OCTOBER 1974 MATT-1076

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BY

J. CANOSA, J. KROMMES,
C. OBERMAN, H. OKUDA, K. TSANG,
J. M. DAWSON AND T. KAMIMURA

PLASMA PHYSICS LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

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Theory and Numerical Simulation of Collective Transport of Plasma in Magnetic Fields*

J. Canosa,† J. Krommes, C. Oberman, H. Okuda, and K. Tsang‡
Princeton University, Plasma Physics Laboratory
Princeton, New Jersey 08540 USA
and
J. M. Dawson and T. Kamimura*
University of California, Los Angeles, California 90024 USA

ABSTRACT

Numerical and theoretical studies have been made on the transport of particles and heat in collisionless plasmas in magnetic fields for both quiescent and turbulent states. The following problems have been investigated: (1) A systematic kinetic theory of hydrodynamic fluctuations in a strong magnetic field has been developed to study the particle diffusion due to thermal convective cells. (2) Effects of density and temperature gradients on thermal convective cells; they are found to destabilize both the ion flute and the convective cells. (3) Effects of mirroring magnetic field on convective diffusion; they are found to increase the diffusion significantly even for open field lines of force. (4) Turbulent electron heat tranfer due to short wavelength ion fluctuations near the lower hybrid frequency; this may be important in a collisionless plasma. (5) Heat transfer due to emission and absorption of electrostatic plasma waves are studied theoretically and by simulation; agreement has been found for a strong magnetic field case. (6) Particle diffusion in a toroidal magnetic field due to collisions (neo-classical diffusion) and due to collective effects is studied by numerical simulations. Neoclassical theory agrees well with the numerical results when the long wavelength collective modes are suppressed.

1. KINETIC THEORY OF HYDRODYNAMIC FLUCTUATIONS

We have initiated a study of the kinetic theory of hydrodynamic fluctuations in a strong magnetic field. It is convenient to begin from the exact non-Markovian equation for the momentum distribution function, as given by Baus [1] :

$$\frac{\partial \rho(\vec{p}\,t)}{\partial t} = \int_0^t d\tau \, \frac{\partial}{\partial \vec{p}} \cdot \left\{ \overleftrightarrow{D} \cdot \frac{\partial}{\partial \vec{p}} + \vec{A} \right\} \rho\,(\vec{p}, t-\tau)$$

(we neglect the destruction term). Both $\overleftrightarrow{D}$ and $\vec{A}$ are defined in terms of the generalized diffusion tensor $\overleftrightarrow{D}^{ij} \langle q_i \vec{e}_i(t) q_j \vec{e}_j(t-\tau) \rangle_{ij}^{t-\tau}$, the ensemble average being taken with respect to the singular distribution $\prod_{\ell \neq ij} [\rho(p_\ell, t-\tau)/V] \times \delta(x_i - \hat{x}_i)\,\delta(x_j - \hat{x}_j)$. The generalization to weak inhomogeneity is straightforward. Conventional plasma kinetic equations are obtained by first solving for the effective field on the Vlasov time scale, then making the Markovian approximation. It is clear that this short time approximation neglects the hydrodynamic fluctuations. We improve the treatment by introducing a cluster expansion for $\overleftrightarrow{D}^{ij}$; approximately, we may express it in terms of the test particle propagator Ω and the density-density correlation function $\Gamma \equiv \langle \delta n(t)\,\delta n(0) \rangle$. We ultimately solve for these quantities in the hydrodynamic limit. We expect the test particle propagator to diffuse, while Williams [2] has shown that in the hydrodynamic regime Γ obeys a fluid equation, depending on certain transport coefficients. We identify these with the actual transport coefficients of the plasma. Solving for Γ, the low-frequency dielectric of Okuda and Dawson [3] appears naturally. Using these results, we compute the friction and drag terms and are led for strong $\vec{B}$ to an enhanced collision operator, expressed as a functional of the transport coefficients. This operator is in general still non-Markovian. A self-consistent solution for the transport coefficients can now be obtained by performing the Chapman-Enskog closure. The details of this calculation are in progress.

2. EFFECTS OF DENSITY AND TEMPERATURE GRADIENTS ON THERMAL CONVECTIVE CELLS

It has been shown both by analytic theory and by extensive numerical simulations, that there is a large enhanced diffusion of particles due to thermally excited convective cells [3]. The convective diffusion has been applied to anomalous diffusion observed in a multipole [4], stellarator [5], and a solid state plasma [6].

In this section the effects of density and temperature gradients on convective cells are studied using a linear theory and numerical simulations.

For a uniform plasma, the dielectric function and hence the dispersion relation for low-frequency ($\omega \ll \Omega_i$) flute modes ($\vec{k} \cdot \vec{B} = 0$) are given by [3]

$$\epsilon_\perp(k_\perp, \omega) = 1 + \frac{\omega_{pi}^2}{\Omega_i^2} \, \frac{\omega + i k_\perp^2 D_\perp}{\omega}, \tag{1}$$

and

$$\omega = -i \, (k_\perp^2 D_\perp^i) \Big/ (1 + \Omega_i^2/\omega_{pi}^2),$$

where $D^i_\perp = \mu_i = (3/10)\rho_i^2 \nu_i$; μ_i and ρ_i are the ion viscosity and the gyro radius.

In a non-uniform plasma, the dielectric function takes the form [7]

$$\epsilon_\perp(k_\perp, \omega) = 1 + \frac{\omega_{pi}^2}{\Omega_i^2} \; \frac{\omega + ik_\perp^2 D_\perp^i}{\omega} \; \frac{\omega - \omega^*}{\omega} ,$$

where

$$\omega^* = -\frac{k_\perp Tc}{eB} \; \frac{\partial \ln N}{\partial x} ,$$

is the ion diamagnetic drift frequency. The dispersion relation then takes the form of

$$\omega = -i \; (k_\perp^2 D_\perp^i) \Big/ (1 + \Omega_i^2/\omega_{pi}^2)$$

and

$$\omega = \frac{\omega^*}{1 + \Omega_i^2/\omega_{pi}^2} + i \; \frac{k_\perp^2 D_\perp^i}{1 + \Omega_i^2/\omega_{pi}^2} .$$

We find that the convective mode is unchanged while the ion flute mode is destabilized by the ion viscosity. For a collisionless plasma where $\omega^* \gg k_\perp^2 D_\perp^i$ for $k_\perp \rho_i < 1$, the thermal fluctuation energy for the convective modes in an inhomogeneous plasma is much less than that for a uniform plasma, and therefore the diffusion due to convective modes will be smaller for an inhomogeneous plasma. However, since the ion flute mode is always unstable, the diffusion due to this mode may be large.

The instability and the associated particle diffusion are investigated using a two-dimensional model with the density gradient in the x-direction [3] .

Figure 1 shows the growth of the most unstable mode propagating in the y-direction. The instability grows until the initial density gradient is completely wiped out. The associated particle diffusion is shown in Fig. 2 together with the convective and collisional diffusion in a homogeneous plasma for comparison. The convective diffusion in an inhomogeneous plasma is several times greater than that for homogeneous plasma, which in turn is much greater than the corresponding collisional diffusion.

When a temperature gradient is introduced, it can be shown [7] that the ion flute mode is stable while the convective mode becomes unstable with $\omega \sim ik_\perp^2 D_\perp^i$. Numerical simulation shows that the enhanced diffusion for this case is smaller than the corresponding density gradient case.

It is interesting to realize that the instabilities discussed above are destabilized by the viscosity given by $k_\perp^2 D_\perp^i$. It is possible that the turbulent diffusion may increase the growth rates.

3. EFFECTS OF MIRRORING ON CONVECTIVE TRANSPORT

Earlier investigations on three-dimensional plasma models [8] showed that systems with closed magnetic field lines exhibited strong enhanced diffusion, whereas systems with ergodic field lines showed diffusion which was close to that predicted from two-particle collisions. If a closed flux tube acquires an excess charge it persits for a long time since the charge cannot move freely across the field lines. On the other hand if the lines are ergodic, regions of positive charge are connected to regions of negative charge by field lines and these charge fluctuations are quickly dissipated by charge flow along the B field. If this shorting is inhibited then the lifetime of the charge fluctuation is increased and the convective transport again becomes important. One such inhibiting effect is the trapping of charges in magnetic mirrors. Such mirrors occur in almost all fusion devices.

We have constructed a $2\frac{1}{2}$-dimensional particle model in which magnetic mirroring is included. The model is shown in Fig. 3. The magnetic field points in an arbitrary direction determined by the angle θ it makes with the z axis and the angle ϕ its x, y projection makes with the x axis. The rods have three components of velocity, but the electric field has only x and y components. The system is taken to be doubly periodic. A mirror region is placed along the line y equals y_0. When a particle enters this region the ratio of $v_{\parallel}$ to $v_{\perp}$ (parallel and perpendicular to B) is checked to see if it passes through mirror or not.

$$(v_{\parallel}/v_{\perp}) \lessgtr (R - 1)^{1/2} , \qquad R = B_{max}/B_{min} .$$

If $v_{\parallel}$ exceeds $(R-1)^{1/2}$ the particle is allowed to pass through the mirror, If it is less $v_{\parallel}$ is reversed.

A number of computer simulations were carried out with this model. All runs were made with thermal equilibrium plasmas. Some results are shown in Figs. 4 and 5. Figure 4 shows the position of the guiding center of a particle at equally spaced time intervals for the case of no mirror and for a mirror ratio of 5 respectively. The points show that in the case of no mirror the particle visits every region of the square whereas for the case of the mirror the particle excursions are quite limited. This exhibits how the mirror inhibits communication between various regions of the plasma. Figure 5 shows the mean square displacement of the guiding centers of a set of test particles from their original lines of force as a function of time for the case of no mirror and a mirror ratio of 2. The slope of these curves at large t are the diffusion rates. The diffusion rate with the mirror is three times that for no mirror exhibiting the expected effect.

4. TURBULENT ELECTRON HEAT TRANSPORT DUE TO ION FLUCTUATIONS

It is well known [9] that the electron heat conduction is almost always anomalous in many tokamak devices.

Since the electron mobility is much larger than that of the ions, electrons are very easily subject to low-frequency fluctuations ($\omega \ll \Omega_e$) which may cause enhanced electron transport, if an appropriate irreversibility (collisions, Landau damping, turbulent damping) is accompanied with the collective motion.

Consider the motion of test electrons in the presence of low frequency, long wavelength ($k_\perp \rho_e \lesssim 1$) fluctuations. The cross field diffusion coefficient may be calculated following the orbits in the presence of such fluctuations [10, 11]. The diffusion coefficient is

$$D_\perp = \lim_{t \to \infty} \frac{\langle (\Delta x)^2 \rangle}{t} = \frac{c^2}{B^2} \sum_k (E^2)_k \frac{k_\perp^2}{k^2} \, \mathrm{Im} \frac{1}{\omega + i\gamma} , \qquad (2)$$

where $(E^2)_k$ is the fluctuation spectrum associated with the wave of frequency ω, and γ is its damping rate. For $k_\perp \gg k_\parallel$ which is important for the cross-field transport, the low-frequency waves are lower hybrid oscillations given by

$$\omega = (\omega_{pi}) \Big/ \left(\sqrt{1 + \omega_{pe}^2/\Omega_e^2}\right)$$

$$\gamma = -\sqrt{\pi/8} \, \frac{\omega^4}{k^3 v_i^3} \exp(-3/2) \exp(-\omega^2/2k^2 v_i^2) - \frac{4k_\perp^2 D_\perp}{1 + \Omega_e^2/\omega_{pe}^2} ,$$

for $\omega_{pi}^2 \gg \Omega_i^2$, and $k_\perp \rho_i \gg 1$; v_i is the ion thermal speed. The first term in γ is ion Landan damping, while the second term is viscous damping associated with turbulent electron transport [3, 12].

Assuming that turbulent damping dominates over the Landau damping for $k\lambda_D < 1$, we find

$$D_\perp = \frac{1}{4} \sqrt{T/L} \, \frac{c}{B} \, \frac{\Omega_e}{\omega_{pe}} \left(\ln \frac{\omega^2 + [4k_{max}^2 D_\perp/(1 + \Omega_e^2/\omega_{pe}^2)]^2}{\omega^2} \right)^{1/2} \qquad (3)$$

where only $k_\parallel = 0$ modes are kept, and L is the length of the system along B field [3]. $(E^2)_k$ is assumed at thermal level [3];

$$\left(E^2\right)_k / 8\pi = \frac{T}{2} \frac{1}{1 + \omega_{pe}^2/\Omega_e^2} ,$$

k_{max} may be λ_D^{-1} or ρ_e^{-1}.

Several three-dimensional simulations were carried out to check the heat transfer for both a uniform field [13] and a sheared field [14]. The magnetic field is in z-direction and the electron temperature varied with x as $T_e(x) \sim \sin \pi x/L$ while the ion temperature was assumed uniform. The heat conductivity, $\kappa_\perp$, was measured from the decay of $T_e(x)$.

Figure 6 shows the results with a uniform field for both closed and open field lines of force [13] . For closed lines, heat conductivity is appreciably enhanced above the classical value for stronger fields. When the field lines are open, then the diffusion is reduced, however, the enhancement above the classical level is appreciable.

Figure 7 shows the heat conductivity for a sheared magnetic field of the form

$$B = B_o \hat{z} + B_s \sin(2\pi x/L)\, \hat{y} \ .$$

It is clear that the electron heat conduction is rather insensitive to shear.

Finally, it should be emphasized that for the simulation plasma, the fluctuation field energy relative to the thermal energy is much higher than for the real plasma, and therefore, the effects of fluctuations on transport is also enhanced. Therefore, it is always necessary to derive a theoretical estimate in agreement with the simulation results, which then may be applied to a real plasma.

5. CROSS FIELD ENERGY TRANSPORT BY PLASMA WAVES

It has been shown recently [15] that the energy transport across a magnetic field by the emission and absorption of plasma waves can be appreciable for a strong magnetic field in a large system. While the particles are not able to diffuse easily across a strong magnetic field, plasma waves can carry energy across field due to the long mean free path for absorption.

Consider a plasma slab extending from $x = -L$ to L, with the magnetic field in the z-direction. Assuming that the electron temperature is given by

$$T_e(x) = T_o(1 - x^2/L^2) \ ,$$

the energy flux due to emission and absorption of plasma waves is obtained from the transport equation. The result is [15,16]

$$A(0) = 3.34\times 10^{-2} (\omega_{pe} T_o/L\lambda_D)(1 - \omega_{pe}^2/\Omega_e^2)/(\ln^2 L/\lambda_D) \ \cdot$$

$$\{1 + 2.25 \ln^{-1}(L/\lambda_D) + O[\ln^{-2}(L/\lambda_D)]\} \quad \text{for } \Omega_e > \omega_{pe} \ .$$

The energy flux for weak magnetic field ($\omega_{pe} > \Omega_e$) is smaller than for strong fields, because in a weak magnetic field the group velocity becomes smaller while the Landau damping increases appreciably.

A $2\frac{1}{2}$-dimensional model shown in Fig. 8 has been developed to study the energy transport by waves. The magnetic field is in the y-direction while the temperature gradient is in the x-direction. It is clear that this model contains the plasma waves propagating in an arbitrary angle with respect to magnetic field.

Figure 9 shows the results of the temperature relaxation due to waves for a strong magnetic field, $\omega_{pe}/\Omega_e = 1/4$. (Ions form a fixed background.)

In order to separate the relaxation due to longitudinal waves from collisional relaxation, parallel and perpendicular decays were obtained separately. It is clearly seen that the decay is due to wave transport because it occurs only in the parallel temperature; this results in an anisotropic velocity distribution.

The simulation result agrees with the theory within 6% for this case. Good agreement was also found for weaker fields ($\omega_{pe}/\Omega_e = 1/2, 1, 2.5$). It is possible that the wave transport can dominate the classical transport in turbulent plasmas.

6. NEO-CLASSICAL DIFFUSION AND CONVECTION IN TOROIDAL SYSTEM

Neoclassical theory of plasma diffusion has been developed quite extensively in the past few years [17]. Some experimental evidence of neoclassical diffusion has also been reported [18].

In this section, results of numerical simulation are given using a model magnetic field which simulates a tokamak geometry. Two different models have been constructed; the first model uses a model Fokker-Planck collision operator [19] for the small angle Coulomb scattering. The second model, which includes the contributions from long wavelength fluctuations as well as short range interactions, is used to study the effects of collective modes on neoclassical diffusion. The model magnetic field used in the simulation is [20],

$$B_z = B_o/(1+x/R)$$
$$B_x = -\Theta B_o y/a$$
$$B_y = \Theta B_o x/a$$

where the x-y plane is the poloidal cross section, z is taken along the local toroidal direction, R is the distance from the magnetic axis to the major axis of the tokamak, Θ is a small parameter characterizing the poloidal field, and a is the size of the plasma column. This model neglects toroidal curvature, but keeps the gradient of the main magnetic field which is essential for the presence of trapped particles [21].

Figure 10 summarizes the results of a simulation using the first model. The diffusion coefficient measured from the test electrons due to electron-ion collisions are shown for both straight and toroidal magnetic fields. For a toroidal field, we choose $a/R = 1/4$, $\Theta = 0.1$. For ν_{ei}/ω_{pe} less than 10^{-3}, the observed diffusion is roughly $D_\perp \approx 45\,\nu_{ei}$ which is quite close to the prediction for the banana regime [22].

$$D_\perp \simeq 1.6\,\nu_{ei}\rho_e^2\,(rB_T/RB_p)^2\,(R/r)^{3/2} .$$

For ν_{ei} greater than 4×10^{-2}, the observed diffusion is $D_\perp \approx 1.8\,\nu_{ei}$ which is also close to the theoretical prediction for the Pfirsch-Schlüter regime;

$$D_\perp \cong \nu_{ei}\rho_e^2 \, [1+(rB_T/RB_p)] \ .$$

Between the Pfirsch-Schlüter and the banana regimes, there is a smooth transition in agreement with the theoretical prediction [17] . The diffusion for uniform field shows the expected B^{-2} dependence for all the collision frequencies.

After confirming the neoclassical diffusion, a self-consistent electric field is turned on in the model to see the effects of collective modes using the second model. Because of the presence of trapped particles, an enhancement over the neoclassical diffusion appears due to the convective cells similar to that studied in Sec. III. It is not clear at this stage, however, how important the convective cells are in a real tokamak where the fluctuation level associated with the convective motion may be smaller than that in simulation plasmas.

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* Work supported jointly by U. S. Atomic Energy Commission Contracts AT(11-1)-3073, AT(04-3)-34PA 157(MOD6), and U. S. Air Force Office of Scientific Research Contract F44620-70-C-0033; use was made of computer facilities supported in part by National Science Foundation Grant NSF-579.

† Permanent address: IBM Scientific Center, Palo Alto, California 94304.

‡ Present address: Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830.

⁑ Permanent address: Institut of Plasma Physics, Nagoya, Japan.

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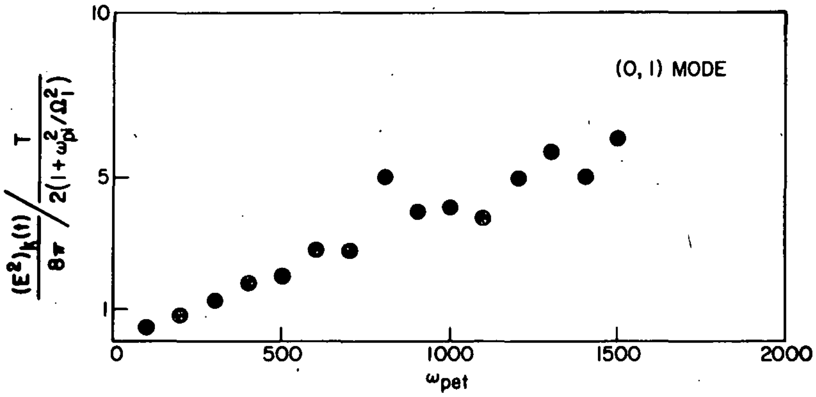

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Fig. 1. Growth of the instability associated with the ion flute mode propagating in the y-direction.

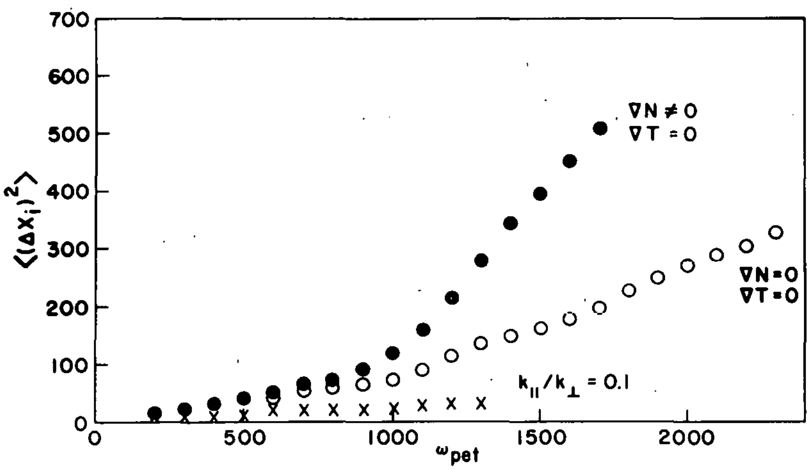

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Fig. 2. Test particle diffusion due to classical collisions (cross marks), due to thermal convective cells (open circles), and due to flute mode (solid circles). Simulation parameters are 64^2 grid, 128^2 particles, and $\Omega_e/\omega_{pe} = 2$.

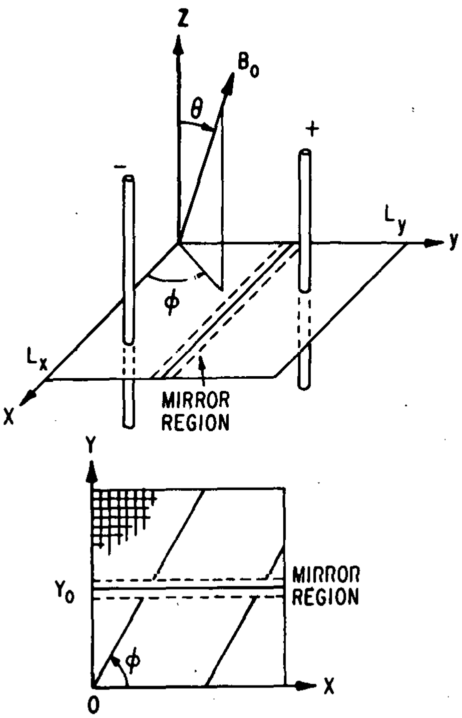

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Fig. 3. Sketch of the model with a mirror magnetic field at $y = y_o$ plane.

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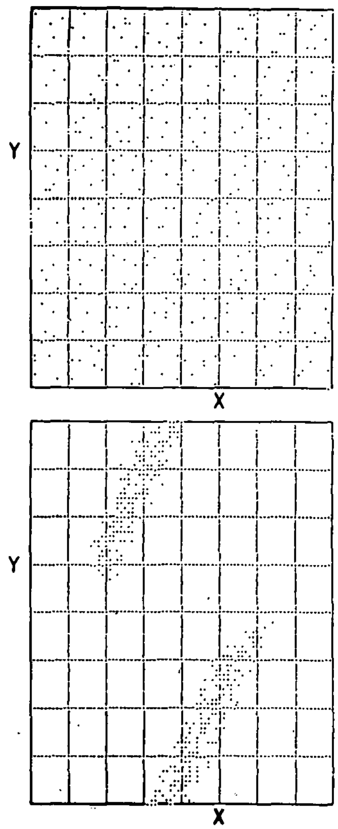
Fig. 4. Particle orbits in a uniform (top) and in a mirror magnetic (down) fields.



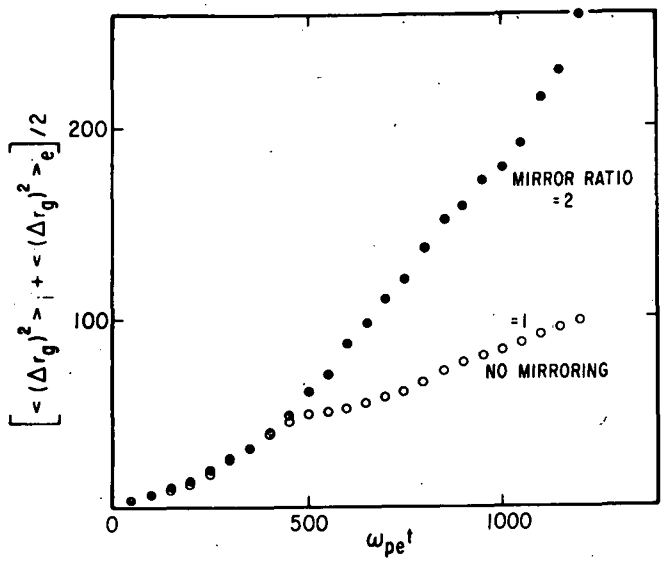


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Fig. 5. Test particle diffusion in a uniform and in a mirror magnetic fields.

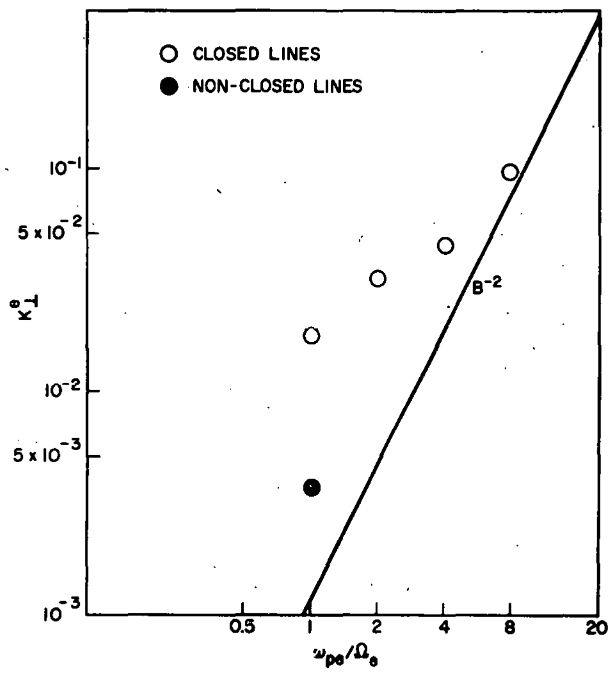

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Fig. 6. Measurement of electron heat conductivity for closed and open field lines of force. $m_i/m_e = 400$ and 32^3 grid and particles.

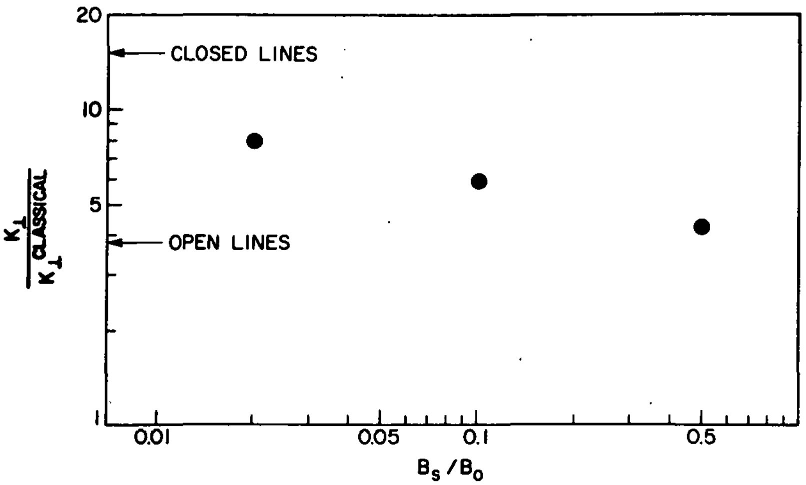

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Fig. 7. Electron heat conductivity in a sheared magnetic field.

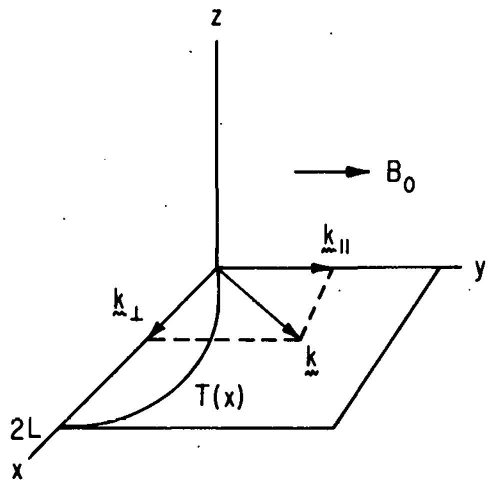

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Fig. 8. Sketch of the model used for simulating wave heat transport.

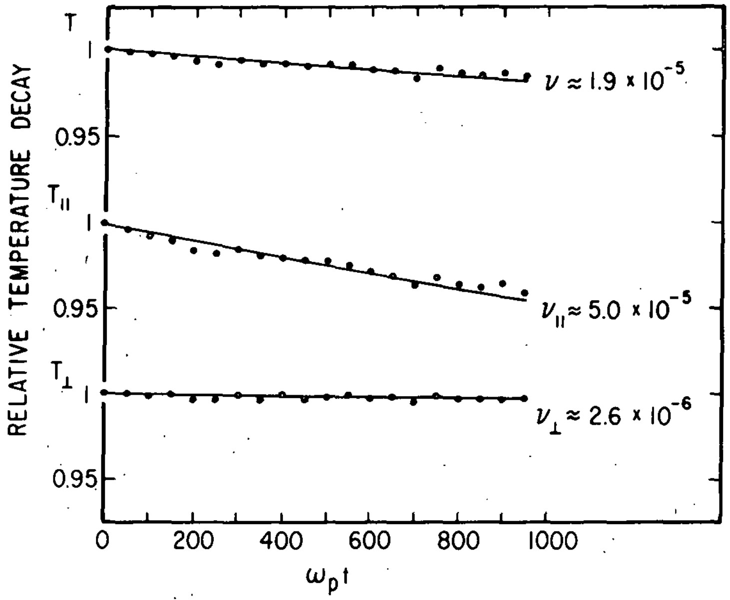

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Fig. 9. Plots of the decay of the temperature due to wave transport. A 64^2 grid and 256^2 particles were used.

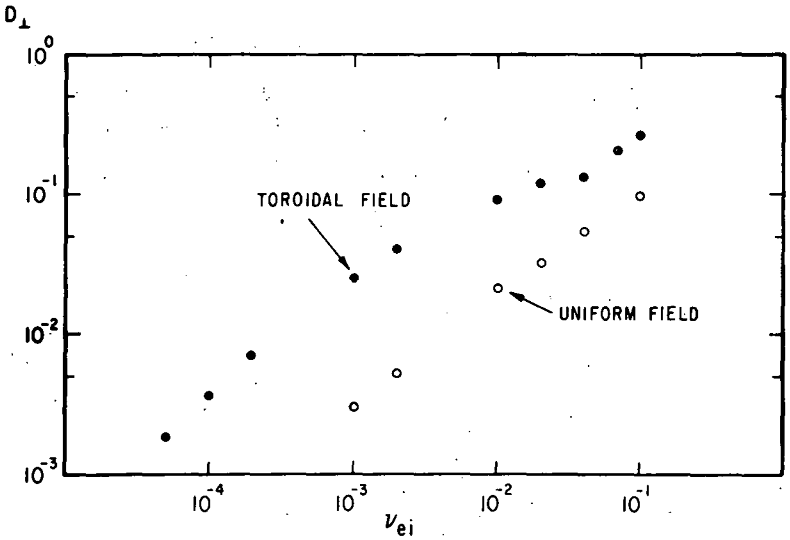

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Fig. 10. Diffusion coefficients for uniform and toroidal magnetic fields.