makes for a slightly lower Reynolds number. There are very slight differences in the aspiration efficiency.

The baseline model was repeated for 740 Torr:

Temp, C	Pressure, Torr	Reynolds No.
25	740	2958

> report3

Orientation	tube diameter, cm	Flow Rate, lpm
h	2.54	56.6

> report4

#	Particle Size, $\mu\text{m}$	Stokes Number	Aspiration Eff.
1	1	0.000256	0.9984
2	3	0.002093	0.9954
3	5	0.005695	0.9925
4	7	0.011065	0.9898
5	10	0.022431	0.9859
6	15	0.050205	0.9798
7	20	0.08902	0.9742
8	25	0.138874	0.9688

Comments: The slightly lower pressure makes for a slightly lower Reynolds number. There are very slight differences in the aspiration efficiency.

The baseline model was repeated for a smaller tube diameter 1.95 cm:

Temp, C	Pressure, Torr	Reynolds No.
25	760	4050

> report3

Orientation	tube diameter, cm	Flow Rate, lpm
h	1.905	56.6

> report4

#	Particle Size, $\mu\text{m}$	Stokes Number	Aspiration Eff.
1	1	0.000604	0.9989
2	3	0.004953	0.9969
3	5	0.013489	0.9953
4	7	0.026212	0.9939
5	10	0.053147	0.9925
6	15	0.118973	0.9913
7	20	0.210967	0.9914
8	25	0.329129	0.992

Comments: The smaller tube diameter caused greater turbulence shown with the higher Reynolds number. The aspiration efficiency increased because of the higher flow rate at the probe opening.

The baseline model was repeated for a smaller tube diameter 3.81 cm:

```
> report1
Temp, C   Pressure, Torr   Reynolds No.
    25         760         2025

> report3
Orientation  tube diameter, cm  Flow Rate, lpm
    h           3.81           56.6

> report4
# Particle Size, µm  Stokes Number  Aspiration Eff.
1         1          7.56E-05    0.9976
2         3          6.19E-04    0.9929
3         5          1.69E-03    0.9882
4         7          3.28E-03    0.9835
5        10          6.64E-03    0.9766
6        15          1.49E-02    0.9651
7        20          2.64E-02    0.9537
8        25          4.11E-02    0.942
```

Comments: The larger diameter caused a decrease in turbulence and aspiration efficiency.

Constant R Charts to compare to Su (2004)

The model was tested for constant R values as defined by Su (2004) where R is the particle settling velocity relative to the air velocity at the sampler inlet.

The script is the same as used for the Thin-Walled Probe in Calm Air, except that a variety of values are generated at constant R values. The particle diameter was generated in a sequence and the velocity was tied to the particle diameter for constant R:

```
n<-seq(from=-1,by=.05,to=2.5)
x<-10^n #generated values up to about 300, but the equation only worked up to between
100 to 150.
D.p<-1e-4*x[1:61] #for R=0.1
D.p<-1e-4*x[1:64] #for R=0.01
R<-0.01
U.o<-D.p^2*g/(18*viscosity.air*R)
```

The resulting charts, below, match Su (2004).

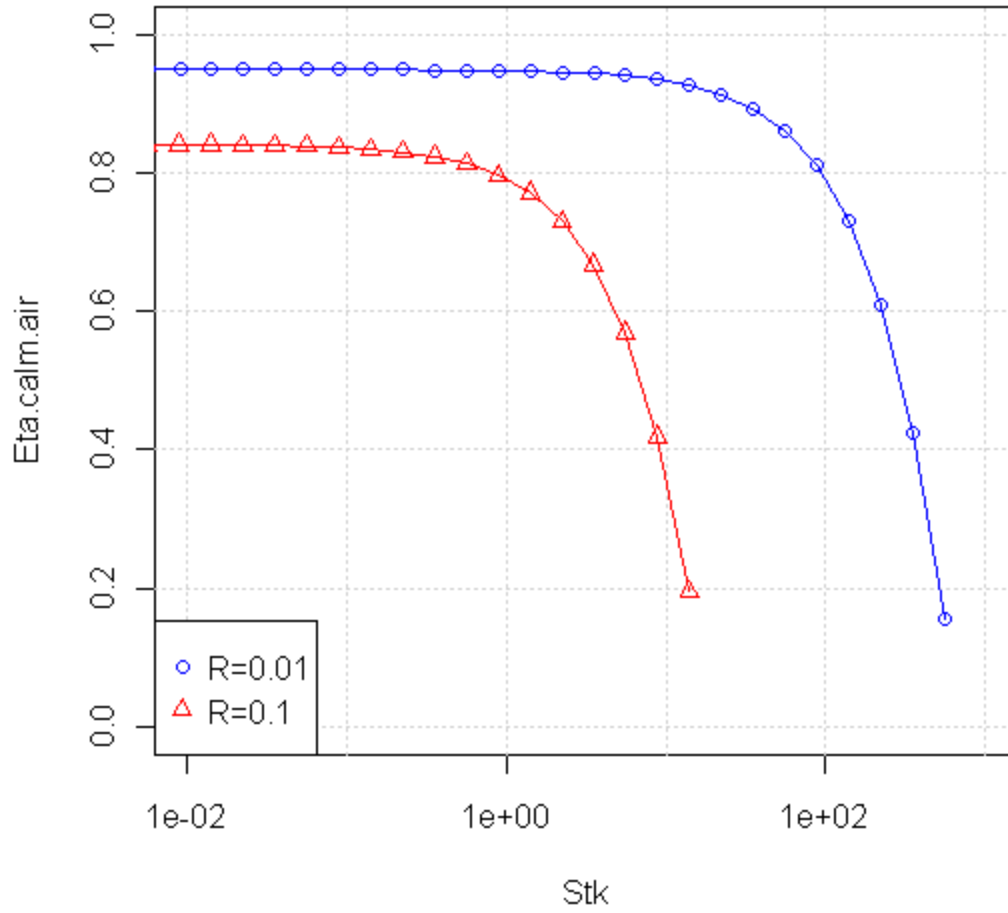


Figure 7: Aspiration efficiency for a downwards-facing thin-walled tube, for two values of R

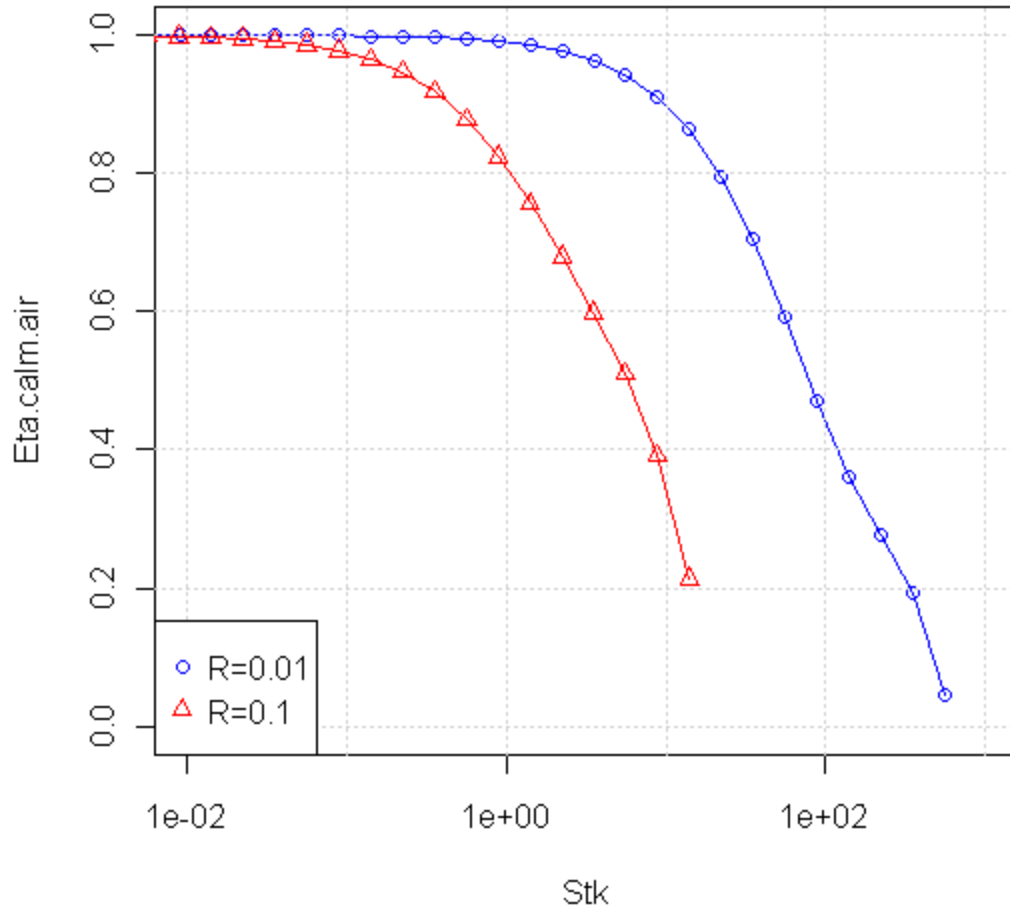


Figure 8: Aspiration efficiency for a upwards-facing thin-walled tube, for two values of R

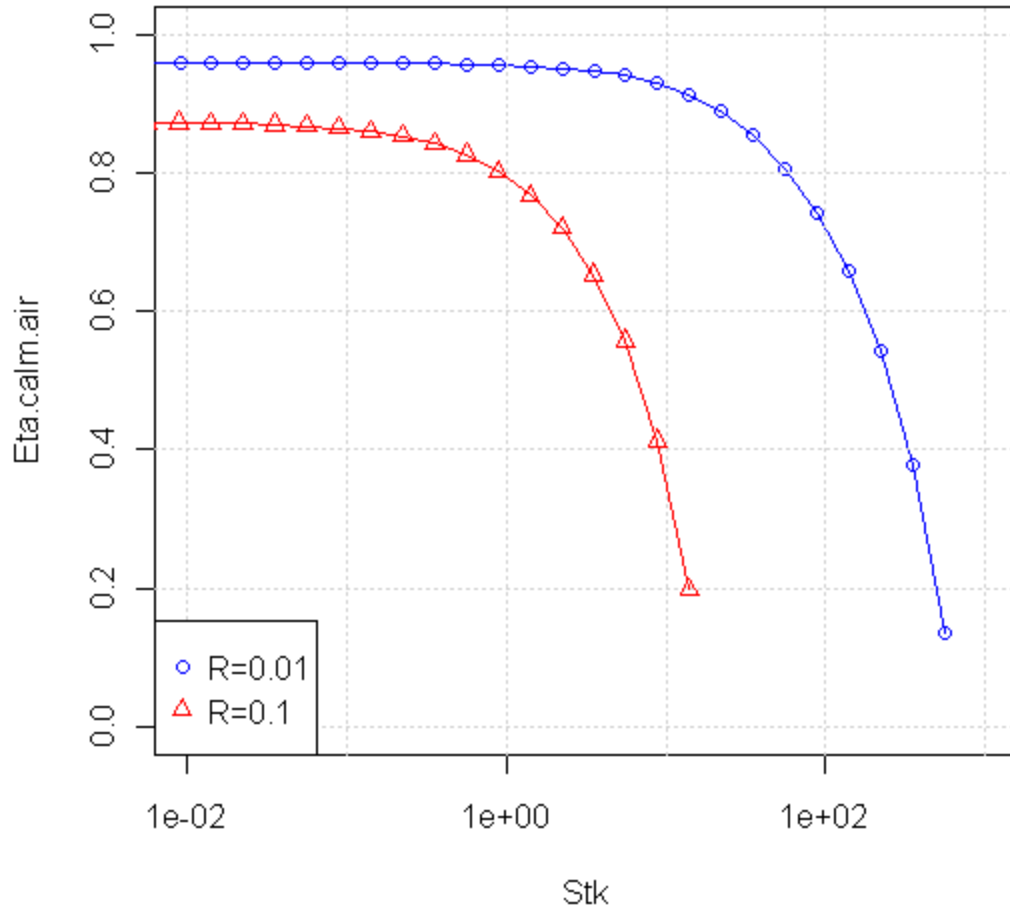


Figure 9: Aspiration efficiency for a horizontal thin-walled tube, for two values of R

---

***Anisokinetic Probe***

The anisokinetic probe model was run for a set of particle sizes. The baseline model has an angle of 0 degrees to the wind speed and will sample at a flow rate near the limit of its use, (where the Reynolds number is 2300) of 42.45 lpm (1.5 cfm).

> report1

Temp, C	Pressure, Torr	Reynolds No.
25	760	2170

> report3

angle to horizontal	tube diameter, cm	Flow Rate, lpm
0	2.667	42.45

> report5

#	wind speed, cm/s	sample speed, cm/s	ratio
1	300	126.65	0.42215

> report6

#	Particle Size, $\mu\text{m}$	angle	TAMU asp eff
1	1	0	0.9998
2	3	0	0.9984
3	5	0	0.9956
4	7	0	0.9914
5	10	0	0.9829
6	15	0	0.9631
7	20	0	0.9376
8	25	0	0.9082

The baseline model is repeated for an angle of 90 degrees to the flow. This only works for the TAMU reading.

> report1

Temp, C	Pressure, Torr	Reynolds No.
25	760	2170

> report3

angle to horizontal	tube diameter, cm	Flow Rate, lpm
90	2.667	42.45

> report5

#	wind speed, cm/s	sample speed, cm/s	ratio
1	300	126.65	0.42215



> report6

#	Particle Size, $\mu\text{m}$	angle	TAMU asp eff
1	1	90	0.9992
2	3	90	0.9932
3	5	90	0.9818
4	7	90	0.9651
5	10	90	0.9313
6	15	90	0.8565
7	20	90	0.7673
8	25	90	0.6728

Comments: As expected, the aspiration efficiency drops off significantly for the probe oriented perpendicular to the flow in the sample stream.

### **Bends**

In the Bend script, a primary transmission efficiency based on Zhang (Zhang 2012) is provided. Two other models are provided for comparison as shown in Figure 3. A value of 10 was used for the curvature ratio.

The baseline model is 90°.

Temp, C	Pressure, Torr	Reynolds No.
25	760	7594

> report2

Angle	tube diameter, cm	Flow Rate, lpm
90	2.54	141.5

> report3

#	Particle Size, $\mu\text{m}$	Transmission Eff.	McFarland	Von der Weiden	Pui 90	Pui 45
1	3	0.9927	0.999	0.95472	0.9771	NA
2	5	0.9762	0.9883	0.88146	0.9389	NA
3	7	0.9483	0.9713	0.78256	0.8846	NA
4	10	0.8843	0.9307	0.60828	0.7799	NA
5	15	0.7258	0.8108	0.32863	0.5732	NA
6	20	0.5308	0.6174	0.13899	0.3728	NA
7	25	0.3414	0.3844	0.04602	0.2145	NA

> report4

#	Particle Size, $\mu\text{m}$	Stokes Number	Transmission Eff.
1	3	0.01045	0.9927
2	5	0.02845	0.9762
3	7	0.05529	0.9483
4	10	0.11211	0.8843

5	15	0.25096	0.7258
6	20	0.44501	0.5308
7	25	0.69426	0.3414

The baseline is changed to 45 °.

> report1

Temp, C	Pressure, Torr	Reynolds No.
25	760	7594

> report2

Angle	tube diameter, cm	Flow Rate, lpm
45	2.54	141.5

> report3

#	Particle Size, $\mu\text{m}$	Transmission Eff.	McFarland	Von der Weiden	Pui 90	Pui 45
1	3	0.9964	1	0.9771	NA	0.9885
2	5	0.988	1	0.9389	NA	0.9689
3	7	0.9738	1	0.8846	NA	0.9405
4	10	0.9404	1	0.7799	NA	0.883
5	15	0.8519	0.9888	0.5733	NA	0.7569
6	20	0.7286	0.9302	0.3728	NA	0.6102
7	25	0.5843	0.8052	0.2145	NA	0.4628

> report4

#	Particle Size, $\mu\text{m}$	Stokes Number	Transmission Eff.
1	3	0.01045	0.9964
2	5	0.02845	0.988
3	7	0.05529	0.9738
4	10	0.11211	0.9404
5	15	0.25096	0.8519
6	20	0.44501	0.7286
7	25	0.69426	0.5843

Comments: There is a large variation in the transmission efficiency by different calculations. The author of this report thinks that the selected method by Zhang (Zhang 2012) is the most accurate. The Pui (Pui 1987) results match reasonably well.

### **Tubes**

The baseline straight tube is horizontal at 28.3 lpm.

> report1

Temp, C	Pressure, Torr	Reynolds No.
25	760	1519

- using laminar formula

> report3

Tube Length, cm	angle of inclination	tube diameter, cm	Flow Rate, lpm
100	0	2.54	28.3

> report4

Particle Size, $\mu\text{m}$	Stokes No.	Turb. Inert. Eff.	Grav. Set. Eff.	Total Eff.
1	0.00025	1	0.9982	0.9982
3	0.00209	1	0.9851	0.9851
5	0.00569	1	0.96	0.96
7	0.01106	1	0.9235	0.9235
10	0.02242	1	0.849	0.849
15	0.05019	1	0.6799	0.6799
20	0.089	1	0.4713	0.4713
25	0.13885	1	0.2489	0.2489

The baseline is changed to 56.6 lpm:

Temp, C    Pressure, Torr    Reynolds No.  
25            760            3037            - using laminar formula

> report3

Tube Length, cm    angle of inclination    tube diameter, cm    Flow Rate, lpm  
100                    0                    2.54                    56.6

> report4

Particle Size, $\mu\text{m}$	Stokes No.	Turb. Inert. Eff.	Grav. Set. Eff.	Total Eff.
1	0.00051	1	0.9991	0.9991
3	0.00418	1	0.9925	0.9925
5	0.01138	1	0.9798	0.9798
7	0.02212	1	0.9612	0.9612
10	0.04484	1	0.9225	0.9225
15	0.10038	1	0.8319	0.8319
20	0.17800	1	0.7132	0.7132
25	0.27770	1	0.5727	0.5727

---

The baseline is changed to 141.5 lpm:

> report1

Temp, C	Pressure, Torr	Reynolds No.	
25	760	7594	- using turbulent formula

> report3

Tube Length, cm	angle of inclination	tube diameter, cm	Flow Rate, lpm
100	0	2.54	141.5

> report4

Particle Size, $\mu\text{m}$	Stokes No.	Turb. Inert. Eff.	Grav. Set. Eff.	Total Eff.
1	1	0.00127	1	0.9999
2	3	0.01045	1	0.9988
3	5	0.02845	1	0.9968
4	7	0.05529	1	0.9937
5	10	0.11211	1	0.9874
6	15	0.25096	0.9998	0.9719
7	20	0.44501	0.999	0.9508
8	25	0.69426	0.997	0.9242

The baseline is changed to an angle of 45 degrees (flow back to 28.3 lpm):

Temp, C	Pressure, Torr	Reynolds No.	
25	760	1519	- using laminar formula

> report3

Tube Length, cm	angle of inclination	tube diameter, cm	Flow Rate, lpm
100	0	2.54	28.3

> report4

Particle Size, $\mu\text{m}$	Stokes No.	Turb. Inert. Eff.	Grav. Set. Eff.	Total Eff.
1	0.00025	1	0.9987	0.9987
3	0.00209	1	0.9895	0.9895
5	0.00569	1	0.9716	0.9716
7	0.01106	1	0.9454	0.9454
10	0.02242	1	0.8916	0.8916
15	0.05019	1	0.7672	0.7672
20	0.08900	1	0.608	0.608
25	0.13885	1	0.4264	0.4264

---

The baseline is at an angle of 45 degrees with flow at 141.5 lpm:

> report1

Temp, C	Pressure, Torr	Reynolds No.	
25	760	7594	- using turbulent formula

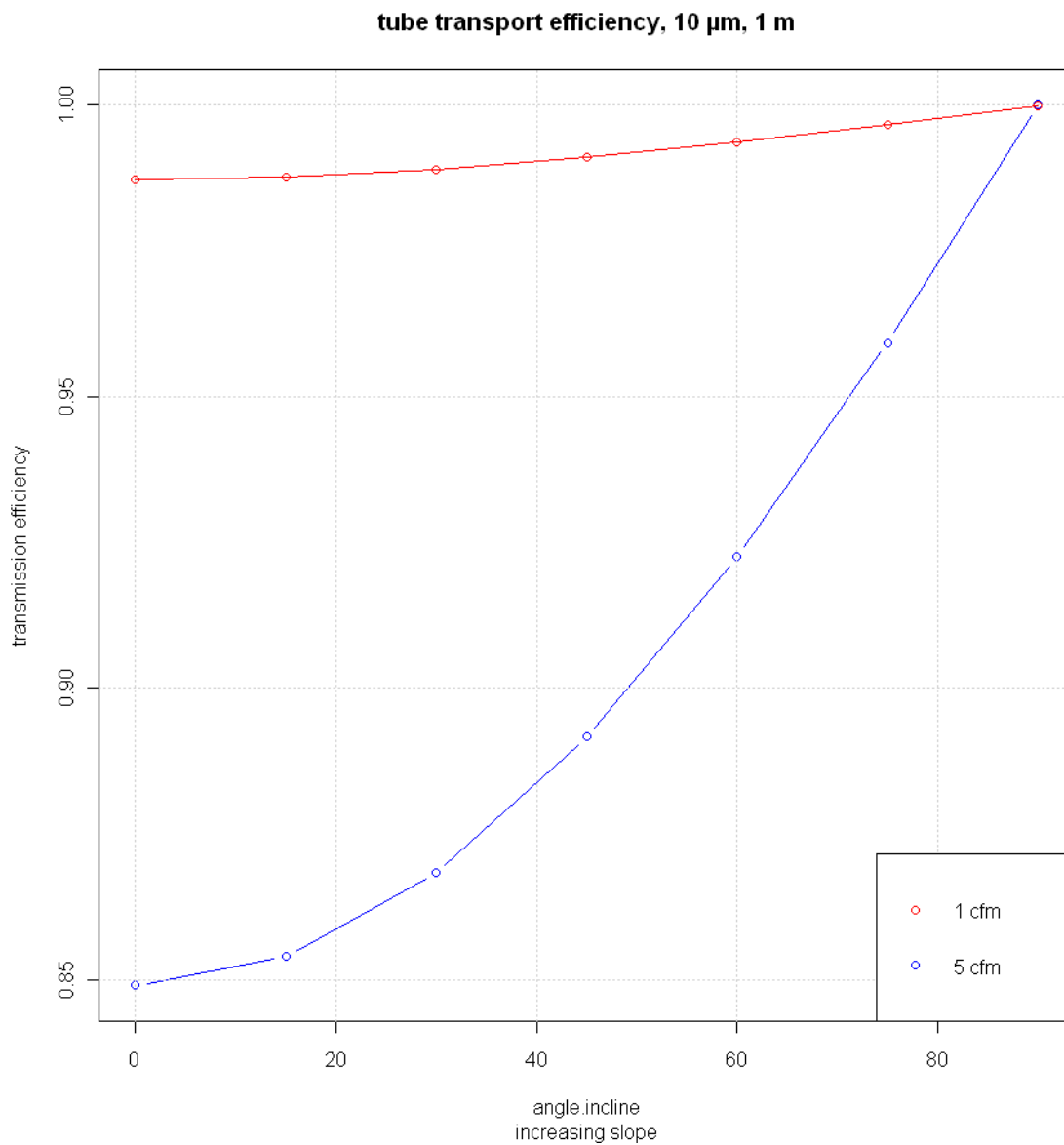
> report3

Tube Length, cm	angle of inclination	tube diameter, cm	Flow Rate, lpm
100	45	2.54	141.5

> report4

Particle Size, $\mu\text{m}$	Stokes No.	Turb. Inert. Eff.	Grav. Set. Eff.	Total Eff.
1	0.00127	1	0.9999	0.9999
3	0.01045	1	0.9992	0.9992
5	0.02845	1	0.9977	0.9977
7	0.05529	1	0.9956	0.9956
10	0.11211	1	0.991	0.991
15	0.25096	0.9998	0.9801	0.9798
20	0.44501	0.999	0.9649	0.964
25	0.69426	0.997	0.9458	0.943

Comments: The test runs show increased losses with particle size due to gravitational settling. This is expected as larger particles settle faster as explained by (Baron 2001 ). Increasing the slope of the tube allows the settling velocity due to gravity to be countered by the upward drag of the gas flow. This effect increases with higher flow rates. This is shown in Figure 10.



**Figure 10: Effect of Increasing Angle on Tube Transport Efficiency**

**Model Example**

For an example air sampling system, we'll start with an isometric drawing of a pipe extending through a shield wall. Figure 11 shows the line, starting with an open pipe in a normally unoccupied processing area. A couple of details not shown in the figure include that the pipe bending ratio on the bends is 5 pipe diameters (10 pipe radii) and that the pipe extends 0.5 inches into the sample area.

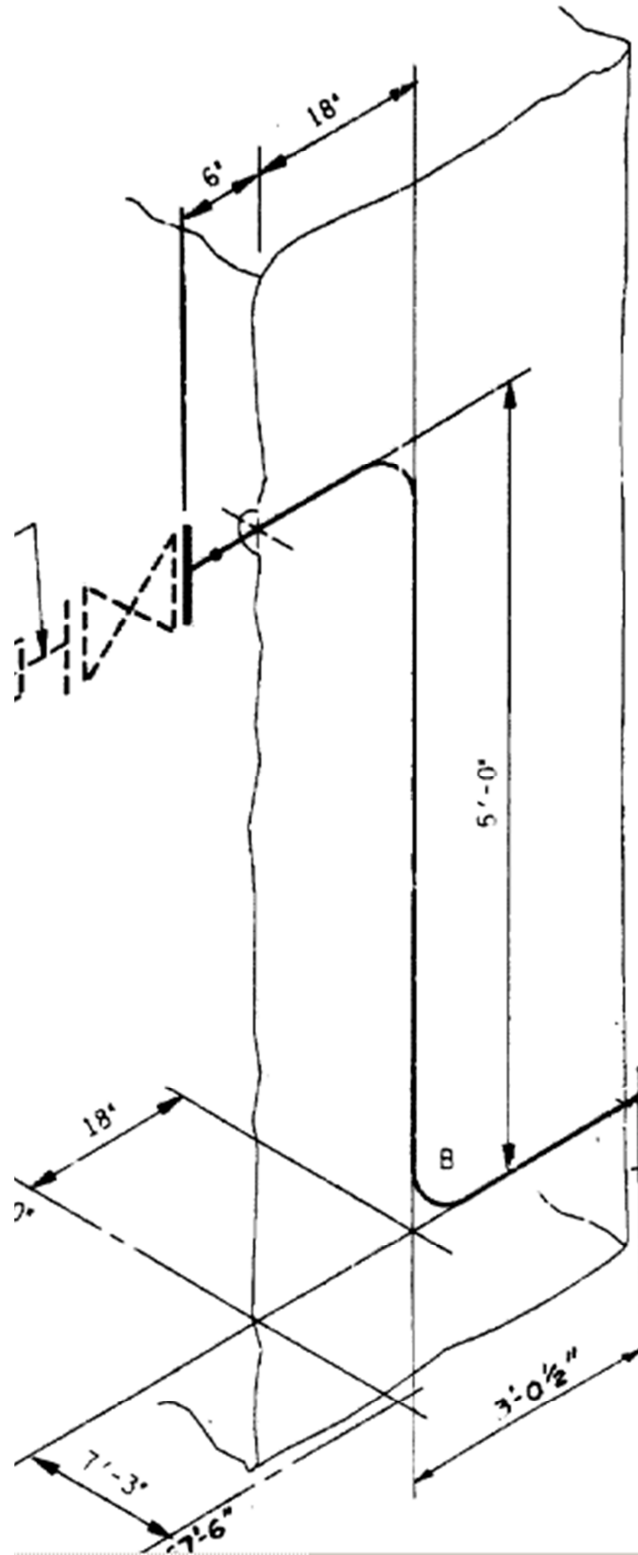


Figure 11: Isometric Drawing of a Sample Line Extending Through a Shield Wall

**Table 4: Elements for Model Air Sample System**

Element No.	Element	Measurements, comments
1	Probe	0.5 inch diameter - 1.27 cm diameter. Assumed to behave as a thin walled probe, but it may actually be too close to the wall to obtain a representative sample.
2	Tube	1.27 cm diameter. 3'-0.5" - 92.71 cm. Horizontal
3	Bend	1.27 cm diameter, 90°, curvature ratio 10
4	Tube	1.27 cm diameter. 5' - 152.4 cm. Vertical
5	Bend	1.27 cm diameter, 90°, curvature ratio 10
6	Tube	1.27 cm diameter. 2' - 60.96 cm. Horizontal

It is assumed that after the last tube, there will be an air sample like the one shown in Figure 5 and that the valve shown schematically will open with a smooth path (as some ball valves do). To optimize the sample flow rate, a series of flow rates is run for the monodispersed cases.

Results of analysis per the hand calculations are provided below and summarized in Table 5. The revised R scripts for the hand calculations for are provided as attachments.

Monodispersed Cases

Probe

1 µm

> report1

Temp, C	Pressure, Torr	tube diameter, cm	particle size, µm
25	760	1.27	1

> report3

#	orientation	Flow Rate, lpm	Reynolds No.
1	h	14.15	1519
2	h	28.3	3037
3	h	56.6	6075
4	h	84.9	9112

> report4

#	Flow Rate, lpm	Stokes Number	Aspiration Eff.
1	14.15	0.00051	0.9985
2	28.3	0.00102	0.999
3	56.6	0.00204	0.9995
4	84.9	0.00306	0.9997



Probe, continued

10 µm

> report1

Temp, C	Pressure, Torr	tube diameter, cm	particle size, µm
25	760	1.27	10

> report3

#	orientation	Flow Rate, lpm	Reynolds No.
1	h	14.15	1519
2	h	28.3	3037
3	h	56.6	6075
4	h	84.9	9112

> report4

# Flow Rate, lpm Stokes Number Aspiration Eff.

#	Flow Rate, lpm	Stokes Number	Aspiration Eff.
1	14.15	0.04484	0.9877
2	28.3	0.08969	0.9963
3	56.6	0.17937	1.0078
4	84.9	0.26906	1.0176

Tube, Element 2

1 µm

> report1

Temp, C	Pressure, Torr	tube diameter, cm	particle size, µm
25	760	1.27	1

> report3

#	Tube Length, cm	angle of incl.	Flow Rate, lpm	Reynolds No.	
1	92.71	0	14.15	1519	- using laminar formula
2	92.71	0	28.3	3037	- using laminar formula
3	92.71	0	56.6	6075	- using turbulent formula
4	92.71	0	84.9	9112	- using turbulent formula

> report4

#	Flow Rate, lpm	Stokes	Turb. Inert.	Grav. Set.	Total Eff.
1	14.15	0.00102	1	0.9983	0.9983
2	28.3	0.00204	1	0.9983	0.9983
3	56.6	0.00408	1	0.9997	0.9997
4	84.9	0.00612	1	0.9998	0.9998

---

Tube , Element 2 continued

10  $\mu\text{m}$

Temp, C	Pressure, Torr	tube diameter, cm	particle size, $\mu\text{m}$
25	760	1.27	10

> report3

#	Tube Length, cm	angle of incl.	Flow Rate, lpm	Reynolds No.	
1	92.71	0	14.15	1519	- using laminar formula
2	92.71	0	28.3	3037	- using laminar formula
3	92.71	0	56.6	6075	- using turbulent formula
4	92.71	0	84.9	9112	- using turbulent formula

> report4

#	Flow Rate, lpm	Stokes	Turb. Inert.	Grav. Set.	Total Eff.
1	14.15	0.08969	1	0.8595	0.8595
2	28.3	0.17937	1	0.928	0.928
3	56.6	0.35874	1	0.9709	0.9709
4	84.9	0.53811	1	0.9805	0.9805

Bend, Element 3

1  $\mu\text{m}$

> report1

#	Temp, C	Pressure, Torr	Flow Rate, lpm	Reynolds No.
1	25	760	14.15	1519
2	25	760	28.3	3037
3	25	760	56.6	6075
4	25	760	84.9	9112

> report2

#	Particle Size, $\mu\text{m}$	angle	tube diameter, cm
1	1	90	1.27

> report3

#	Flow Rate, lpm	Transmission Eff.
1	14.15	0.9984
2	28.3	0.9966
3	56.6	0.9928
4	84.9	0.9889

Bend, Element 3, continued

> report1

#	Temp, C	Pressure, Torr	Flow Rate, lpm	Reynolds No.
1	25	760	14.15	1519
2	25	760	28.3	3037
3	25	760	56.6	6075
4	25	760	84.9	9112

> report2

Particle Size,  $\mu\text{m}$  angle tube diameter, cm

#	Particle Size, $\mu\text{m}$	angle	tube diameter, cm
1	10	90	1.27

> report3

#	Flow Rate, lpm	Transmission Eff.
1	14.15	0.8205
2	28.3	0.6598
3	56.6	0.4172
4	84.9	0.2593

Tube, Element 4

1  $\mu\text{m}$

> report1

Temp, C	Pressure, Torr	tube diameter, cm	particle size, $\mu\text{m}$
25	760	1.27	1

> report3

#	Tube Length, cm	angle of incl.	Flow Rate, lpm	Reynolds No.	
1	152.5	90	14.15	1519	- using laminar formula
2	152.5	90	28.3	3037	- using laminar formula
3	152.5	90	56.6	6075	- using turbulent formula
4	152.5	90	84.9	9112	- using turbulent formula

turbulent formula

> report4

#	Flow Rate, lpm	Stokes	Turb. Inert.	Grav. Set.	Total Eff.
1	14.15	0.00102	1	1	1
2	28.3	0.00204	1	1	1
3	56.6	0.00408	1	1	1
4	84.9	0.00612	1	1	1

Tube , Element 4 continued

10  $\mu\text{m}$

> report1

Temp, C	Pressure, Torr	tube diameter, cm	particle size, $\mu\text{m}$
25	760	1.27	10

> report3

#	Tube Length, cm	angle of incl.	Flow Rate, lpm	Reynolds No.	
1	152.5	90	14.15	1519	- using laminar formula
2	152.5	90	28.3	3037	- using laminar formula
3	152.5	90	56.6	6075	- using turbulent formula
4	152.5	90	84.9	9112	- using turbulent formula

> report4

#	Flow Rate, lpm	Stokes	Turb. Inert.	Grav. Set.	Total Eff.
1	14.15	0.08969	1	1	1
2	28.3	0.17937	1	1	1
3	56.6	0.35874	1	1	1
4	84.9	0.53811	1	1	1

Bend, Element 5

The monodisperse reports for element 5 are identical to element 3 and are therefore not repeated.

Tube, Element 6

The monodisperse reports for element 6 are identical to element 2 and are therefore not repeated.

Comments: The various flow rates show that the bend is the most significant element. The total activity extracted from a 10  $\mu\text{m}$  distribution peaks at around the 56.6 lpm flow rate. For the summary table, the lowest flow rate, 14.15 lpm (0.5 cfm), is used.

Lognormally dispersed Case geometric mean = 5  $\mu\text{m}$ , sd=2.5

Probe, 14.15 lpm

> report1

Temp, C	Pressure, Torr	Tube diameter, cm	Reynolds No.
25	760	1.27	1519

> report2

D.geomean	D.geosd
5	2.5

---

```
> summary(Stk)
  Min.   1st Qu.  Median   Mean   3rd Qu.  Max.
0.000044 0.003717 0.01327 0.2645 0.1135 3.458
> print(report9,quote=FALSE)
aspiration eff <1 µm:      0.9987
aspiration eff 1-5 µm:     0.9944
aspiration eff 5-10 µm:    0.9898
aspiration eff overall:    0.9453
```

### Tube, Element 2

```
> report1
  Temp, C  Pressure, Torr  Reynolds No.
    25      760          1519
> report2
  Tube length, cm  Tube diameter, cm  angle to horizontal
    92.71          1.27              0
> print(report8,quote=FALSE)
cumulative eff, mass <1 µm:      0.9977
cumulative eff, mass 1-5 µm:     0.9755
cumulative eff, mass 5-10 µm:    0.9169
cumulative eff, mass overall:    0.3248
> print(report9,quote=FALSE)
efficiency, mass <1 µm:          0.999
efficiency, mass 1-5 µm:         0.981
efficiency, mass 5-10 µm:        0.9271
efficiency, mass overall:        0.3436
```

### Bend, Element 3

```
> report1
  Temp, C  Pressure, Torr  Flow Rate, lpm  Reynolds No.
    25      760          14.15      1519
> report2
  angle tube  diameter, cm
    90        1.27
> print(report8,quote=FALSE)
cumulative eff, mass <1 µm:      0.9965
cumulative eff, mass 1-5 µm:     0.9483
cumulative eff, mass 5-10 µm:    0.8092
cumulative eff, mass overall:    0.1406
> print(report9,quote=FALSE)
efficiency, mass <1 µm:          0.9988
efficiency, mass 1-5 µm:         0.9721
efficiency, mass 5-10 µm:        0.885
efficiency, mass overall:        0.4328
```

---

Tube, Element 4

> report1

Temp, C	Pressure, Torr	Reynolds No.
---------	----------------	--------------

25	760	1519
----	-----	------

> report2

Tube length, cm	Tube diameter, cm	angle to horizontal
-----------------	-------------------	---------------------

92.71	1.27	0
-------	------	---

> print(report8,quote=FALSE)

cumulative eff, mass <1 $\mu\text{m}$ :	0.9956
---	--------

cumulative eff, mass 1-5 $\mu\text{m}$ :	0.9303
--	--------

cumulative eff, mass 5-10 $\mu\text{m}$ :	0.7484
---	--------

cumulative eff, mass overall:	0.1067
-------------------------------	--------

> print(report9,quote=FALSE)

efficiency, mass <1 $\mu\text{m}$ :	0.999
-------------------------------------	-------

efficiency, mass 1-5 $\mu\text{m}$ :	0.9811
--------------------------------------	--------

efficiency, mass 5-10 $\mu\text{m}$ :	0.9286
---------------------------------------	--------

efficiency, mass overall:	0.7589
---------------------------	--------

Bend, Element 5

> report1

Temp, C	Pressure, Torr	Flow Rate, lpm	Reynolds No.
---------	----------------	----------------	--------------

25	760	14.15	1519
----	-----	-------	------

> report2

angle	tube diameter, cm
-------	-------------------

90	1.27
----	------

> print(report8,quote=FALSE)

cumulative eff, mass <1 $\mu\text{m}$ :	0.9944
---	--------

cumulative eff, mass 1-5 $\mu\text{m}$ :	0.9045
--	--------

cumulative eff, mass 5-10 $\mu\text{m}$ :	0.6625
---	--------

cumulative eff, mass overall:	0.07294
-------------------------------	---------

> print(report9,quote=FALSE)

efficiency, mass <1 $\mu\text{m}$ :	0.9988
-------------------------------------	--------

efficiency, mass 1-5 $\mu\text{m}$ :	0.9722
--------------------------------------	--------

efficiency, mass 5-10 $\mu\text{m}$ :	0.8875
---------------------------------------	--------

efficiency, mass overall:	0.6837
---------------------------	--------

Tube, Element 6

```
> report1
Temp, C   Pressure, Torr   Reynolds No.
    25         760         1519
> report2
Tube length, cm   Tube diameter, cm   angle to horizontal
    92.71         1.27         0
> print(report8,quote=FALSE)
cumulative eff, mass <1 µm:      0.9934
cumulative eff, mass 1-5 µm:     0.8876
cumulative eff, mass 5-10 µm:    0.6139
cumulative eff, mass overall:    0.06138
> print(report9,quote=FALSE)
efficiency, mass <1 µm:          0.999
efficiency, mass 1-5 µm:         0.9812
efficiency, mass 5-10 µm:        0.9301
efficiency, mass overall:        0.8415
```

Comments: Logarithmic Distributions are mass corrected as described above. The R scripts for accomplishing this are in the attachments. Note that the scripts provide a cumulative efficiency which helps assure that the distribution of particles is being handled correctly from element to element. Table 5 shows that the lognormal distribution has a lower overall efficiency than even the 10 µm monodispersed distribution. This is because the lognormal distribution selected has a lot of mass higher than 10 µm which is transported very inefficiently.

Deposition code for this distribution results in an overall efficiency of 0.769. The code may be cutting off the evaluation at some size; it provides a note that Beal's equation (which Deposition uses for tubes) is valid for particle sizes from 0.01 to 30 µm.

Table 5: Results for Model Air Sample System, 14.15 lpm

Element No.	Element	monodispersed 1 $\mu\text{m}$	monodispersed 10 $\mu\text{m}$	Comparison Run in Deposition 2001a for 10 $\mu\text{m}$	lognormal mean=5 $\mu\text{m}$ , std dev=2.5
1	Probe (thin walled calm air)	0.9985	0.9877	0.991 <sup>iv</sup>	0.9873
2	Tube	0.9983	0.8595	0.860	0.8832
3	Bend	0.9984	0.8205	0.904	0.9122
4	Tube	1	1	0.995	1
5	Bend	0.9984	0.8205	0.904	0.9308
6	Tube	0.9989	0.9061	0.905	0.9685
Total		0.9925	0.5178	0.628	0.7172

***Models in published reports***

ANSI/HPS N13.1-1999

ANSI/HPS N13.1-1999 (ANSI 1999) provides a model with results based on Deposition version 4.0. The model in Deposition 2001a is reproduced and the results compared with hand calculations. The model has a flow rate of 56.6 lpm in a stack with 10  $\text{m}\cdot\text{s}^{-1}$  free stream velocity. It starts with a shrouded probe and since this kind of probe is not used for occupational air modeling at SRS, this portion of the model is skipped in the hand calculations. As shown in Table 6, the hand calculations match very well with the comparison models.

<sup>iv</sup> Using anisokinetic probe with 1.27 cm diameter, 0 wind speed.



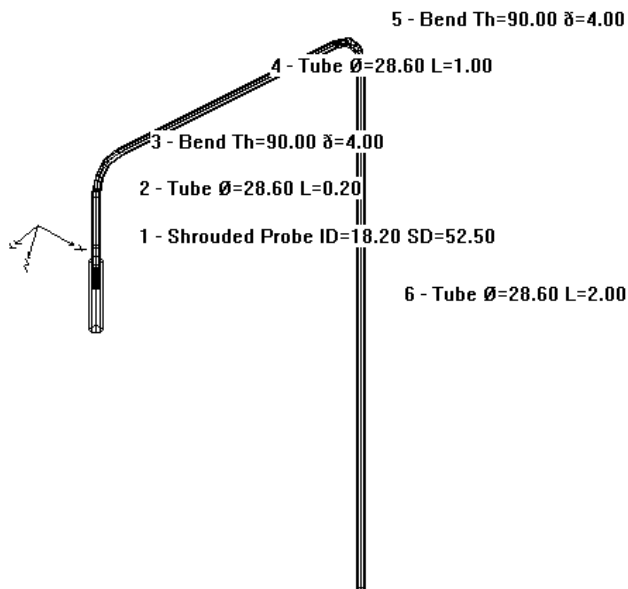


Figure 12: ANSI 13.1-1999 System Model from Deposition 2001a

Table 6: ANSI/HPS N13.1-1999 Model

Element	ANSI Result	Deposition 2001a Result	Hand Calculation
Probe diameter: 18.20 mm, Shroud diameter: 52.50 mm, Velocity reduction ratio 3.021.	0.974	0.916	NA
Length: 0.200 m, At 0.000 degrees from horizontal.	1.00	1.00	1.00
Bend angle: 90.000 degrees. Curvature ratio assumed = 4	0.933	0.987	0.973
Length: 1.000 m, At 0.000 degrees from horizontal.	0.913	0.913	0.913
Bend angle: 90.000 degrees. Curvature ratio assumed = 4	0.933	0.987	0.973
Length: 2.000 m, At 90.000 degrees from horizontal.	0.999	0.999	1
Overall Penetration Not Including Probe	0.794	0.873	0.865
Overall Penetration	0.773	0.813	NA

Aerosol Sampling Models Survey (Metz 1993)

An extensive set of comparisons was performed by Metz and Harvey. They used several models to estimate particle transport efficiency and ran an experiment on their model. In the interest of brevity, comparisons are presented in chart form only.

Of the models presented, only the Texas Agricultural and Mechanical University (TAMU) model results are included in the charts that follow because the other models either were incomplete or were identified in the report as being inconsistent with the experimental data. To the TAMU results, two model results are added, hand calculations and Deposition 2001a. For the complete, combined results, the experimental results are included, which are available only for throughput of the entire test assembly.

The two models being compared to the hand calculations are closely related. TAMU developed the Deposition code. So the differences between these two sets of results are based on changes made over the years between the TAMU model and the present version of the code, and possibly differences in inputs (or of course errors). Checking for consistency between the three helps to verify that there are no gross errors.

Figure 13 shows the model used in Deposition 2001a. This matches the schematic in Metz (1993) which provides the added details:

- The probe is an open pipe, not a probe with a tapered edge as shown
- The flow is  $3 \text{ m}\cdot\text{s}^{-1}$  at  $0^\circ$  to the probe. (Metz 1993 provides also a  $90^\circ$  flow.)
- The first tube is horizontal
- The second tube is vertical
- The third tube is  $45^\circ$  from horizontal.
- As shown, the bends are  $90^\circ$  and  $45^\circ$ .
- Metz (1993) provides contradictory data on the piping diameter. It states that the piping is  $\frac{3}{4}$ " schedule 40 PVC. It also at one point identifies it as 1" schedule 40 PVC. In a couple of places, the inside diameter is identified as 1.05 inches. In fact, 1.05 inches is the outside diameter of schedule 40 PVC piping (ASTM 2012). The inside diameter can vary based on the density of the PVC used and manufacturer discretion to meet the pressure requirements. While the minimum wall thickness, per the ASTM is 2.87 mm, which would result in an inner diameter of 23.8 mm, an inner diameter of 20.42 is a reasonable value based on a vendor specification providing average internal diameter (Georg Fisher 2013). In the hand calculations, it becomes evident that the models reported in Metz (1993) actually used the larger diameter. The evidence is that the reported aspiration efficiencies become lower as particle sizes increase, and this happens when the ratio of sample speed to wind speed is less than 1, but the smaller diameter provides a ratio of sample speed to wind speed greater than 1, which provides an increasing aspiration efficiency with particle size. Hand calculations were tried with both diameters discussed here, but the match to both experimental and model results generally got worse with the smaller diameter, so only the larger diameter results are reported.

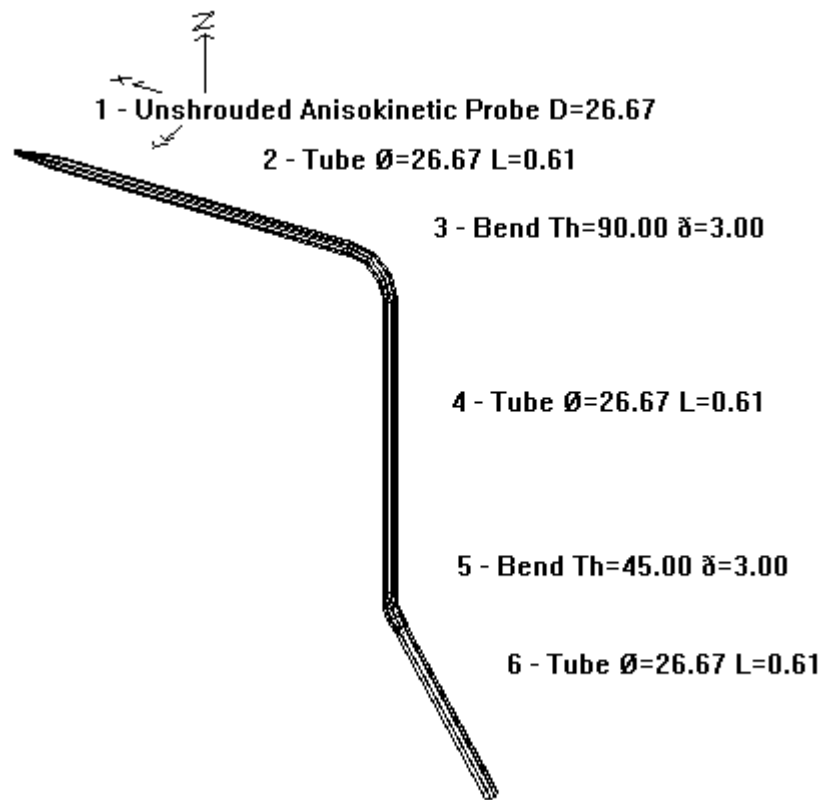


Figure 13: Metz et. al. System Model from Deposition 2001a

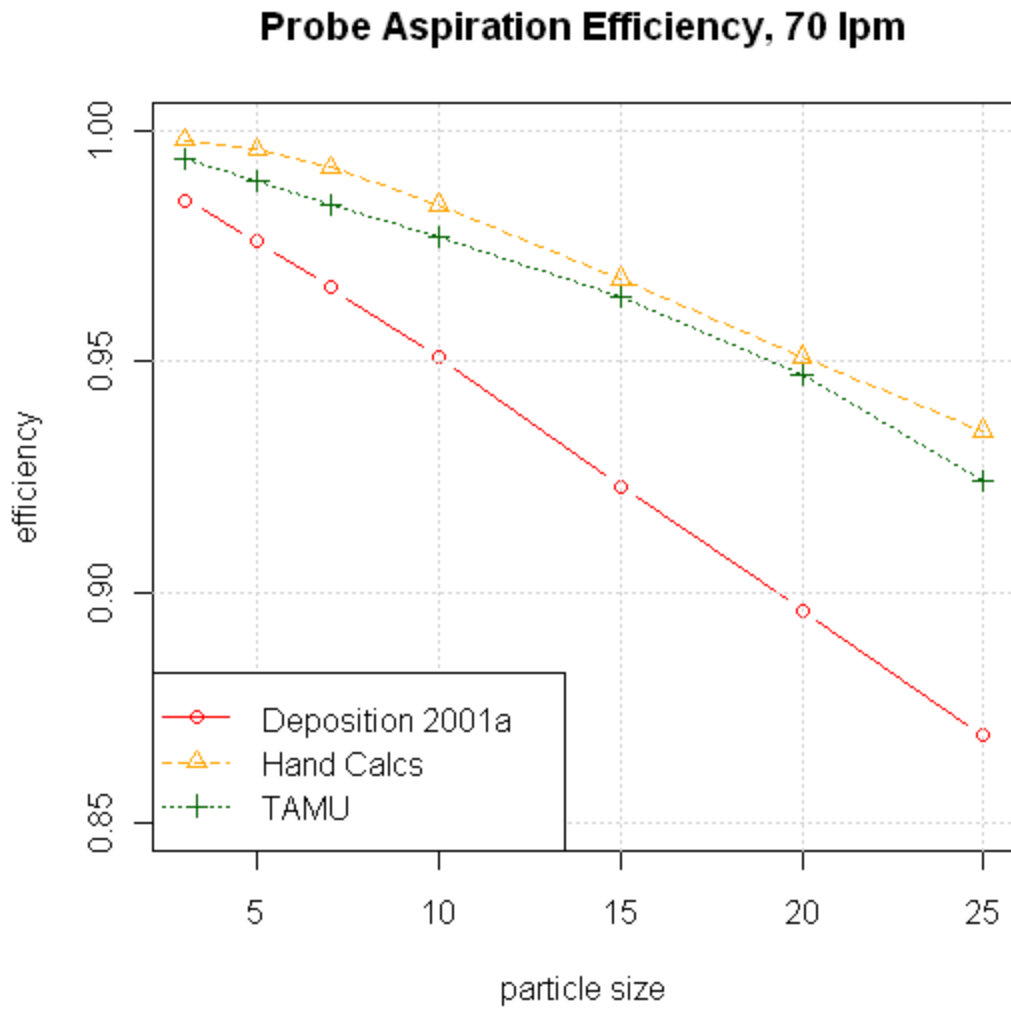


Figure 14: Probe, 70 lpm

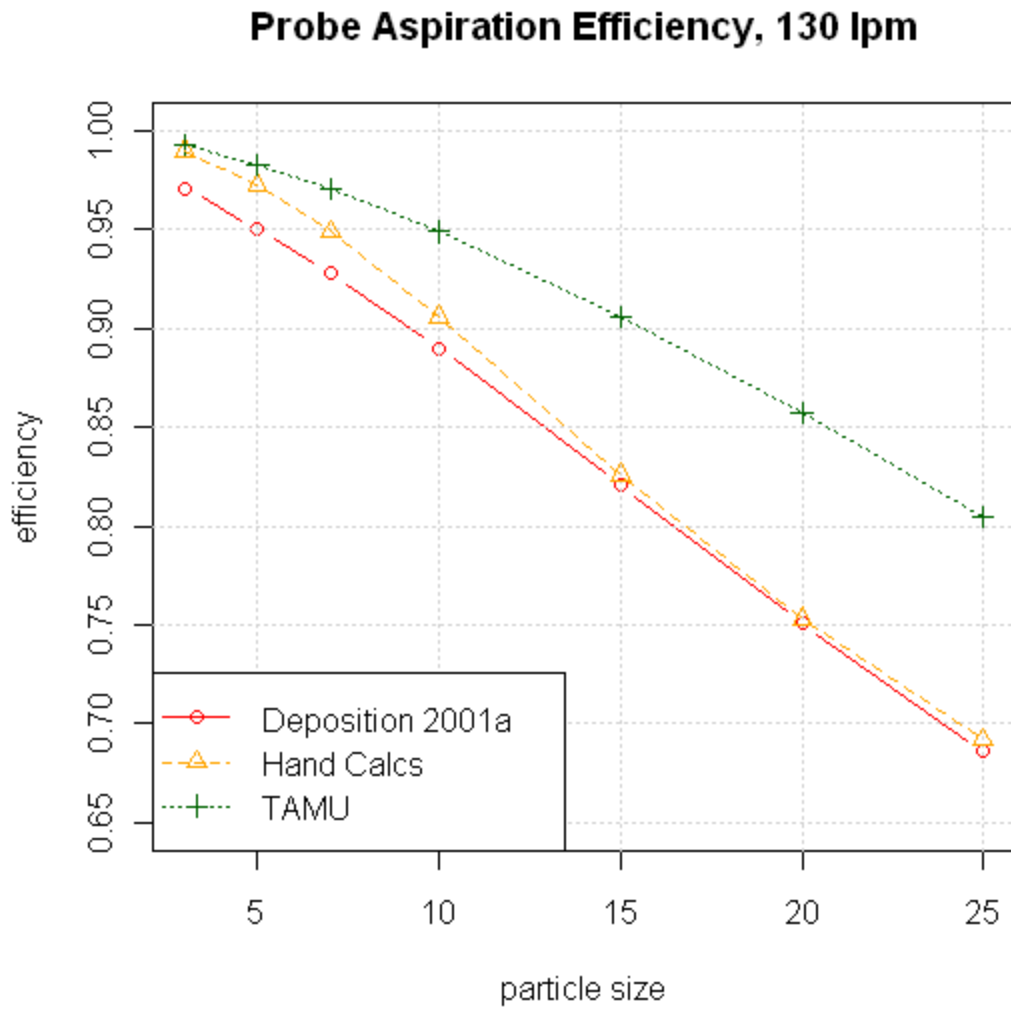


Figure 15: Probe, 130 lpm

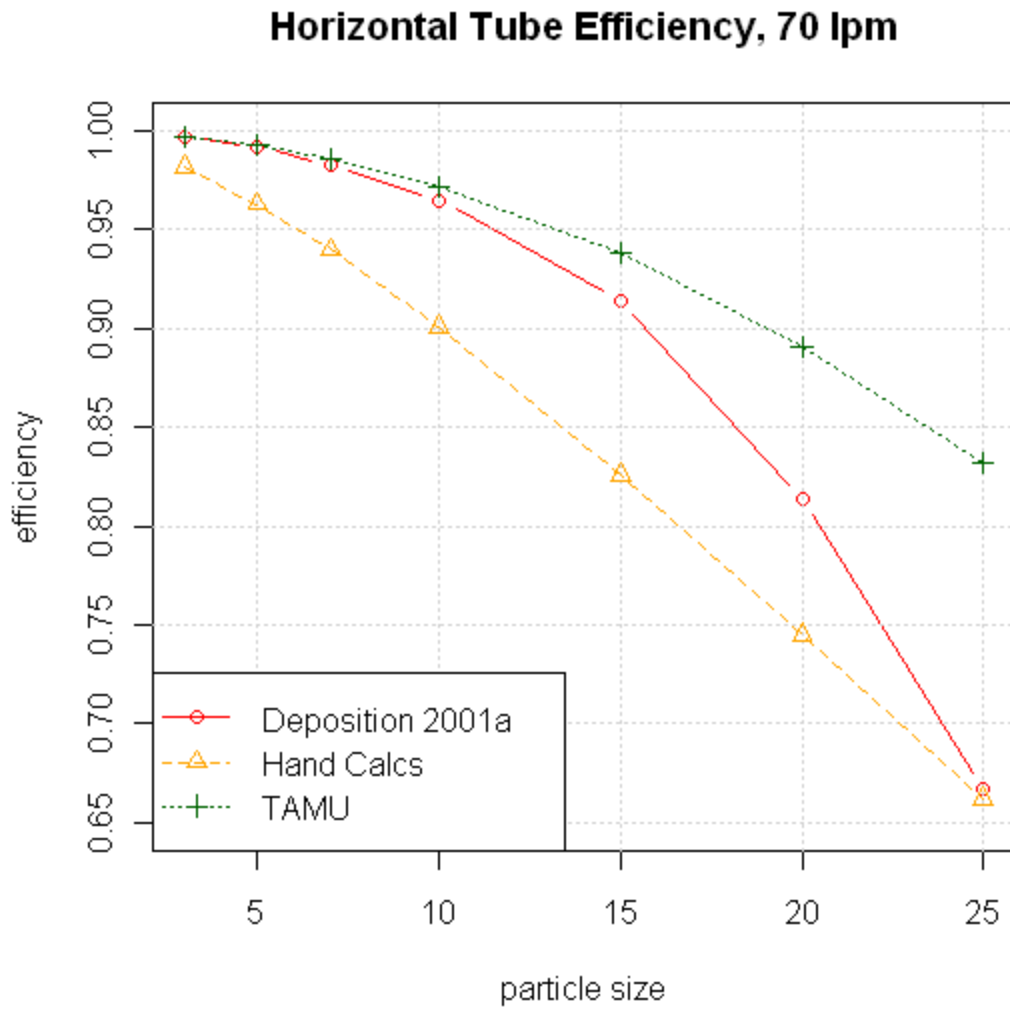


Figure 16: Straight Tube, 70 lpm

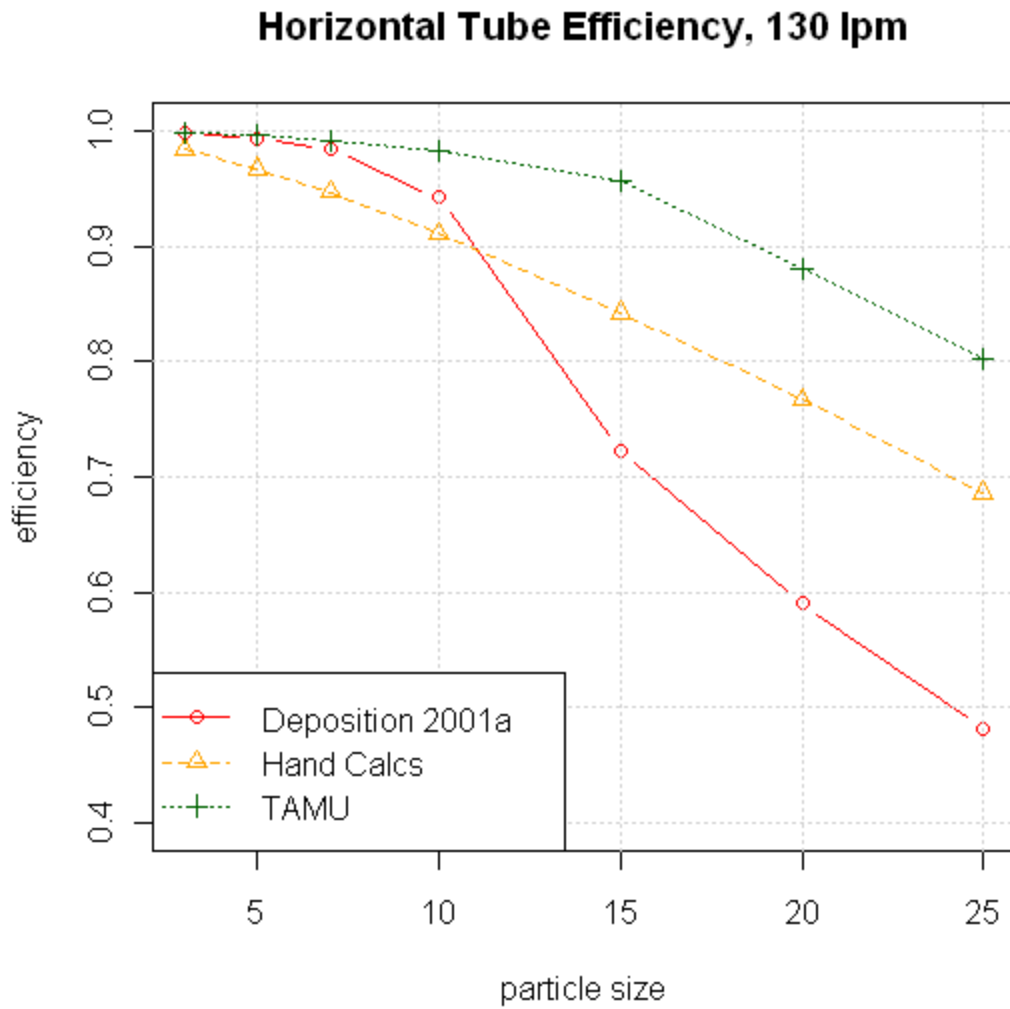


Figure 17: Straight Tube, 130 lpm

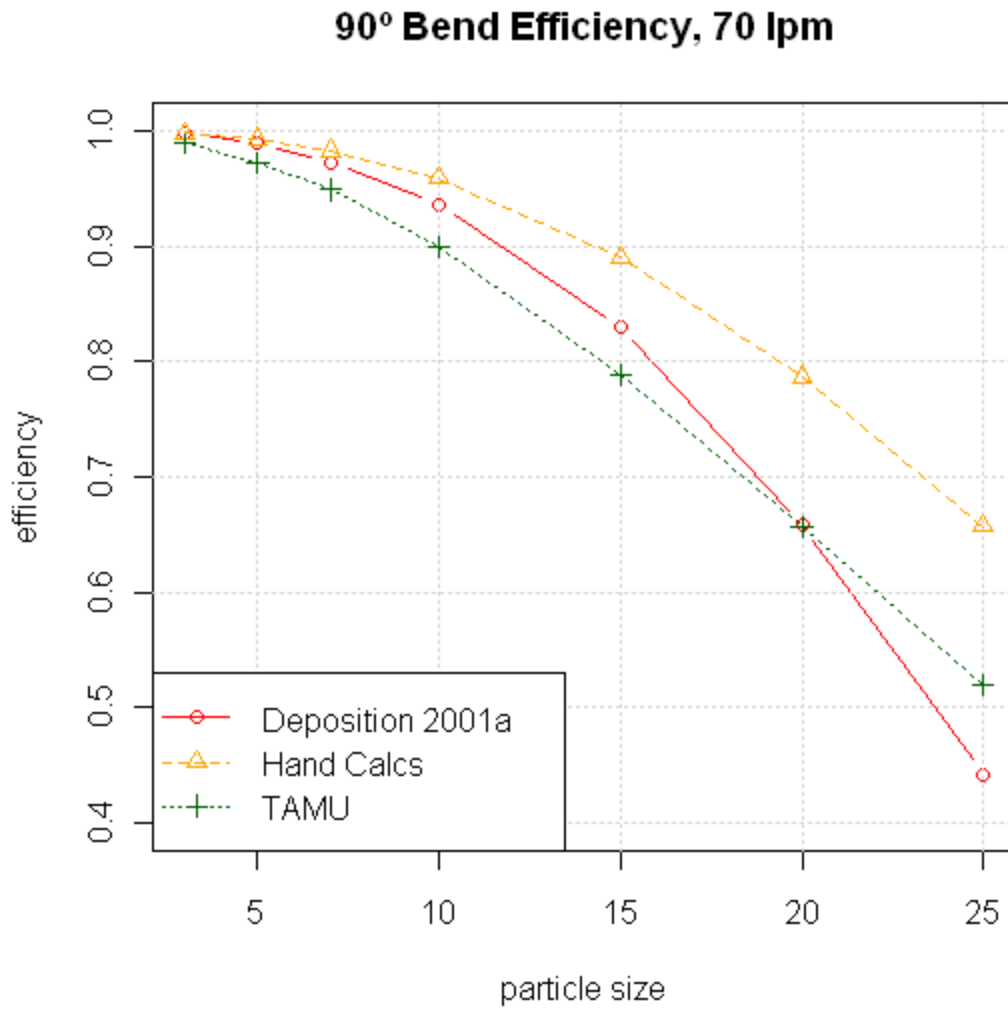


Figure 18: Bend, 90° angle, 70 lpm



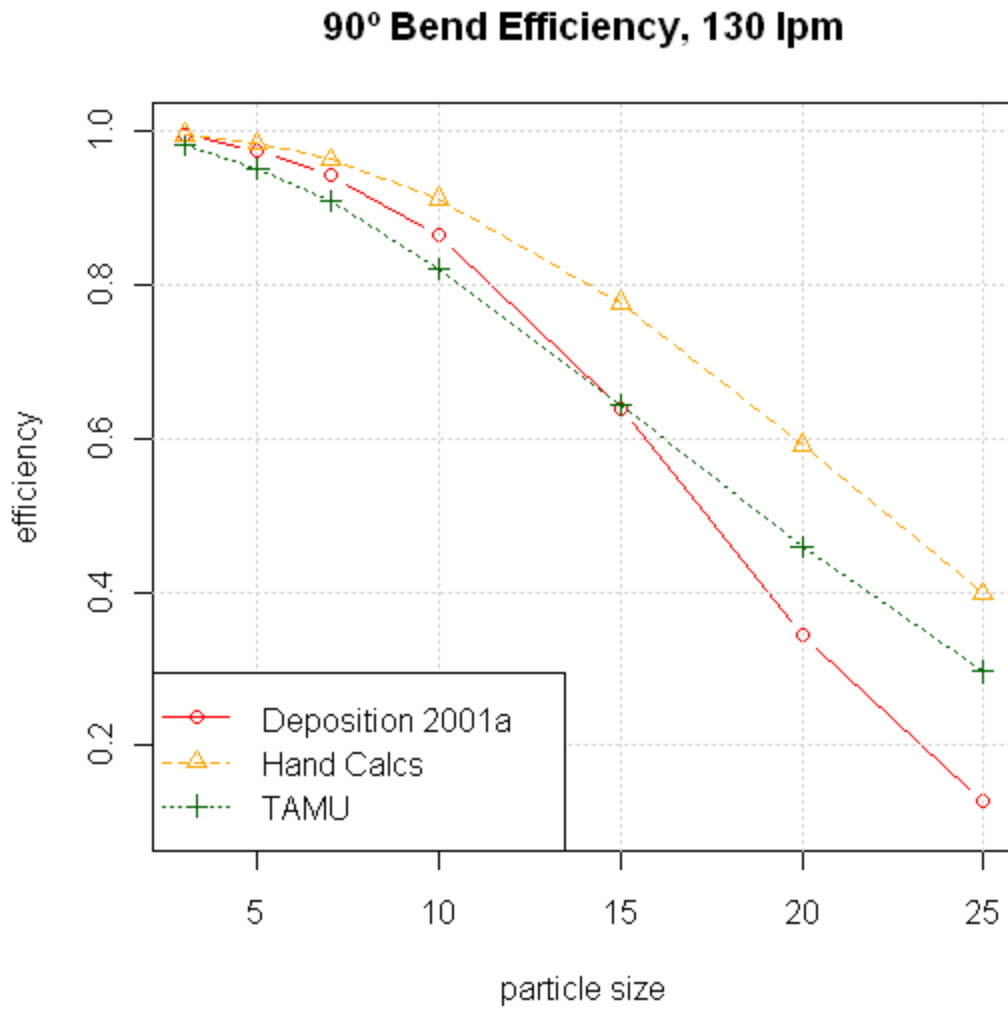


Figure 19: Bend, 90° angle, 130 lpm

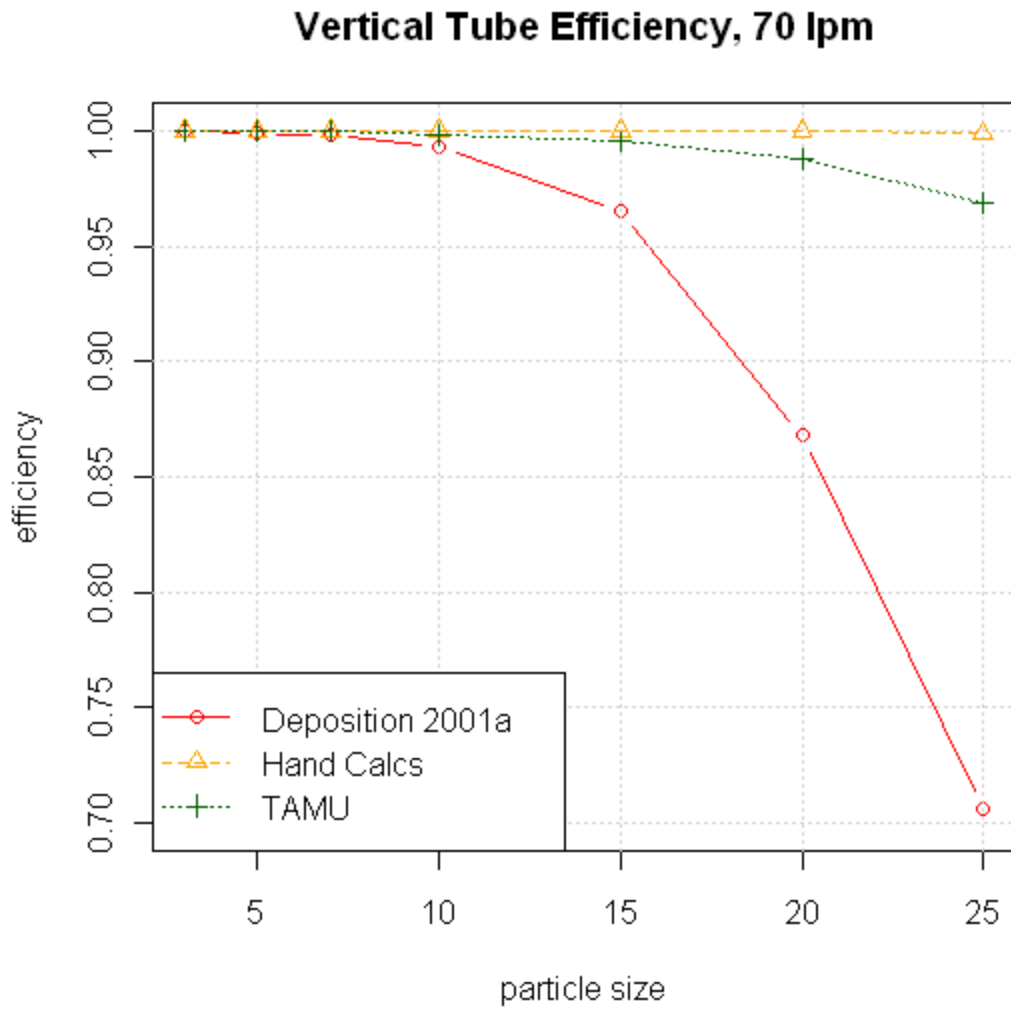


Figure 20: Vertical Tube, 70 lpm

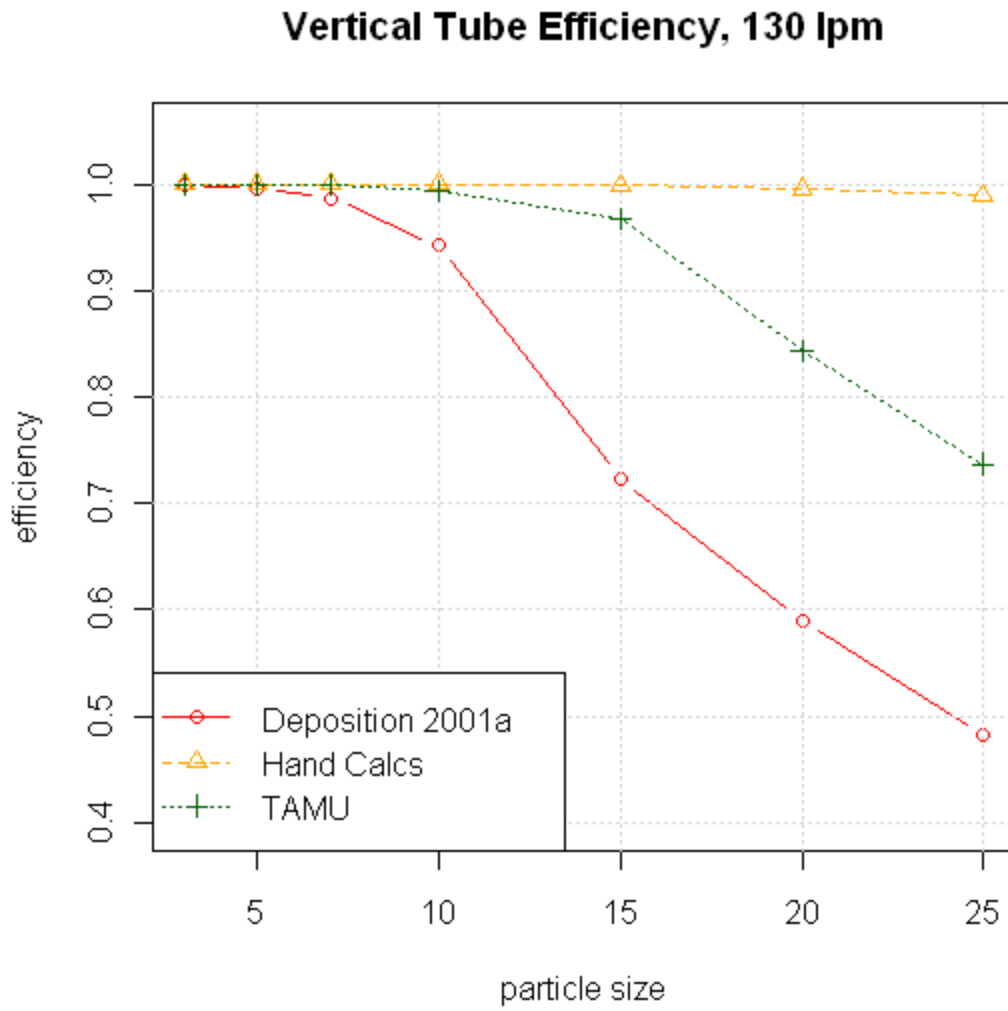


Figure 21: Vertical Tube, 130 lpm

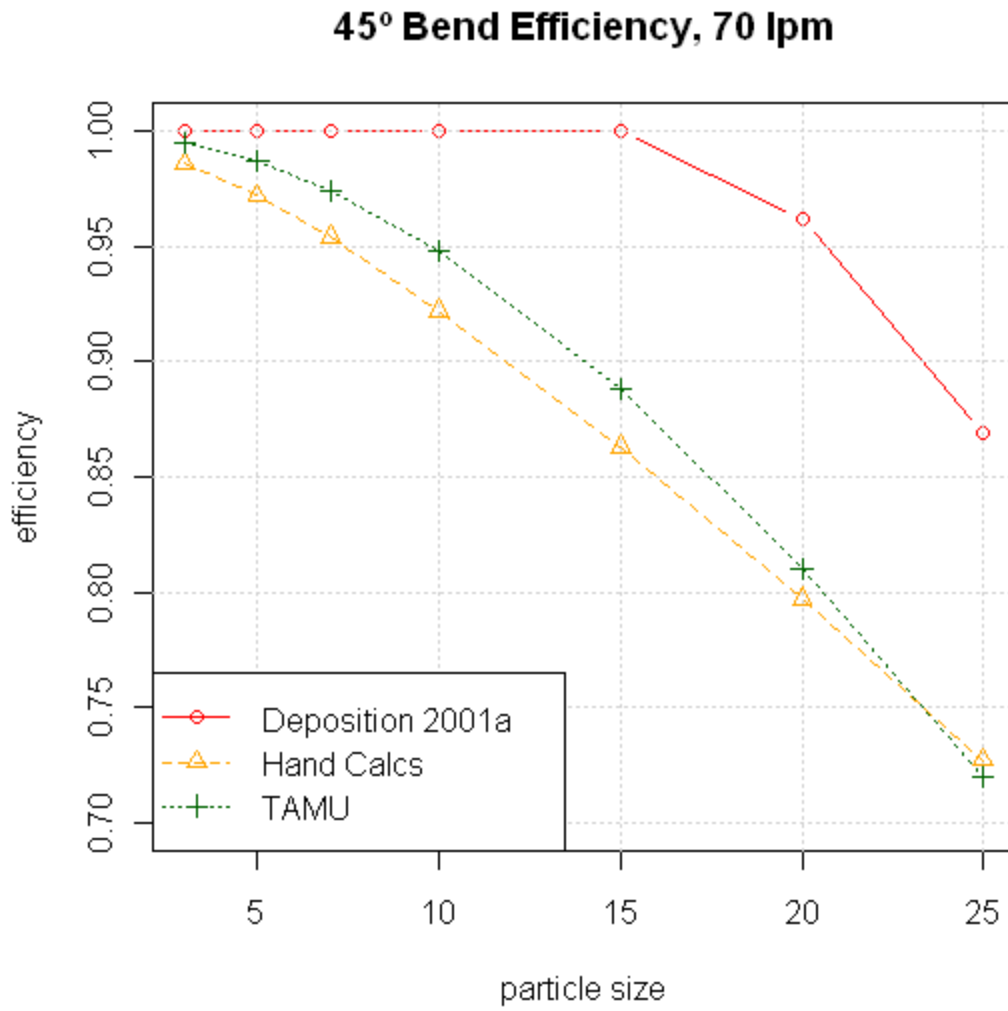


Figure 22: 45° Bend, 70 lpm

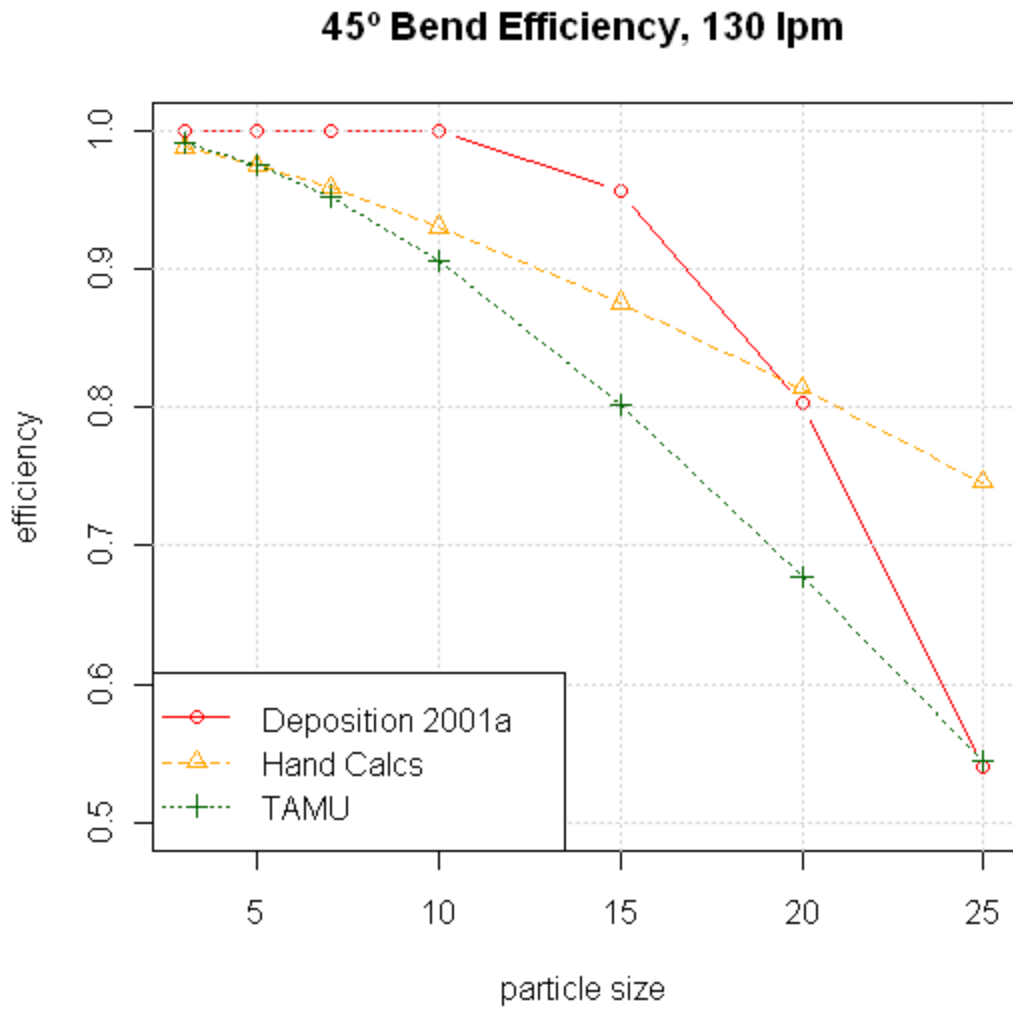


Figure 23: 45° Bend, 130 lpm

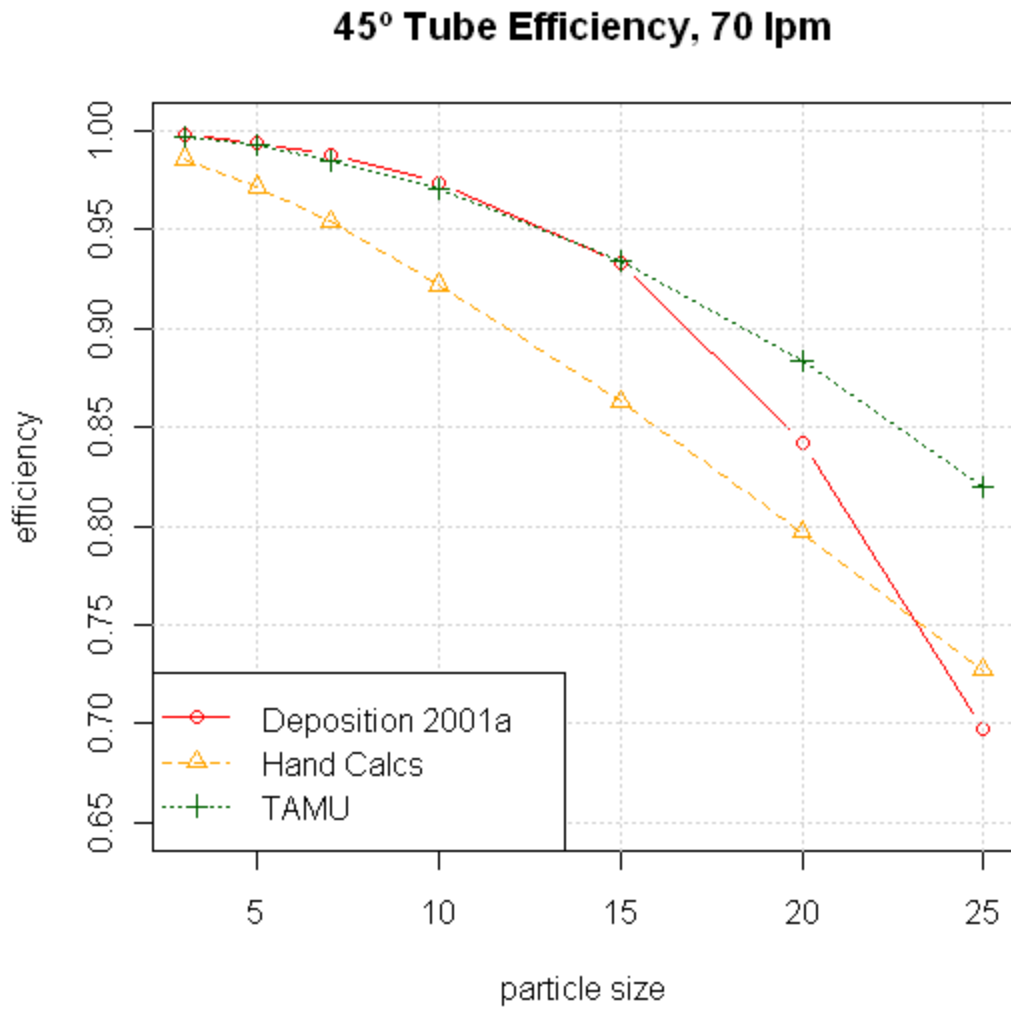


Figure 24: Sloped 45° Tube, 70 lpm

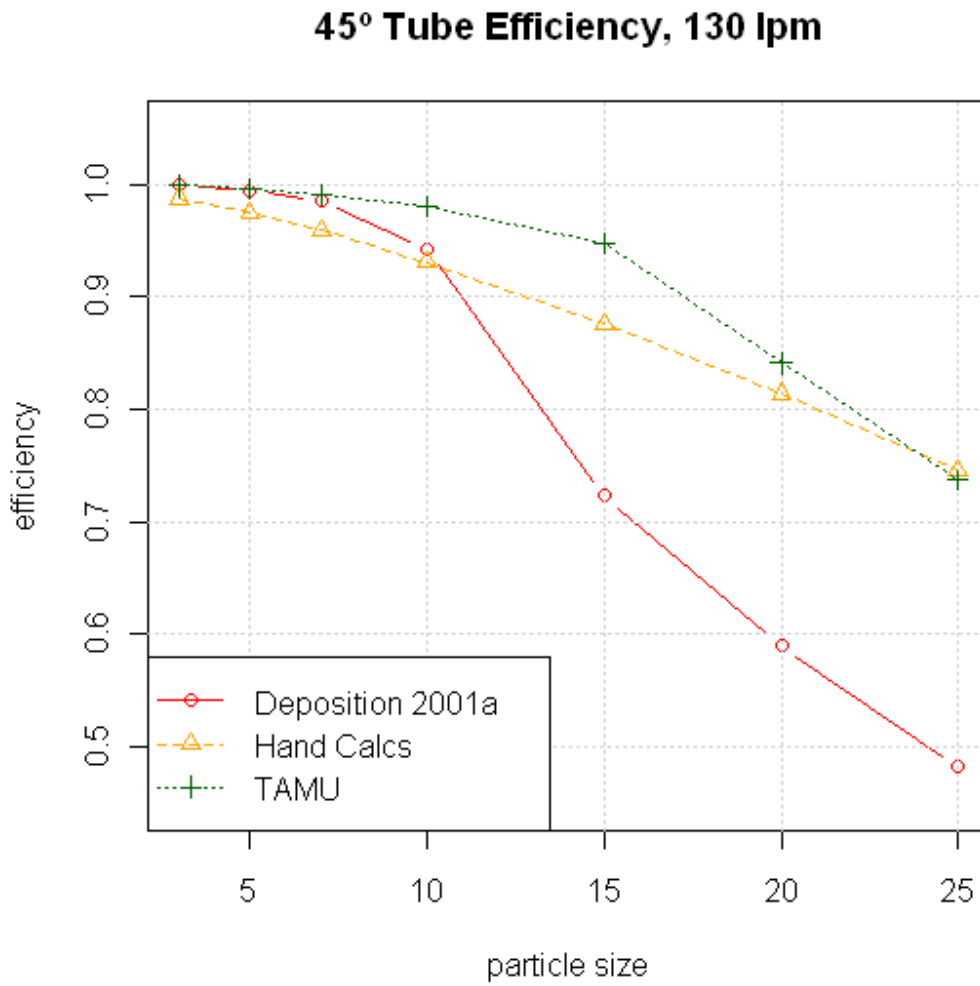


Figure 25: Sloped 45° Tube 130 lpm

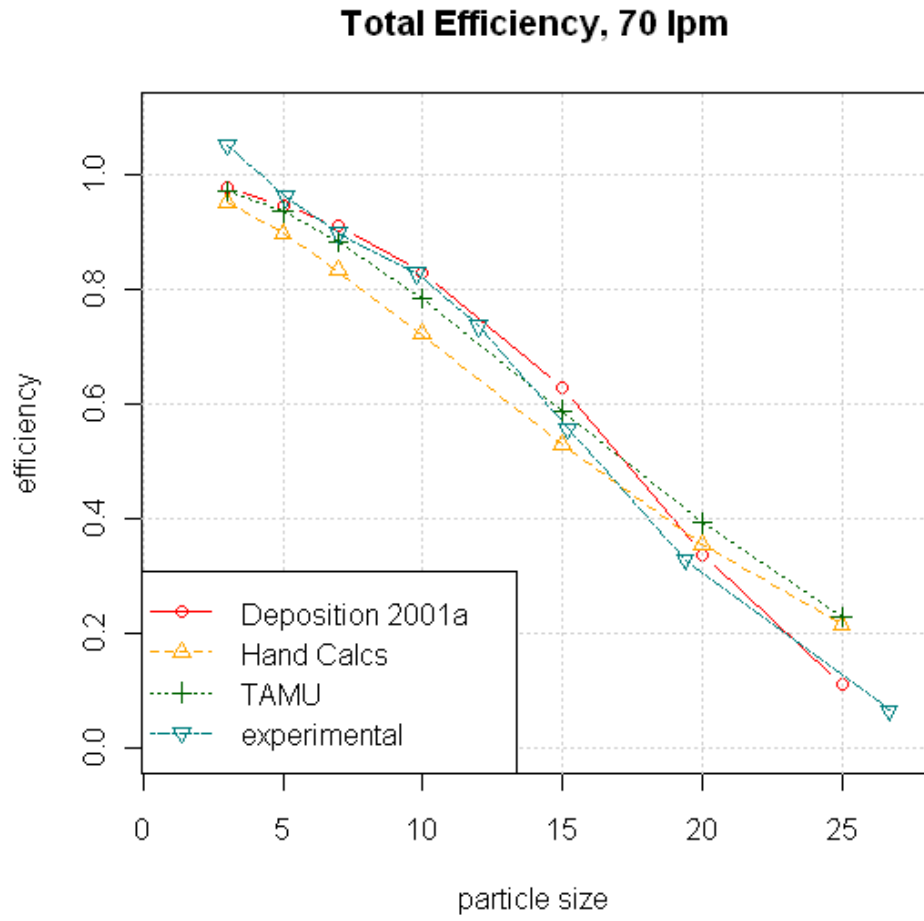


Figure 26: Total for Model Survey, 70 lpm



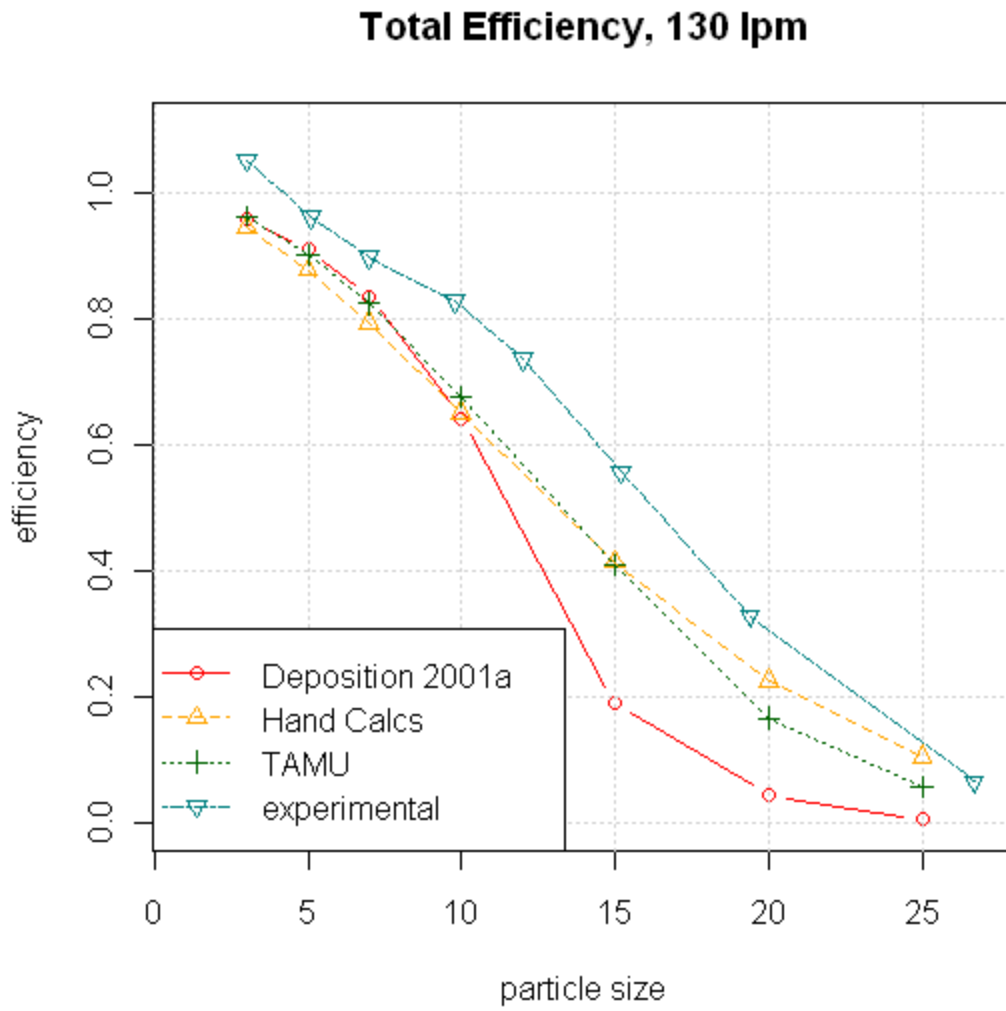


Figure 27: Total for Model Survey, 130 lpm

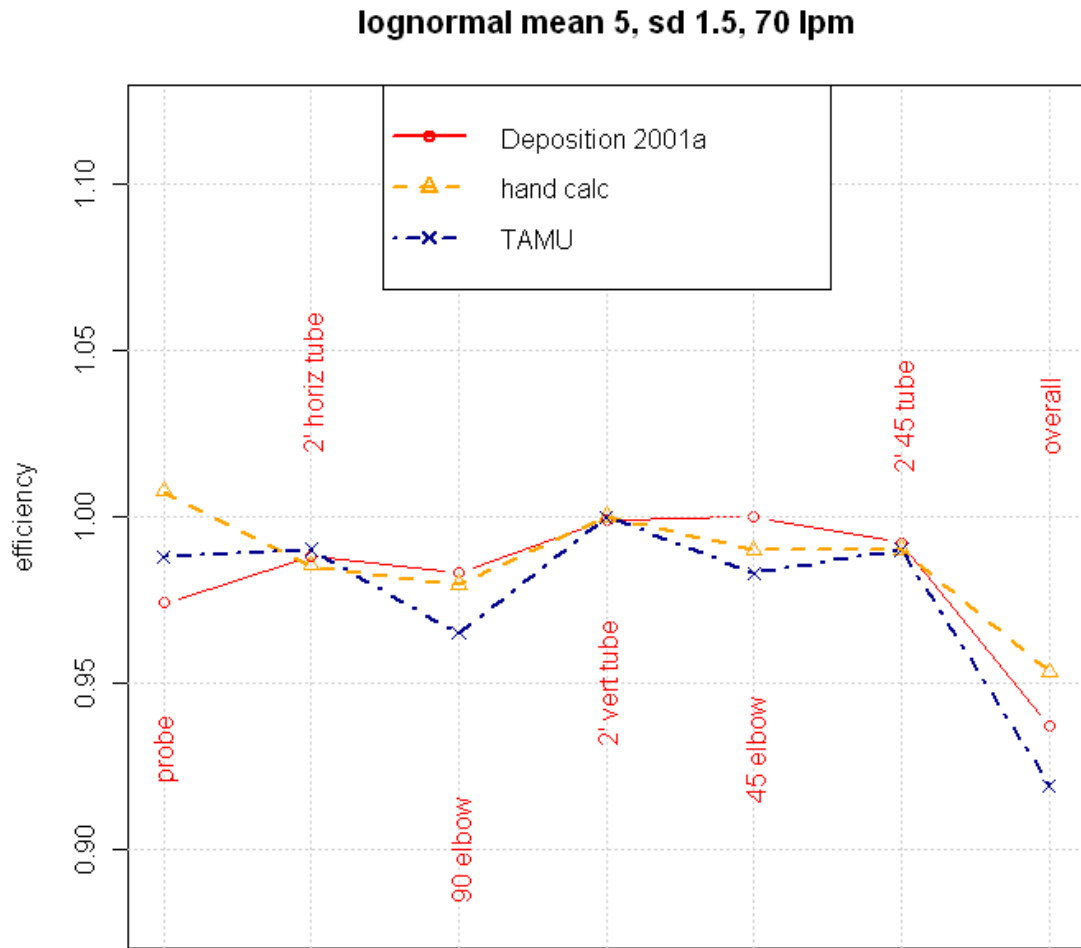


Figure 28: Lognormal Model Suvey Results 5, 1.5, 70

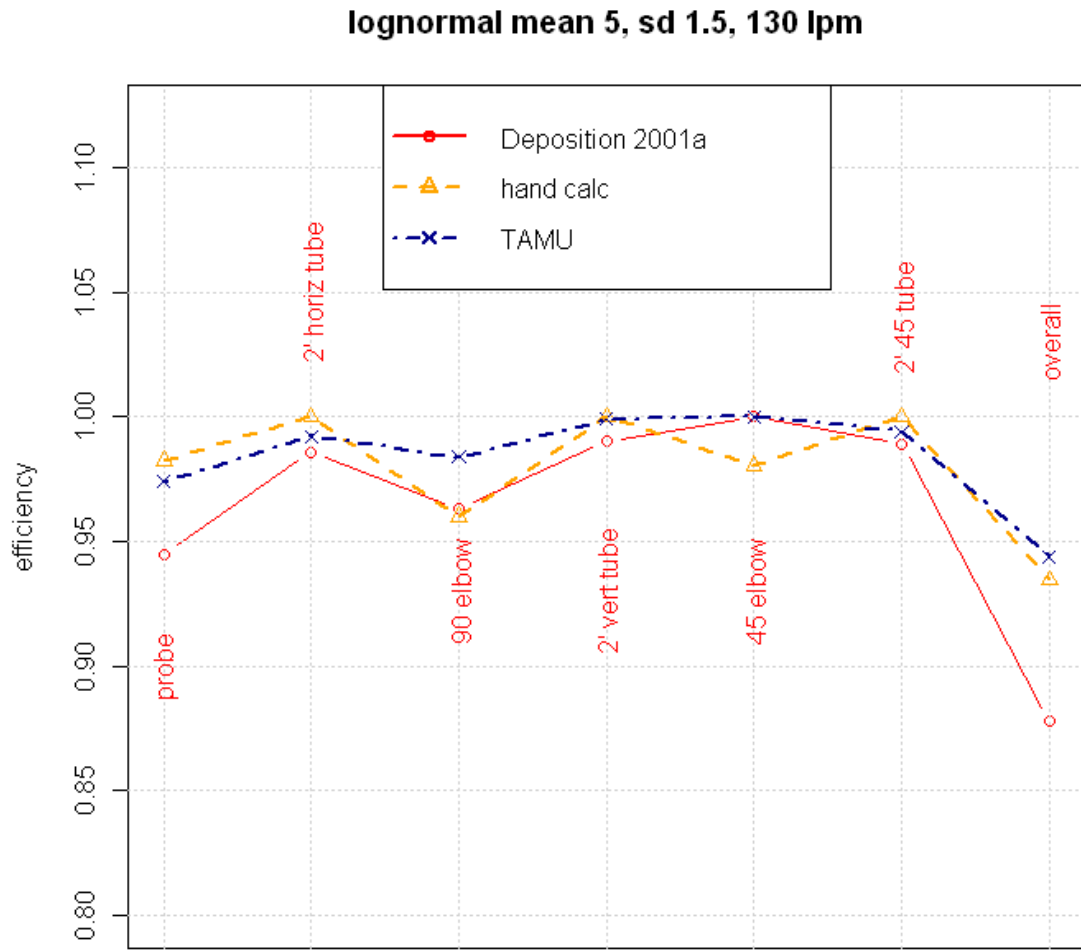


Figure 29: Lognormal Model Survey Results 5, 1.5, 130

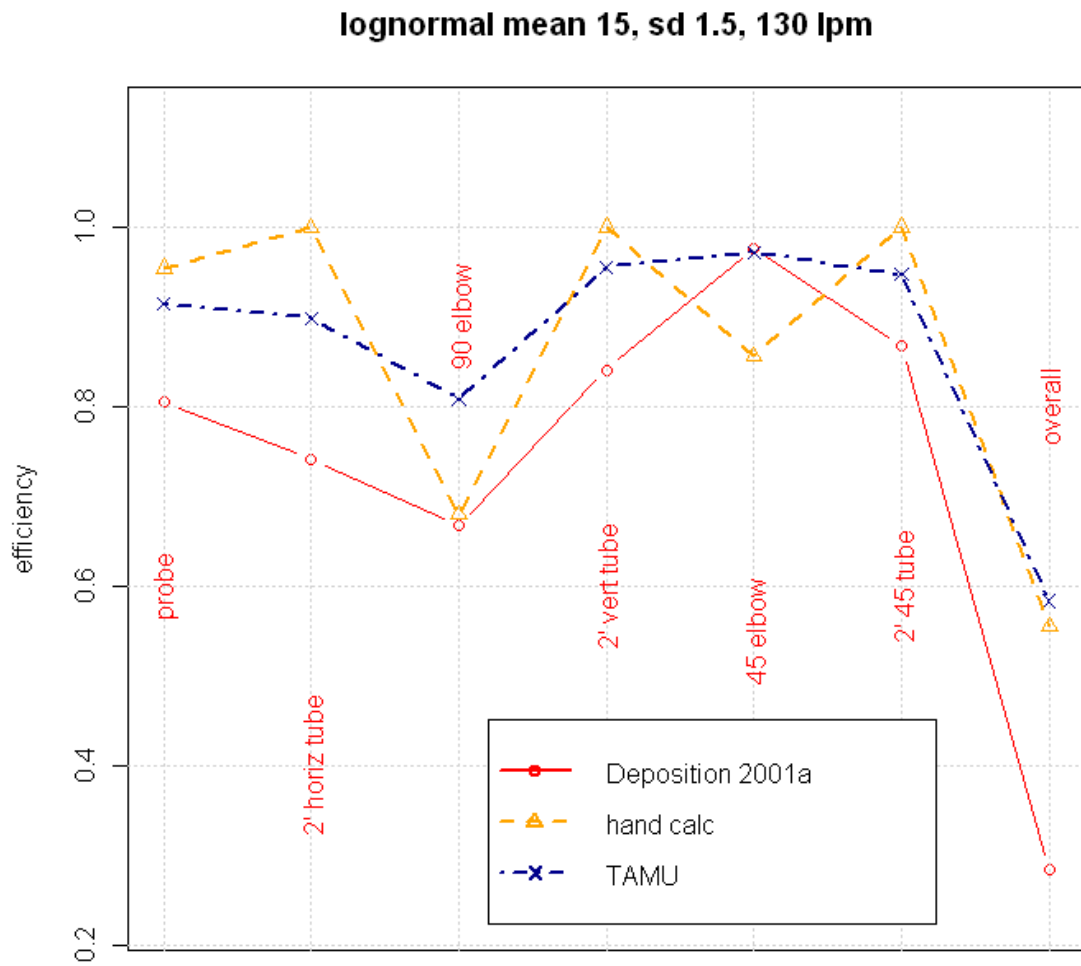
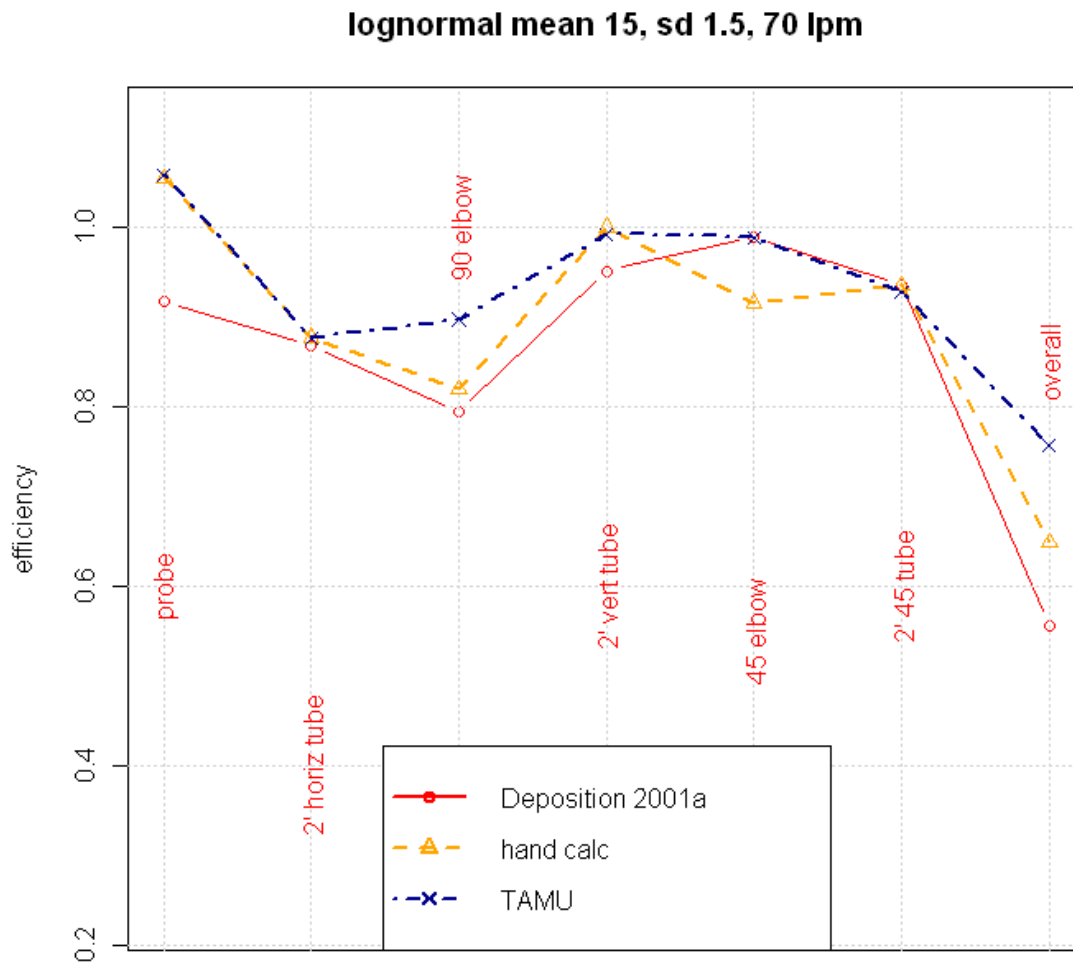


Figure 30: Lognormal Model Survey Results 15, 1.5, 70



**Figure 31: Lognormal Model Survey Results 15, 1.5, 130**

**Discussion of Results**

The hand calculations follow reasonably closely with the TAMU and Deposition models. The most striking results are:

- All three models followed the total penetration experimental results within about 10% penetration for all particle sizes at 70 lpm. Uncertainty range on the experimental data is not available, but it is qualitatively encouraging.
- At 130 lpm, the TAMU and hand calculation results started at around 10% of full penetration lower than the experimental data, but diverged from experimental to about 20% lower at the larger particle sizes. The Deposition model started with the other two models but diverged sharply lower after 10 micrometers.

- Similar divergence of the Deposition code from the other models was seen on the lognormal calculations.
- The Deposition Code has an inflection point around 12.5 micrometers on the tubing models. The Deposition Code development references stress 10 micrometers as the reference diameter of interest, so it is possible that they chose a model that was not as accurate at the larger diameters.
- At 70 lpm, the hand calculations predicted less efficient transport in tubes than the other two models, due to using the conservative laminar flow model in the intermediate flow region (Figure 4).

### CONCLUSION

Hand calculations have been identified that are consistent with their source documents, with other models and with experimental results. Hand calculations were used repeatedly in different situations and produce reasonable results. Differences with other models were expected because development of aerosol science over the years has progressed and various concurrent models are available for use.

The hand calculations identified are valid for use as benchmarks for occupational air sampling designs. Actual application of results to assigned internal dose should be avoided because of uncertainties both in ideal transport models and the vagaries of the real world situation such as variations in aerosol concentrations, particle size, flow and sampling system conditions.

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### Attachment 1: R Script for Anisokinetic Probes – Logarithmic Distribution

```
#####  
# Probes section of Sampling Train Particle Penetration Analysis  
# Using TAMU and Duisburg equations from Ref. 5  
#####  
# Using cgs units except where noted  
#####  
# References  
# 1 Maiello ML and Hoover MD (eds) Chapter 15 Glissmeyer J,  
# Radioactive Air Sampling Methods, CRC Press NY 2011  
# 2 Kulkarni P, Baron PA, Willeke K, Fundamentals of  
# Single Particle Transport in Aerosol Measurement:  
# Principles, Techniques, and  
# Applications, Third Ed, Chapter 2  
# Wiley-InterScience, Inc. 2011  
# 3 Von der Weiden SL, Drewnick F and Borrmann S  
# Atmos. Meas. Tech., 2, 479494, 2009  
# 4 Brockman JE Sampling and Transport of Aerosols  
# in Aerosol Measurement: Principles, Techniques, and  
# Applications, Third Ed, Chapter 6  
# Wiley-InterScience, Inc. 2011  
# 5 Metz D, Harvey P, AEROSOL SAMPLING MODELS SURVEY,  
# GENERAL MANAGEMENT ASSOCIATES, Abingdon, MD 21009, March 1993.  
#  
#####  
density.air.ntp<-1.20484e-3 #g/cm^3  
P.ntp<-1013250.1 #1 atm = this many barye  
# 1 pascal = 0.00001 bar  
viscosity.air.ref<-18.203*1e-5 #Table 4-1 Ref. 2  
Boltzmann<-1.38e-16 # g*cm*(molecule)^-1*(K)^-1*(s)^-2 #boltzmann constant  
mfp.air<-0.0665*1e-4 #Table 4-1 Ref. 2 NTP  
g<-980.7 # acceleration due to gravity cm/s^2  
T.ntp<-293.15 #kelvin = 20 C  
#####  
# Needed parameters  
#####  
T.C<-25  
T<-T.C+273.15 #Enter temperature in Kelvin  
density.particle<-1 #g*cm^-3 okay if using AMAD for particle diameter  
D.tube<-2.667 #1.05" diameter tube  
P.torr<-760  
P<-P.torr*1013250.1/760 #1 atm = this many barye (easy to convert from mb, etc)  
Q.lpm<-130 #LPM (non-cgs)  
Q<-Q.lpm*1000/60 #in tube (cm^3/s) LPM *1000/60 (cgs)  
D.geomean.um<-15 #Enter geometric mean um size  
D.geomean<-D.geomean.um*1e-4  
D.geosd<-1.5 #Enter geometric standard deviation  
angle<-0 # wrt wind  
angle.radians<-angle*pi/180  
wind.speed<-300 #300 cm/s  
#####  
# Generating Lognormal Samples
```

```
#####  
D.p.mu<-seq(from=1e-5,to=7e-3,by=1e-5)*1e4  
D.p.prob.cum<-plnorm(D.p.mu,meanlog=log(D.geomean.mu),sdlog=log(D.geosd))  
D.p<-D.p.mu*1e-4  
D.p.prob<-c(0,0,0) #initiate term  
D.p.prob[1]<-D.p.prob.cum[1]  
for(i in 2:(length(D.p))) {  
  D.p.prob[i]<-D.p.prob.cum[i]-D.p.prob.cum[i-1]  
}  
# plots - run generating sample multiple times for extra points  
# plot(x=D.p.mu,y=D.p.prob,col="red",cex=0.4,  
#   main="Lognormal Particle Distribution",ylim=c(0,0.022))  
# leg<-c("geometric mean 15, sd 1.5","geometric mean 5, sd 1.5")  
# points(x=D.p.mu,y=D.p.prob,log="y",col="blue",cex=0.4)  
# legend("topright",leg,pch=1,col=c("red","blue"))  
#####  
#           General stuff  
#####  
U.o<-4*Q/(pi*(D.tube)^2) #particle speed in sample stream cm/s  
Sutherland<-110.4  
viscosity.air<-viscosity.air.ref*(T.ntp+Sutherland)/(T+Sutherland)*  
  (T/T.ntp)^1.5 #Ref. 2 (4-10)  
#####  
#           Knudsen Number - Ref: 2 Eq. 4-6, 4-7  
#####  
mfp<-mfp.air*(1010000/P)*(T/293)*(1+110/293)/(1+110/T)  
K.n<-2*mfp/D.p #  
#####  
#           Cunningham Slip Correction Factor - Ref: 2 Eq. 4-8  
#####  
#constants valid for the air mfp for solid particles  
alpha.C<-1.142  
beta.C<-0.558  
gamma.C<-0.999  
C.c<-1+K.n*(alpha.C+beta.C*exp(-gamma.C/K.n))  
#####  
#           Stokes - Ref: 1 (15.6)  
#           - Ref: 2 (4-39)  
#####  
Stk<-density.particle*D.p^2*U.o*C.c/(9*viscosity.air*D.tube)  
#####  
#####  
#           Reynolds - Ref: 2 (4-1)  
#####  
density.air.correction<-(P/P.ntp)*(T.ntp/T)  
density.air<-density.air.ntp*density.air.correction  
Re<-density.air*U.o*D.tube/viscosity.air #Re of gas  
Re.p<-6.5*U.o*D.p #Re of particle (Relaxation time valid when Re.p<1)  
#####  
#           Probe Efficiency - Ref 5  
#####  
R<-wind.speed/U.o  
Eta.TAMU<-1+(1-1/(1+1.05*Stk*(cos(angle)+4*(R*sin(angle))^0.5)))*(R*cos(angle)-1)
```

```
Eta.Duisburg<-1+(R-1)*((2*R+0.62)/(R*Stk^-1+2*R+0.62))
#####
#          Summary
#####
aspirated.TAMU.lt1<-sum(Eta.TAMU[which(D.p.um<1)]*D.p.prob[which(D.p.um<1)]/
  sum(D.p.prob[which(D.p.um<1)]))
aspirated.TAMU.1to5<-((sum(Eta.TAMU[which(D.p.um<5)]*D.p.prob[which(D.p.um<5)]))
  -sum(Eta.TAMU[which(D.p.um<1)]*D.p.prob[which(D.p.um<1)]))/
  (sum(D.p.prob[which(D.p.um<5)]))
  -sum(D.p.prob[which(D.p.um<1)]))
aspirated.TAMU.5to10<-((sum(Eta.TAMU[which(D.p.um<10)]*D.p.prob[which(D.p.um<10)]))
  -sum(Eta.TAMU[which(D.p.um<5)]*D.p.prob[which(D.p.um<5)]))/
  (sum(D.p.prob[which(D.p.um<10)]))
  -sum(D.p.prob[which(D.p.um<5)]))
aspirated.TAMU.lt30<-((sum(Eta.TAMU[which(D.p.um<30)]*D.p.prob[which(D.p.um<30)])/
  sum(D.p.prob[which(D.p.um<30)]))
aspirated.TAMU.total<-sum(Eta.TAMU*D.p.prob)/sum(D.p.prob)

aspirated.Duisburg.lt1<-sum(Eta.Duisburg[which(D.p.um<1)]*D.p.prob[which(D.p.um<1)]/
  sum(D.p.prob[which(D.p.um<1)]))
aspirated.Duisburg.1to5<-((sum(Eta.Duisburg[which(D.p.um<5)]*D.p.prob[which(D.p.um<5)]))
  -sum(Eta.Duisburg[which(D.p.um<1)]*D.p.prob[which(D.p.um<1)]))/
  (sum(D.p.prob[which(D.p.um<5)]))
  -sum(D.p.prob[which(D.p.um<1)]))
aspirated.Duisburg.5to10<-((sum(Eta.Duisburg[which(D.p.um<10)]*D.p.prob[which(D.p.um<10)]))
  -sum(Eta.Duisburg[which(D.p.um<5)]*D.p.prob[which(D.p.um<5)]))/
  (sum(D.p.prob[which(D.p.um<10)]))
  -sum(D.p.prob[which(D.p.um<5)]))
aspirated.Duisburg.lt30<-((sum(Eta.Duisburg[which(D.p.um<30)]*D.p.prob[which(D.p.um<30)])/
  sum(D.p.prob[which(D.p.um<30)]))
aspirated.Duisburg.total<-sum(Eta.Duisburg*D.p.prob)/sum(D.p.prob)
#####
#          Write Output
#####
setwd("d:/jobs/mgh151_aerosol_particle_transport_hand_calculations/models survey article
testing/1.05 inch/")
param.df<-data.frame(T.C,P.torr,D.tube,Q.lpm,density.particle,D.geomean.um,D.geosd)
write.table(param.df,file="param.dat",row.names=FALSE,col.names=FALSE)
next.element.df<-data.frame(D.p,Eta.TAMU*D.p.prob)
write.table(next.element.df,file="pass.distribution.1.dat",row.names=FALSE,col.names=FALSE)
# original is to allow a check at each element back to original air conc.
original.df<-data.frame(D.p,D.p.prob)
write.table(original.df,file="original.dat",row.names=FALSE,col.names=FALSE)
#####
#          Plot
#####
main.text<-as.character(paste("lognormal",D.geomean.um,"+/- ",D.geosd,Q.lpm," lpm"))
plot(x=D.p*10000,y=Eta.TAMU*D.p.prob,
  col="blue",cex=0.4,ylab="activity",
  xlab="particle diameter, micrometer",
  main=main.text)
points(x=D.p*10000,y=D.p.prob,
  col="red",pch=2,cex=0.2)
```

---

```
# abline(v=30,col="blue")
leg<-c("ambient","aspirated")
legend("topright",leg,pch=c(1,2),col=c("red","blue"))
#####
#          Report
#####
parameters1<-data.frame(T.C,P.torr,D.tube,Q.lpm,format(Re,digits=4),D.geomean.µm,D.geosd)
colnames(parameters1)<-c("Temp, C","Pressure, Torr","Tube diameter, cm",
"flow rate, lpm","Reynolds","Geomean","Geo sd")
rownames(parameters1)<-c("")
parameters2<-data.frame(D.geomean.µm,D.geosd)

report.limits<-c("<1 µm","1 to 5 µm","5 to 10 µm",<30 µm","total")
report.numbers.TAMU.out<-c(aspirated.TAMU.lt1,aspirated.TAMU.1to5,
aspirated.TAMU.5to10,aspirated.TAMU.lt30,
aspirated.TAMU.total)
report.out.TAMU.df<-data.frame(report.limits,report.numbers.TAMU.out)
report.numbers.Duisburg.out<-c(aspirated.Duisburg.lt1,aspirated.Duisburg.1to5,
aspirated.Duisburg.5to10,aspirated.Duisburg.lt30,
aspirated.Duisburg.total)
report.out.Duisburg.df<-data.frame(report.limits,report.numbers.Duisburg.out)

parameters1
parameters2
"This element"
report.out.TAMU.df
report.out.Duisburg.df
```

**Attachment 2: R Script for Probes in Calm Air (multiple particle sizes)**

```
#####  
# Probes section of Sampling Train Particle Penetration Analysis  
#####  
# Using cgs units except where noted  
#####  
# References  
# 1 Maiello ML and Hoover MD (eds) Chapter 15 Glissmeyer J,  
# Radioactive Air Sampling Methods, CRC Press NY 2011  
# 2 Kulkarni P, Baron PA, Willeke K, Fundamentals of  
# Single Particle Transport in Aerosol Measurement:  
# Principles, Techniques, and  
# Applications, Third Ed, Chapter 2  
# Wiley-InterScience, Inc. 2011  
# 6 Su WC and Vincent JH, Towards a general semi-empirical  
# model for the aspiration efficiencies of aerosol samplers  
# in perfectly calm air, Aerosol Science 35 (2004) 1119-1134  
#####  
density.air.ntp<-1.20484e-3 #g/cm^3  
P.ntp<-1013250.1 #1 atm = this many barye  
# 1 pascal = 0.00001 bar  
viscosity.air.ref<-18.203*1e-5 #Table 4-1 Ref. 2  
Boltzmann<-1.38e-16 # g*cm*(molecule)^-1*(K)^-1*(s)^-2 #Boltzmann constant  
mfp.air<-0.0665*1e-4 #Table 4-1 Ref. 2 NTP  
g<-980.7 # acceleration due to gravity cm/s^2  
T.ntp<-293.15 #kelvin = 20 C  
#####  
# Needed parameters  
#####  
T.C<-25  
T<-T.C+273.15 #Enter temperature in Kelvin  
density.particle<-1 #g*cm^-3 okay if using AMAD for particle diameter  
D.tube<-2.54 #1" diameter tube  
P.torr<-760  
P<-P.torr*1013250.1/760 #1 atm = this many barye (easy to convert from mb,  
etc)  
Q.lpm<-56.6 #LPM (non-cgs)  
Q<-Q.lpm*1000/60 #in tube (cm^3/s) LPM *1000/60 (cgs)  
D.um<-c(1,3,5,7,10,15,20,25)  
D.p<-D.um*1e-4 #particle size, cm (um*e-4), mean AMAD if density set to 1  
#####  
# General stuff  
#####
```

```
U.o<-4*Q/(pi*(D.tube)^2) #particle speed in sample stream cm/s
Sutherland<-110.4
viscosity.air<-viscosity.air.ref*(T.ntp+Sutherland)/(T+Sutherland)*
(T/T.ntp)^1.5 #Ref. 2 (4-10)
#####
#           Knudsen Number - Ref: 2 Eq. 4-6, 4-7
#####
mfp<-mfp.air*(1010000/P)*(T/293)*(1+110/293)/(1+110/T)
K.n<-2*mfp/D.p #
#####
#           Cunningham Slip Correction Factor - Ref: 2 Eq. 4-8
#####
#constants valid for the air mfp for solid particles
alpha.C<-1.142
beta.C<-0.558
gamma.C<-0.999
C.c<-1+K.n*(alpha.C+beta.C*exp(-gamma.C/K.n))
#####
#           Stokes - Ref: 6 (6)
#####
Stk<-density.particle*D.p^2*U.o*C.c/(9*viscosity.air*D.tube)
Stk2<-Stk/2 #different formula in ref 6, using 18 in denominator vs. 9.
#####
#####
#           Reynolds - Ref: 2 (4-1)
#####
density.air.correction<-(P/P.ntp)*(T.ntp/T)
density.air<-density.air.ntp*density.air.correction
Re<-density.air*U.o*D.tube/viscosity.air #Re of gas
Re.p<-6.5*U.o*D.p #Re of particle (Relaxation time valid when Re.p<1)
#####
#           Probe Efficiency - Ref 6 (30)
#####
R<-D.p^2*g/(18*viscosity.air*U.o)
v.settling<-U.o*R
p<-2.2*R^(1/3)*Stk2
q<-75*R^1.7*Stk2
#***** alpha, beta and B are from Ref 6:
# Thin-walled probe facing downwards: alpha = 1; beta = 0; B = 1
# Thin-walled probe facing upwards: alpha = 0; beta = 1; B = 1
# Horizontal thin-walled probe: alpha = 0.8; beta = 0.2; B = 1

orient<-"h" #type in either u for up, d for down or h for horizontal in quotes
```

```
if(orient=="u")alpha<-0; if(orient=="d")alpha<-1;
if(orient=="h")alpha<-0.8
if(orient=="u")beta<-1; if(orient=="d")beta<-0;
if(orient=="h")beta<-0.2
B<-1 #for all thin-walled probes
Eta.calm.air<-(1-0.8*(4*Stk2*R^(3/2)) + 0.08*(4*Stk2*R^(3/2))^2
- alpha*((0.5*R^(1/2))-R*(B^2-1))
- beta*(0.12*R^0.4*(exp(-p)-exp(-q))
- R^(3/2)*(B^(1/2)-1)))
Eta.calm.air[which(Eta.calm.air=="NaN")]<-0
Eta.calm.air[which(Eta.calm.air<0)]<-0 #correct for negative
Eta.calm.air
#####
#          Report
#####
report1<-data.frame(T.C,P.torr,format(Re,digits=4))
colnames(report1)<-c("Temp, C", "Pressure, Torr", "Reynolds No.")
rownames(report1)<-c("")
report3<-data.frame(orient,D.tube,Q/(1000/60))
colnames(report3)<-c("orientation", "tube diameter, cm", "Flow Rate, lpm")
rownames(report3)<-c("")
report4<-data.frame(D.um,format(Stk2,digits=4),format(Eta.calm.air,digits=4))
colnames(report4)<-c("Particle Size, µm", "Stokes Number", "Aspiration Eff.")

report1
report3
report4
```

Attachment 3: Anisokinetic Probe Aspiration Efficiency (multiple particle sizes)

```
0#####
# Probes section of Sampling Train Particle Penetration Analysis
# using cgs units except where noted
#####
# References
# 1 Maiello ML and Hoover MD (eds) Chapter 15 Glissmeyer J,
# Radioactive Air Sampling Methods, CRC Press NY 2011
# 2 Kulkarni P, Baron PA, Willeke K, Fundamentals of
# Single Particle Transport in Aerosol Measurement:
# Principles, Techniques, and
# Applications, Third Ed, Chapter 2
# Wiley-InterScience, Inc. 2011
# 3 Von der Weiden SL, Drewnick F and Borrmann S
# Atmos. Meas. Tech., 2, 479494, 2009
# 4 Brockman JE Sampling and Transport of Aerosols
# in Aerosol Measurement: Principles, Techniques, and
# Applications, Third Ed, Chapter 6
# Wiley-InterScience, Inc. 2011
# 5 Metz D, Harvey P, AEROSOL SAMPLING MODELS SURVEY,
# GENERAL MANAGEMENT ASSOCIATES, Abingdon, MD 21009, March 1993.
# 6 28. Vincent JH, Stevens DC, Mark D, Marshall M, Smith TA,
# On the aspiration characteristics of large-diameter,
# thin-walled aerosol sampling probes at yaw orientations
# with respect to the wind, Journal of Aerosol Science, 17(2) :211-224, 1986.
#####
density.air.ntp<-1.20484e-3 #g/cm^3
P.ntp<-1013250.1 #1 atm = this many barye
# 1 pascal = 0.00001 bar
viscosity.air.ref<-18.203*1e-5 #Table 4-1 Ref. 2
Boltzmann<-1.38e-16 # g*cm*(molecule)^-1*(K)^-1*(s)^-2 #boltzmann constant
mfp.air<-0.0665*1e-4 #Table 4-1 Ref. 2 NTP
g<-980.7 # acceleration due to gravity cm/s^2
T.ntp<-293.15 #kelvin = 20 C
#####
# Needed parameters
#####
T.C<-25
T<-T.C+273.15 #Enter temperature in Kelvin
density.particle<-1 #g*cm^-3 okay if using AMAD for particle diameter
D.tube<-2.54*1.05 #1.05" diameter tube
P.torr<-760
P<-P.torr*1013250.1/760 #1 atm = this many barye (easy to convert from mb, etc)
Q.lpm<-1.5*28.3 #LPM (non-cgs)
Q<-Q.lpm*1000/60 #in tube (cm^3/s) LPM *1000/60 (cgs)
D.mu<-c(1,3,5,7,10,15,20,25)
D.p<-D.mu*1e-4 #particle size, cm (mu*e-4), mean AMAD if density set to 1
angle<-0 # wrt wind
angle.radians<-angle*pi/180
wind.speed<-300 #300 cm/s
#####
# General stuff
```



```
#####  
U.o<-4*Q/(pi*(D.tube)^2) #particle speed in sample stream cm/s  
Sutherland<-110.4  
viscosity.air<-viscosity.air.ref*(T.ntp+Sutherland)/(T+Sutherland)*  
  (T/T.ntp)^1.5 #Ref. 2 (4-10)  
#####  
#      Knudsen Number - Ref: 2 Eq. 4-6, 4-7  
#####  
mfp<-mfp.air*(1010000/P)*(T/293)*(1+110/293)/(1+110/T)  
K.n<-2*mfp/D.p #  
#####  
#      Cunningham Slip Correction Factor - Ref: 2 Eq. 4-8  
#####  
#constants valid for the air mfp for solid particles  
alpha.C<-1.142  
beta.C<-0.558  
gamma.C<-0.999  
C.c<-1+K.n*(alpha.C+beta.C*exp(-gamma.C/K.n))  
#####  
#      Stokes - Ref: 1 (15.6) - Ref: 2 (4-39)  
#####  
Stk<-density.particle*D.p^2*U.o*C.c/(9*viscosity.air*D.tube)  
#####  
#      Reynolds - Ref: 2 (4-1)  
#####  
density.air.correction<-(P/P.ntp)*(T.ntp/T)  
density.air<-density.air.ntp*density.air.correction  
Re<-density.air*U.o*D.tube/viscosity.air #Re of gas  
Re.p<-6*U.o*D.p #Re of particle (Relaxation time valid when Re.p<1)  
#####  
#      Anisokinetic Probe Efficiency - Ref 5  
#####  
R<-U.o/wind.speed  
ifelse(Re<2300,  
Eta.TAMU<-1+(1-1/(1+1.05*Stk*(cos(angle)+4*(R*sin(angle))^0.5)))*(R*cos(angle)-1),  
Eta.TAMU<-NA)  
#####  
#      Report  
#####  
report1<-data.frame(T.C,P.torr,format(Re,digits=4))  
colnames(report1)<-c("Temp, C","Pressure, Torr","Reynolds No.")  
rownames(report1)<-c("")  
report3<-data.frame(angle,D.tube,Q/(1000/60))  
colnames(report3)<-c("angle to horizontal","tube diameter, cm", "Flow Rate, lpm")  
rownames(report3)<-c("")  
report5<-data.frame(wind.speed,U.o,U.o/wind.speed)  
colnames(report5)<-c("wind speed, cm/s","sample speed, cm/s","ratio")  
report6<-data.frame(D.um, angle, format(Eta.TAMU,digits=4))  
colnames(report6)<-c("Particle Size, µm","angle","TAMU asp eff")  
report1  
report3  
report5  
report6
```

#### Attachment 4: R Script for Bends (multiple particle sizes)

```
#####  
# Elbow (Bend) section of Sampling Train Particle Penetration Analysis  
#####  
# Using cgs units except where noted  
#####  
# References  
# 1 Maiello ML and Hoover MD (eds) Chapter 15 Glissmeyer J,  
# Radioactive Air Sampling Methods, CRC Press NY 2011  
# 2 Kulkarni P, Baron PA, Willeke K, Fundamentals of  
# Single Particle Transport in Aerosol Measurement:  
# Principles, Techniques, and  
# Applications, Third Ed, Chapter 2  
# Wiley-InterScience, Inc. 2011  
# 3 Von der Weiden SL, Drewnick F and Borrmann S  
# Atmos. Meas. Tech., 2, 479494, 2009  
# 4 Brockman JE Sampling and Transport of Aerosols  
# in Aerosol Measurement: Principles, Techniques, and  
# Applications, Third Ed, Chapter 6  
# Wiley-InterScience, Inc. 2011  
# 5 Metz D, Harvey P, AEROSOL SAMPLING MODELS SURVEY,  
# GENERAL MANAGEMENT ASSOCIATES, Abingdon, MD 21009, March 1993.  
# 6 McFarland AR et. al. Aerosol Deposition in Bends with  
# Turbulent Flow  
# 7 Zhang P, Roberts RM, Bénard A, Computational  
# guidelines and an empirical model for particle deposition  
# in curved pipes using an Eulerian-Lagrangian approach,  
# Journal of Aerosol Science, 53:1-20, November 2012.  
# 8 Pui DYH, Romay-Novas F and Liu BYH, Experimental Study  
# of Particle Deposition in Bends of Circular Cross Section,  
# Aerosol, Science and Technology, 7:3, 301-315, 1987.  
#####  
density.air.ntp<-1.20484e-3 #g/cm^3  
P.ntp<-1013250.1 #1 atm = this many barye  
# 1 pascal = 0.00001 bar  
viscosity.air.ref<-18.203*1e-5 #Table 4-1 Ref. 2  
Boltzmann<-1.38e-16 # g*cm*(molecule)^-1*(K)^-1*(s)^-2 #boltzmann constant  
mfp.air<-0.0665*1e-4 #Table 4-1 Ref. 2 NTP  
g<-980.7 # acceleration due to gravity cm/s^2  
T.ntp<-293.15 #kelvin = 20 C  
#####  
# Needed parameters  
#####  
T.C<-25  
T<-T.C+273.15 #Enter temperature in Kelvin  
density.particle<-1 #g*cm^-3 okay if using AMAD for particle diameter  
D.tube<-2.54 #1" diameter tube  
P.torr<-760  
P<-P.torr*1013250.1/760 #1 atm = this many barye (easy to convert from mb, etc)  
Q.lpm<-28.3*5 #LPM (non-cgs)  
Q<-Q.lpm*1000/60 #in tube (cm^3/s) LPM *1000/60 (cgs)  
D.um<-c(3,5,7,10,15,20,25)
```

```
D.p<-D.um*1e-4 #particle size, cm (um*e-4), mean AMAD if density set to 1
angle.bend<-90 # bend angle in degrees
angle.bend.radians<-angle.bend*pi/180
ratio.curvature<-4
#####
#           General stuff
#####
U.o<-4*Q/(pi*(D.tube)^2) #particle speed in sample stream cm/s
Sutherland<-110.4
viscosity.air<-viscosity.air.ref*(T.ntp+Sutherland)/(T+Sutherland)*
(T/T.ntp)^1.5 #Ref. 2 (4-10)
#####
#           Knudsen Number - Ref: 2 Eq. 4-6, 4-7
#####
mfp<-mfp.air*(1010000/P)*(T/293)*(1+110/293)/(1+110/T)
K.n<-2*mfp/D.p #
#####
#           Cunningham Slip Correction Factor - Ref: 2 Eq. 4-8
#####
#constants valid for the air mfp for solid particles
alpha<-1.142
beta<-0.558
gamma<-0.999
C.c<-1+K.n*(alpha+beta*exp(-gamma/K.n))
#####
#           Stokes - Ref: 1 (15.6)
#           - Ref: 2 (4-39)
#####
Stk<-density.particle*D.p^2*U.o*C.c/(9*viscosity.air*D.tube)
Stk2<-Stk/2 #different formula in ref 3, using 18 in denominator vs. 9.
#####
#           Reynolds - Ref: 2 (4-1)
#####
density.air.correction<-(P/P.ntp)*(T.ntp/T)
density.air<-density.air.ntp*density.air.correction
Re<-density.air*U.o*D.tube/viscosity.air #Re of gas
Re.p<-6.5*U.o*D.p #Re of particle (Relaxation time valid when Re.p<1)
#####
#           Bend Efficiency - Ref: 1 (15.24)
#           - Ref: 6 (12)
#           - Ref: 3 (30)
#           - Ref: 7 (15)
#####
Eta.bend<-exp(-0.528*angle.bend.radians*Stk^((2)^(1/ratio.curvature))*
ratio.curvature^0.5) # Zhang Ref 7 (15)
# From Pui and McFarland
#comp numbers for 90 degree only
ifelse(angle.bend==90, ifelse(Re<3200,Eta.bend.90<-NA,Eta.bend.90<-10^(-0.963*Stk)),
Eta.bend.90<-NA)
#comp numbers for 45 degree only
ifelse(angle.bend==45, ifelse(Re<3200,
Eta.bend.45<-NA,
```

```
Eta.bend.45<-10^(-0.482*Stk)), Eta.bend.45<-NA)
#General from PLC (Ref 3 (30))
ifelse(Re<2300,
  Eta.bend.plc<-(1+(Stk2/0.171)^(0.452*(Stk2/0.171)+2.242))^(2*angle.bend.radians/pi),
  Eta.bend.plc<-exp(-2.823*Stk2*angle.bend.radians))
# for comparison - McFarland 1997
a<--0.9526-0.05686*ratio.curvature
b<-(-0.297-0.0174*ratio.curvature)/(1-0.07*ratio.curvature+0.0171*ratio.curvature^2)
c<--0.306+1.895/sqrt(ratio.curvature)-2/ratio.curvature
d<-(-0.131-0.0132*ratio.curvature+0.000383*ratio.curvature^2)/
  (1-0.129*ratio.curvature+0.0136*ratio.curvature^2)
ifelse(ratio.curvature>2,
  lnP<-(4.61+a*angle.bend.radians*Stk)/(1+b*angle.bend.radians*Stk+
    c*angle.bend.radians*Stk^2+d*angle.bend.radians^2*Stk),
  NA)
ifelse(ratio.curvature>2,
  ifelse(Re<2300,Eta.bend.McFarland<-NA,
    Eta.bend.McFarland<-exp(lnP/100),Eta.bend.McFarland<-NA)
  Eta.bend.McFarland[which(Eta.bend.McFarland>1)]<-1
  #####
  #          Report
  #####
  report1<-data.frame(T,C,P.torr,format(Re,digits=4))
  colnames(report1)<-c("Temp, C","Pressure, Torr","Reynolds No.")
  rownames(report1)<-c("")
  report2<-data.frame(angle.bend,D.tube,Q/(1000/60))
  colnames(report2)<-c("angle","tube diameter, cm", "Flow Rate, lpm")
  rownames(report2)<-c("")
  report3<-data.frame(D.um,format(Eta.bend,digits=4),format(Eta.bend.McFarland,digits=4),
    format(Eta.bend.plc,digits=4),format(Eta.bend.90,digits=4),
    format(Eta.bend.45,digits=4))
  colnames(report3)<-c("Particle Size, um","Transmission Eff.",
    "McFarland","Von der Weiden","Pui 90","Pui 45")
  report4<-data.frame(D.um,format(Stk,digits=4),format(Eta.bend,digits=4))
  colnames(report4)<-c("Particle Size, um","Stokes Number","Transmission Eff.")
  report1
  report2
  report3
  report4

  #####
  #          Plots
  #####
  plot(x=Stk,y=Eta.bend,col="#0000ff",
    ylab="transmission efficiency",main="bend transport efficiency",type="b",
    ylim=range(0,1))
  points(x=Stk,y=Eta.bend.plc,col="#ff0000",pch=2)
  lines(x=Stk,y=Eta.bend.plc,col="#ff0000")
  points(x=Stk,y=Eta.bend.McFarland,col="#00ff00",pch=3)
  lines(x=Stk,y=Eta.bend.McFarland,col="#00ff00")
  grid()
  leg<-c("Zhang 2012","VdW 2012 (Pui 1987)","McFarland 1997")
  legend("topright",leg,pch=c(1,2,3),col=c("#0000ff","#ff0000","#00ff00"))
```

Attachment 5: R Script for Bends (model comparisons)

```
#####  
# Elbow (Bend) section of Sampling Train Particle Penetration Analysis  
#####  
# Using cgs units except where noted  
#####  
# References  
# 1 Maiello ML and Hoover MD (eds) Chapter 15 Glissmeyer J,  
# Radioactive Air Sampling Methods, CRC Press NY 2011  
# 2 Kulkarni P, Baron PA, Willeke K, Fundamentals of  
# Single Particle Transport in Aerosol Measurement:  
# Principles, Techniques, and  
# Applications, Third Ed, Chapter 2  
# Wiley-InterScience, Inc. 2011  
# 3 Von der Weiden SL, Drewnick F and Borrmann S  
# Atmos. Meas. Tech., 2, 479494, 2009  
# 4 Brockman JE Sampling and Transport of Aerosols  
# in Aerosol Measurement: Principles, Techniques, and  
# Applications, Third Ed, Chapter 6  
# Wiley-InterScience, Inc. 2011  
# 5 Metz D, Harvey P, AEROSOL SAMPLING MODELS SURVEY,  
# GENERAL MANAGEMENT ASSOCIATES, Abingdon, MD 21009, March 1993.  
# 6 McFarland AR et. al. Aerosol Deposition in Bends with  
# Turbulent Flow  
# 7 Zhang P, Roberts RM, Bénard A, Computational  
# guidelines and an empirical model for particle deposition  
# in curved pipes using an Eulerian-Lagrangian approach,  
# Journal of Aerosol Science, 53:1-20, November 2012.  
# 8 Pui DYH, Romay-Novas F and Liu BYH, Experimental Study  
# of Particle Deposition in Bends of Circular Cross Section,  
# Aerosol, Science and Technology, 7:3, 301-315, 1987.  
#####  
density.air.ntp<-1.20484e-3 #g/cm^3  
P.ntp<-1013250.1 #1 atm = this many barye  
# 1 pascal = 0.00001 bar  
viscosity.air.ref<-18.203*1e-5 #Table 4-1 Ref. 2  
Boltzmann<-1.38e-16 # g*cm*(molecule)^-1*(K)^-1*(s)^-2 #boltzmann constant  
mfp.air<-0.0665*1e-4 #Table 4-1 Ref. 2 NTP  
g<-980.7 # acceleration due to gravity cm/s^2  
T.ntp<-293.15 #kelvin = 20 C  
#####  
# Needed parameters  
#####  
T.C<-25  
T<-T.C+273.15 #Enter temperature in Kelvin  
density.particle<-1 #g*cm^-3 okay if using AMAD for particle diameter  
D.tube<-2.54 #1" diameter tube  
P.torr<-760  
P<-P.torr*1013250.1/760 #1 atm = this many barye (easy to convert from mb, etc)  
Q.lpm<-28.3*5 #LPM (non-cgs)  
Q<-Q.lpm*1000/60 #in tube (cm^3/s) LPM *1000/60 (cgs)  
D.um<-c(3,5,7,10,15,20,25)
```

```
D.p<-D.um*1e-4 #particle size, cm (um*e-4), mean AMAD if density set to 1
angle.bend<-90 # bend angle in degrees
angle.bend.radians<-angle.bend*pi/180
ratio.curvature<-4
#####
#           General stuff
#####
U.o<-4*Q/(pi*(D.tube)^2) #particle speed in sample stream cm/s
Sutherland<-110.4
viscosity.air<-viscosity.air.ref*(T.ntp+Sutherland)/(T+Sutherland)*
(T/T.ntp)^1.5 #Ref. 2 (4-10)
#####
#           Knudsen Number - Ref: 2 Eq. 4-6, 4-7
#####
mfp<-mfp.air*(1010000/P)*(T/293)*(1+110/293)/(1+110/T)
K.n<-2*mfp/D.p #
#####
#           Cunningham Slip Correction Factor - Ref: 2 Eq. 4-8
#####
#constants valid for the air mfp for solid particles
alpha<-1.142
beta<-0.558
gamma<-0.999
C.c<-1+K.n*(alpha+beta*exp(-gamma/K.n))
#####
#           Stokes - Ref: 1 (15.6)
#           - Ref: 2 (4-39)
#####
Stk<-density.particle*D.p^2*U.o*C.c/(9*viscosity.air*D.tube)
Stk2<-Stk/2 #different formula in ref 3, using 18 in denominator vs. 9.
#####
#           Reynolds - Ref: 2 (4-1)
#####
density.air.correction<-(P/P.ntp)*(T.ntp/T)
density.air<-density.air.ntp*density.air.correction
Re<-density.air*U.o*D.tube/viscosity.air #Re of gas
Re.p<-6.5*U.o*D.p #Re of particle (Relaxation time valid when Re.p<1)
#####
#           Bend Efficiency - Ref: 1 (15.24)
#           - Ref: 6 (12)
#           - Ref: 3 (30)
#           - Ref: 7 (15)
#####
Eta.bend<-exp(-0.528*angle.bend.radians*Stk^((2)^(1/ratio.curvature))*
ratio.curvature^0.5) # Zhang Ref 7 (15)
# From Pui and McFarland
#comp numbers for 90 degree only
ifelse(angle.bend==90, ifelse(Re<3200,Eta.bend.90<-NA,Eta.bend.90<-10^(-0.963*Stk)),
Eta.bend.90<-NA)
#comp numbers for 45 degree only
ifelse(angle.bend==45, ifelse(Re<3200,
Eta.bend.45<-NA,
```

```
Eta.bend.45<-10^(-0.482*Stk)), Eta.bend.45<-NA)
#General from PLC (Ref 3 (30))
ifelse(Re<2300,
  Eta.bend.plc<-(1+(Stk2/0.171)^(0.452*(Stk2/0.171)+2.242))^(2*angle.bend.radians/pi),
  Eta.bend.plc<-exp(-2.823*Stk2*angle.bend.radians))

# for comparison - McFarland 1997
a<--0.9526-0.05686*ratio.curvature
b<-(-0.297-0.0174*ratio.curvature)/(1-0.07*ratio.curvature+0.0171*ratio.curvature^2)
c<--0.306+1.895/sqrt(ratio.curvature)-2/ratio.curvature
d<-(0.131-0.0132*ratio.curvature+0.000383*ratio.curvature^2)/
  (1-0.129*ratio.curvature+0.0136*ratio.curvature^2)
ifelse(ratio.curvature>2,
  lnP<-(4.61+a*angle.bend.radians*Stk)/(1+b*angle.bend.radians*Stk+
    c*angle.bend.radians*Stk^2+d*angle.bend.radians^2*Stk),
  NA)
ifelse(ratio.curvature>2,
  ifelse(Re<2300,Eta.bend.McFarland<-NA,
    Eta.bend.McFarland<-exp(lnP/100),Eta.bend.McFarland<-NA)
  Eta.bend.McFarland[which(Eta.bend.McFarland>1)]<-1
  #####
#      Report
#####
report1<-data.frame(T.C,P.torr,format(Re,digits=4))
colnames(report1)<-c("Temp, C", "Pressure, Torr", "Reynolds No.")
rownames(report1)<-c("")
report2<-data.frame(angle.bend,D.tube,Q/(1000/60))
colnames(report2)<-c("angle", "tube diameter, cm", "Flow Rate, lpm")
rownames(report2)<-c("")
report3<-data.frame(D.um,format(Eta.bend,digits=4),format(Eta.bend.McFarland,digits=4),
  format(Eta.bend.plc,digits=4),format(Eta.bend.90,digits=4),
  format(Eta.bend.45,digits=4))
colnames(report3)<-c("Particle Size, um", "Transmission Eff.",
  "McFarland", "Von der Weiden", "Pui 90", "Pui 45")
report4<-data.frame(D.um,format(Stk,digits=4),format(Eta.bend,digits=4))
colnames(report4)<-c("Particle Size, um", "Stokes Number", "Transmission Eff.")
report1
report2
report3
report4
#####
#      Plots
#####
plot(x=Stk,y=Eta.bend,col="#0000ff",
  ylab="transmission efficiency",main="bend transport efficiency",type="b",
  ylim=range(0,1))
points(x=Stk,y=Eta.bend.plc,col="#ff0000",pch=2)
lines(x=Stk,y=Eta.bend.plc,col="#ff0000")
points(x=Stk,y=Eta.bend.McFarland,col="#00ff00",pch=3)
lines(x=Stk,y=Eta.bend.McFarland,col="#00ff00")
grid()
leg<-c("Zhang 2012", "VdW 2012 (Pui 1987)", "McFarland 1997")
legend("topright",leg,pch=c(1,2,3),col=c("#0000ff", "#ff0000", "#00ff00"))
```

### Attachment 6: R Script for Tubes (multiple particle sizes)

```
#####  
# Tube section of Sampling Train Particle Penetration Analysis  
# for straight tubes  
# for monodisperse particle sizes  
#####  
# Using cgs units except where noted  
#####  
# References  
# 1 Maiello ML and Hoover MD  
# Radioactive Air Sampling Methods  
# CRC Press NY 2011  
# Eq (15.6)  
# 2 Kulkarni P, Baron PA, Willeke K, Fundamentals of  
# Single Particle Transport in Aerosol Measurement:  
# Principles, Techniques, and  
# Applications, Third Ed, Chapter 2  
# Wiley-InterScience, Inc. 2011  
# 3 Von der Weiden SL, Drewnick F and Borrmann S  
# Atmos. Meas. Tech., 2, 479494, 2009  
#####  
# Constants  
#####  
density.air.ntp<-1.20484e-3 #g/cm^3  
P.ntp<-1013250.1 #1 atm = this many barye  
# 1 pascal = 0.00001 bar  
viscosity.air.ref<-18.203*1e-5 #Table 4-1 Ref. 2  
Boltzmann<-1.38e-16 # g*cm*(molecule)^-1*(K)^-1*(s)^-2 #Boltzmann constant  
mfp.air<-0.0665*1e-4 #Table 4-1 Ref. 2 NTP  
g<-980.7 # acceleration due to gravity cm/s^2  
T.ntp<-293.15 #kelvin = 20 C  
#####  
# Needed parameters  
#####  
T.C<-25  
T<-T.C+273.15 #Enter temperature in Kelvin  
density.particle<-1 #g*cm^-3 okay if using AMAD for particle diameter  
D.tube<-2.54 #1" diameter tube  
angle.incline<-45 #angle of incline for straight tube  
angle.incline.radians<-angle.incline*pi/180  
P.torr<-760  
P<-P.torr*1013250.1/760 #1 atm = this many barye (easy to convert from mb, etc)  
Q.lpm<-28.3*5 #LPM (non-cgs)  
Q<-Q.lpm*1000/60 #in tube (cm^3/s) LPM *1000/60 (cgs)  
D.um<-c(1,3,5,7,10,15,20,25)  
D.p<-D.um*1e-4 #particle size, cm (um*e-4), mean AMAD if density set to 1  
L<-100 #tube length, cm  
#####  
# General stuff  
#####  
U.o<-4*Q/(pi*(D.tube)^2) #particle speed in sample stream cm/s
```



```
Sutherland<-110.4
viscosity.air<-viscosity.air.ref*(T.ntp+Sutherland)/(T+Sutherland)*
(T/T.ntp)^1.5 #Ref. 2 (4-10)
#####
#           Knudsen Number - Ref: 2 Eq. 4-6, 4-7
#####
mfp<-mfp.air*(1010000/P)*(T/293)*(1+110/293)/(1+110/T)
K.n<-2*mfp/D.p #
#####
#           Cunningham Slip Correction Factor - Ref: 2 Eq. 4-8
#####
#constants valid for the air mfp for solid particles
alpha<-1.142
beta<-0.558
gamma<-0.999
C.c<-1+K.n*(alpha+beta*exp(-gamma/K.n))
#####
#           Stokes - Ref: 1 (15.6)
#           - Ref: 2 (4-39)
#####
Stk<-density.particle*D.p^2*U.o*C.c/(9*viscosity.air*D.tube)
#####
#           Reynolds - Ref: 2 (4-1)
#####
density.air.correction<-(P/P.ntp)*(T.ntp/T)
density.air<-density.air.ntp*density.air.correction
Re<-density.air*U.o*D.tube/viscosity.air #Re of gas
Re.p<-6.5*U.o*D.p #Re of particle (Relaxation time valid when Re.p<1)
#####
#           Sedimentation - Ref: 3 (24 & 25)
#####
t.relax<-density.particle*D.p^2*C.c/(18*viscosity.air) #Ref. 2 (4-34) Particle Relaxation Time
v.term.settling<-t.relax*g
if(Re<=4000) {
Z=L*v.term.settling/(D.tube*U.o)
e.grav.dep<-(3/4*Z)*cos(angle.incline.radians) #,
Eta.grav<-1-(2/pi)*
(2*e.grav.dep*sqrt(1-e.grav.dep^(2/3))
-e.grav.dep^(1/3)*sqrt(1-e.grav.dep^(2/3))
+asin(e.grav.dep^(1/3))) }
if(Re>4000) Eta.grav<-exp(-L*v.term.settling*cos(angle.incline.radians)/Q)
ifelse(Re>4000,form.txt<-" - using turbulent formula",
form.txt<-" - using laminar formula")
#####
#           Diffusion - Ref: 3 (21)
#           - Ref: 2 (4-13)
# ifelse esures that this only applies to particles<100 nm (0.1 µm)
#####
# Diff<-Boltzmann*T*C.c/(3*pi*viscosity.air*D.p) #diffusion coefficient
# Xi<-pi*Diff*L/Q #noted under Ref. 3 (21)
# Schmidt<-viscosity.air/(density.air)/Diff #Schmidt Number
# ifelse(Re<2000,Sherwood<-3.66+(0.2672/(Xi+0.10079*Xi^(1/3))),
```

```
# Sherwood<-0.0118*Re^(7/8)*Schmidt^(1/3)
# ifelse(D.p<1e-5,Eta.diff<-exp(-Xi*Schmidt),Eta.diff<-1)
#####
# Turbulent Inertial Deposition - Ref: 3 (28)
# ifelse esures that this only applies to high Reynolds number
#####
V.t<-(6e-4*(0.0395*Stk*Re^(3/4))^2+2e-8*Re)*U.o/(5.03*Re^(1/8))
ifelse(Re>4000,Eta.turb.inert<-exp(-pi*D.p*L*V.t/Q),Eta.turb.inert<-1)
#####
# Total Efficiency
Eta.total<-Eta.grav*Eta.turb.inert
#####
# Report
#####
report1<-data.frame(T.C,P.torr,format(Re,digits=4),form.txt)
colnames(report1)<-c("Temp, C","Pressure, Torr","Reynolds No.", "")
rownames(report1)<-c("")
report3<-data.frame(L,angle.incline,D.tube,Q/(1000/60))
colnames(report3)<-c("Tube Length, cm", "angle of inclination",
"tube diameter, cm", "Flow Rate, lpm")
rownames(report3)<-c("")
report4<-data.frame(D.um,Stk,format(Eta.turb.inert,digits=4),
format(Eta.grav,digits=4),format(Eta.total,digits=4))
colnames(report4)<-c("Particle Size, µm","Stokes Number","Turb. Inert. Eff.",
"Grav. Set. Eff.,"Total Eff.")

report1
report3
report4
```

**Attachment 7: R Script for Thin-Walled Probe in Calm Air (multiple flow rates)**

```
#####  
# Probes section of Sampling Train Particle Penetration Analysis  
#####  
# Using cgs units except where noted  
#####  
# References  
# 1 Maiello ML and Hoover MD (eds) Chapter 15 Glissmeyer J,  
# Radioactive Air Sampling Methods, CRC Press NY 2011  
# 2 Kulkarni P, Baron PA, Willeke K, Fundamentals of  
# Single Particle Transport in Aerosol Measurement:  
# Principles, Techniques, and  
# Applications, Third Ed, Chapter 2  
# Wiley-InterScience, Inc. 2011  
# 6 Su WC and Vincent JH, Towards a general semi-empirical  
# model for the aspiration efficiencies of aerosol samplers  
# in perfectly calm air, Aerosol Science 35 (2004) 1119-1134  
#####  
density.air.ntp<-1.20484e-3 #g/cm^3  
P.ntp<-1013250.1 #1 atm = this many barye  
# 1 pascal = 0.00001 bar  
viscosity.air.ref<-18.203*1e-5 #Table 4-1 Ref. 2  
Boltzmann<-1.38e-16 # g*cm*(molecule)^-1*(K)^-1*(s)^-2 #Boltzmann constant  
mfp.air<-0.0665*1e-4 #Table 4-1 Ref. 2 NTP  
g<-980.7 # acceleration due to gravity cm/s^2  
T.ntp<-293.15 #kelvin = 20 C  
#####  
# Needed parameters  
#####  
T.C<-25  
T<-T.C+273.15 #Enter temperature in Kelvin  
density.particle<-1 #g*cm^-3 okay if using AMAD for particle diameter  
D.tube<-1.27 # diameter tube  
P.torr<-760  
P<-P.torr*1013250.1/760 #1 atm = this many barye (easy to convert from mb, etc)  
Q.lpm<-28.3*c(0.5,1,2,3) #LPM (non-cgs)  
Q<-Q.lpm*1000/60 #in tube (cm^3/s) LPM *1000/60 (cgs)  
D.um<-1  
D.p<-D.um*1e-4 #particle size, cm (um*e-4), mean AMAD if density set to 1  
#####  
# General stuff  
#####  
U.o<-4*Q/(pi*(D.tube)^2) #particle speed in sample stream cm/s  
Sutherland<-110.4  
viscosity.air<-viscosity.air.ref*(T.ntp+Sutherland)/(T+Sutherland)*  
(T/T.ntp)^1.5 #Ref. 2 (4-10)  
#####  
# Knudsen Number - Ref: 2 Eq. 4-6, 4-7  
#####  
mfp<-mfp.air*(1010000/P)*(T/293)*(1+110/293)/(1+110/T)  
K.n<-2*mfp/D.p #  
#####
```

```
# Cunningham Slip Correction Factor - Ref: 2 Eq. 4-8
#####
#constants valid for the air mfp for solid particles
alpha.C<-1.142
beta.C<-0.558
gamma.C<-0.999
C.c<-1+K.n*(alpha.C+beta.C*exp(-gamma.C/K.n))
#####
# Stokes - Ref: 6 (6)
#####
Stk<-density.particle*D.p^2*U.o*C.c/(9*viscosity.air*D.tube)
Stk2<-Stk/2 #different formula in ref 6, using 18 in denominator vs. 9.
#####
#####
# Reynolds - Ref: 2 (4-1)
#####
density.air.correction<-(P/P.ntp)*(T.ntp/T)
density.air<-density.air.ntp*density.air.correction
Re<-density.air*U.o*D.tube/viscosity.air #Re of gas
Re.p<-6.5*U.o*D.p #Re of particle (Relaxation time valid when Re.p<1)
#####
# Probe Efficiency - Ref 6 (30)
#####
R<-D.p^2*g/(18*viscosity.air*U.o)
v.settling<-U.o*R
p<-2.2*R^(1/3)*Stk2
q<-75*R^1.7*Stk2
#***** alpha, beta and B are from Ref 6:
# Thin-walled probe facing downwards: alpha = 1; beta = 0; B = 1
# Thin-walled probe facing upwards: alpha = 0; beta = 1; B = 1
# Horizontal thin-walled probe: alpha = 0.8; beta = 0.2; B = 1

orient<-"h" #type in either u for up, d for down or h for horizontal in quotes

if(orient=="u")alpha<-0; if(orient=="d")alpha<-1;
if(orient=="h")alpha<-0.8
if(orient=="u")beta<-1; if(orient=="d")beta<-0;
if(orient=="h")beta<-0.2
B<-1 #for all thin-walled probes
Eta.calm.air<-(-1-0.8*(4*Stk2*R^(3/2)) + 0.08*(4*Stk2*R^(3/2)))^2
- alpha*((0.5*R^(1/2))-R*(B^2-1))
- beta*(0.12*R^0.4*(exp(-p)-exp(-q))
- R^(3/2)*(B^(1/2)-1)))
Eta.calm.air[which(Eta.calm.air=="NaN")]<-0
Eta.calm.air[which(Eta.calm.air<0)]<-0 #correct for negative
Eta.calm.air
#####
# Report
#####
report1<-data.frame(T.C,P.torr,D.tube,D.um)
colnames(report1)<-c("Temp, C","Pressure, Torr","tube diameter, cm","particle size, um")
rownames(report1)<-c("")
report3<-data.frame(orient,Q/(1000/60),format(Re,digits=4))
```

```
colnames(report3)<-c("orientation", "Flow Rate, lpm", "Reynolds No.")  
report4<-data.frame(Q/(1000/60),format(Stk2,digits=4),format(Eta.calm.air,digits=4))  
colnames(report4)<-c("Flow Rate, lpm", "Stokes Number", "Aspiration Eff.")
```

```
report1  
report3  
report4
```

Attachment 8: R Script for Tubes (multiple flow rates)

```
#####  
# Tube section of Sampling Train Particle Penetration Analysis  
# for straight tubes  
# for monodisperse particle sizes  
#####  
# Using cgs units except where noted  
#####  
# References  
# 1 Maiello ML and Hoover MD  
# Radioactive Air Sampling Methods  
# CRC Press NY 2011  
# Eq (15.6)  
# 2 Kulkarni P, Baron PA, Willeke K, Fundamentals of  
# Single Particle Transport in Aerosol Measurement:  
# Principles, Techniques, and  
# Applications, Third Ed, Chapter 2  
# Wiley-InterScience, Inc. 2011  
# 3 Von der Weiden SL, Drewnick F and Borrmann S  
# Atmos. Meas. Tech., 2, 479494, 2009  
#####  
# Constants  
#####  
density.air.ntp<-1.20484e-3 #g/cm^3  
P.ntp<-1013250.1 #1 atm = this many barye  
# 1 pascal = 0.00001 bar  
viscosity.air.ref<-18.203*1e-5 #Table 4-1 Ref. 2  
Boltzmann<-1.38e-16 # g*cm*(molecule)^-1*(K)^-1*(s)^-2 #Boltzmann constant  
mfp.air<-0.0665*1e-4 #Table 4-1 Ref. 2 NTP  
g<-980.7 # acceleration due to gravity cm/s^2  
T.ntp<-293.15 #kelvin = 20 C  
#####  
# Needed parameters  
#####  
T.C<-25  
T<-T.C+273.15 #Enter temperature in Kelvin  
density.particle<-1 #g*cm^-3 okay if using AMAD for particle diameter  
D.tube<-1.27 #.5" diameter tube  
angle.incline<-0 #angle of incline for straight tube  
angle.incline.radians<-angle.incline*pi/180  
P.torr<-760  
P<-P.torr*1013250.1/760 #1 atm = this many barye (easy to convert from mb, etc)  
Q.lpm<-28.3*c(0.5,1,2,3) #LPM (non-cgs)  
Q<-Q.lpm*1000/60 #in tube (cm^3/s) LPM *1000/60 (cgs)  
D.um<-10  
D.p<-D.um*1e-4 #particle size, cm (um*e-4), mean AMAD if density set to 1  
L<-92.71 #tube length, cm  
#####  
# General stuff  
#####  
U.o<-4*Q/(pi*(D.tube)^2) #particle speed in sample stream cm/s  
Sutherland<-110.4
```

```
viscosity.air<-viscosity.air.ref*(T.ntp+Sutherland)/(T+Sutherland)*  
  (T/T.ntp)^1.5 #Ref. 2 (4-10)  
#####  
#      Knudsen Number - Ref: 2 Eq. 4-6, 4-7  
#####  
mfp<-mfp.air*(1010000/P)*(T/293)*(1+110/293)/(1+110/T)  
K.n<-2*mfp/D.p #  
#####  
#      Cunningham Slip Correction Factor - Ref: 2 Eq. 4-8  
#####  
#constants valid for the air mfp for solid particles  
alpha<-1.142  
beta<-0.558  
gamma<-0.999  
C.c<-1+K.n*(alpha+beta*exp(-gamma/K.n))  
#####  
#      Stokes - Ref: 1 (15.6)  
#      - Ref: 2 (4-39)  
#####  
Stk<-density.particle*D.p^2*U.o*C.c/(9*viscosity.air*D.tube)  
#####  
#####  
#      Reynolds - Ref: 2 (4-1)  
#####  
density.air.correction<-(P/P.ntp)*(T.ntp/T)  
density.air<-density.air.ntp*density.air.correction  
Re<-density.air*U.o*D.tube/viscosity.air #Re of gas  
Re.p<-6.5*U.o*D.p #Re of particle (Relaxation time valid when Re.p<1)  
#####  
#      Sedimentation - Ref: 3 (24 & 25)  
#####  
t.relax<-density.particle*D.p^2*C.c/(18*viscosity.air) #Ref. 2 (4-34) Particle Relaxation Time  
v.term.settling<-t.relax*g  
#initiate Eta.grav, form.txt  
Eta.grav<-c(9,9,9,9)  
form.txt<-c(NA,NA,NA,NA)  
e.grav.dep<-c(1,1,1,1)  
Z<-c(1,1,1,1)  
for(i in 1:4) {  
  if(Re[i]<=4000) {  
    Z[i]=L*v.term.settling/(D.tube*U.o[i])  
    e.grav.dep[i]<-(3/4*Z[i])*cos(angle.incline.radians) #  
    e.grav.dep[i]<-(3/4*Z[i])*cos(angle.incline.radians) #  
    Eta.grav[i]<-1-(2/pi)*  
      (2*e.grav.dep[i]*sqrt(1-e.grav.dep[i]^(2/3))  
      -e.grav.dep[i]^(1/3)*sqrt(1-e.grav.dep[i]^(2/3))  
      +asin(e.grav.dep[i]^(1/3)))  
  }  
  if(Re[i]>4000) {  
    Eta.grav[i]<-exp(-L*v.term.settling*cos(angle.incline.radians)/Q[i]) }  
  ifelse(Re[i]>4000,form.txt[i]<-" - using turbulent formula",  
    form.txt[i]<-" - using laminar formula") }  
#####
```

```
# Diffusion - Ref: 3 (21)
# - Ref: 2 (4-13)
# ifelse esures that this only applies to particles<100 nm (0.1 µm)
#####
# Diff<-Boltzmann*T*C.c/(3*pi*viscosity.air*D.p) #diffusion coefficient
# Xi<-pi*Diff*L/Q #noted under Ref. 3 (21)
# Schmidt<-viscosity.air/(density.air)/Diff #Schmidt Number
# ifelse(Re<2000,Sherwood<-3.66+(0.2672/(Xi+0.10079*Xi^(1/3))),
# Sherwood<-0.0118*Re^(7/8)*Schmidt^(1/3))
# ifelse(D.p<1e-5,Eta.diff<-exp(-Xi*Schmidt),Eta.diff<-1)
#####
# Turbulent Inertial Deposition - Ref: 3 (28)
# ifelse esures that this only applies to high Reynolds number
#####
V.t<-(-6e-4*(0.0395*Stk*Re^(3/4))^2+2e-8*Re)*U.o/(5.03*Re^(1/8))
ifelse(Re>4000,Eta.turb.inert<-exp(-pi*D.p*L*V.t/Q),Eta.turb.inert<-1)
#####
# Total Efficiency
Eta.total<-Eta.grav*Eta.turb.inert
#####
# Report
#####
report1<-data.frame(T.C,P.torr,D.tube,D.µm)
colnames(report1)<-c("Temp, C","Pressure, Torr","tube diameter, cm","particle size, µm")
rownames(report1)<-c("")
report3<-data.frame(L,angle.incline,Q/(1000/60),format(Re,digits=4),form.txt)
colnames(report3)<-c("Tube Length, cm", "angle of incl.",
"Flow Rate, lpm","Reynolds No.", "")
report4<-data.frame(Q/(1000/60),format(Stk,digits=4),format(Eta.turb.inert,digits=4),
format(Eta.grav,digits=4),format(Eta.total,digits=4))
colnames(report4)<-c("Flow Rate, lpm","Stokes","Turb. Inert.",
"Grav. Set.,"Total Eff.")

report1
report3
report4
```



Attachment 9: R Script for Bends (multiple flow rates)

```
#####  
# Elbow (Bend) section of Sampling Train Particle Penetration Analysis  
#####  
# Using cgs units except where noted  
#####  
# References  
# 1 Maiello ML and Hoover MD (eds) Chapter 15 Glissmeyer J,  
# Radioactive Air Sampling Methods, CRC Press NY 2011  
# 2 Kulkarni P, Baron PA, Willeke K, Fundamentals of  
# Single Particle Transport in Aerosol Measurement:  
# Principles, Techniques, and  
# Applications, Third Ed, Chapter 2  
# Wiley-InterScience, Inc. 2011  
# 3 Von der Weiden SL, Drewnick F and Borrmann S  
# Atmos. Meas. Tech., 2, 479494, 2009  
# 4 Brockman JE Sampling and Transport of Aerosols  
# in Aerosol Measurement: Principles, Techniques, and  
# Applications, Third Ed, Chapter 6  
# Wiley-InterScience, Inc. 2011  
# 5 Metz D, Harvey P, AEROSOL SAMPLING MODELS SURVEY,  
# GENERAL MANAGEMENT ASSOCIATES, Abingdon, MD 21009, March 1993.  
# 6 McFarland AR et. al. Aerosol Deposition in Bends with  
# Turbulent Flow  
# 7 Zhang P, Roberts RM, Bénard A, Computational  
# guidelines and an empirical model for particle deposition  
# in curved pipes using an Eulerian-Lagrangian approach,  
# Journal of Aerosol Science, 53:1-20, November 2012.  
# 8 Pui DYH, Romay-Novas F and Liu BYH, Experimental Study  
# of Particle Deposition in Bends of Circular Cross Section,  
# Aerosol, Science and Technology, 7:3, 301-315, 1987.  
#####  
density.air.ntp<-1.20484e-3 #g/cm^3  
P.ntp<-1013250.1 #1 atm = this many barye  
# 1 pascal = 0.00001 bar  
viscosity.air.ref<-18.203*1e-5 #Table 4-1 Ref. 2  
Boltzmann<-1.38e-16 # g*cm*(molecule)^-1*(K)^-1*(s)^-2 #Boltzmann constant  
mfp.air<-0.0665*1e-4 #Table 4-1 Ref. 2 NTP  
g<-980.7 # acceleration due to gravity cm/s^2  
T.ntp<-293.15 #kelvin = 20 C  
#####  
# Needed parameters  
#####  
T.C<-25  
T<-T.C+273.15 #Enter temperature in Kelvin  
density.particle<-1 #g*cm^-3 okay if using AMAD for particle diameter  
D.tube<-1.27 #.5" diameter tube  
P.torr<-760  
P<-P.torr*1013250.1/760 #1 atm = this many barye (easy to convert from mb, etc)  
Q.lpm<-28.3*c(0.5,1,2,3) #LPM (non-cgs)  
Q<-Q.lpm*1000/60 #in tube (cm^3/s) LPM *1000/60 (cgs)  
D.um<-1
```

```
D.p<-D.um*1e-4 #particle size, cm ( $\mu\text{m} \times 10^{-4}$ ), mean AMAD if density set to 1
angle.bend<-90 # bend angle in degrees
angle.bend.radians<-angle.bend*pi/180
ratio.curvature<-10
#####
#           General stuff
#####
U.o<-4*Q/(pi*(D.tube)^2) #particle speed in sample stream cm/s
Sutherland<-110.4
viscosity.air<-viscosity.air.ref*(T.ntp+Sutherland)/(T+Sutherland)*
(T/T.ntp)^1.5 #Ref. 2 (4-10)
#####
#           Knudsen Number - Ref: 2 Eq. 4-6, 4-7
#####
mfp<-mfp.air*(1010000/P)*(T/293)*(1+110/293)/(1+110/T)
K.n<-2*mfp/D.p #
#####
#           Cunningham Slip Correction Factor - Ref: 2 Eq. 4-8
#####
#constants valid for the air mfp for solid particles
alpha<-1.142
beta<-0.558
gamma<-0.999
C.c<-1+K.n*(alpha+beta*exp(-gamma/K.n))
#####
#           Stokes - Ref: 1 (15.6)
#           - Ref: 2 (4-39)
#####
Stk<-density.particle*D.p^2*U.o*C.c/(9*viscosity.air*D.tube)
#####
#           Reynolds - Ref: 2 (4-1)
#####
density.air.correction<-(P/P.ntp)*(T.ntp/T)
density.air<-density.air.ntp*density.air.correction
Re<-density.air*U.o*D.tube/viscosity.air #Re of gas
Re.p<-6.5*U.o*D.p #Re of particle (Relaxation time valid when Re.p<1)
#####
#           Bend Efficiency - Ref: 1 (15.24)
#           - Ref: 6 (12)
#           - Ref: 3 (30)
#           - Ref: 7 (15)
#####
Eta.bend<-exp(-0.528*angle.bend.radians*Stk^((2)^(1/ratio.curvature))*
ratio.curvature^0.5) # Zhang Ref 7 (15)
#####
#           Report
#####
report1<-data.frame(T.C,P.torr,Q/(1000/60),format(Re,digits=4))
colnames(report1)<-c("Temp, C", "Pressure, Torr", "Flow Rate, lpm", "Reynolds No.")
report2<-data.frame(D.um,angle.bend,D.tube)
colnames(report2)<-c("Particle Size,  $\mu\text{m}$ ", "angle", "tube diameter, cm")
report3<-data.frame(Q/(1000/60),format(Eta.bend,digits=4))
```

```
colnames(report3)<-c("Flow Rate, lpm","Transmission Eff.")
# report4<-data.frame(D.µm,format(Stk,digits=4),format(Eta.bend,digits=4))
# colnames(report4)<-c("Particle Size, µm","Stokes Number","Transmission Eff.")

report1
report2
report3
#####
#          Plots
#####
plot(x=Q.lpm,y=Eta.bend,col="#0000ff",
     ylab="transmission efficiency",
     main="bend transport efficiency",type="b",ylim=c(.2,1))
points(x=Q.lpm,y=Eta.bend,col="#ff0000",pch=2)
lines(x=Q.lpm,y=Eta.bend,col="#ff0000")
grid()
leg<-c("1 µm","10 µm")
legend("right",leg,pch=c(1,2),col=c("#0000ff","#ff0000"))

#total1<-Eta.bend #did this for 1 and 10 micron runs
#total1<-total1*Q.lpm #so we have total activity on the filter assuming 100% eff. for filter

plot(x=Q.lpm,y=total1,col="#0000ff",
     ylab="total activity through bend",
     main="bend total transport",type="b",ylim=range(total1,total10))
points(x=Q.lpm,y=total10,col="#ff0000",pch=2)
lines(x=Q.lpm,y=total10,col="#ff0000")
grid()
leg<-c("1 µm","10 µm")
legend("right",leg,pch=c(1,2),col=c("#0000ff","#ff0000"))
```

## Attachment 10: R Script for Tubes – Logarithmic Distribution

```
#####  
# Tube section of Sampling Train Particle Penetration Analysis  
# for straight tubes  
# for monodisperse particle sizes  
#####  
# Using cgs units except where noted  
#####  
# References  
# 1 Maiello ML and Hoover MD  
# Radioactive Air Sampling Methods  
# CRC Press NY 2011  
# Eq (15.6)  
# 2 Baron PA and Willeke K  
# Aerosol Measurement: Principles, Techniques, and  
# Applications, Second Ed, Chapter 4  
# Wiley-InterScience, Inc. 2001  
# 3 Von der Weiden SL, Drewnick F and Borrmann S  
# Atmos. Meas. Tech., 2, 479494, 2009  
#####  
# Constants  
#####  
density.air.ntp<-1.20484e-3 #g/cm^3  
P.ntp<-1013250.1 #1 atm = this many barye  
# 1 pascal = 0.00001 bar  
viscosity.air.ref<-18.203*1e-5 #Table 4-1 Ref. 2  
Boltzmann<-1.38e-16 # g*cm*(molecule)^-1*(K)^-1*(s)^-2 #boltzmann constant  
mfp.air<-0.0665*1e-4 #Table 4-1 Ref. 2 NTP  
g<-980.7 # acceleration due to gravity cm/s^2  
T.ntp<-293.15 #kelvin = 20 C  
#####  
# Needed parameters  
# Read Input  
#####  
setwd("d:/jobs/mgh151_aerosol_particle_transport_hand_calculations/models survey article  
testing/1.05 inch")  
# Read in temperature, tube diameter, pressure, flow, density, geometric mean diameter, sd  
params<-read.table(file="param.dat")  
T.C<-as.numeric(params[1])  
P.torr<-as.numeric(params[2])  
D.tube<-as.numeric(params[3])  
Q.lpm<-as.numeric(params[4])  
density.particle<-as.numeric(params[5])  
D.geomean.um<-as.numeric(params[6])  
D.geosd<-as.numeric(params[7])  
# read in particle size and fraction input to this element  
next.element<-read.table(file="pass.distribution.1.dat")  
D.p<-next.element[,1] #input to this element, diameter  
D.p.prob<-next.element[,2] #input to this element, probability  
original<-read.table(file="original.dat") #needed for cumulative eff. calc.  
D.p.original<-original[,1]
```

```
D.p.prob.original<-original[,2]
#####
#      Conversions of Input and other Parameters
#####
T<-T.C+273.15 #Enter temperature in Kelvin
density.particle<-1 #g*cm^-3 okay if using AMAD for particle diameter
P<-P.torr*1013250.1/760 #1 atm = this many barye (easy to convert from mb, etc)
Q<-Q.lpm*1000/60 #in tube (cm^3/s) LPM *1000/60 (cgs)
D.p.um<-D.p/1e-4 #particle size, cm (um*e-4), mean AMAD if density set to 1
#####
#      Element Specific Parameters
#####
angle.incline<-0 #angle of incline for straight tube
angle.incline.radians<-angle.incline*pi/180
L<-61 #tube length
#####
#      General stuff
#####
U.o<-4*Q/(pi*(D.tube)^2) #particle speed in sample stream cm/s
Sutherland<-110.4
viscosity.air<-viscosity.air.ref*(T.ntp+Sutherland)/(T+Sutherland)*
(T/T.ntp)^1.5 #Ref. 2 (4-10)
#####
#      Knudsen Number - Ref: 2 Eq. 4-6, 4-7
#####
mfp<-mfp.air*(1010000/P)*(T/293)*(1+110/293)/(1+110/T)
K.n<-2*mfp/D.p #
#####
#      Cunningham Slip Correction Factor - Ref: 2 Eq. 4-8
#####
#constants valid for the air mfp for solid particles
alpha<-1.142
beta<-0.558
gamma<-0.999
C.c<-1+K.n*(alpha+beta*exp(-gamma/K.n))
#####
#      Stokes - Ref: 1 (15.6)
#      - Ref: 2 (4-39)
#####
Stk<-density.particle*D.p^2*U.o*C.c/(9*viscosity.air*D.tube)
#####
#      Reynolds - Ref: 2 (4-1)
#####
density.air.correction<-(P/P.ntp)*(T.ntp/T)
density.air<-density.air.ntp*density.air.correction
Re<-density.air*U.o*D.tube/viscosity.air #Re of gas
Re.p<-6.5*U.o*D.p #Re of particle (Relaxation time valid when Re.p<1)
#####
#      Sedimentation - Ref: 3 (24 & 25)
#####
t.relax<-density.particle*D.p^2*C.c/(18*viscosity.air) #Ref. 2 (4-34) Particle Relaxation Time
v.term.settling<-t.relax*g
```

```
#initiate
Eta.grav<-c(9,9,9,9)
e.grav.dep<-c(1,1,1,1)
Z<-c(1,1,1,1)
V.turb<-c(9,9,9,9)
for(i in 1:length(t.relax)) {
  Z[i]=L*v.term.settling[i]/(D.tube*U.o)
  V.turb[i]<-(6e-4*(0.0395*Stk[i]*Re^(3/4))^2+2e-8*Re)*U.o/(5.03*Re^(1/8))
  e.grav.dep[i]<-(3/4*Z[i])*cos(angle.incline.radians) #k in ref 3
  ifelse(Re<=4000,Eta.grav[i]<-1-(2/pi)*
    (2*e.grav.dep[i]*sqrt(1-e.grav.dep[i]^(2/3))
    -e.grav.dep[i]^(1/3)*sqrt(1-e.grav.dep[i]^(2/3))
    +asin(e.grav.dep[i]^(1/3))),
    Eta.grav[i]<-exp(-D.tube*L*v.term.settling*cos(angle.incline.radians)/Q)) }
if(Re<=4000) for(i in 1:length(Eta.grav)) {
  if(1-e.grav.dep[i]^(2/3)<0) Eta.grav[i]<-0 }
#####
#           Diffusion   - Ref: 3 (21)
#           - Ref: 2 (4-13)
#   ifelse esures that this only applies to particles<100 nm (0.1 µm)
#####
# Diff<-Boltzmann*T*C.c/(3*pi*viscosity.air*D.p) #diffusion coefficient
# Xi<-pi*Diff*L/Q #noted under Ref. 3 (21)
# Schmidt<-viscosity.air/(density.air)/Diff #Schmidt Number
# ifelse(Re<2000,Sherwood<-3.66+(0.2672/(Xi+0.10079*Xi^(1/3))),
#   Sherwood<-0.0118*Re^(7/8)*Schmidt^(1/3))
# ifelse(D.p<1e-5,Eta.diff<-exp(-Xi*Schmidt),Eta.diff<-1)
#####
#           Turbulent Inertial Deposition - Ref: 3 (28)
#   ifelse esures that this only applies to high Reynolds number
#####
V.turb<-c(1,1,1,1)
V.t<-(6e-4*(0.0395*Stk*Re^(3/4))^2+2e-8*Re)*U.o/(5.03*Re^(1/8))
Eta.turb.inert<-c(1,1,1,1)
for(i in 1:length(V.t)) {
  ifelse(Re>4000,Eta.turb.inert[i]<-exp(-pi*D.p[i]*L*V.t[i]/Q),
    Eta.turb.inert[i]<-1)}
#####
#           Total Efficiency
Eta.total<-Eta.grav*Eta.turb.inert
#####
#           Summary
#####
# this element
el2.lt1<-sum(D.p.prob[which(D.p.µm<1)]*Eta.total[which(D.p.µm<1)])/
  sum(D.p.prob[which(D.p.µm<1)])
el2.1to5<-((sum(D.p.prob[which(D.p.µm<5)]*Eta.total[which(D.p.µm<5)])-
  sum(D.p.prob[which(D.p.µm<1)]*Eta.total[which(D.p.µm<1)]))/
  (sum(D.p.prob[which(D.p.µm<5)])-sum(D.p.prob[which(D.p.µm<1)])))
el2.5to10<-((sum(D.p.prob[which(D.p.µm<10)]*Eta.total[which(D.p.µm<10)])-
  sum(D.p.prob[which(D.p.µm<5)]*Eta.total[which(D.p.µm<5)]))/
  (sum(D.p.prob[which(D.p.µm<10)])-sum(D.p.prob[which(D.p.µm<5)])))
el2.lt30<-((sum(D.p.prob[which(D.p.µm<30)]*Eta.total[which(D.p.µm<30)]))
```

```
sum(D.p.prob)))
el2.total<-sum(D.p.prob*Eta.total)/sum(D.p.prob)
# cumulative
el2.lt1.cum<-sum(D.p.prob[which(D.p.um<1)]*Eta.total[which(D.p.um<1)]/
sum(D.p.prob.original[which(D.p.um<1)])
el2.1to5.cum<-((sum(D.p.prob[which(D.p.um<5)]*Eta.total[which(D.p.um<5)])-
sum(D.p.prob[which(D.p.um<1)]*Eta.total[which(D.p.um<1)]))/
(sum(D.p.prob.original[which(D.p.um<5)]-sum(D.p.prob.original[which(D.p.um<1)])))
el2.5to10.cum<-((sum(D.p.prob[which(D.p.um<10)]*Eta.total[which(D.p.um<10)])-
sum(D.p.prob[which(D.p.um<5)]*Eta.total[which(D.p.um<5)]))/
(sum(D.p.prob.original[which(D.p.um<10)]
-sum(D.p.prob.original[which(D.p.um<5)])))
el2.lt30.cum<-((sum(D.p.prob[which(D.p.um<30)]*Eta.total[which(D.p.um<30)])/
sum(D.p.prob.original)))
el2.total.cum<-sum(D.p.prob*Eta.total)/sum(D.p.prob.original)
#####
# Write Output
#####
next.element.df<-data.frame(D.p,Eta.total*D.p.prob)
write.table(next.element.df,file="pass.distribution.2.dat",row.names=FALSE,col.names=FALSE)
#####
# Plot
#####
main.text<-as.character(paste("Lognormal",D.geomean.um,"+/- ",D.geosd,"",
Q.lpm," lpm"))
plot(x=D.p*10000,y=D.p.prob.original,
col="blue",cex=0.4,ylab="activity",
xlab="particle diameter, micrometer",
main=main.text)
leg<-c("ambient", "through element 2")
legend(35,0.005,leg,pch=c(1,2),col=c("blue","red"),bty="n")
points(x=D.p*10000,y=D.p.prob*Eta.total,
col="red",pch=2,cex=0.4)
grid()
# log y scale
plot(x=D.p*10000,y=D.p.prob.original,
col="blue",cex=0.4,ylab="activity",
xlab="particle diameter, micrometer",
main=main.text,log="y")
leg<-c("ambient", "through element 2")
legend(15,1e-14,leg,pch=c(1,2),col=c("blue","red"),bty="n")
points(x=D.p*10000,y=D.p.prob*Eta.total,
col="red",pch=2,cex=0.4)
grid()
#####
# Report
#####
parameters1<-data.frame(T.C,P.torr,D.tube,Q.lpm,format(Re,digits=4),D.geomean.um,D.geosd)
colnames(parameters1)<-c("Temp, C", "Pressure, Torr", "Tube diameter, cm",
"flow rate, lpm", "Reynolds", "Geomean", "Geo sd")
rownames(parameters1)<-c("")
element.type<-data.frame(L,angle.incline)
```

```
colnames(element.type)<-c("tube length, cm", "degrees from horizontal")  
rownames(element.type)<-c("")
```

```
report.limits<-c("<1 µm", "1 to 5 µm", "5 to 10 µm", "<30 µm", "total")  
report.numbers.cum.el2<-c(el2.lt1.cum, el2.1to5.cum,  
                           el2.5to10.cum, el2.lt30.cum, el2.total.cum)  
report.el2.cum.df<-data.frame(report.limits, report.numbers.cum.el2)
```

```
report.numbers.el2<-c(el2.lt1, el2.1to5, el2.5to10,  
                     el2.lt30, el2.total)  
report.el2.df<-data.frame(report.limits, report.numbers.el2)
```

```
element.type  
parameters1  
"cumulative"  
report.el2.cum.df  
"This element"  
report.el2.df
```



### Attachment 11: R Script for Bends – Logarithmic Distribution

```
#####  
# Elbow (Bend) section of Sampling Train Particle Penetration Analysis  
#####  
# Using cgs units except where noted  
#####  
# References  
# 1 Maiello ML and Hoover MD (eds) Chapter 15 Glissmeyer J,  
# Radioactive Air Sampling Methods, CRC Press NY 2011  
# 2 Kulkarni P, Baron PA, Willeke K, Fundamentals of  
# Single Particle Transport in Aerosol Measurement:  
# Principles, Techniques, and  
# Applications, Third Ed, Chapter 2  
# Wiley-InterScience, Inc. 2011  
# 3 Von der Weiden SL, Drewnick F and Borrmann S  
# Atmos. Meas. Tech., 2, 479494, 2009  
# 4 Brockman JE Sampling and Transport of Aerosols  
# in Aerosol Measurement: Principles, Techniques, and  
# Applications, Third Ed, Chapter 6  
# Wiley-InterScience, Inc. 2011  
# 5 Metz D, Harvey P, AEROSOL SAMPLING MODELS SURVEY,  
# GENERAL MANAGEMENT ASSOCIATES, Abingdon, MD 21009, March 1993.  
# 6 McFarland AR et. al. Aerosol Deposition in Bends with  
# Turbulent Flow  
# 7 Zhang P, Roberts RM, Bénard A, Computational  
# guidelines and an empirical model for particle deposition  
# in curved pipes using an Eulerian-Lagrangian approach,  
# Journal of Aerosol Science, 53:1-20, November 2012.  
# 8 Pui DYH, Romay-Novas F and Liu BYH, Experimental Study  
# of Particle Deposition in Bends of Circular Cross Section,  
# Aerosol, Science and Technology, 7:3, 301-315, 1987.  
#####  
density.air.ntp<-1.20484e-3 #g/cm^3  
P.ntp<-1013250.1 #1 atm = this many barye  
# 1 pascal = 0.00001 bar  
viscosity.air.ref<-18.203*1e-5 #Table 4-1 Ref. 2  
Boltzmann<-1.38e-16 # g*cm*(molecule)^-1*(K)^-1*(s)^-2 #boltzmann constant  
mfp.air<-0.0665*1e-4 #Table 4-1 Ref. 2 NTP  
g<-980.7 # acceleration due to gravity cm/s^2  
T.ntp<-293.15 #kelvin = 20 C  
#####  
# Needed parameters  
# Read Input  
#####  
setwd("d:/jobs/mgh151_aerosol_particle_transport_hand_calculations/models survey article  
testing/1.05 inch")  
# param.df<-data.frame(T.C,D.tube,P.torr,Q.lpm,D.um)  
params<-read.table(file="param.dat")  
T.C<-as.numeric(params[1])  
P.torr<-as.numeric(params[2])  
D.tube<-as.numeric(params[3])
```

```
Q.lpm<-as.numeric(params[4])
density.particle<-as.numeric(params[5])
D.geomean.um<-as.numeric(params[6])
D.geosd<-as.numeric(params[7])
next.element<-read.table(file="pass.distribution.2.dat")
D.p<-next.element[,1] #input to this element, diameter
D.p.prob<-next.element[,2] #input to this element, probability
original<-read.table(file="original.dat")
D.p.original<-original[,1]
D.p.prob.original<-original[,2]
#####
#      Conversions of Input and other Parameters
#####
T<-T.C+273.15 #Enter temperature in Kelvin
density.particle<-1 #g*cm^-3 okay if using AMAD for particle diameter
P<-P.torr*1013250.1/760 #1 atm = this many barye (easy to convert from mb, etc)
Q<-Q.lpm*1000/60 #in tube (cm^3/s) LPM *1000/60 (cgs)
D.p.um<-D.p/1e-4 #particle size, cm (um*e-4), mean AMAD if density set to 1
#####
#      Element Specific Parameters
#####
angle.bend<-90 # bend angle in degrees
angle.bend.radians<-angle.bend*pi/180
ratio.curvature<-3*2.54/(D.tube/2) #(3" centerlines see Ref5 p. 27)
#####
#      General stuff
#####
U.o<-4*Q/(pi*(D.tube)^2) #particle speed in sample stream cm/s
Sutherland<-110.4
viscosity.air<-viscosity.air.ref*(T.ntp+Sutherland)/(T+Sutherland)*
(T/T.ntp)^1.5 #Ref. 2 (4-10)
#####
#      Knudsen Number - Ref: 2 Eq. 4-6, 4-7
#####
mfp<-mfp.air*(1010000/P)*(T/293)*(1+110/293)/(1+110/T)
K.n<-2*mfp/D.p #
#####
#      Cunningham Slip Correction Factor - Ref: 2 Eq. 4-8
#####
#constants valid for the air mfp for solid particles
alpha<-1.142
beta<-0.558
gamma<-0.999
C.c<-1+K.n*(alpha+beta*exp(-gamma/K.n))
#####
#      Stokes - Ref: 1 (15.6)
#      - Ref: 2 (4-39)
#####
Stk<-density.particle*D.p^2*U.o*C.c/(9*viscosity.air*D.tube)
#####
#      Reynolds - Ref: 2 (4-1)
#####
```

```
density.air.correction<-(P/P.ntp)*(T.ntp/T)
density.air<-density.air.ntp*density.air.correction
Re<-density.air*U.o*D.tube/viscosity.air #Re of gas
Re.p<-6.5*U.o*D.p #Re of particle (Relaxation time valid when Re.p<1)
#####
#           Bend Efficiency - Ref: 1 (15.24)
#           - Ref: 6 (12)
#           - Ref: 3 (30)
#           - Ref: 7 (15)
#####
Eta.bend<-exp(-0.528*angle.bend.radians*Stk^((2)^(1/ratio.curvature))*
              ratio.curvature^0.5) # Zhang Ref 7 (15)
#####
#           Summary
#####
# this element
el3.lt1<-sum(D.p.prob[which(D.p.um<1)]*Eta.bend[which(D.p.um<1)])/
  sum(D.p.prob[which(D.p.um<1)])
el3.1to5<-((sum(D.p.prob[which(D.p.um<5)]*Eta.bend[which(D.p.um<5)])-
  sum(D.p.prob[which(D.p.um<1)]*Eta.bend[which(D.p.um<1)]))/
  (sum(D.p.prob[which(D.p.um<5)])-sum(D.p.prob[which(D.p.um<1)])))
el3.5to10<-((sum(D.p.prob[which(D.p.um<10)]*Eta.bend[which(D.p.um<10)])-
  sum(D.p.prob[which(D.p.um<5)]*Eta.bend[which(D.p.um<5)]))/
  (sum(D.p.prob[which(D.p.um<10)])-sum(D.p.prob[which(D.p.um<5)])))
el3.lt30<-((sum(D.p.prob[which(D.p.um<30)]*Eta.bend[which(D.p.um<30)])/
  sum(D.p.prob)))
el3.total<-sum(D.p.prob*Eta.bend)/sum(D.p.prob)
# cumulative
el3.lt1.cum<-sum(D.p.prob[which(D.p.um<1)]*Eta.bend[which(D.p.um<1)])/
  sum(D.p.prob.original[which(D.p.um<1)])
el3.1to5.cum<-((sum(D.p.prob[which(D.p.um<5)]*Eta.bend[which(D.p.um<5)])-
  sum(D.p.prob[which(D.p.um<1)]*Eta.bend[which(D.p.um<1)]))/
  (sum(D.p.prob.original[which(D.p.um<5)])-sum(D.p.prob.original[which(D.p.um<1)])))
el3.5to10.cum<-((sum(D.p.prob[which(D.p.um<10)]*Eta.bend[which(D.p.um<10)])-
  sum(D.p.prob[which(D.p.um<5)]*Eta.bend[which(D.p.um<5)]))/
  (sum(D.p.prob.original[which(D.p.um<10)])
  -sum(D.p.prob.original[which(D.p.um<5)])))
el3.lt30.cum<-((sum(D.p.prob[which(D.p.um<30)]*Eta.bend[which(D.p.um<30)])/
  sum(D.p.prob.original)))
el3.total.cum<-sum(D.p.prob*Eta.bend)/sum(D.p.prob.original)
#####
#           Write Output
#####
next.element.df<-data.frame(D.p,Eta.bend*D.p.prob)
write.table(next.element.df,file="pass.distribution.3.dat",row.names=FALSE,col.names=FALSE)
#####
#           Plot
#####
main.text<-as.character(paste("Lognormal",D.geomean.um,"+/- ",D.geosd,"",
  Q.lpm," lpm"))
plot(x=D.p*10000,y=D.p.prob.original,
  col="blue",cex=0.4,ylab="activity",
  xlab="particle diameter, micrometer",
```

```
main=main.text)
leg<-c("ambient","through element 3")
legend(35,0.004,leg,pch=c(1,2),col=c("blue","red"),bty="n")
points(x=D.p*10000,y=D.p.prob*Eta.bend,
       col="red",pch=2,cex=0.4)
grid()
# log y scale
plot(x=D.p*10000,y=D.p.prob.original,
     col="blue",cex=0.4,ylab="activity",
     xlab="particle diameter, micrometer",
     main=main.text,log="y")
leg<-c("ambient","through element 3")
legend(15,1e-10,leg,pch=c(1,2),col=c("blue","red"),bty="n")
points(x=D.p*10000,y=D.p.prob*Eta.bend,
       col="red",pch=2,cex=0.4)
grid()
#####
#       Report
#####
parameters1<-data.frame(T.C,P.torr,D.tube,Q.lpm,format(Re,digits=4),D.geomean.um,D.geosd)
colnames(parameters1)<-c("Temp, C","Pressure, Torr","Tube diameter, cm",
                        "flow rate, lpm","Reynolds","Geomean","Geo sd")
rownames(parameters1)<-c("")

element.type<-data.frame(angle.bend)
colnames(element.type)<-c("elbow, degrees bend")
rownames(element.type)<-c("")

report.limits<-c("<1 µm","1 to 5 µm","5 to 10 µm","<30 µm","total")
report.numbers.cum.el3<-c(el3.lt1.cum,el3.1to5.cum,
                          el3.5to10.cum,el3.lt30.cum,el3.total.cum)
report.el3.cum.df<-data.frame(report.limits,report.numbers.cum.el3)

report.numbers.el3<-c(el3.lt1,el3.1to5,el3.5to10,
                      el3.lt30, el3.total)
report.el3.df<-data.frame(report.limits,report.numbers.el3)
report.el3.df<-data.frame(report.limits,report.numbers.el3)

element.type
parameters1
"cumulative"
report.el3.cum.df
"This element"
report.el3.df
```