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Erratum: Time-step Considerations in Particle Simulation Algorithms for Coulomb Collisions in Plasmas [B.I. Cohen, A. M. Dimits, A. Friedman, and R. E. Caflisch, IEEE Trans. Plasma Sci. 38, 2394 (2010)]

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Erratum: Time-step Considerations in Particle Simulation Algorithms for Coulomb Collisions in Plasmas [B.I. Cohen, A. M. Dimits, A. Friedman, and R. E. Caflisch, IEEE Trans. Plasma Sci. **38**, 2394 (2010)]

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***Index Terms*— collision processes, plasmas, particle collisions, algorithms, computer applications, numerical analysis**

In Eqs.(12-17) of [1], a next higher-order correction in powers of $\Delta t^{1/2}$ to the Langevin equations for Coulomb collisions was derived. In Eq.(16) of [1] the one-dimensional drag-diffusion equation is given as

$$y_{n+1} = y_n + a(y_n)\Delta t + b(y_n)\Delta W + \tfrac{1}{2} b \left.\frac{db}{dy}\right|_{y_n} (\Delta W^2 - \Delta t) \qquad (1)$$

where a is the drag coefficient, b is the diffusion coefficient, y_n represents a velocity variable at time step with index n, and ΔW is a Gaussian random variable with zero mean and variance Δt. The last term in Eq.(1) is the so-called Milstein correction [2]. The ΔW^2 in the Milstein term is the square of the ΔW in the diffusion term.

In Eq.(17) an error was made by us in the Milstein term: namely, the same random number N_1 should have been used rather than a new random number N_4 in keeping with Eq.(1) given here. The corrected version of Eq.(17) in [1] is thus

$$\begin{aligned} \mathrm{v}_z^{t+\Delta t} &= \mathrm{v}_z^t + F_d \Delta t + g\Delta t^{1/2} N_1 + \tfrac{1}{2} g \tfrac{dg}{d\,\mathrm{v}} \Delta t (N_1^2 - 1) \\ \mathrm{v}_{\perp 1,2}^{t+\Delta t} &= \Delta \mathrm{v}_{\perp 1,2} \end{aligned} \qquad (2)$$

Furthermore, N_4^2 should be replaced in the text of [1] with N_1^2 wherever N_4^2 appears after Eq.(17). The error in Eq.(17) of [1] propagated into the simulations whose results are reported in Fig. 7 of [1]. We have corrected the simulation algorithm and repeated the simulations shown in Fig. 7 of [1]. The new numerical results differ very little from the previous results (<1% difference) and are not detectable to the eye in the revised Fig. 7 of [1]. This is a consequence of the fact that the Milstein correction has little effect on moments of the velocity distribution owing to $< N_1^2 - 1 > = 0$ and the smallness of the coefficient $dg/d\mathrm{v}$ in the Milstein term in Eq.(2) over most of the velocity distribution as discussed in Sec. III of [1].

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