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# HARMONIC RATCHETING FOR FAST ACCELERATION\*

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## Abstract

From medical synchrotrons to accelerator driven systems, there is a need for fast acceleration of protons and light ions. This can be a costly undertaking, requiring specially designed ferrite loaded cavities to be tuned across a wide range of frequencies. Ferromagnetic materials allow for the precise adjustment of cavity resonant frequency, but rapid tuning changes and operation outside material specific frequencies result in significant Q-loss. This leads to a considerable increase in power requirements. We introduce an acceleration scheme known as harmonic ratcheting which reduces the cavity frequency range when accelerating an ion beam in a synchrotron. This scheme addresses the needs of high repetition rate machines for applications in which low beam intensity is needed. We demonstrate with simulations the type of ramps achievable with ratcheting and consider its advantages.

## INTRODUCTION

The potential for fast acceleration of low energy ion beams remains rich, with a host of applications proposed to take advantage of rapid cycling synchrotrons or FFAGs. Nonetheless, conventional acceleration of low  $\beta$  charged particles remains expensive and inefficient. The primary challenge of accelerating such particles lies in achieving robust and flexible tuning. Using speciality ferrites, tuning can be obtained at the cost of efficiency. Ferrite materials suffer dramatic loss effects when driven at high bias fields and high magnetic flux [1] [2]. These difficulties have stemmed the advancement of fast and efficient accelerators for low energy ion beams.

The integer harmonic number  $h$  relates the RF and revolution frequencies through the relationship  $f_{RF} = h f_{rev}$ . The relative RF frequency range can be significantly reduced below the revolution frequency range by decreasing  $h$  in steps as the ions accelerate and  $f_{rev}$  increases. This is the motivation and basic method of *harmonic ratcheting*.

## HARMONIC RATCHETING

Suppose that two (or more) RF cavities take turns accelerating the beam – one turns on when the other turns off, at different RF frequencies – so that the RF frequency is always constrained to remain in the range

$$f_{min} \leq f_{RF} \leq f_{max} \quad (1)$$

where  $f_{min}$  and  $f_{max}$  are externally determined design parameters. It is possible to make the transition back or forth

between harmonic numbers

$$\begin{aligned} h &= n \\ \text{and } h &= n + \Delta \end{aligned} \quad (2)$$

where  $n$  and  $\Delta$  are positive integers, if

$$\frac{f_{max}}{n + \Delta} > \frac{f_{min}}{n} \quad (3)$$

Equivalently, we have

$$\Delta < n r \quad (4)$$

where

$$r \equiv \frac{f_{max}}{f_{min}} - 1 > 0 \quad (5)$$

is the “ratcheting parameter”. Equation 4 shows that  $\Delta$  has a maximum permissible value, which must be greater than 1 if a harmonic transition is to be possible. A ratcheting transition is possible if the following conditions are met.

$$\begin{aligned} n &> 1, & r &> 1 \\ n &> \frac{1}{r}, & r &< 1 \end{aligned} \quad (6)$$

Several harmonic ratchets can take place during one acceleration ramp. Figure 1 show an example with  $r = 0.50$ , where the revolution frequency increases from 0.61 MHz to 3.35 MHz, while the RF frequency is constrained to lie between 5.5 MHz and 8.25 MHz.

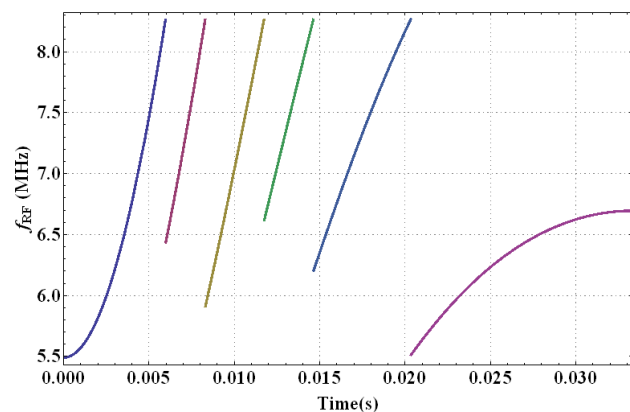


Figure 1: An example of harmonic ratcheting with  $r = 0.50$ . The plot shows one possible solution – with the harmonic number sequence  $h = 9, 7, 5, 4, 3, 2$  – that maximizes RF frequency while minimizing ratcheting number.

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## Designing a Ratcheting Ramp

The selection of the initial harmonic number in the ratcheting ramp is determined by considering the minimum required bucket length,  $t_{inj}$  to accept particles into a single bucket at injection. This length sets a maximum possible initial harmonic. Moreover, the energy acceptance at injection also scales inversely with  $h_{max}^2$ . These requirements can be summarized by

$$h_{max} \leq \frac{1}{t_{inj} f_{rev}(0)} \quad (7)$$

and  $h_{max}^2 \leq \frac{2QV_g \beta^2 E_s}{\pi |\eta_s| \Delta E_0^2}$

Here,  $Q$  is particle charge,  $E_s$  is synchronous energy, and  $\eta_s$  is the phase slip factor. As the initial harmonic increases, a corresponding increase in voltage is necessary to keep the acceptance the same. Ferrite tuning time poses an additional constraint on the ratcheting permutation. Tuning the inactive cavity from a high frequency down to a low frequency requires circumnavigating the hysteresis curve. This tuning time,  $\tau_t$ , is on the order of a few ms, which sets a lower limit on the active time for any cavity between ratcheting transitions  $\tau_{active} > \tau_t$ .

## Emittance growth at ratcheting transition

During the harmonic transition, the voltage on the cavity operating near  $f_{max}$  at harmonic  $h = n + \Delta$  is reduced while a second cavity operating at harmonic  $h = n$  has its voltage raised from a feedback level to provide the desired gap voltage at the new harmonic. Assume that this change happens quickly and that both the synchronous phase and total accelerating voltage are smoothly varying when the ratcheting takes place.

The result of the harmonic ratcheting is a relaxation of the bucket potential. As a result, large amplitude particles stray further in phase from the design particle in order to feel the same restoring force, thereby increasing longitudinal emittance  $S$ . Similarly, the bucket area  $A$  will grow as  $h$  is reduced. Both quantities grow as

$$\frac{S_2}{S_1} \propto \frac{A_2}{A_1} \propto \left( \frac{n + \Delta}{n} \right)^{\frac{3}{2}} \quad (8)$$

The emittance growth due to the harmonic number transition matches with the bucket area growth during the ratcheting process. Thus, for small or slow changes in harmonic number, the ratcheting process is inherently stable, as illustrated in figure 4. This transition is calculated non-adiabatically. The transfer of voltage takes place over some time period  $\tau_r$ . A particle will make  $n_t = \beta c \tau_r / C$  turns in that time. If  $\tau_r = 100 \mu s$ , a  $\beta = 0.1$  particle makes 50 turns of a 60 m synchrotron.

## FERRITE-LOADED CAVITIES

The resonant frequency of a ferrite-loaded cavity is  $f = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$  where  $L$ , the inductance of the cavity, is ad-

justable between minimum and maximum values  $L_{min}$  and  $L_{max}$ . Consider a cavity with a total ferrite length of  $l$ , made of many rings with inner and outer radii  $r_a$  and  $r_b$ . The inductance is  $L = \frac{l}{2\pi} \mu' \ln \frac{r_b}{r_a}$  where the *incremental* complex permeability of the ferrite is written  $\mu = \mu' + j\mu''$ . The component  $\mu'$  and the inductance of the cavity are tuned by biasing the ferrite with a pseudo-constant azimuthal magnetic field driven by a tuning current. Ignoring the stray inductance, the required dynamic range of  $L$  and of  $\mu'$  is

$$\frac{L_{min}}{L_{max}} = \frac{\mu'_{min}}{\mu'_{max}} = \frac{1}{(1+r)^2} \quad (9)$$

The voltage across the gap of a cavity is

$$V_{gap} = 2\pi f B_{max} l r_a \ln \frac{r_b}{r_a} \sim f l \quad (10)$$

where  $B_{max} \approx 0.01$  T is the maximum RF magnetic field that is allowed at the inner radius of the ferrite rings. This shows that for a fixed maximum field  $B_{max}$ , the ferrite length  $l$  can be reduced if the RF frequency in the cavity is increased. Thus, harmonic ratcheting allows a given acceleration waveform to be achieved with shorter cavities.

The total length of ferrite required with a ratcheting parameter  $r$ , relative to a non-ratcheting scheme in which ions are accelerated over the full dynamic range is given approximately by

$$\frac{l_{ratchet}}{l_{non-ratchet}} = \frac{f_{min,non-ratchet}}{f_{min,ratchet}} \approx 2 \frac{1+r}{D} \quad (11)$$

where  $D = \frac{\beta_{max}}{\beta_{min}}$ , and the factor of 2 recognizes that only half the cavities are active at any one time. For example, Figure 1 illustrates acceleration over a dynamic range of  $D = 5.5$ , using a ratcheting parameter of  $r = 0.50$ . In this case ratcheting makes it possible to decrease the total length of ferrite by a factor of 0.55. Conversely, the gap voltage can be nearly doubled for a fixed cavity length, almost doubling the potential repetition rate of a rapid cycling synchrotron.

## EXAMPLE: A RAPID CYCLING MEDICAL SYNCHROTRON

Consider a rapid cycling synchrotron for acceleration of  $C^{6+}$  ions for radiation therapy applications. Assume a 15 Hz repetition rate and a circumference  $C = \sim 65$  m. Ions must be accelerated through a range 8 to 400 MeV/ $\mu$ .

Once the initial harmonic is determined, the initial RF frequency is set at  $f_{RF}(0) = h_0 \beta c / C$ . Appropriate selection of a frequency ceiling limits the ferrite to operate within a more efficient permeability range. In this instance, we choose a ratcheting parameter of  $r = 0.5$  via a final harmonic transition of  $h = 3$  to  $h = 2$ . Figure 1 shows the RF frequency and figure 2 plots the active harmonic.

When the frequency ceiling is reached at the current harmonic, RF power is transferred to the second cavity system at a lower harmonic number. Each time the harmonic

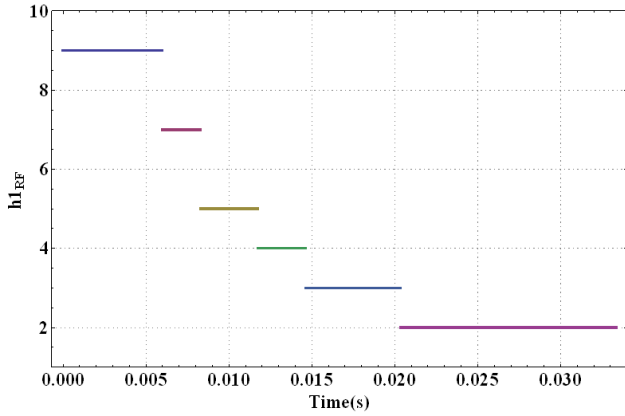


Figure 2: Active cavity harmonic for the  $\sim 65$  m Carbon synchrotron harmonic ratcheting ramp.

number is reduced, the RF frequency is reduced by a factor  $\frac{h_n}{h_{n-1}}$  where  $h_n$  is the new harmonic, and  $h_{n-1}$  is the previous value. The maximum frequency swing during the ramp is determined by the maximum value of this ratio, or equivalently the ratcheting parameter  $r$ , according to

$$\frac{\Delta f}{f_0} = r \quad (12)$$

For example, if the largest transition in a particular ratcheting scheme is from  $h = 3$  to  $h = 2$ , then the ratcheting parameter is  $r = 0.5$ , and that particular ramp is limited to a 50% frequency swing.

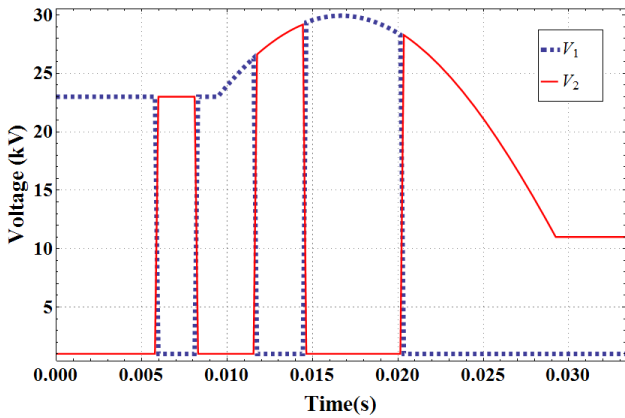


Figure 3: Output voltage for each cavity system. The voltage is calculated to meet the constraint  $\phi_s \leq 33^\circ$ .

Figure 3 outlines the output voltage needed for each cavity system in our example ramp. The harmonic transition and transfer of power to the inactive cavity cannot be performed instantaneously; the voltage may be raised over  $\sim 100\mu s$ . Detailed feedforward and diagnostic systems are required in order to maintain the proper voltage. The emittance evolution for our example case is shown in 4.

Ratcheting may also reduce losses due to anomalous effects in ferrite. At sustained high fields, ferrites demonstrate a Q-loss effect (QLE) which can result in a reduction of 20% of maximum field. High bias fields can increase

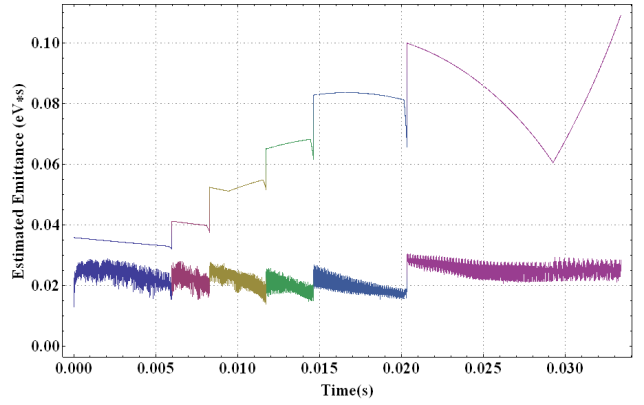


Figure 4: Estimated emittance and bucket area is shown for a simulated ramp with fixed synchronous phase.

this effect by reducing the threshold of Q-loss onset. However, at high bias rates the threshold for this effect is significantly raised [1]. A ratcheting scheme should mitigate QLE by reducing total biasing. Rapid bias field changes may also induce a dynamic loss effect in the ferrite. These losses can lower a cavity's Q value by up to 40%. Unlike the QLE, this effect appears to be more dependent on the temperature and duration of the biasing. By decreasing the time spent biasing at high fields, this effect can be reduced considerably [2].

## CONCLUSION

A new scheme for accelerating low energy ions quickly and over a large frequency range is presented. By adjusting the harmonic number during acceleration in a manner specifically tailored to the desired acceleration cycle, a significant reduction in the frequency range is obtained, easing the ferrite's tuning requirement. Moreover, the frequency increase allows potentially several times larger gap voltages to be obtained for the same amount of ferrite, making higher machine repetition rates possible. Alternatively, cavity length could be reduced by an equivalent factor. This approach is particularly suited to low intensity ion beams which do not require filling a ring but necessitate fast acceleration across a range of energies.

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## REFERENCES

- [1] K. Kaspar, H. G. König, and T. Winnefeld., "Studies on the maximum RF voltages in ferrite-tuned accelerating cavities," TUPKF013, Proceedings of EPAC 2004, 985-7.
- [2] J.E. Griffin and G. Nicholls. IEEE Trans, Nucl. Sci. 26 (1979) 3965-7.
- [3] Y. Zhao, "A Study of Ferrite Cavity," BNL Informal Report CAP-341-RF-R, April 2002
- [4] N. Cook, Dissertation, Stony Brook University. To Be Published.