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A TEMPERATURE DISTRIBUTION ANALYSIS ALONG A THERMAL RADIATING FIN OF NONUNIFORM THICKNESS

by

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A TEMPERATURE DISTRIBUTION ANALYSIS ALONG A THERMAL RADIATING FIN OF NONUNIFORM THICKNESS

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I. ABSTRACT

The general differential equation of a thermally radiating fin is presented for the situation of a uniform heat source along the base of the fin.

An electronic analog solution of a fin of nonuniform thickness with one-dimensional steady-state radiant heat transfer is presented. The specific example is concerned with a constant rate of variation of thickness; however, any general configuration could have been substituted.

II. NOMENCLATURE

a	scale factor for time
b	scale factor for temperature
dq	element of heat
ds	element of surface
e	fin efficiency
h	fin end half-thickness
H	fin base half-thickness
k	thermal conductivity
L	length
q	rate of heat dissipation
q^0	ideal rate of heat dissipation
t	time
t'	scaled time
T	absolute temperature

T_0	absolute temperature at the fin base
T_s	absolute temperature of the sink
T'	scaled absolute temperature
W	unit width
x	variable distance parallel to abscissa
y_x or $y(x)$	variable distance parallel to ordinate
σ	Stefan-Boltzman Constant
ϵ	emissivity

III. INTRODUCTION

Designers of power plants for space vehicles are confronted with the problem of rejecting waste heat from their thermodynamic cycle by radiation. A radiator of minimum weight is the objective in power plant designs, and it is a measure of the optimization achieved in designing a specific radiator.

Space radiators are generally composed of coolant-carrying tubes or channels which have affixed to them extended surfaces to convey the heat away from the coolant more efficiently. Finning lowers the number of coolant tubes required for heat rejection, and a reduction in the number of working fluid passages reduces the probability of a coolant loss as a result of meteor puncture. Fins penetrated by meteors have very little of their heat transfer area removed. The principal objective of finning, then, is to obtain a maximum heat flux per unit weight for a minimum probable operating time between meteor punctures. Before the fin length thickness and effectiveness may be evaluated for a given heat load and probable meteor density, however, the temperature distribution along the surface of the fin must be determined. The general configuration of the fin may not necessarily be of uniform thickness for obtaining a radiator of overall minimum weight. Therefore, a generalized differential equation was required.

IV. EQUATION DEVELOPMENT

In the generalized differential equation, the difference in heat conducted into a slice dx thick is

$$\frac{d}{dx}\left(2kWy_x \frac{dT}{dx}\right) dx = 2\, Wky_x \frac{d^2T}{dx^2}\, dx + 2\, Wk \frac{dT}{dx} \frac{dy_x}{dx}\, dx \quad . \qquad (1)$$

In order for the above equation to be true, the following assumptions were made:

(1) the temperature gradient in the y and z directions was negligible;

(2) radiation exchange between source and fin was of minor importance; and

(3) the surface emissivity and thermal conductivity were constant.

A general heat balance requires the differential equation for heat conducted down the fin to be equal to the heat dq rejected by radiation:

$$dq = 2\,\sigma\epsilon L\,ds\,(T^4 - T_s^4) \quad . \tag{2}$$

By assuming that ds on the arbitrary surface is equivalent to dx, the generalized differential equation for T is represented by

$$y_x \frac{d^2T}{dx^2} + \frac{dT}{dx}\frac{dy_x}{dx} - \frac{\sigma\epsilon}{k}(T^4 - T_s^4) = 0 \quad . \tag{3}$$

For a specific example, consider the geometry shown in Figure 1, where

$$y_x = -\frac{(H - h)}{L}\,x + H \quad . \tag{4}$$

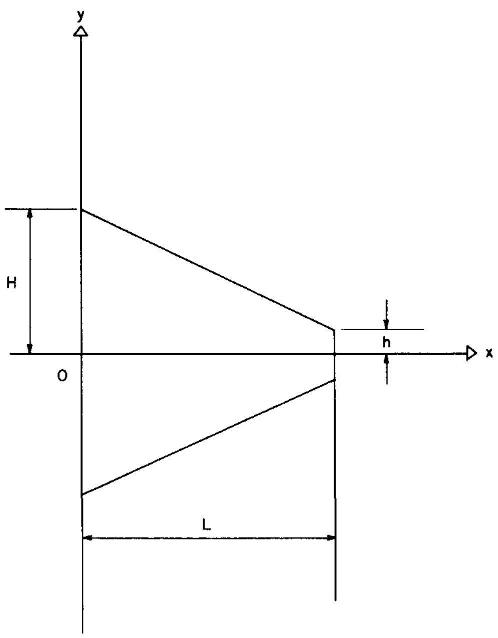

Figure 1. Thermal Model of Fin with Constantly Varying Thickness.

Substituting equation (4) into equation (3), there is obtained

$$\frac{d^2T}{dx^2} = \frac{1}{HL - (H - h)x}\left[(H - h)\frac{dT}{dx} + \frac{L\sigma\epsilon}{k}(T^4 - T_s^4)\right] \quad . \tag{5}$$

If $h = H$, $y_x = H$. Equation (5) is then reduced to the equation for a straight rectangular fin, which is given by

$$\frac{d^2T}{dx^2} = \frac{\sigma\epsilon}{kH}(T^4 - T_s^4) \quad . \tag{6}$$

Once the temperature distribution is resolved, the rate of heat dissipation from the surface is computed according to the equation

$$q = 2\ \sigma\epsilon W \int_0^L (T^4 - T_s^4)\ dx \quad . \tag{7}$$

If the entire fin surface, however, were at the base temperature T_0, then the ideal rate of radiation heat dissipation would be given by

$$q^0 = 2\sigma\epsilon(T_0^4 - T_s^4)\,LW \quad . \tag{8}$$

From equation (7) and equation (8), fin performance may be calculated as the ratio of total heat dissipated by the fin (with temperature gradient) to that which would be dissipated if the entire fin surface were at T_0.

Fin efficiency e is defined by

$$e \equiv q/q^0 \quad . \tag{9}$$

V. ANALOG EQUATION DEVELOPMENT

The transformation of equation (5) into a form suitable for analog representation is made by the following substitutions:
$x = t$, $t' = at$, and $T' = bT$. An additional assumption which need not have been made was that $T_s = 0$. The equation, however, is now represented by

$$\frac{d^2T'}{dt'^2} = \frac{10^2}{[10^3 aHL - 10^3(H - h)\,t']}\left[10\,(H - h)\frac{dT'}{dt'} + \frac{10\,L\sigma\epsilon 10^6}{kab^3}\left(\frac{T'^4}{10^6}\right)\right] \quad .$$

Potentiometers shown on the analog diagram, Figure 2, are set as listed below.

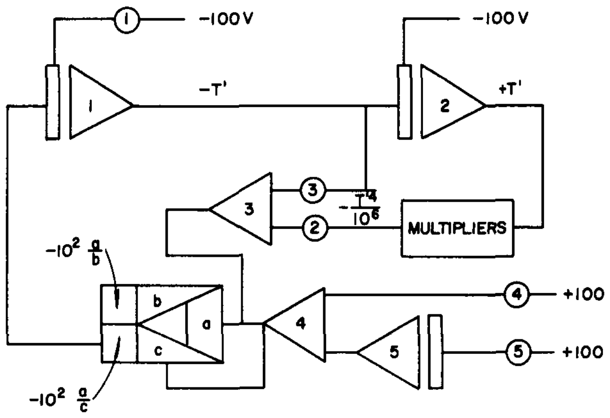

Figure 2. Analog Diagram of Solution

Pot one was used to insure that $dT/x = 0$ at $x = L$,

Pot two was used for representing $\frac{L\sigma\epsilon\, 10^7}{kab^3}$,

Pot three for $10(H - h)$,

Pot four for $(10^3\, aHL)$ volts,

and Pot five for $10^3(H - h)$ volts.

The constants supplied below were those used for obtaining the curves given in Figures 3 and 4:

$a = 10^2$

$b = 5 \times 10^{-2}$

$h = 0.25 \times 10^{-3}$ ft

$H = 1.25 \times 10^{-3}$ ft

$k = 25.0$ BTU/hr-ft^2 - ^{0}R

$k = 100.0$ BTU/hr-ft^2 - ^{0}R

$L = 0.25$ ft

$T_0 = 2000^0$R

$W =$ one ft

$\sigma = 0.173 \times 10^{-8}$ BTU/hr-ft^2 - ^{0}R^4

$\epsilon = 0.90$.

The boundary conditions

$T = T_0$ at $x = 0$ or $T = T_0$ at $t' = 0$

and

$$\frac{dT}{dx} = 0 \text{ at } x = L \text{ or } \frac{dT}{dt'} = 0 \text{ at } t' = t \text{ (final)}$$

are imposed upon the differential equation in order that the physical situation be correctly represented. It is also noted that when $h = 0$ both the mathematical and the physical situation become impractical to achieve. Therefore, another requirement that must be satisfied is that

$$0 < h \leq H$$

for practical systems. When $h > H$, the practicability of the designs becomes questionable.

The solution of equation (5) for any general geometry has also demonstrated its ability to be accurate, quick, and economical.

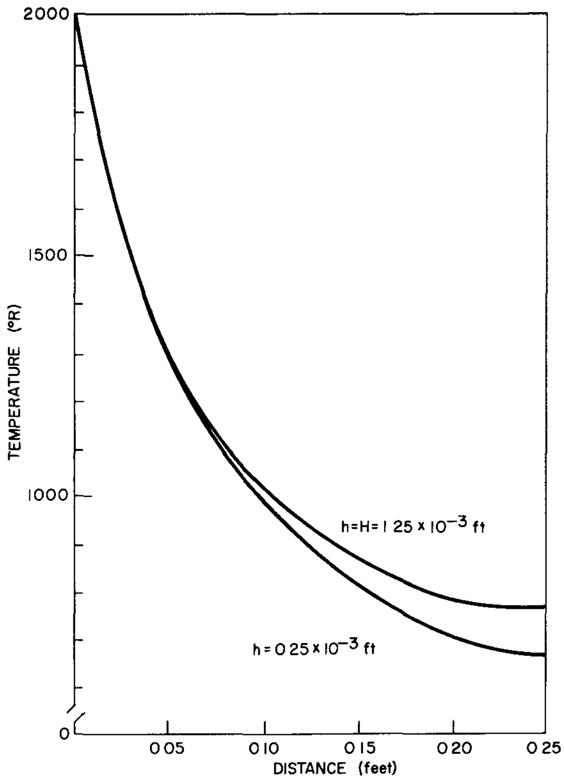

Figure 3. Temperature Distributions of a Uniform and a Constantly Varying Thickness Fin (k = 25 BTU/hr - ft - °R)

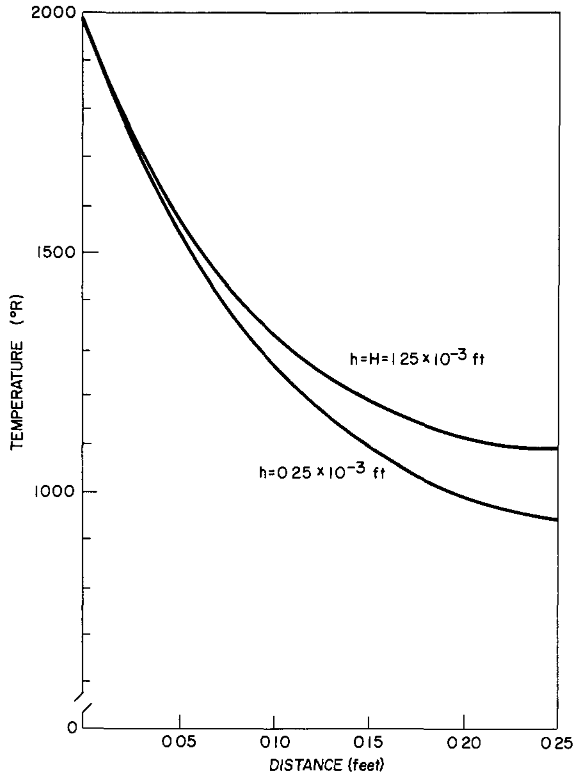

Figure 4. Temperature Distributions of a Uniform and a Constantly Varying Thickness Fin (k = 100 BTU/hr - ft - °R)

VI. BIBLIOGRAPHY

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VII. ACKNOWLEDGMENT

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