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Validation of COG10 and ENDFB6R7 on the Auk Workstation for General Application to Plutonium Systems

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August 16, 2011

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This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

December 29, 2010
 CSM 1602

TO: David Heinrichs

FROM: Catherine

Cc: CSM Files (L-198)

SUBJECT: Validation of COG 10 and ENDFB6R7 on the *Auk* Workstation for General Application to Plutonium Systems

1.0 Introduction

The COG 10 code package¹ on the *Auk* workstation is now validated with the ENDFB6R7 neutron cross section library for general application to plutonium (Pu) systems by comparison of the calculated $k_{\text{effective}}$ to the expected $k_{\text{effective}}$ of several relevant experimental benchmarks. This validation is supplemental to the installation and verification of COG 10 on the *Auk* workstation².

2.0 Benchmark Experiments

Computational models for 143 experimental benchmarks are derived from the specifications of *Volume I, Plutonium Systems* of the International Handbook of Evaluated Criticality Safety Benchmark Experiments.² The basis for their selection is the availability of models in the proper code input format as developed and collected from previous efforts. Such a large number ensures the broadest possible Area-of-Applicability of the validation. The distribution of selected benchmarks by material form and characteristic fission energy as defined in the Handbook is shown below in Table 1. The number of experimental benchmark models from any given evaluation is indicated in Table 2.

Table 1. Selected Pu Benchmarks by Material Form and Characteristic Fission Energy from Volume I of the ICSBEP Handbook

Material Form and Characteristic Fission Energy	Number Selected
METAL Systems (Fast, Intermediate, Thermal, Mixed)	37 (36, 1, 0, 0)
COMPOUND Systems (Fast, Intermediate, Thermal, Mixed)	35 (0, 1, 0, 34)
SOLUTION Systems (Intermediate, Thermal, Mixed)	71 (0, 71, 0)
TOTAL (Fast, Intermediate, Thermal, Mixed)	143 (36, 2, 71, 34)



Table 2. Selected Pu Benchmarks by Evaluation from Volume I of the ICSBEP Handbook

Evaluation ID Number	Title	Number Selected
PU-MET-FAST-001	Bare Sphere of Plutonium-239 Metal (²³⁹ Pu Jezebel)	1
PU-MET-FAST-002	Bare Sphere of Plutonium-239 Metal (²⁴⁰ Pu Jezebel)	1
PU-MET-FAST-003	Unmoderated Plutonium Metal Button Array	1
PU-MET-FAST-005	Benchmark Critical Experiment of a Plutonium Sphere Reflected by Tungsten	1
PU-MET-FAST-006	Plutonium Sphere Reflected by Normal Uranium Using Flattop	1
PU-MET-FAST-008	Benchmark Critical Experiment of a Thorium Reflected Plutonium Sphere	1
PU-MET-FAST-009	Benchmark Critical Experiment of a Plutonium Sphere Reflected by Aluminum	1
PU-MET-FAST-010	Benchmark Critical Experiment of a Delta-Phase Plutonium Sphere Reflected by Normal Uranium	1
PU-MET-FAST-011	Benchmark Critical Experiment of a Water Reflected Alpha-Phase Plutonium Sphere	1
PU-MET-FAST-012	Uranium-Reflected Array of Plutonium Fuel Rods	1
PU-MET-FAST-013	Copper-Reflected Array of Plutonium Fuel Rods	1
PU-MET-FAST-014	Nickel-Reflected Array of Plutonium Fuel Rods	1
PU-MET-FAST-015	Iron-Reflected Array of Plutonium Fuel Rods	1
PU-MET-FAST-018	Benchmark Critical Experiment of a Delta-Phase Plutonium Sphere Reflected by Beryllium	1
PU-MET-FAST-019	Sphere of Plutonium Reflected by Beryllium	1
PU-MET-FAST-020	Sphere of Plutonium Reflected by Depleted Uranium	1
PU-MET-FAST-021	Beryllium- and Beryllium Oxide-Reflected Cylinders of Plutonium	2
PU-MET-FAST-033	ZPPR-21 Phase A: A Cylindrical Assembly of Pu Metal Reflected by Graphite	1
PU-MET-FAST-035	Spherical Assembly of ²³⁹ Pu(δ,98%) with 3.15-cm Lead Reflector	1
PU-MET-FAST-036	Spherical Assembly of ²³⁹ Pu(δ,98%) with Compound Reflector of 0.05-cm Cadmium and 2.3-cm Polyethylene	1
PU-MET-FAST-039	Spherical Assembly of ²³⁹ Pu(δ,98%) with a 4.25-cm Duralumin Reflector	1
PU-MET-FAST-040	Spherical Assembly of ²³⁹ Pu(δ,98%) with a 1.6-cm Copper	1

Table 2. Selected Pu Benchmarks by Evaluation from Volume I of the ICSBEP Handbook

Evaluation ID Number	Title	Number Selected
	Reflector	
PU-MET-FAST-041	Spherical Assembly of ^{239}Pu (α ,88%) with a 20.98-cm Depleted-Uranium Reflector	1
PU-MET-FAST-044	Plutonium (5.1 wt.% ^{240}Pu) Metal Sphere Beryllium, Graphite, Aluminum, Iron, and Molybdenum Tampers and Polyethylene Reflectors	5
PU-MET-FAST-045	Critical Experiments Performed for LAMPRE, The Los Alamos Molten Plutonium Reactor	7
PU-MET-INTER-002	ZPR-6 Assembly 10: A Cylindrical Plutonium/Carbon/Stainless Steel Assembly with Stainless Steel and Iron Reflectors	1
PU-COMP-INTER-001	k-Infinity Experiments in Intermediate Neutron Spectra for ^{239}Pu	1
PU-COMP-MIXED-001	Unreflected Slabs of Polystyrene-Moderated Plutonium Oxide	5
PU-COMP-MIXED-002	Plexiglas Reflected Slabs of Polystyrene-Moderated Plutonium Oxide	29
PU-SOL-THERM-001	Water Reflected 11.5" Diameter Spheres of Plutonium Nitrate Solutions	6
PU-SOL-THERM-002	Water Reflected 12" Diameter Spheres of Plutonium Nitrate Solutions	7
PU-SOL-THERM-007	Water Reflected 11.5" Diameter Spheres Partly Filled with Plutonium Nitrate Solutions	8
PU-SOL-THERM-008	Concrete-Reflected 14-Inch Diameter Spheres of Plutonium Nitrate Solutions	15
PU-SOL-THERM-010	Water Reflected 9", 10", 11", and 12" Diameter Cylinders of Plutonium Nitrate Solutions	14
PU-SOL-THERM-026	Unreflected Slabs of Plutonium Nitrate Solutions	21
	TOTAL	143

3.0 Selected Code Options

Numerous code options are available to the user and default values are employed more often than not with a few notable exceptions that are relevant to validation and use. The total number of neutron histories simulated for each benchmark is specified in the basic data block as follows:

```
npart=5000  
nbatch=1100  
sdt=0.0001  
nfirst=100
```

Also, the ENDFB6R7 neutron cross section library is employed for all benchmarks as is the available $S(\alpha,\beta)$ treatment for selected elements where appropriate. To this end, neutron cross section data is specified in the mix data block as follows:

```
nlib=ENDFB6R7  
sablib=COGSAB (as necessary)
```

4.0 Validation Methodology

Each benchmark result is normalized to the delayed critical condition ($k_{\text{effective}} = 1$) as follows:

$$k_{\text{normalized}} = (k_{\text{calculated}} - k_{\text{expected}}) + 1$$

The associated uncertainty (σ_{combined}) for each benchmark result becomes:

$$\sigma_{\text{combined}} = \sqrt{(\sigma_{\text{calculated}})^2 + (\sigma_{\text{expected}})^2}$$

The determination of an Upper Subcritical Limit (USL) to differentiate subcritical and critical conditions to a high probability by computation requires an assessment of the bias, the total uncertainty, and a margin of safety as follows:

$$\text{USL} = \{1 + \text{bias}\} - \{\text{total uncertainty}\} - \{\text{margin of safety}\}$$

Thus, a conservative criterion for general application is:

$$k_{\text{calculated}} + 3\sigma_{\text{calculated}} \leq \text{USL} = \{1 + \text{bias}\} - \{3\sigma_{\text{total}}\} - \{0.02\}$$

This criterion ensures better than 99.8% confidence that the USL lays below the selected benchmark results with an additional 0.02 margin of safety.

In practice, a simple estimate of {1+ bias} is the un-weighted average (k_{average}) of the calculated $k_{\text{normalized}}$ benchmark results:

$$k_{\text{average}} = \frac{\sum_{i=1}^n (k_{i, \text{normalized}})}{n}$$

The total uncertainty ($3\sigma_{\text{total}}$) is determined from the combination of the uncertainty of the bias estimate (σ_{bias}) and the average of the combined uncertainties of the individual benchmark results (σ_{average}) according to the following relations:

$$\sigma_{\text{total}} = \sqrt{(\sigma_{\text{bias}})^2 + (\sigma_{\text{average}})^2}$$

$$\sigma_{\text{bias}} = \sqrt{\frac{\sum_{i=1}^n (k_{i, \text{normalized}} - k_{\text{average}})^2}{n}}$$

$$\sigma_{\text{average}} = \sqrt{\frac{n}{\sum_{i=1}^n \frac{1}{(\sigma_{i, \text{combined}})^2}}}$$

Results for all 143 selected Pu benchmarks are provided in Table 3 and illustrated by Figure 1 on the pages that follow. Not unexpectedly, the high values of $k_{\text{normalized}}$ for the 36 intermediate and mixed fission energy systems increase the {1 + bias} term and the value of σ_{bias} . The net effect may be an unnecessarily low estimate of the USL for some applications that can be improved with additional benchmarks or their separate treatment, as evident by the summary data below.

	THERMAL	INTERMEDIATE + MIXED	FAST	ALL
Number of cases	71	36	36	143
{1 + bias}	1.0002	1.0216	1.0021	1.0060
σ_{bias}	0.0105	0.0091	0.0042	0.0127
σ_{average}	0.0045	0.0058	0.0022	0.0034
{total uncertainty} = $3\sigma_{\text{total}}$	0.0344	0.0324	0.0143	0.0393
{margin of safety}	0.02	0.02	0.02	0.02
USL	0.9458	0.9692	0.9678	0.9467

**Table 3. Plutonium Benchmark Results on the *Auk* Workstation with
 COG 10 and the ENDFB6R7 Neutron Cross-Section Library**

Benchmark ID	Experimental Benchmark k-effective (k_{expected})	Experimental Benchmark Uncertainty (σ_{expected})	Calculated k-effective ($k_{\text{calculated}}$)	Calculational Uncertainty ($\sigma_{\text{calculated}}$)	Calculated Median Energy of the Neutrons Causing Fission (MeV)	Normalized Calculated k-effective ($k_{\text{normalized}}$)	Combined Uncertainty (σ_{combined})
pmf001	1.0000	0.0020	0.9981	0.0007	1.47E+00	0.9981	0.0021
pmf002	1.0000	0.0020	0.9975	0.0007	1.48E+00	0.9975	0.0021
pmf003-1	1.0000	0.0030	1.0009	0.0007	1.47E+00	1.0009	0.0031
pmf005	1.0000	0.0013	1.0097	0.0007	1.18E+00	1.0097	0.0015
pmf006	1.0000	0.0030	1.0039	0.0007	1.53E+00	1.0039	0.0031
pmf008	1.0000	0.0006	1.0078	0.0007	1.29E+00	1.0078	0.0009
pmf009	1.0000	0.0027	1.0024	0.0007	1.36E+00	1.0024	0.0028
pmf010	1.0000	0.0018	1.0008	0.0007	1.47E+00	1.0008	0.0019
pmf011	1.0000	0.0010	0.9992	0.0007	1.06E+00	0.9992	0.0012
pmf012	1.0009	0.0021	1.0049	0.0007	1.43E+00	1.0040	0.0022
pmf013	1.0034	0.0023	1.0088	0.0007	9.89E-01	1.0054	0.0024
pmf014	1.0037	0.0031	1.0061	0.0007	1.04E+00	1.0024	0.0032
pmf015	1.0041	0.0026	1.0008	0.0007	1.17E+00	0.9967	0.0027
pmf018	1.0000	0.0030	1.0017	0.0007	1.22E+00	1.0017	0.0031
pmf019	0.9992	0.0015	1.0026	0.0007	1.17E+00	1.0034	0.0017
pmf020	0.9993	0.0017	0.9996	0.0007	1.49E+00	1.0003	0.0018
pmf021-1	1.0000	0.0026	1.0057	0.0007	1.21E+00	1.0057	0.0027
pmf021-2	1.0000	0.0026	0.9945	0.0007	1.23E+00	0.9945	0.0027
pmf033	0.9967	0.0026	1.0007	0.0007	6.86E-01	1.0040	0.0027
pmf035	1.0000	0.0016	1.0089	0.0007	1.40E+00	1.0089	0.0017
pmf036	1.0000	0.0031	1.0050	0.0007	1.28E+00	1.0050	0.0032
pmf039	1.0000	0.0022	0.9893	0.0007	1.36E+00	0.9893	0.0023
pmf040	1.0000	0.0038	0.9948	0.0007	1.34E+00	0.9948	0.0039
pmf041	1.0000	0.0016	1.0064	0.0007	1.58E+00	1.0064	0.0017
pmf044-1	0.9977	0.0021	1.0023	0.0007	1.21E+00	1.0046	0.0022
pmf044-2	0.9980	0.0022	1.0003	0.0007	1.22E+00	1.0023	0.0023

**Table 3. Plutonium Benchmark Results on the *Auk* Workstation with
 COG 10 and the ENDFB6R7 Neutron Cross-Section Library**

Benchmark ID	Experimental Benchmark k-effective (k_{expected})	Experimental Benchmark Uncertainty (σ_{expected})	Calculated k-effective ($k_{\text{calculated}}$)	Calculational Uncertainty ($\sigma_{\text{calculated}}$)	Calculated Median Energy of the Neutrons Causing Fission (MeV)	Normalized Calculated k-effective ($k_{\text{normalized}}$)	Combined Uncertainty (σ_{combined})
pmf044-3	0.9977	0.0021	0.9992	0.0007	1.22E+00	1.0015	0.0022
pmf044-4	0.9978	0.0026	0.9984	0.0007	1.16E+00	1.0006	0.0027
pmf044-5	0.9977	0.0024	0.9976	0.0007	1.21E+00	0.9999	0.0025
pmf045s-1	1.0000	0.0034	0.9968	0.0007	1.15E+00	0.9968	0.0035
pmf045s-2	1.0000	0.0034	1.0045	0.0007	1.13E+00	1.0045	0.0035
pmf045s-3	1.0000	0.0035	1.0068	0.0007	1.16E+00	1.0068	0.0036
pmf045s-4	1.0000	0.0035	1.0059	0.0007	1.17E+00	1.0059	0.0036
pmf045s-5	1.0000	0.0036	1.0038	0.0007	1.23E+00	1.0038	0.0037
pmf045s-6	1.0000	0.0038	1.0045	0.0007	1.20E+00	1.0045	0.0039
pmf045s-7	1.0000	0.0038	1.0014	0.0007	1.16E+00	1.0014	0.0039
pmi002	0.9869	0.0026	1.0278	0.0006	7.70E-03	1.0409	0.0027
pci001	1.0000	0.0110	1.0116	0.0006	1.06E-04	1.0116	0.0110
pcm001-1	0.9986	0.0041	1.0222	0.0007	1.27E+00	1.0236	0.0042
pcm001-2	1.0000	0.0068	1.0280	0.0007	4.82E-04	1.0280	0.0068
pcm001-3	0.9990	0.0067	1.0258	0.0007	1.15E-05	1.0268	0.0067
pcm001-4	1.0000	0.0066	0.9952	0.0007	1.34E-05	0.9952	0.0066
pcm001-5	0.9989	0.0072	1.0136	0.0007	3.21E-07	1.0147	0.0072
pcm002-01	0.9990	0.0046	1.0310	0.0007	3.48E-01	1.0320	0.0047
pcm002-02	0.9990	0.0046	1.0298	0.0007	3.16E-01	1.0308	0.0047
pcm002-03	0.9990	0.0046	1.0245	0.0007	2.75E-01	1.0255	0.0047
pcm002-04	0.9990	0.0046	1.0191	0.0007	2.05E-01	1.0201	0.0047
pcm002-05	0.9990	0.0046	1.0163	0.0007	1.31E-01	1.0173	0.0047
pcm002-06	1.0000	0.0075	1.0275	0.0007	5.54E-05	1.0275	0.0075
pcm002-07	1.0000	0.0075	1.0253	0.0007	4.97E-05	1.0253	0.0075
pcm002-08	1.0000	0.0075	1.0238	0.0007	2.43E-05	1.0238	0.0075
pcm002-09	1.0000	0.0075	1.0240	0.0006	1.88E-05	1.0240	0.0075

**Table 3. Plutonium Benchmark Results on the *Auk* Workstation with
 COG 10 and the ENDFB6R7 Neutron Cross-Section Library**

Benchmark ID	Experimental Benchmark k-effective (k_{expected})	Experimental Benchmark Uncertainty (σ_{expected})	Calculated k-effective ($k_{\text{calculated}}$)	Calculational Uncertainty ($\sigma_{\text{calculated}}$)	Calculated Median Energy of the Neutrons Causing Fission (MeV)	Normalized Calculated k-effective ($k_{\text{normalized}}$)	Combined Uncertainty (σ_{combined})
pcm002-10	1.0000	0.0073	1.0345	0.0007	4.39E-07	1.0345	0.0073
pcm002-11	1.0000	0.0073	1.0316	0.0006	4.62E-07	1.0316	0.0073
pcm002-12	1.0000	0.0073	1.0322	0.0006	5.05E-07	1.0322	0.0073
pcm002-13	1.0000	0.0073	1.0296	0.0007	5.25E-07	1.0296	0.0073
pcm002-14	1.0000	0.0073	1.0350	0.0006	5.35E-07	1.0350	0.0073
pcm002-15	1.0000	0.0073	1.0309	0.0007	5.31E-07	1.0309	0.0073
pcm002-16	1.0000	0.0073	1.0272	0.0006	5.05E-07	1.0272	0.0073
pcm002-17	0.9988	0.0055	1.0100	0.0006	4.67E-07	1.0112	0.0055
pcm002-18	0.9988	0.0055	1.0129	0.0006	5.63E-07	1.0141	0.0055
pcm002-19	0.9988	0.0055	1.0126	0.0007	5.88E-07	1.0138	0.0055
pcm002-20	0.9988	0.0055	1.0127	0.0006	6.03E-07	1.0139	0.0055
pcm002-21	0.9988	0.0055	1.0142	0.0007	6.03E-07	1.0154	0.0055
pcm002-22	0.9988	0.0055	1.0164	0.0006	5.84E-07	1.0176	0.0055
pcm002-23	1.0000	0.0068	1.0130	0.0006	2.32E-07	1.0130	0.0068
pcm002-24	1.0000	0.0068	1.0150	0.0007	2.34E-07	1.0150	0.0068
pcm002-25	1.0000	0.0068	1.0144	0.0006	2.36E-07	1.0144	0.0068
pcm002-26	1.0000	0.0068	1.0144	0.0007	2.38E-07	1.0144	0.0068
pcm002-27	1.0000	0.0068	1.0155	0.0006	2.40E-07	1.0155	0.0068
pcm002-28	1.0000	0.0068	1.0155	0.0006	2.41E-07	1.0155	0.0068
pcm002-29	1.0000	0.0068	1.0154	0.0006	2.42E-07	1.0154	0.0068
pst001-1	1.0000	0.0050	1.0002	0.0007	6.58E-08	1.0002	0.0050
pst001-2	1.0000	0.0050	1.0009	0.0006	7.91E-08	1.0009	0.0050
pst001-3	1.0000	0.0050	1.0036	0.0007	9.27E-08	1.0036	0.0050
pst001-4	1.0000	0.0050	0.9972	0.0007	1.02E-07	0.9972	0.0050
pst001-5	1.0000	0.0050	1.0019	0.0007	1.07E-07	1.0019	0.0050
pst001-6	1.0000	0.0050	1.0008	0.0006	1.97E-07	1.0008	0.0050

**Table 3. Plutonium Benchmark Results on the *Auk* Workstation with
COG 10 and the ENDFB6R7 Neutron Cross-Section Library**

Benchmark ID	Experimental Benchmark k-effective (k_{expected})	Experimental Benchmark Uncertainty (σ_{expected})	Calculated k-effective ($k_{\text{calculated}}$)	Calculational Uncertainty ($\sigma_{\text{calculated}}$)	Calculated Median Energy of the Neutrons Causing Fission (MeV)	Normalized Calculated k-effective ($k_{\text{normalized}}$)	Combined Uncertainty (σ_{combined})
pst002-1	1.0000	0.0047	1.0020	0.0006	5.67E-08	1.0020	0.0047
pst002-2	1.0000	0.0047	1.0026	0.0006	5.75E-08	1.0026	0.0047
pst002-3	1.0000	0.0047	1.0016	0.0006	6.03E-08	1.0016	0.0047
pst002-4	1.0000	0.0047	1.0039	0.0006	6.22E-08	1.0039	0.0047
pst002-5	1.0000	0.0047	1.0068	0.0007	6.44E-08	1.0068	0.0048
pst002-6	1.0000	0.0047	1.0013	0.0007	6.88E-08	1.0013	0.0048
pst002-7	1.0000	0.0047	1.0031	0.0006	7.30E-08	1.0031	0.0047
pst007-02	1.0000	0.0047	1.0015	0.0007	1.69E-07	1.0015	0.0048
pst007-03	1.0000	0.0047	0.9953	0.0006	1.62E-07	0.9953	0.0047
pst007-05	1.0000	0.0047	1.0033	0.0007	7.94E-08	1.0033	0.0048
pst007-06	1.0000	0.0047	0.9965	0.0006	8.04E-08	0.9965	0.0047
pst007-07	1.0000	0.0047	0.9996	0.0007	7.97E-08	0.9996	0.0048
pst007-08	1.0000	0.0047	0.9932	0.0006	8.08E-08	0.9932	0.0047
pst007-09	1.0000	0.0047	0.9908	0.0007	8.05E-08	0.9908	0.0048
pst007-10	1.0000	0.0047	0.9943	0.0006	7.62E-08	0.9943	0.0047
pst008-16	1.0000	0.0033	1.0128	0.0007	5.35E-08	1.0128	0.0034
pst008-17	1.0000	0.0040	1.0160	0.0007	6.18E-08	1.0160	0.0041
pst008-18	1.0000	0.0040	1.0187	0.0007	6.23E-08	1.0187	0.0041
pst008-19	1.0000	0.0028	1.0152	0.0006	5.07E-08	1.0152	0.0029
pst008-20	1.0000	0.0028	1.0156	0.0006	4.90E-08	1.0156	0.0029
pst008-21	1.0000	0.0040	0.9953	0.0006	6.59E-08	0.9953	0.0040
pst008-22	1.0000	0.0061	0.9891	0.0007	2.58E-07	0.9891	0.0061
pst008-23	1.0000	0.0037	1.0172	0.0007	6.05E-08	1.0172	0.0038
pst008-24	1.0000	0.0031	1.0129	0.0007	5.42E-08	1.0129	0.0032
pst008-25	1.0000	0.0042	1.0151	0.0007	7.36E-08	1.0151	0.0043
pst008-26	1.0000	0.0041	1.0159	0.0006	6.12E-08	1.0159	0.0041

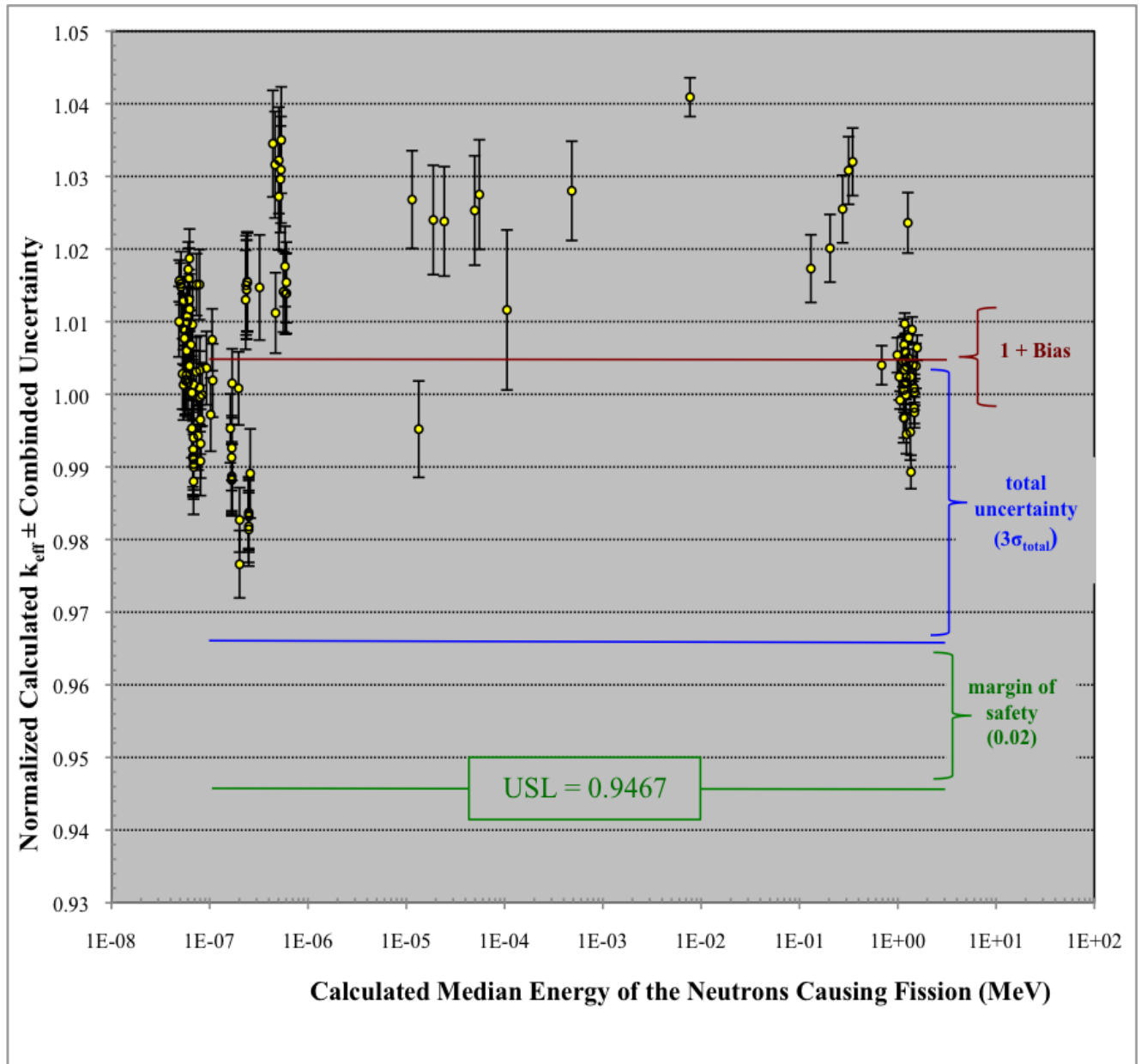
**Table 3. Plutonium Benchmark Results on the *Auk* Workstation with
 COG 10 and the ENDFB6R7 Neutron Cross-Section Library**

Benchmark ID	Experimental Benchmark k-effective (k_{expected})	Experimental Benchmark Uncertainty (σ_{expected})	Calculated k-effective ($k_{\text{calculated}}$)	Calculational Uncertainty ($\sigma_{\text{calculated}}$)	Calculated Median Energy of the Neutrons Causing Fission (MeV)	Normalized Calculated k-effective ($k_{\text{normalized}}$)	Combined Uncertainty (σ_{combined})
pst008-27	1.0000	0.0042	1.0075	0.0007	1.06E-07	1.0075	0.0043
pst008-28	1.0000	0.0042	0.9999	0.0007	8.37E-08	0.9999	0.0043
pst008-29	1.0000	0.0041	1.0130	0.0007	6.19E-08	1.0130	0.0042
pst008-30	1.0000	0.0041	1.0107	0.0007	5.90E-08	1.0107	0.0042
pst010-01	1.0000	0.0048	1.0151	0.0006	7.87E-08	1.0151	0.0048
pst010-02	1.0000	0.0048	1.0096	0.0007	6.68E-08	1.0096	0.0049
pst010-03	1.0000	0.0048	1.0060	0.0006	5.83E-08	1.0060	0.0048
pst010-04	1.0000	0.0048	1.0099	0.0007	5.91E-08	1.0099	0.0049
pst010-05	1.0000	0.0048	1.0089	0.0007	5.59E-08	1.0089	0.0049
pst010-06	1.0000	0.0048	1.0077	0.0007	5.53E-08	1.0077	0.0049
pst010-07	1.0000	0.0048	1.0013	0.0007	5.40E-08	1.0013	0.0049
pst010-08	1.0000	0.0048	1.0028	0.0007	5.29E-08	1.0028	0.0049
pst010-09	1.0000	0.0048	1.0117	0.0007	6.22E-08	1.0117	0.0049
pst010-10	1.0000	0.0048	1.0019	0.0006	5.62E-08	1.0019	0.0048
pst010-11	1.0000	0.0048	1.0088	0.0006	5.59E-08	1.0088	0.0048
pst010-12	1.0000	0.0048	1.0090	0.0006	5.34E-08	1.0090	0.0048
pst010-13	1.0000	0.0048	1.0148	0.0006	5.09E-08	1.0148	0.0048
pst010-14	1.0000	0.0048	1.0100	0.0007	4.87E-08	1.0100	0.0049
pst026-01	1.0000	0.0052	0.9914	0.0006	6.73E-08	0.9914	0.0052
pst026-02	1.0000	0.0052	0.9911	0.0007	6.78E-08	0.9911	0.0052
pst026-03	1.0000	0.0051	0.9924	0.0007	6.77E-08	0.9924	0.0051
pst026-04	1.0000	0.0045	0.9880	0.0006	6.85E-08	0.9880	0.0045
pst026-05	1.0000	0.0044	0.9900	0.0006	6.85E-08	0.9900	0.0044
pst026-06	1.0000	0.0043	0.9904	0.0007	6.87E-08	0.9904	0.0044
pst026-07	1.0000	0.0041	0.9910	0.0007	6.83E-08	0.9910	0.0042
pst026-08	1.0000	0.0039	0.9940	0.0007	6.84E-08	0.9940	0.0040

**Table 3. Plutonium Benchmark Results on the *Auk* Workstation with
 COG 10 and the ENDFB6R7 Neutron Cross-Section Library**

Benchmark ID	Experimental Benchmark k-effective (k_{expected})	Experimental Benchmark Uncertainty (σ_{expected})	Calculated k-effective ($k_{\text{calculated}}$)	Calculational Uncertainty ($\sigma_{\text{calculated}}$)	Calculated Median Energy of the Neutrons Causing Fission (MeV)	Normalized Calculated k-effective ($k_{\text{normalized}}$)	Combined Uncertainty (σ_{combined})
pst026-09	1.0000	0.0049	0.9882	0.0006	1.68E-07	0.9882	0.0049
pst026-10	1.0000	0.0048	0.9883	0.0007	1.67E-07	0.9883	0.0049
pst026-11	1.0000	0.0047	0.9884	0.0007	1.67E-07	0.9884	0.0048
pst026-12	1.0000	0.0047	0.9887	0.0007	1.68E-07	0.9887	0.0048
pst026-13	1.0000	0.0045	0.9913	0.0007	1.67E-07	0.9913	0.0046
pst026-14	1.0000	0.0044	0.9926	0.0007	1.68E-07	0.9926	0.0045
pst026-15	1.0000	0.0046	0.9766	0.0006	2.01E-07	0.9766	0.0046
pst026-16	1.0000	0.0044	0.9827	0.0006	2.01E-07	0.9827	0.0044
pst026-17	1.0000	0.0050	0.9814	0.0007	2.50E-07	0.9814	0.0050
pst026-18	1.0000	0.0049	0.9818	0.0007	2.49E-07	0.9818	0.0049
pst026-19	1.0000	0.0049	0.9837	0.0006	2.49E-07	0.9837	0.0049
pst026-20	1.0000	0.0049	0.9832	0.0006	2.50E-07	0.9832	0.0049
pst026-21	1.0000	0.0049	0.9835	0.0007	2.50E-07	0.9835	0.0049
143 total cases							

Figure 1. Plutonium Benchmark Results on the *Auk* Workstation with COG 10 and the ENDFB6R7 Neutron Cross Section Library



5.0 Recommended Upper Subcritical Limit

A conservative limit to differentiate subcritical from critical conditions by computation with the COG 10 code system and the ENDFB6R7 neutron cross section library on the *Auk* workstation for general application to Pu systems is:

$$k_{\text{calculated}} + 3\sigma_{\text{calculated}} \leq \text{USL} = 0.9467$$

which satisfies the criterion:

$$k_{\text{calculated}} + 3\sigma_{\text{calculated}} \leq \text{USL} = \{1 + \text{bias}\} - \{3\sigma_{\text{total}}\} - \{0.02\}$$

This criterion ensures better than 99.8% confidence that the USL lays below the 143 selected Pu benchmark results with an additional 0.02 margin of safety.

Some applications may warrant a lower limit if a higher confidence or a larger margin of safety is desired. Conversely, some applications may warrant a higher limit if a lower confidence or smaller margin of safety is justified. Results may also vary with the statistical methodology or the number and combination of selected Pu benchmarks.

6.0 References

1. *COG: A Multiparticle Monte Carlo Transport Code, Version 10, CCC-724*, contributed by the Lawrence Livermore National Laboratory to the Radiation Safety Information Computational Center at the Oak Ridge National Laboratory, released February 2006.
2. C. Lee and P. Chou, *Verification of the Installation of COG10 on Auk*, CSAM10-144, Lawrence Livermore National Laboratory, November 18, 2010.
3. *International Handbook of Evaluated Criticality Safety Benchmark Experiments, NEA/NSC/DOC(95)03*, Organization for Economic Cooperation and Development, Nuclear Energy Agency, Paris, France (September 2008 edition).