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QUARK FOCUSING AT HADRONIC VERTICES

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ABSTRACT

A basis for a dual theory of processes that involve external and/or intermediate exotic hadrons is found in the form of a quark focusing principle. This principle, derived from the conformal invariance of the theory, requires that at a hadronic vertex all active quarks be focused to a point of their world sheet and act together (as a point-like particle) in the transfer of momentum. It is found that all hadronic couplings (including the coupling of any three exotic hadrons) can be expressed in terms of a single (say the $\rho\pi\pi$-) coupling constant. Applications to $B\bar{B}$- scattering are made.

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1. Introduction

The factorized Veneziano model[1)-4)] serves as a basis for the dynamics of nonexotic (i.e. $q\bar{q}$-type) mesons. It provides one with a vertex and a propagator from which the amplitude for any process involving only nonexotic mesons can be calculated. In nature, in addition to nonexotic mesons, there exist baryons and if strong interactions are dual there must[5)] also exist exotic hadrons.[*)] A complete basis for hadrodynamics requires the specification of all hadronic (i.e. mesonic, baryonic and exotic) vertices and propagators.

In the present paper and in a forthcoming paper[6)] we consider all these vertices and propagators. Our main guide in attacking this problem is duality. The solution has to answer three basic problems:

A) The number of degrees of freedom of a hadron increases as its total quark number (number of quarks + number of antiquarks) increases. How does one describe structures with an ever increasing number of degrees of freedom?

B) In the usual Veneziano model the process of a meson emitting another meson can be visualized[1),3)] as one of the quarks (the active quark) of the meson doing the emission and

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*) As pointed out in ref. 5, duality in $B\bar{B} \rightarrow B\bar{B}$ requires the existence of $qq\bar{q}\bar{q}$-mesons M_4. Similarly duality in $BM_4 \rightarrow BM_4$ requires the existence of $qqqq\bar{q}$-baryons B_5. Then $B\bar{B}_5 \rightarrow B\bar{B}_5$ requires $q^3\bar{q}^3$-mesons M_6 and dibaryons D_6, etc...One thus concludes that in a dual theory there must exist exotic hadrons with arbitrarily high total quark number.

executing the transfer of momentum while the other quark acts as a "spectator". At vertices involving hadrons with total quark number $N \geqslant 3$, two or more quarks get transfered from one hadron to another. Therefore two or more quarks may become active (i.e. participate in the transfer of momentum). The question then is what is the share of each of these active quarks in the transfer of momentum?

C) Given any three hadrons A, B and C such that the vertex ABC is allowed by the usual selection rules[7] of hadronic couplings, how does the ABC coupling constant*) depend on the total quark numbers N_A, N_B, N_C? Thus, for instance, what is the ratio of the MMM and $\bar{B}BM_4$ ($M = q\bar{q}$, $B = qqq$, $M_4 = qq\bar{q}\bar{q}$) coupling constants?

In this paper we shall answer questions B and C. Our answer to question B can be formulated in the form of a quark focusing principle: at a hadronic vertex all active quarks are focused to a point of their world sheet**) and act together (as a point-like particle) in the transfer of momentum.

A typical illustration of the quark focusing principle is given in Fig. 1. Here a baryon emits a baryon to become

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*) We mean here the coupling constant of the ground states of N_A, N_B and N_C quarks and/or antiquarks. The factorized dual resonance model then automatically produces the couplings of orbital and radial excitations.

**) i.e. the manifold described by the propagation in proper time of the constituent "rubber bands" of the hadrons. See ref. 8.

a meson. The three active quarks a, b and c focus at the point V of the world sheet. In other words the quark a and b (the active quarks) of baryon 1 get together and act as a point-like diquark that picks up momentum from the antibaryon 2 and (conserving baryon number) becomes the antiquark c of the final meson.

The answer to problem C) will be provided in the form of a universality principle of hadron couplings. The principle states that the matrix element for shifting the momentum of the focused active quarks of the initial hadron to that of the final hadron in the process of hadron emission is independent on the number of quark lines being focused and on the number of spectator quarks. This universality principle will be derived from duality. It fixes all hadronic coupling constants once one - say the $\rho\pi\pi$ coupling constant - is given.

Problem A will be considered in paper II[5)] of this series.

2. Determination of the Dual Baryon Emission Vertex

We start by considering nonexotic meson-meson scattering (fig. 2). At the vertex A the meson 2 is emitted by the quark of meson 1, leading to the "intermediate" meson of the process. This meson emission amounts to a shift by k_2 of the momentum of the quark. Therefore the vertex operator is given by the expression*

$$\Gamma_{(M \rightarrow M)_M} = G_2 : e^{i k_{2\mu} \phi^{\mu} (\xi_q, \lambda_2)} : \qquad (1)$$

where $\phi^{\mu}(\xi_q, \lambda_2)$ is the generator of momentum shifts for the quark of meson 1 at the proper time $\tau_2 = i\lambda_2$ at which the emission occurs. This is precisely the quark position operator, i.e. the well known Nambu-Susskind field evaluated at the quark ($\xi_q = 0$). G_2 in eq. (1) is a constant that normalizes the overall strength of meson couplings.

To construct the vertex for baryon emission consider meson-baryon scattering. The baryon is viewed[8),9)] as a set of three quarks in the polar positions $\xi_1 = 0$, $\xi_2 = {}^{2\pi}/3$ and $\xi_3 = {}^{4\pi}/3$ on a "rubber band". In proper time (and therefore in λ) this rubber band describes a cylinder which for the sake of simplicity we cut open along the trajectory of the first quark whereupon the "world sheet" of

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* Here and below we use the notation $\Gamma_{(A \rightarrow B)_C}$ for the vertex at which the initial hadron A becomes the final hadron B by emitting the hadron C.

the baryon gets mapped onto the strip $0 \leqslant \xi \leqslant 2\pi$ of the complex $z = \lambda + i\xi$ plane. Meson-baryon scattering is now pictured (fig. 3a) as an incoming baryon 1($z_1 = -\infty$) that emits a meson 2 at $z_2 = 0$, then emits another meson at some $z_3 = \mathrm{Re}\, z_3 > 0$ to end up as the outgoing baryon 4 ($z_4 = +\infty$). The corresponding quark diagram is 3b. The diagrams 3c and 3d will be discussed later. One can easily calculate[9),10)] the amplitude V_{3a} corresponding to fig. 3a:

$$V_{3a} = G_3^2 \int_0^\infty d\lambda_3\, f(\lambda_3) \langle 0 | e^{ik_3\phi(0,\lambda_3)} e^{ik_2\phi(0,0)} | 0 \rangle =$$

$$= G_3^2 \int_0^1 dx\; x^{-1-\alpha_B(s)} (1-x)^{-1-\alpha_M(t)} \qquad (2)$$

V_{3a} takes the usual Veneziano form. In eq. (2)

$$\phi_\mu(\xi,\lambda) = \sum_{r=1}^\infty \left(\frac{2}{r}\right)^{1/2} \Big[\left(a_{\mu r}\, e^{-r\lambda} + a_{\mu r}^\dagger\, e^{r\lambda}\right) \cos r\xi$$

$$+ \left(b_{\mu r}\, e^{-r\lambda} + b_{\mu r}^\dagger\, e^{r\lambda}\right) \sin r\xi \Big], \qquad (3)$$

$f(\lambda_3)$ is the usual intercept adjusting factor, the detailed and for us irellevant manipulations of which have been omitted for simplicity's sake. To extract the baryon emission vertex we recall that duality is a consequence of the conformal invariance of the theory. We therefore perform the conformal mapping

$$w = \ln (e^{z} - 1) \tag{4}$$

It maps the points $z_1, \ldots, z_4$ into the points $w_1 = i\pi$, $w_2 = -\infty$, $w_3 = \xi$ where ξ is a real number and $w_4 = z_4 = +\infty$ (fig. 4a). The trajectories Im $z = i\,\frac{2\pi}{3}$ and Im $z = i\,\frac{4\pi}{3}$ of the passive quarks of fig. 3a get mapped into the curves $C_{\pm}$ of fig. 4a. With the notation

$$w = \mu + i\eta \tag{5}$$

the curves C_+ and C_- are the solutions of the equation

$$\cos \eta_{\pm}(\mu) = - \frac{3 + (4e^{2\mu} - 3)^{1/2}}{4e^{2\mu}} \tag{6}$$

selected by the asymptotic conditions

$$\eta_{\pm}(\infty) = \pi \pm \pi/3 \tag{7}$$

The quark trajectories C_+ and C_- that originally intersected at $z = z_1 = -\infty$ and at $z = z_4 = +\infty$, now, in view of the conformal nature of the mapping (4) intersect at $z_4 = +\infty$ and at $z_1 = i\pi$. As was already mentioned in the introduction this means that the two quarks that get transfered from the baryon 1 to the baryon 4 are focused to a point at exactly $w = i\pi$. One can thus visualize the vertex as the baryon 1 "kicking" the antiquark of the meson 2 with momentum k_1. At the same time this antiquark becomes a point-like diquark that as time goes on defocuses along the paths C_+ and C_- until asymptotically at $\mu = +\infty$ the two quarks are back in the positions $\eta_2 = 2\pi/3$ and $\eta_3 = \frac{4\pi}{3}$ of the baryon 4. The quark diagram corresponding to fig. 4a is fig. 4b. It is now clear that the vertex for baryon emission is

$$\Gamma_{(M\to B)_B} = G_3 : e^{ik_{1\mu}\phi^\mu(\xi_F, \lambda_F)} : \tag{8}$$

where ξ_F and λ_F (in the case of fig. 4a $\xi_F = \pi$, $\lambda_F = 0$) are the coordinates of the focusing point. As a check we now calculate diagram 4a,b using this vertex in conjunction with the usual meson emission vertex (1) and meson propagator, summing of course over both time orderings of the lines 2 and 3 in fig. 4a. We then obtain

$$V_{4a} = G_3 G_2 \int_{-\infty}^{\infty} d\mu\, f(\mu) \langle 0| T\, e^{ik_3\phi(0,\mu)}\, e^{ik_4\phi(\pi,0)} |0\rangle =$$
$$= G_3 G_2 \int_0^1 dx\; x^{-1-\alpha_B(s)}\; (1-x)^{-1-\alpha_M(t)} \tag{9}$$

so that

$$V_{4a} = V_{3a} \tag{10}$$

provided $$G_3 = G_2 \tag{11}$$

This is of course nothing more than a reconfirmation of the theory's conformal invariance. Fig. 3c gets mapped onto fig. 4c and a similar discussion applies. A less trivial case is that of the sum of the two diagrams of fig. 3d. This sum maps into the one diagram 4d. The integration is over μ_3 along the curve C_+. We find (M = baryon ground state mass, m = meson ground state mass)

$$V_{4d} = G_2 G_3 \int_{C_+} d\ell \langle 0| e^{ik_3\phi(\mu,\eta_+(\mu))} e^{ik_1\phi(0,\pi)} |0\rangle$$

$$= G_2 G_3 \int_0^\infty d\mu \left[1+\left(\frac{d\eta_+(\mu)}{d\mu}\right)^2\right]^{\frac{1}{2}} \left(e^{-\mu}\right)^{-s-1+M^2} \left(1+e^{-2\mu}+2e^{-\mu}\cos\eta_+(\mu)\right)^{-k_1.k_3}$$

$$= G_2 G_3 \int \frac{d\mu\; 2}{(4-3e^{-2\mu})^{\frac{1}{2}}} \left(e^{-\mu}\right)^{-s-1+M^2} \left(1-\frac{1}{2}e^{-2\mu}-\frac{1}{2}e^{-\mu}(4-3e^{-2\mu})^{\frac{1}{2}}\right)^{-\frac{u+m^2+M^2}{2}} \tag{12a}$$

Introducing the new variables

$$U = e^{-\mu} \text{ and } V = -\frac{1}{2}U + \frac{1}{2}\ (4-3U^2)^{\frac{1}{2}}$$

and then observing that the last parenthesis in (12a) is just $(V^2)^{-\frac{u+m^2+M^2}{2}}$, that $(4-3e^{-2\mu})^{\frac{1}{2}} = U + 2V$ and that $U^2 + V^2 + UV - 1 = 0$ we can cast V_{4d} in the final form

$$V_{4d} = 2G_2G_3 \int_0^1 dU \int_0^1 dV\, \delta\left(U^2+V^2+UV-1\right) U^{-1-(s-M^2)} V^{-1-(u-M^2)+(1+m^2)} \tag{12b}$$

which as usual becomes crossing symmetric for the mesonic intercept $\alpha_2(0) = -m^2 = 1$. For $\alpha_2(0) \neq 1$ one has to adjust the intercept using a "fifth dimension" in the oscillators. As was to be expected this is the same result as was obtained by Frye, Lee and Susskind[9] and by Mandelstam[10] for the diagram 3d. Thus conformal invariance again ensures

$$V_{4d} = V_{3d} \tag{13}$$

provided eq. (11) holds.

The essence of this section is that in meson-baryon scattering the quark focusing principle is obeyed. Indeed all active quarks focus at the point $z' = i\pi$ on the world sheet in all diagrams of fig. 4. They act in common, as explained above, in the transfer of momentum.

3. $B\bar{B}$-Scattering, Exotic Meson Resonances

With the baryon emission vertex (8) we can now calculate the amplitude for $B\bar{B}$-scattering. This process has no external mesons and therefore could not be calculated with the usual rules of ref. 1-3. Corresponding to the quark diagram 5a we have the z-plane diagram 5b from which we read off the amplitude

$$V_{5a} = G_3^2 \int_{-\infty}^{0} d\lambda\, f(\lambda) \langle 0 |\, e^{ik_3\phi(\pi,0)}\, e^{ik_2\phi(\pi,\lambda_4)} | 0 \rangle$$
$$= G_3^2 \int_0^1 dx\; x^{-1-\alpha_2(s)}\, (1-x)^{-1-\alpha_4(t)} \tag{14}$$

which is of the Veneziano type. Here $\alpha_2(s)$ and $\alpha_4(t)$ are the trajectories of the $q\bar{q}$ and $qq\bar{q}\bar{q}$ mesons respectively. Observe that as far as these considerations are concerned, the intercept $\alpha_4(0)$ is not fixed. Yet, as has been pointed out previously[11] there are arguments in favor of a quadratic dependence of $\alpha_N(0)$ on N and specifically in favor of

$$\alpha_N(0) = \frac{N}{2} - \frac{N^2}{8} \tag{15}$$

which would imply $\alpha_4(0) = 0$ so that the lowest $qq\bar{q}\bar{q}$-meson is expected to lie in the mass region 1500 Mev $\lesssim m \lesssim$ 2000 MeV. We can now perform the conformal mapping

$$v = \ln(e^z + 1) \tag{16}$$

This is precisely the inverse of the mapping (4) so that

$v_1 = 0$, v_2 = negative real number, $v_3 = -\infty$, $v_4 = +\infty$. For the same reason the curves $C_{\pm}$ of fig. 5b, which because of factorization are identical with the curves $C_{\pm}$ on fig. 4a, map onto the straight lines $\tilde{C}_+$ and $\tilde{C}_-$ of fig. 5c the equations of which are

$$\mathrm{Im}\, v_{\pm} = \pi \pm \frac{\pi}{3} \tag{17}$$

The lines D_+, D_- of fig. 5b map onto the lines $\tilde{D}_+$, $\tilde{D}_-$ of fig. 5c which focus twice as required by the principle of quark focusing. From fig. 5c we read off the vertex

$$\Gamma_{(B \to M_4)_B} = G_4 : e^{ik^r \phi_\mu(\xi_{\tilde{F}}, \lambda_{\tilde{F}})} : \tag{18}$$

(here $M_4 = qq\bar{q}\bar{q}$ meson and $\tilde{F}$ is the focusing point fig. 5c). As we did in section 2 we may now calculate the amplitude V_{5c} using the vertex (18). At this level a question arises. In nonexotic mesonic and baryonic processes ϕ_μ was given by eq. (3). In the case of purely nonexotic mesonic processes we had a single infinity of active modes, those described by operators a_r. The b_r modes in this case were decoupled. As soon as baryons are included extra degrees of freedom appear and one activates the b_r modes. Exotic hadrons in general and M_4-mesons in particular have additional degrees of freedom. One has to include these modes in the propagator of these hadrons. Because of duality however, the $\bar{B}BM_4$ vertex is fixed in the form (18) and these new modes do <u>not</u> get excited in $\bar{B}B$ scattering. (They get excited however in other processes as will be shown in II). Thus one should get the same result whether or not one explicitly includes them in the propagator. It is indeed straightforward to check that with the vertex (18) and a propagator corresponding to the Hamiltonian

$$H = \sum r(a_r^+ a_r + b_r^+ b_r) \qquad (19)$$

one obtains the duality relation

$$V_{5c} = V_{5b} \qquad (20)$$

provided

$$G_4 = G_3 \qquad (21)$$

It is also interesting to note that the BB -channel being "intrinsically exotic" there are no s-u and t-u type $B\bar{B}$-amplitudes. Thus the whole $\bar{B}B$ amplitude is given by the sum of diagrams 4a and 4c both of which are of the Veneziano-form. Thus, even though it involves baryons, $B\bar{B}$ scattering, unlike MB scattering, does not have Mandelstam type terms but only usual Veneziano-type terms. Our result also once and for ever settles the problem of what nonexotic meson exchange in $\bar{B}B$ scattering is dual to. Factorization and duality unambiguously imply that it has to be dual to exotic (M_4) mesons and not to background.

4. Principle of Quark Focusing

Figures 4 and 5 offer some typical illustrations of a rather general principle: quark focusing at hadronic vertices. It is the purpose of this section to establish this principle for any hadronic vertex. The crucial ingredients that enter our proof are the following:

i) Any hadronic process, no matter how exotic the hadrons involved (i.e. no matter how large their total quark number) can be described in terms of one or more world sheets (i.e. complex $\lambda + i\xi_n$ planes).

For processes involving only nonexotic mesons and baryons this is obvious. For processes involving exotic hadrons this presumes a solution of the "geometric" problem of quark arrangement within the hadron. As was mentioned in the introduction this will be presented in II. While the details of the "geometry" derived there are irrelevant for our present discussion, we shall use the fact that statement i) holds.

ii) Duality

Duality will only be used in the sense that all hadronic amplitudes are invariant under the mapping (4) in each of the world sheets. Consider an arbitrary hadronic process "initiated" by a given hadron H of total quark number N. By initiated we mean here that on the world sheet $z_H = -\infty$. Otherwise the process can involve any finite number of hadron emissions and any given final (i.e. $z = +\infty$) state compatible with the usual selection rules. In this configuration all quark "trajectories" in H as $z \to -\infty$ will approach parallel lines of Im z = const. and correspondingly intersect at $z = -\infty$. Now perform the conformal mapping (4). H now becomes an emitted hadron and H emission occurs at $w_H = \ln(e^{z_H} - 1) = i\pi$. Because of the conformal nature of the mapping all quark lines that in the z-plane intersected at z_H will now intersect at w_H. These N quark lines will now be coming from the two "neighboring" intermediate hadrons which by a suitable choice of the process can be made to be <u>any</u> two preassigned hadrons A and B for which a vertex HAB is allowed by the hadrodynamic selection rules.[7)]Thus the focusing of active quarks occurs at <u>all</u> hadronic vertices. We repeat here that this proof is independent of any detailed knowledge of the "geometry" of the hadron H. All that matters is the existence of one or more world sheets for the process. In case there are more world sheets we can repeat the above reasoning on each of these sheets.

5. Universality of Hadron Couplings

Along with quark focusing another very important feature of hadron couplings emerges from the argument of the previous sections. In order for duality to hold, the various ways - connected by conformal mappings - of calculating an amplitude must not only give the same s- and t- dependence for the amplitude but also the same normalization. This has led us to "universality" relations of the type $G_2 = G_3 = G_4$ (eqs. (11) and (20)). Now we can continue discussing higher total quark numbers. As a first step we consider the process $MM_4 \rightarrow MM_4$. An argument completely similar to that in MB scattering now tells us that the MM_4M_4 and MMM couplings must be equal. Then $BM_4 \rightarrow BM_4$ scattering tells us that the MM_4M_4 and B_5M_4B couplings have to be the same. We have seen that no matter how large the total quark numbers of all those hadrons A,B,C that are involved, any hadronic vertex has the form

$$G_{ABC} : e^{ik\phi(\xi_F, \lambda_F)} : \qquad (22)$$

where G_{ABC} is the appropriate "coupling constant" and $z_F = \lambda_F + i\xi_F$ the position of the focusing point in the world sheet. Continuing the recursion from M_4 and B_5 to arbitrarily large quark numbers we find that all G_{ABC}'s must be equal,

$$G_{ABC} = G = \text{universal constant} \qquad (23)$$

In other words the following general universality principle holds: the matrix element for shifting by k_i the momentum of $\tilde{N}_i$ quarks (and/or antiquarks) focused at a point, while at the same time emitting N_e quarks and/or antiquarks, and transiting to a final state of $\tilde{N}_f$ quarks and/or antiquarks, is independent of $\tilde{N}_i$, N_e, and $\tilde{N}_f$ and independent of the number of spectator quarks and/or antiquarks $(N_i-\tilde{N}_i = N_f-\tilde{N}_f)$.

The most remarkable feature of this result is that once one strong coupling constant is given - say $g_{\rho\pi\pi}$ - all other strong coupling constants are determined. Partial universality principles have been known in the past. For example ρ-universality[12)] is known to predict that

$$\frac{g_{\rho^\circ \bar{H}H}}{g_{\rho^\circ \pi^-\pi^+}} = (I_3)_H \tag{24}$$

but ρ-universality cannot predict

$$\frac{g_{\rho^\circ \bar{H}_1 H_2}}{g_{\rho^\circ \pi^-\pi^+}} \tag{25}$$

where H_1 and H_2 are two different hadrons. Similarly PCAC, tensor-universality[13)], scalar-universality[13),14)] are limited universality principles. Our general universality principle solves the problem of finding all hadron couplings. Thus it predicts the ratios (24) of ρ-universality and relations ($g_{\sigma\pi\pi}/g_{\sigma\bar{p}p} = 2/3$) of scalar universality, etc...But it also predicts the ratios (25) that ρ-universality was not able to predict. It moreover predicts ratios of the type

$$g_{f\pi\pi} : \quad g_{\rho\pi\pi} : g_{\sigma\pi\pi} \tag{26}$$

that neither ρ-nor tensor- nor scalar-universality could predict. Another important special case is the following. It is readily checked that as a consequence of the general universality principle the residua $\beta_{K^+p}^{(M_2)}$ and $\beta_{pp}^{(M_2)}(t)$ of the leading M-trajectory (the ρ-f-ω-A_2 trajectory) in K^+p- and pp-scattering are proportional to each other.

$$\frac{\beta_{K^+p}^{(M_2)}(t)}{\beta_{pp}^{(M_2)}(t)} = \text{constant}$$

This is the so called reggeized vector + tensor universality principle.[15] While the essence of the general universality principle that all strong coupling constants can be expressed in terms of one single coupling constant is clear, in order to derive realistic ratios, of coupling constants, from the principles one first has to include quark spin and unitary spin. If one does this along the lines proposed in the paper of Carlitz, Ellis, Freund and Matsuda[16] one recovers the usual results for ρ-universality, f-universality, etc.... Of course in deriving relations between coupling constants of individual particles one has to sum over the contributions of all possible systems of active quarks.

6. Conclusions

In this paper we have shown how duality relates all hadron couplings. In terms of a quark substructure all these couplings can be viewed as the $\tilde{N}_i$ active quarks of the original hadron focusing to a point being kicked in common by the momentum of the absorbed (or emitted) hadron (of total quark number N_e) and at the same time becoming $\tilde{N}_f$ quarks of the final hadron. The matrix element for this process turned out to be independent of $\tilde{N}_i$, $\tilde{N}_f$, N_e and of the number of spectator quarks ($N_i-\tilde{N}_i = N_f-\tilde{N}_f$). Thus once we accept the existence of $M_2M_2M_2$ couplings we cannot "pretend" that there existed no baryons or that there existed no exotic hadrons. Not only is the existence of these other hadrons predicted by duality but all their couplings are fixed. Thus second and higher order unitarity corrections to dual narrow-resonance models must include the effects of baryons and exotic hadrons. One does not have the liberty to fix their couplings so as to supress these effects. All attempts[17)] at calculating unitarity corrections including only nonexotic mesons must be viewed as imcomplete. The effects of baryons and exotic hadrons in unitarity corrections is expected to be most important at higher energies. $E_{cm} \gtrsim 2.5$ GeV. A most important unitarity effect: the appearance of a Pomeranchuk singularity has thus to be reconsidered along the lines suggested in ref. 11b.

Our results give a simple physical picture which fixes the ratio of any two strong coupling constants, in particular reproduces all known universality principles (ρ-universality,

ρ-universality, PCAC, etc.). We thus found as a byproduct of our results that all these limited universality principles have a common origin in the quark substructure of hadrons and in the way the quarks get activated at hadronic vertices.

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Figure Captions

Fig. 1: Quark focusing at meson-baryon vertex.

Fig. 2: Quark diagram for meson-meson scattering.

Fig. 3: Quark diagrams for meson-baryon scattering.

Fig. 4: Focused quark diagrams for meson-baryon scattering.

Fig. 5: Focused quark diagrams for baryon-antibaryon scattering.

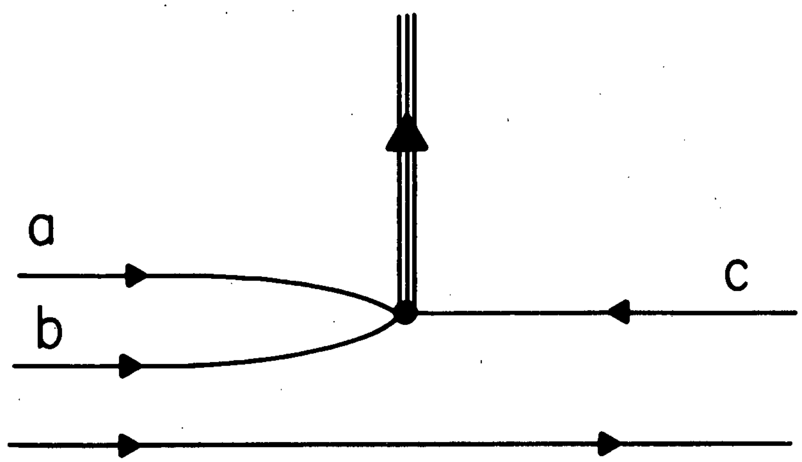

Fig. I

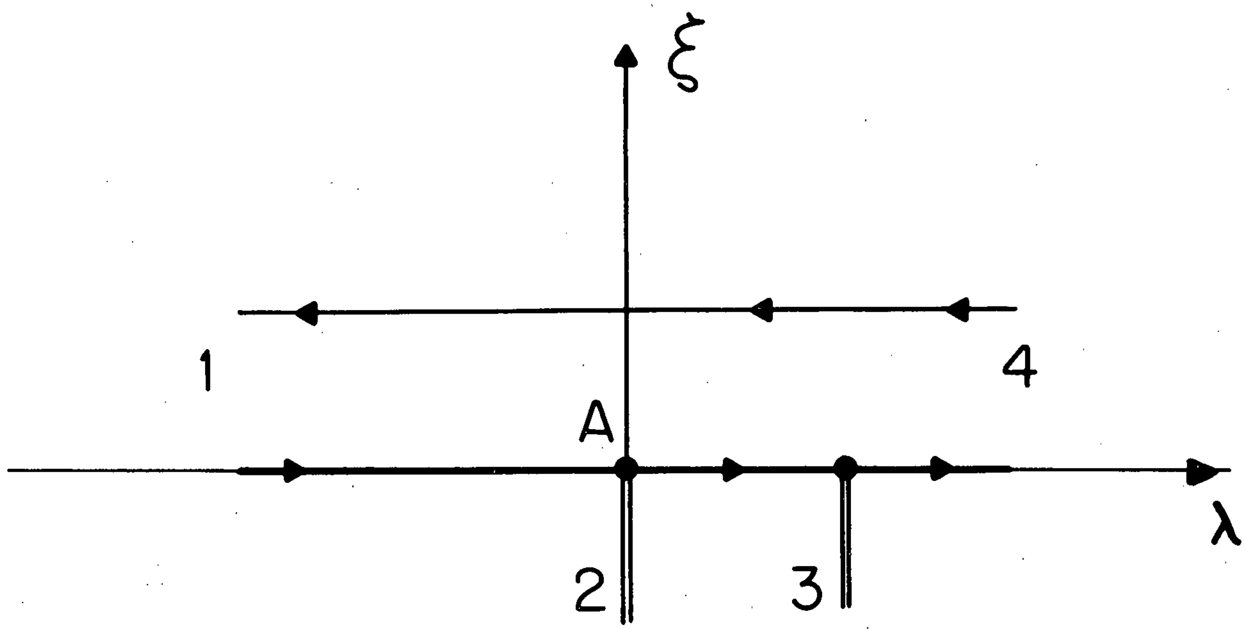

Fig.2

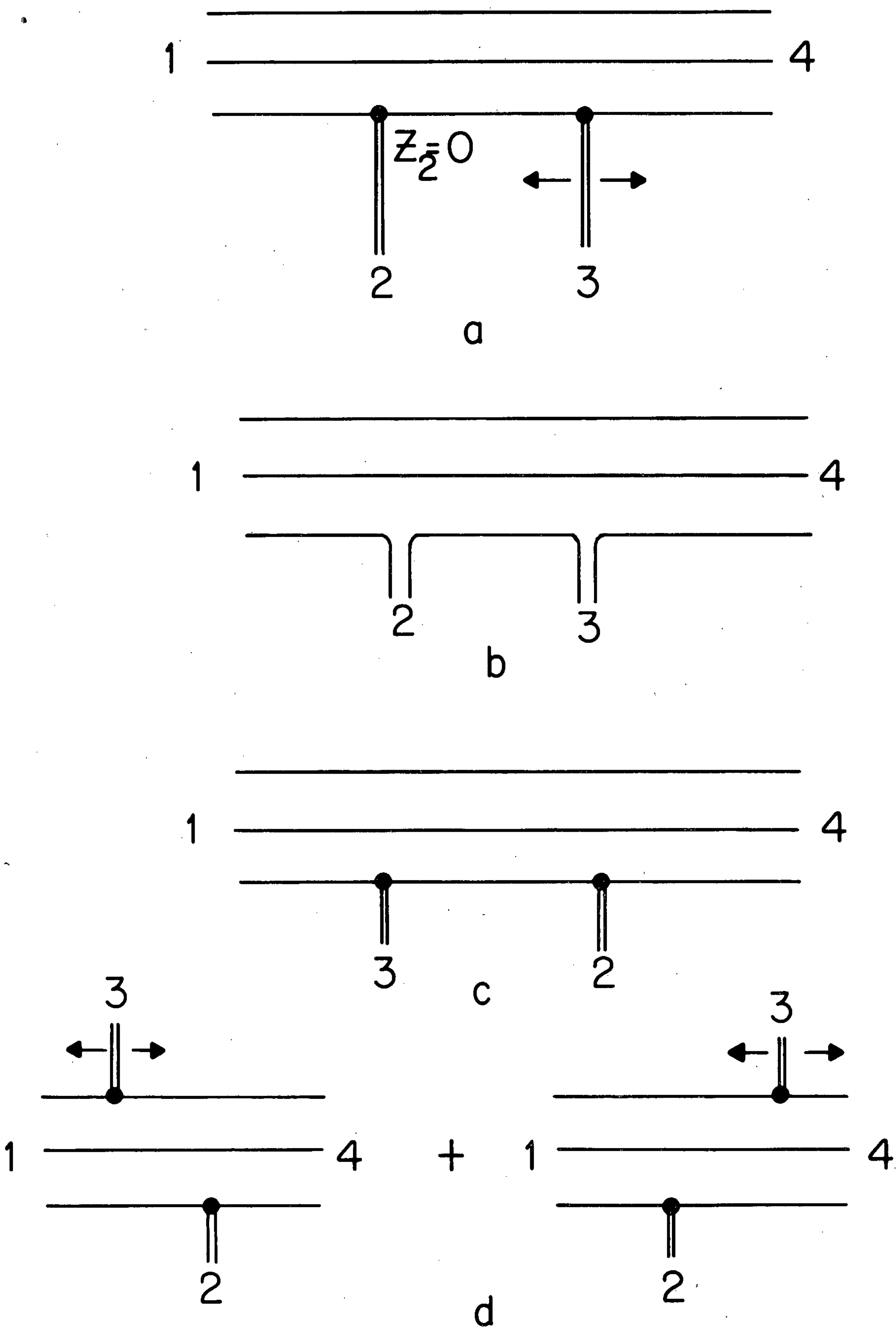

Fig.3

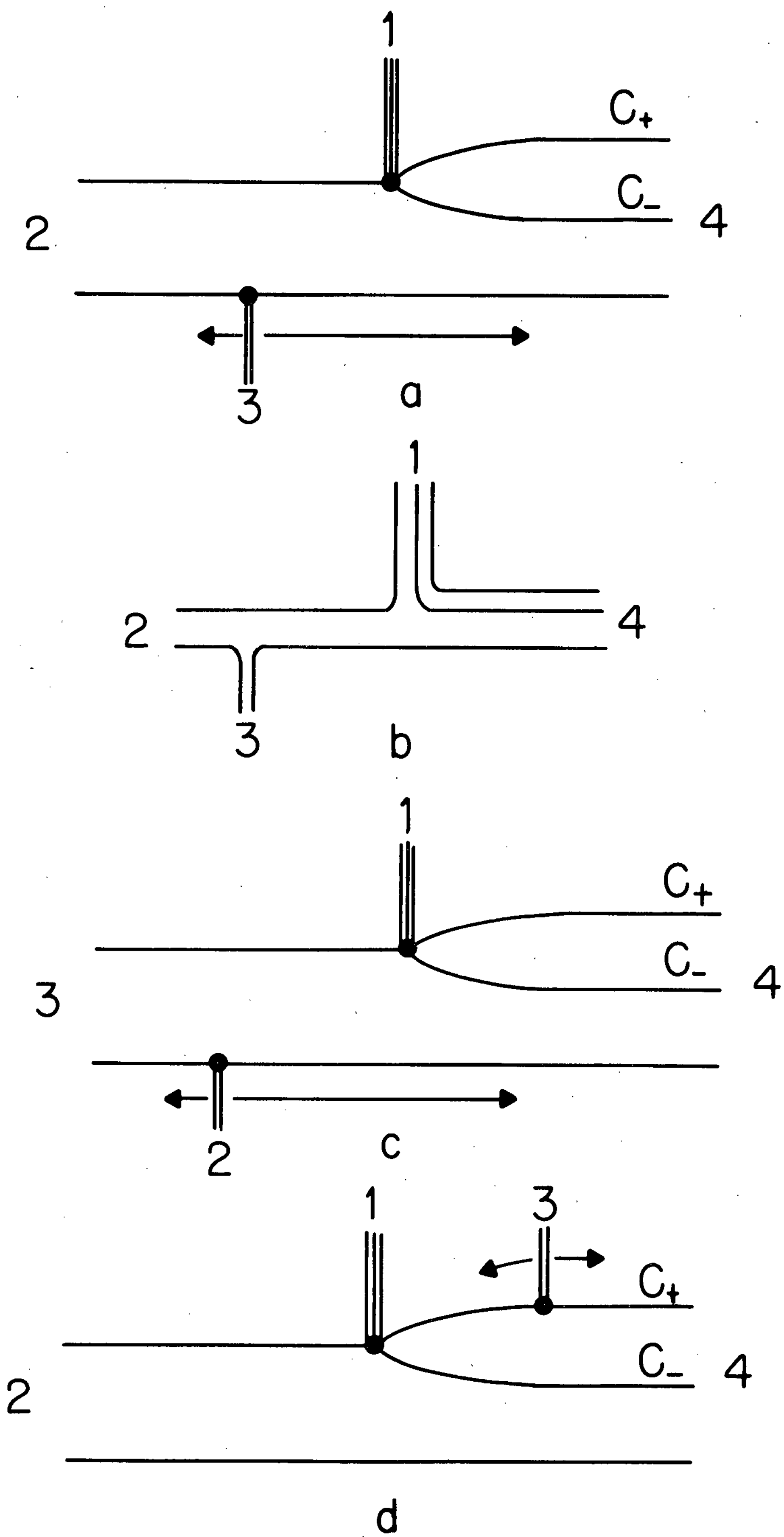

Fig.4

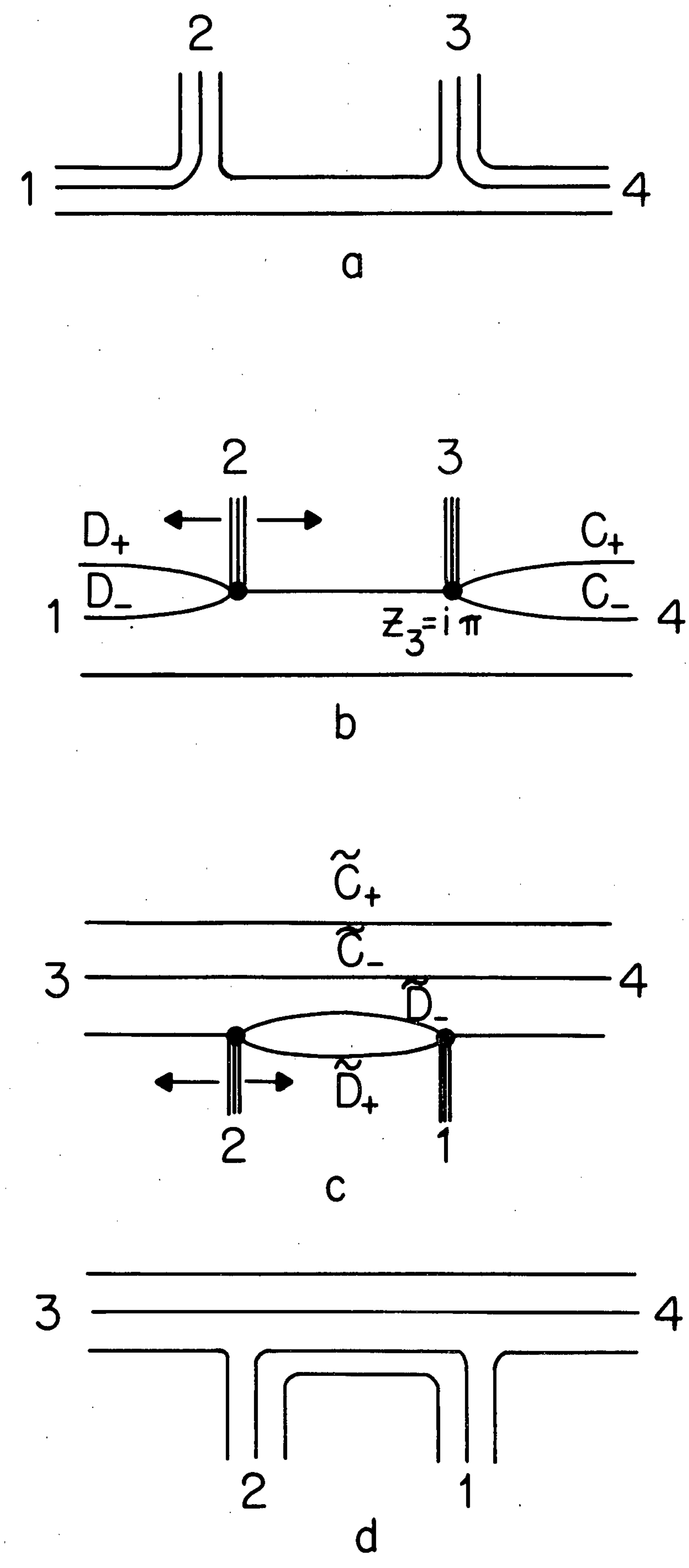

Fig.5

Errata

QUARK FOCUSING AT HADRONIC VERTICES

Paul H. Frampton and Peter G. O. Freund

i) The denominator on the right hand side of Equation (6) should read $4e^{\mu}$, not $4e^{2\mu}$

ii) The lower limit of the first integral on the right-hand-side of Equation (9) should be 0, not $-\infty$