STUDY OF N + α SYSTEM WITH THE

RESONATING – GROUP METHOD

I. Reichstein and Y. C. Tang[*]

School of Physics, University of Minnesota,
Minneapolis, Minnesota

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Abstract

The n + α and p + α systems are considered with the resonating-group method in the one-channel approximation. The nucleon-nucleon potential used has both a central part and a spin-orbit component. From the results it is found that very good agreement with experiment can be obtained in the energy region below the reaction thresholds. In fact, even above the reaction thresholds where the one-channel approximation is expected to be less valid, the results are quite satisfactory. An effective potential between the clusters is also constructed. From this potential it is determined that the requirement of antisymmetry for the total wave function is very important. In particular, it creates an odd-even feature, wherein the effective central and the effective one-body spin-orbit potentials in the odd-ℓ states are appreciably different from those in the even-ℓ states.

1. Introduction

Single-channel resonating-group calculations have hitherto been performed for a number of nuclear systems[1] using a rather simple central nucleon-nucleon potential which does not contain a repulsive core and which has equal range in the triplet and singlet states. These calculations have generally yielded satisfactory but not close agreement with the experimental results. In those cases where the single-channel approximation is expected to be good, such as $\alpha + \alpha$ and $\alpha + {}^3\text{He}$ scattering in the energy region below or close to the reaction thresholds, it is our belief that the lack of a close agreement is not caused by the use of a potential without repulsive core[2], but is mainly a consequence of the simplified nature in the long-range part. Indeed, it was our finding that the calculated level spacings, which depend rather sensitively on the range of the nucleon-nucleon potential, were in general too large[3,4]. Motivated by this, we have introduced in our most recent calculation[5] an improved potential which has different triplet and singlet ranges and which gives a very good fit to the low-energy two-nucleon scattering data. This potential was used to study the $\alpha + \alpha$ scattering problem[5] and excellent results were in fact obtained. Thus it seems worthwhile to use this improved potential to study other systems with the resonating-group method. In this calculation, we consider the $\alpha + n$ and $\alpha + p$ systems; these are chosen since the low compressibility of the α cluster and the high reaction thresholds enable us again to employ the one-channel approximation over a wide energy range. As will be seen below, such a calculation does yield results which are significantly better than those obtained using the potential of our older calculation[6].

Also, a two-nucleon spin-orbit potential will be used in the calculation, with the depth and range adjusted to give a best fit to the experimental $\alpha + p$ and $\alpha + n$ data in the low-energy region.

Clearly, the addition of such a noncentral component is necessary
in order to explain the observed splittings in the scattering phase
shifts. It is realized, of course, that the introduction of a
tensor interaction will be a more desirable procedure; but, the
calculation, if carried out properly, will become much more compli-
cated. In any case, the primary purpose of our calculation with
a non-central potential is to extend our previous study on the
effective interaction between the clusters[6]. In particular, we
wish to gain knowledge about the nature of the non-central part of
this interaction, with the hope that one can then use this infor-
mation to constuct more realistic one-body spin-orbit potential in
the optical model. For this latter purpose, it is our opinion
that the use of a simple two-nucleon spin-orbit potential, which
could however lead to satisfactory agreement with the experimental
α + p and α + n results, should be quite sufficient.

In the next section, a brief formulation of the problem will
be given, together with a discussion of the central potential,
the spin-orbit potential, and the approximations which are present
in the calculation. In sect. 3 we discuss the method used to
determine the depth and range of the spin-orbit interaction and
present the results for the phase shift, cross section, polarization
and spin-rotation parameter. Here also, the effect of the exchange
Coulomb interaction will be briefly studied.

In sect. 4 we shall discuss the effective α + N potentials
derived by using the wave functions obtained from the resonating-
group calculation. Finally, in sect. 5, concluding remarks are
made.

2. Formulation

In the one-channel approximation, the wave function for the $\alpha + N$ system is written as

$$\Psi = A \left\{ \varphi_\alpha(\underline{r}_1, \underline{r}_2, \underline{r}_3, \underline{r}_4) \, \zeta_\alpha(\underline{s}_1, \underline{s}_2, \underline{s}_3, \underline{s}_4 ; t_1, t_2, t_3, t_4) \right.$$

$$\left. \times \left[\delta(t_5, \pm\tfrac{1}{2}) \sum_{J=\frac{1}{2}}^{\infty} \sum_{\ell=J-\frac{1}{2}}^{J+\frac{1}{2}} \frac{1}{r} f_{J\ell}(r) \, \mathcal{Y}_{J\ell\frac{1}{2}}^{M} \right] \right\}, \qquad (1)$$

where A is an antisymmetrization operator, and the plus and minus signs are for the $\alpha + p$ and $\alpha + n$ systems, respectively. The function ζ_α is a charge-spin function and has the form

$$\zeta_\alpha(\underline{s}_1, \underline{s}_2, \underline{s}_3, \underline{s}_4 ; t_1, t_2, t_3, t_4)$$

$$= \delta(\underline{s}_1, \tfrac{1}{2}) \, \delta(t_1, \tfrac{1}{2}) \, \delta(\underline{s}_2, \tfrac{1}{2}) \, \delta(t_2, -\tfrac{1}{2})$$

$$\times \delta(\underline{s}_3, -\tfrac{1}{2}) \, \delta(t_3, \tfrac{1}{2}) \, \delta(\underline{s}_4, -\tfrac{1}{2}) \, \delta(t_4, -\tfrac{1}{2}), \qquad (2)$$

with $\delta(\underline{s}, m_\underline{s})$ and $\delta(t, m_t)$ denoting the spin and isobaric spin functions, respectively. The function φ_α describes the spatial behaviour of the α cluster and is given by

$$\varphi_\alpha = \exp\left[-\tfrac{1}{2}\alpha \sum_{i=1}^{4} (\underline{r}_i - \underline{R}_\alpha)^2 \right], \qquad (3)$$

where $\underset{\sim}{R}_\alpha$ is the position vector of the center of mass of the α cluster. The width parameter α is chosen to yield the experimentally determined value of the rms radius of the nucleon distribution in the alpha particle[7]; it is equal to 0.514 fm^{-2}, which corresponds to a rms radius of 1.48 fm[8]. The function $\mathcal{Y}^M_{Jl\frac{1}{2}}$ is a normalized spin-angle function, belonging to a state of total angular momentum J whose z-component is M and which is a combination of an orbital angular momentum l with a spin s = 1/2. Finally, the functions f_{Jl} describe the relative motion of the clusters and are determined from the variational principle

$$\delta \int \Psi^* (H - E') \Psi \, d\tau = 0 , \qquad (4)$$

where H is the Hamiltonian of the system and E' is the total energy, composed of the internal energy of the α cluster and the relative energy E in the c.m. system.

The nucleon-nucleon potential is chosen as

$$V_{ij} = \left[\frac{1+P^\sigma_{ij}}{2} V_t + \frac{1-P^\sigma_{ij}}{2} V_s \right]\left[\frac{u}{2} + \frac{2-u}{2} P^r_{ij} \right]$$

$$- V_\lambda \exp(-\lambda r^2_{ij})(\underset{\sim}{r}_i - \underset{\sim}{r}_j) \times (\underset{\sim}{p}_i - \underset{\sim}{p}_j) \cdot (\underset{\sim}{\sigma}_i + \underset{\sim}{\sigma}_j) \frac{1}{2\hbar}$$

$$+ \frac{e^2}{4 r_{ij}} (1 + \tau_{iz})(1 + \tau_{jz}) , \qquad (5)$$

where V_t and V_s are the S-wave triplet and singlet potentials, given by

$$V_t = - V_{0t} \exp(-K_t r^2) ,$$

$$V_s = - V_{0s} \exp(-K_s r^2) . \qquad (6)$$

The constants V_{ot}, K_t, V_{os} and K_s are adjusted to yield the correct values for the two-nucleon effective-range parameters; in this way, we find that[5]

$$V_{ot} = 66.92 \quad MeV,$$

$$K_t = 0.415 \quad fm^{-2},$$

$$V_{os} = 29.05 \quad MeV, \tag{7}$$

$$K_s = 0.292 \quad fm^{-2}.$$

The parameters u, V_λ and λ, on the other hand, cannot be determined from the low-energy two-nucleon S-wave scattering data. In this calculation, we shall fix them by requiring that an overall good fit to the experimental $\alpha + N$ scattering data be obtained in the energy region below the reaction thresholds, with particular emphasis being given to the behaviour of the P-wave phase shifts in the resonance region. This will be discussed in more detail in the next section.

In our calculation, a one-channel approximation has been adopted. This means that the specific distortion effect, i.e., the distortion effect over and above that already implicitly given by the antisymmetrization procedure, is not properly accounted for. Also, to facilitate computations, we have employed a rather simple nucleon-nucleon potential which contains no repulsive core. It is hoped that, by fixing the value of u as described above, we can partially correct for the defects caused by the adoption of these simplifying assumptions.

It should be mentioned, however, that the value of u determined by the above-mentioned procedure should be close to 1, since it is known that the experimental two-nucleon scattering data favour a near-Serber exchange mixture. If the resultant value for u should turn out to be quite different from 1, then we should take it as a clear indication that this crude procedure of varying u is not accurate enough and more elaborate calculation must be performed.

The nucleon-nucleon potential used here has two important

features not present in most of our previous calculations[1]. First, the central part of the potential has different ranges in the triplet and singlet states. As a consequence of this, the effective-range parameters can be well fitted[5]. Second, a phenomenological two-nucleon spin-orbit term has been introduced. This is necessary since the experimental phase shifts are known to be split and a non-central potential of this type is the simplest one which could account for such splitting.

As was mentioned in the Introduction, the use of this potential in the $\alpha + \alpha$ case has yielded excellent results[5]. In fact, the resultant value for u was equal to 0.925, which is quite reasonable in view of our discussion above.

Even with these improvements, this nucleon-nucleon potential has still no saturating character. Therefore, it is again necessary to adopt the crude procedure of fixing the width parameter α as 0.514 fm^{-2} which yields the correct value for the rms radius of the nucleon distribution in the alpha particle. Using this value, the expectation value of the alpha-particle Hamiltonian obtained with the function φ_α of eq.(3) is -26.6 MeV, which is, fortunately, quite close to the experimental value of -28.3 MeV.

Using eq.(4), integrodifferential equations for the functions f_{Jl} can be derived. They have the form

$$\left\{ \frac{\hbar^2}{2\mu} \left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right] + E - V_N(r) - V_C(r) - \eta_{Jl} V_{SO}(r) \right\} f_{Jl}(r)$$

$$= \int_0^\infty \left[k_l^N(r,r') + k_l^C(r,r') + \eta_{Jl} k_l^{SO}(r,r') \right] f_{Jl}(r') dr', \tag{8}$$

where μ is the reduced mass and the quantities η_{Jl} are given by

$$\eta_{l+\frac{1}{2},l} = l \quad , \quad \eta_{l-\frac{1}{2},l} = -(l+1) \quad , \quad \eta_{\frac{1}{2},0} = 0 \quad . \tag{9}$$

The functions V_N, V_C and V_{SO} are the direct nuclear-central potential, the direct Coulomb potential and the direct spin-orbit potential, respectively, while the functions k_ℓ^N, k_ℓ^C and k_ℓ^{SO} represent the kernels arising from the exchange character in the nucleon-nucleon potential and the antisymmetrization procedure. The expressions for these functions are quite lengthy and will be given in the Appendix.

By solving eq.(8) numerically, phase shifts in the various (J, ℓ) states can be obtained. Using these phase shifts, the differential cross section $d\sigma/d\Omega$, the polarization P and the spin-rotation parameter β are given by the relations

$$\frac{d\sigma}{d\Omega} = |g|^2 + |h|^2 ,$$

$$P = \frac{2\,Re\,(g^* h)}{|g|^2 + |h|^2} ,$$

$$\beta = \tan^{-1}\left[\frac{2\,Im\,(g h^*)}{|g|^2 - |h|^2} \right]. \tag{10}$$

In these equations, the quantities $g(\theta)$ and $h(\theta)$ are the spin-independent and spin-dependent scattering amplitudes; in terms of the phase shifts, they can be expressed as

$$g(\theta) = f_C + \frac{1}{k} \sum_\ell \left[(\ell+1) e^{i\delta_\ell^+} \sin\delta_\ell^+ + \ell\, e^{i\delta_\ell^-} \sin\delta_\ell^- \right]$$

$$\times\, e^{2i\sigma_\ell} P_\ell(\cos\theta) \tag{11}$$

$$h(\theta) = \frac{i}{k} \sum_\ell \left[e^{i\delta_\ell^+} \sin\delta_\ell^+ - e^{i\delta_\ell^-} \sin\delta_\ell^- \right] e^{2i\sigma_\ell}$$

$$\times\, \sin\theta\, \frac{dP_\ell(\cos\theta)}{d(\cos\theta)} , \tag{12}$$

where f_C is the pure-Coulomb scattering amplitude, σ_ℓ is the

Coulomb phase shift, and $\delta_\ell{}^+$ and $\delta_\ell{}^-$ are the nuclear phase shifts for $J = \ell + 1/2$ and $J = \ell - 1/2$, respectively.

3. Results

3.1 COMPARISON WITH PREVIOUS RESULTS

One of the major improvements in this calculation is that a nucleon-nucleon potential with different ranges in the triplet and singlet states is employed. Thus it is interesting to compare the results obtained with this potential with those of our previous calculation[6] where a simpler nucleon-nucleon potential was used. For this purpose, we omit the spin-orbit term in eq. (5) and choose a value for u which yields a good fit to $n + \alpha$ phase shifts calculated using the central part of the optical potential obtained recently by Satchler et al. from a phenomenological study of the experimental data[9]. The comparison is shown in fig. 1, where the solid and dashed lines represent the phase shifts calculated with the potential of this investigation (u = 0.99) and the potential of our previous study[†, 6] (y = 0.95), respectively, while the crosses represent the phase shifts of Satchler et al. as described above. From this figure, it is clear that the introduction of the feature about the ranges of the nucleon-nucleon potential does result in a significantly better fit to the $n + \alpha$ scattering data.

Together with our previous finding that excellent results can also be obtained in the $\alpha + \alpha$ case[5], this leads us to believe that, at least in the low- and medium-energy region, the use of a near-Serber potential which yields a good fit to the low-energy nucleon-nucleon scattering data is quite sufficient in a resonating-group calculation on light nuclear systems. The adoption of a potential with a repulsive core is of course desirable, but, in our opinion, not a really necessary procedure.[††]

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† The phase shifts given here are slightly different from those given in fig. 4 of ref. 6. The reason is that the exchange Coulomb interaction, omitted in Ref. 6, is properly taken into account in this calculation.

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†† See also ref. 2.

3.2 PHASE SHIFTS

To determine the appropriate values for the parameters u, V_λ and λ in eq.(5), the following procedure will be adopted. We choose a value for the spin-orbit range λ and then adjust the values of u and V_λ to attempt to obtain a good fit to the $n + \alpha$ phase shifts in the energy region below the reaction thresholds, as given by the analysis of Morgan and Walters[10]. The results show that it is possible to fit these data very well using a wide range of values for λ. In table 1, four sets of these parameter values are listed; for convenience, these will be referred to as Set I, II, III and IV in the following discussions.

From table 1 it is gratifying to note that while the value of u is somewhat dependent upon the choice of the spin-orbit parameter λ, it always remains close to 1. As was discussed in the previous section, this is an indication that the assumptions used in this calculation are reasonably valid.

The results for δ_0, δ_1^+ and δ_1^- calculated with Set I and Set III are shown in fig. 2. In this figure the crosses represent the empirical values for these phase shifts obtained by Morgan and Walters[10]. Here one sees that the agreement between the calculated and the empirical values is quite good, with Set III perhaps marginally favoured over Set I.

With the same parameter values obtained from fitting $n + \alpha$ data, we have also computed the $p + \alpha$ phase shifts. The results for Set I and Set III are shown in fig. 3, where the experimental data of Brown et al.[11], Satchler et al.[9], and Weitkamp and Haeberli[12] are also shown. Here again, one sees that the agreement is quite satisfactory, with the quality of fit being similar to that in the $n + \alpha$ case.

For future references we have listed in tables 2 and 3 the values of the $n + \alpha$ and $p + \alpha$ phase shifts for $\ell = 0$ to 5. These values are calculated with Set III. With the other sets, the values, especially at higher energies, are somewhat different; for example, at 40 MeV with Set I, they are equal to 60.99°, 61.63°, 36.72°, 31.60°, 4.14°, 16.90°, 4.62° for δ_0, δ_1^+, δ_1^-, δ_2^+, δ_2^-, δ_3^+, δ_3^- in the $n + \alpha$ case.

In table 4 we compare the result of Set III with the real parts of the
empirical phase shifts obtained by Giamati and Thaler[14] at 32 MeV for the
p + α system.   From this table it is seen that even at such an energy where
the one-channel resonating-group calculation is expected to be less valid due
to the presence of various open reaction channels, the agreement is still
quite good.   This indicates that, even with its present simplified formulation,
the resonating-group calculation can yield results which are very useful in
situations where the lack of sufficient experimental information would other-
wise impede the execution of a detailed phase-shift analysis[1].

3.3 CROSS SECTION, POLARIZATION AND SPIN-ROTATION PARAMETER

In this subsection we compare the results of our calculation for the
differential cross section, polarization and spin-rotation parameter with
those from experimental measurements[12,15-20] for the p + α system at
energies both below and above the reaction thresholds.

Comparisons are made for the differential cross sections at 13.94 and
24.77 MeV in figs. 4 and 5, where the solid and dashed lines represent the
results for Set III and I, respectively.   Here it is seen that the calculated
results are quite satisfactory at both energies and no choice can be made
between these two sets.

The polarizations calculated at 7.9, 13.94, 23.04 and 31.96 MeV are shown
in figs. 6-8.   From these figures one notes that Set III, which has a spin-
orbit potential of rather short range, is slightly preferred over Set I.   Also,
there is a trend towards better fits at lower energies.   At 31.96 MeV the be-
haviour of the calculated polarization is certainly correct, but detailed
agreement with experiment is lacking.   As has been observed in other calcu-
lations[1] involving light nuclear systems, this is most likely caused by the
adoption of a one-channel approximation in our calculation.   To remedy this
one would either have to perform the difficult task of taking into account the
various reaction channels explicitly or adopt a simple but less accurate pro-
cedure of including a phenomenological imaginary potential in the formulation.

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† In an optical-model study of ^{3}He + α scattering, Brown[13] has found that
the real part of δ_ℓ is only slightly affected by an adjustment in the ima-
ginary optical potential.   From this we conclude that such a comparison is
in fact quite reasonable.

The calculated and experimental results for the spin-rotation parameter β at 38.3 MeV are shown in fig. 9. Here also one finds that the theory explains the data quite well. But for a more detailed discussion it seems prudent to wait until the results of a better calculation, which takes into consideration the presence of the open reaction channels as described in the above paragraph, become available.

3.4 EFFECTS OF EXCHANGE COULOMB POTENTIAL AND TWO-NUCLEON SPIN-ORBIT POTENTIAL

In our previous calculations[1], we have computed the Coulomb interaction between the clusters by using only the unantisymmetrized part of the wave function. For simplicity, the exchange Coulomb interaction, represented by the quantity $k_\ell^c (r,r')$ in eq. (8), has been set equal to zero. Here we try to determine the importance of this interaction by performing calculations both with and without the quantity k_ℓ^c being included. The result is shown in fig. 10, where the $p + \alpha$ phase shifts, obtained by using $u = 0.99$ and $V_\lambda = 0$, are shown in the energy range of 0 to 18 MeV, with the solid and dashed lines representing the cases with and without the exchange Coulomb interaction, respectively. From this figure, it is seen that the exchange Coulomb interaction plays a relatively minor role. In fact, one can easily compensate for its omission by adjusting the value of u slightly. This is also shown in fig. 10, where the crosses represent the values of the phase shifts calculated by omitting $k_\ell^c (r,r')$, but with u adjusted to be 0.955.

There is, however, one important feature contained in fig. 10 which should be pointed out. The exchange Coulomb interaction is attractive in the $\ell = 0$ state, but repulsive in the $\ell = 1$ state. This indicates that, in general, it may not be possible to omit the exchange Coulomb term by making a compensating adjustment in the value of u. This can be done in the present case of $N + \alpha$ scattering simply because the exchange Coulomb interaction has a very minor influence on δ_0, which happens to be also rather insensitive to a small adjustment in u. In a general case where the phase shifts are more sensitive to the value of u, the exchange Coulomb term may have to be included, if a detailed agreement with experiment is desired.

Next, we briefly discuss the question of whether it is necessary to include a spin-orbit term in the two-nucleon potential, insofar as fitting experimental data is concerned. In the energy region where there are sharp resonance levels, for instance, around 2 MeV in the p + α case, the inclusion of such a potential term is certainly necessary. On the other hand, in the energy region where no sharp level exists, semi-quantitative agreement with experiment can still be obtained even with a purely central nucleon-nucleon potential. This is illustrated with Set III in fig. 11, where the solid and dashed lines represent the results for the differential cross section of p + α scattering at 13.94 MeV with and without the two-nucleon spin-orbit term. As is seen, the main difference between these two curves occurs only in the region of the diffraction minimum at about 110°; at other angles, the difference is in fact quite minor.

4. Effective Potential

In this section we continue our study on the effective potential between the clusters, which was initiated in our previous investigation on the α + N system[6]. To avoid unnecessary complication, we shall omit the exchange Coulomb interaction and define the effective potential as

$$V^{*}_{J\ell}(r) = V_N(r) + \eta_{J\ell} V_{SO}(r) + \frac{1}{f_{J\ell}(r)} \int_0^{\infty} \left(k_{\ell}^{N} + \eta_{J\ell} k_{\ell}^{SO} \right) f_{J\ell}(r') \, dr' \qquad (13)$$

where the functions $f_{J\ell}$ are obtained from solving eq. (8), with k_{ℓ}^{c} set as zero. Clearly, because of the particular way of construction, the potential $V^{*}_{J\ell}$ has rather complicated features, being energy dependent and different in various (J, ℓ) states.

For convenience we shall further express $V^{*}_{J\ell}$ as

$$V_{J\ell}^{*}(r) = V_{ce}(r) + \eta_{J\ell} V_{\sigma e}(r) , \qquad (14)$$

with V_{ce} and $V_{\sigma e}$ being the effective central and effective one-body spin-orbit potentials, respectively. They are given by

$$V_{ce} = \frac{1}{2\ell + 1}\left[(\ell+1) V_{\ell+1/2,\ell}^{*} + \ell V_{\ell-1/2,\ell}^{*}\right] , \qquad (15)$$

and

$$V_{\sigma e} = \frac{1}{2\ell + 1}\left(V_{\ell+1/2,\ell}^{*} - V_{\ell-1/2,\ell}^{*}\right) . \qquad (16)$$

Obviously, these potentials are both energy- and ℓ-dependent. The energy dependence is, however, quite slight, as is found by explicit calculation; hence, the main concern here will be to investigate the nature of the ℓ-dependence of these potentials.

The effective central potential V_{ce}, calculated with Set III, is illustrated at 2 MeV for $\ell = 0$ to 3 in fig. 12[+]. For comparison, the direct central potential V_N is also shown. From this figure one can easily see the odd-even effect[1,3], wherein the potentials in the odd-ℓ states are quite different from those in the even-ℓ states. Also, it is noted that the antisymmetrization procedure plays an important role. The effective potentials in all ℓ-states are very different from the direct potential V_N. In fact, the even-ℓ potentials have only a rather weak attractive component, a feature also found by Swan and Pearce[21] from a phenomenological analysis of the empirical phase-shift data.

The effective spin-orbit potential $V_{\sigma e}$ at 2 MeV with Set III and the direct spin-orbit potential V_{SO} are shown for the $n + \alpha$ case in

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[+] The curve for $\ell = 0$ in fig. 12 is actually for the effective potential $\tilde{V}_{\frac{1}{2},0}$, which is simply related to the effective potential $V_{\frac{1}{2},0}^{*}$, as was explained in ref. 6.

fig. 13. Here one sees again that the antisymmetrization effect need to be properly considered. The effective spin-orbit potential has not only an odd-even feature, but is also more attractive than the direct potential V_{SO} in all orbital-angular-momentum states.

To have a semi-quantitative estimate of the importance of the anti-symmetrization procedure on the effective spin-orbit potential, we have performed a calculation where the kernel k_ℓ^{SO} is set as zero and a multiplicative factor is introduced to the potential V_{SO}. The result shows that if this multiplicative factor is chosen to be around 1.3, the phase shifts given in table 2 can be roughly reproduced. This indicates that, in this particular case, the anti-symmetrization procedure has about a 30% effect and is certainly important enough to warrant careful consideration.

Finally, we have briefly studied the effective exchange Coulomb potential defined as

$$V_{ex}^C = \frac{1}{f_{J\ell}} \int_0^\infty k_\ell^C(r,r')\, f_{J\ell}(r')\, dr' , \qquad (17)$$

and found that it is short-ranged and has also a distinct odd-even feature. In the case of $p + \alpha$ scattering, this potential is attractive in even-ℓ states, but repulsive in odd-ℓ states.

5. Conclusion

In this study we have considered the $\alpha + p$ and $\alpha + n$ systems with the resonating-group method in the one-channel approximation. The nucleon-nucleon potential used has both a central part, which has unequal ranges in the triplet and singlet states, and a spin-orbit component. The results show that with a near-Serber exchange mixture and appropriate values for the depth and range of the spin-orbit potential, very good fit to the empirical phase-shift data can be obtained in the energy region below the reaction thresholds. In fact, even above the reaction thresholds where the result is not expected to be as good, the agreement with the experimental cross-section, polarization and spin-rotation-

parameter data is quite satisfactory.

We should point out that there are phenomenological aspects in our calculation. The depth and range of the nucleon-nucleon spin-orbit potential and the exchange-mixture parameter u are adjusted to yield a best fit with the low-energy $n + \alpha$ scattering data. But, it should be mentioned that the adjustment in the value of u is rather limited due to our insistence that the resultant exchange mixture should be close to that of a Serber potential. Also, it is interesting to observe that in a purely phenomenological analysis by Satchler et al.[9], the number of parameters used to obtain a good fit with the $n + \alpha$ and $p + \alpha$ scattering data below the reaction thresholds is ten. Thus, it seems to us that the present approach, which combines theoretical consideration with a small number of phenomenological parameters, is a very useful way in dealing with nuclear scattering problems.

We have also used the resultant wave function to construct an effective potential between the clusters. The main purpose here is to assess the importance of the requirement that the wave function be antisymmetric with respect to the exchange of the incident nucleon with a nucleon in the target nucleus. From the result it is found that the antisymmetrization effect is indeed quite important. In particular, it creates an odd-even feature, wherein the effective central and the effective one-body spin-orbit potentials in the odd-l states are appreciably different from those in the even-l states. It should be recalled that in the usual analyses on heavier systems with the optical model, this particular feature is not at all considered. Thus, from our viewpoint, the optical potential used presently does not have enough flexibility and further analyses on heavier systems with an optical potential which does take the odd-even feature into account may very well be quite worthwhile.

EXPRESSIONS FOR THE DIRECT POTENTIALS AND THE KERNELS

The expressions for the direct potentials V_N, V_{SO} and V_C, and the kernels k_ℓ^N, k_ℓ^{SO} and k_ℓ^C are as follows:

$$V_N(r) = -\sum_{i=1}^{2} V_{0i} \left(4w_i - m_i + 2b_i - 2h_i\right) \left(\frac{4\alpha}{4\alpha + 3K_i}\right)^{3/2}$$

$$\times \quad \exp\left(-\frac{4\alpha K_i}{4\alpha + 3K_i} r^2\right) \tag{A1}$$

with

$$V_{01} = V_{0t} \quad , \qquad V_{02} = V_{0s} \quad ,$$

$$K_1 = K_t \quad , \qquad K_2 = K_s \quad ,$$

$$w_1 = b_1 = w_2 = -b_2 = u/4 \quad ,$$

$$m_1 = h_1 = m_2 = -h_2 = (2-u)/4 \quad .$$

$$V_{SO}(r) = -\frac{5}{2} V_\lambda \left(\frac{4\alpha}{4\alpha + 3\lambda}\right)^{5/2} \exp\left(-\frac{4\alpha\lambda}{4\alpha + 3\lambda} r^2\right) \tag{A2}$$

$$V_C(r) = \frac{z z' e^2}{r} \Phi\left[\left(\frac{4\alpha}{3}\right)^{\frac{1}{2}} r\right] \tag{A3}$$

with z, z' being the atomic numbers of the two clusters and

$$\Phi(\nu) = \frac{2}{\sqrt{\pi}} \int_0^\nu \exp\left(-t^2\right) dt \quad .$$

$$k_\ell^N(r, r') = -\frac{\hbar^2}{2M} \mathcal{J} - \sum_{i=1}^{2} V_{0i} \mathcal{V}_i + E'\mathcal{E} \tag{A4}$$

where M is the nucleon mass and E' is given by

$$E' = E + E_\alpha \ ,$$

with

$$E_\alpha = \frac{\hbar^2}{2M} \frac{9}{2} \alpha - \sum_{i=1}^{2} 6 (w_i + m_i) V_{0i} \left(\frac{\alpha}{\alpha + 2K_i}\right)^{3/2} + e^2 \left(\frac{2\alpha}{\pi}\right)^{1/2} \ ,$$

$$\mathcal{J} = \left(\frac{4}{5}\right)^3 \left(\frac{4\alpha}{3\pi}\right)^{3/2} \exp\left[-\frac{34}{75}\alpha(r^2 + r'^2)\right]$$

$$\times \left[\frac{47}{5}\alpha S_\ell\left(\frac{32}{75}\alpha\right) - \frac{1216}{1125}\alpha^2(r^2 + r'^2) S_\ell\left(\frac{32}{75}\alpha\right) - \frac{1568}{1125}\alpha^2 rr' T_\ell\left(\frac{32}{75}\alpha\right)\right] ,$$

with

$$S_\ell(\nu) = \frac{4\pi}{\nu} \mathcal{J}_{\ell+\frac{1}{2}}(\nu rr') \ ,$$

$$T_\ell(\nu) = \frac{4\pi}{\nu}\left[\mathcal{J}_{\ell+\frac{3}{2}}(\nu rr') - \frac{\ell}{\nu rr'}\mathcal{J}_{\ell+\frac{1}{2}}(\nu rr')\right] ,$$

and $\mathcal{J}(\nu rr')$ being a hyperbolic spherical Bessel function,

$$\mathcal{V}_i = \left(\frac{4}{5}\right)^3 \left(\frac{4\alpha}{3\pi}\right)^{3/2} \left\{(-w_i + 4m_i - 2b_i + 2h_i) S_\ell\left(\frac{32}{75}\alpha - \frac{32}{25}K_i\right)\right.$$

$$\times \exp\left[-\frac{34\alpha + 48K_i}{75}(r^2 + r'^2)\right]$$

$$+ (-3w_i - 3m_i)\left(\frac{\alpha}{\alpha + 2K_i}\right)^{3/2} S_\ell\left(\frac{32}{75}\alpha\right) \exp\left[-\frac{34}{75}\alpha(r^2 + r'^2)\right]$$

$$+ (-3w_i - 3m_i)\left(\frac{3\alpha}{3\alpha + 2K_i}\right)^{3/2} S_\ell\left(\frac{32\alpha^2 + 64\alpha K_i}{75\alpha + 50K_i}\right)$$

$$\times \left[\exp\left(-\frac{34\alpha^2 + 28\alpha K_i}{75\alpha + 50 K_i} r^2 - \frac{34\alpha^2 + 108\alpha K_i}{75\alpha + 50 K_i} r'^2 \right) \right.$$

$$\left. + \exp\left(-\frac{34\alpha^2 + 108\alpha K_i}{75\alpha + 50 K_i} r^2 - \frac{34\alpha^2 + 28\alpha K_i}{75\alpha + 50 K_i} r'^2 \right) \right] \right\} \quad,$$

$$\mathcal{E} = \left(\frac{4}{5}\right)^3 \left(\frac{4\alpha}{3\pi}\right)^{3/2} S_\ell \left(\frac{32}{75}\alpha\right) \exp\left[-\frac{34}{75}\alpha (r^2 + r'^2) \right]$$

$$k_\ell^{SO}(r,r') = V_\lambda \left(\frac{4}{5}\right)^3 \left(\frac{4\alpha}{3\pi}\right)^{3/2} \left(\frac{5\alpha}{\alpha - 3\lambda}\right) S_\ell \left(\frac{32}{75}\alpha - \frac{32}{25}\lambda\right)$$

$$\times \exp\left[-\frac{34\alpha + 48\lambda}{75} (r^2 + r'^2) \right] \qquad (A5)$$

$$k_\ell^C(r,r') = k_\ell^{Cn} , \qquad \text{for } n+\alpha \text{ system}$$

$$= k_\ell^{Cp} , \qquad \text{for } p+\alpha \text{ system} \qquad (A6)$$

where

$$k_\ell^{Cn} = -\left(\frac{4}{5}\right)^3 \left(\frac{4\alpha}{3\pi}\right)^{3/2} \left(\frac{2\alpha}{\pi}\right)^{\frac{1}{2}} e^2 S_\ell \left(\frac{32}{75}\alpha\right) \exp\left[-\frac{34}{75}\alpha (r^2 + r'^2) \right],$$

and

$$k_\ell^{Cp} = -\left(\frac{4}{5}\right)^3 \left(\frac{4\alpha}{3\pi}\right)^{3/2} e^2 (2\pi rr') \exp\left[-\frac{34}{75}\alpha (r^2 + r'^2) \right]$$

$$\times \left\{ \frac{5}{4} \int_{-1}^1 \frac{\exp\left(-\frac{32}{75}\alpha rr'\mu\right) P_\ell(\mu)}{(r^2 + r'^2 - 2rr'\mu)^{1/2}} \, d\mu \right.$$

$$\left. + \int_{-1}^1 \frac{1}{Q} \exp\left(-\frac{32}{75}\alpha rr'\mu\right) \Phi\left(\sqrt{\frac{3\alpha}{2}}\, Q\right) P_\ell(\mu) \, d\mu \right.$$

$$+ \int_{-1}^{1} \frac{1}{Q'} \exp\left(- \frac{32}{75} \alpha r r' \mu\right) \Phi\left(\sqrt{\frac{3\alpha}{2}}\, Q'\right) P_{\ell}(\mu)\, d\mu \quad ,$$

with

$$Q = \frac{1}{15}\left(256\, r^2 + 16\, r'^2 + 128\, r r' \mu\right)^{\frac{1}{2}} \quad ,$$

$$Q' = \frac{1}{15}\left(16\, r^2 + 256\, r'^2 + 128\, r r' \mu\right)^{\frac{1}{2}} \quad .$$

Table 1

Values for the parameters u, V_λ and λ

Set number	λ (fm^{-2})	V_λ (MeV)	u
I	0.46	11.0	0.97
II	1.0	48.5	0.96
III	2.0	224.8	0.95
IV	3.0	591.1	0.94

Table 2

n + α phase shifts, in degrees, calculated with Set III

E (MeV)	δ_0	δ_1^+	δ_1^-	δ_2^+	δ_2^-	δ_3^+	δ_3^-	δ_4^+	δ_4^-	δ_5^+	δ_5^-
0.5	161.35	20.43	2.60								
0.7	157.99	46.12	4.38								
0.9	155.10	77.65	6.50	0.01	0						
1.0	153.78	89.85	7.67	0.01	0						
1.5	148.07	115.68	14.40	0.02	0						
2.0	143.33	121.13	22.04	0.05	0	0.01	0				
2.5	139.23	122.03	29.74	0.08	0.01	0.01	0.01				
3.0	135.59	121.44	36.77	0.14	0.02	0.02	0.02				
4.0	129.28	118.84	47.45	0.29	0.05	0.05	0.04				
6.0	119.21	112.68	57.28	0.82	0.19	0.18	0.15				
8.0	111.27	107.03	59.41	1.70	0.45	0.41	0.34	0.01	0.01		
10.0	104.70	102.09	58.66	2.93	0.81	0.76	0.62	0.03	0.03		
12.0	99.11	97.76	56.82	4.50	1.26	1.22	0.98	0.07	0.05	0.01	0.01
16.0	89.97	90.52	52.19	8.45	2.32	2.48	1.89	0.20	0.17	0.03	0.03
20.0	82.70	84.63	47.55	13.03	3.38	4.09	2.92	0.44	0.35	0.09	0.08
25.0	75.34	78.53	42.26	18.78	4.44	6.43	4.23	0.90	0.69	0.20	0.18
30.0	69.30	73.42	37.58	23.90	5.09	8.97	5.40	1.54	1.13	0.38	0.33
35.0	64.23	69.04	33.46	28.13	5.36	11.55	6.35	2.35	1.64	0.62	0.53
40.0	59.88	65.24	29.81	31.47	5.34	14.05	7.07	3.29	2.17	0.94	0.78
50.0	52.83	59.02	23.69	36.08	4.79	18.58	7.84	5.42	3.17	1.75	1.36

Table 3

p + α phase shifts, in degrees, calculated with Set III

E (MeV)	δ_0	δ_1^+	δ_1^-	δ_2^+	δ_2^-	δ_3^+	δ_3^-	δ_4^+	δ_4^-	δ_5^+	δ_5^-
0.5	173.27	2.36	0.71								
0.7	170.05	5.53	1.49								
0.9	167.10	10.68	2.52								
1.0	165.72	14.23	3.14								
1.5	159.57	44.81	7.07	0.01	0						
2.0	154.38	82.21	12.26	0.02	0						
2.5	149.85	101.26	18.32	0.05	0	0.01	0				
3.0	145.82	108.96	24.75	0.08	0.01	0.01	0.01				
4.0	138.84	113.17	36.75	0.19	0.03	0.04	0.03				
6.0	127.73	111.13	51.53	0.59	0.12	0.14	0.12				
8.0	118.99	106.79	56.72	1.30	0.32	0.34	0.29	0.01	0.01		
10.0	111.80	102.44	57.57	2.35	0.63	0.64	0.54	0.03	0.02		
12.0	105.71	98.42	56.61	3.72	1.04	1.06	0.87	0.06	0.05	0.01	0.01
16.0	95.79	91.48	52.82	7.32	2.04	2.22	1.71	0.18	0.15	0.03	0.03
20.0	87.82	85.70	48.48	11.65	3.09	3.71	2.72	0.39	0.32	0.07	0.07
25.0	80.08	79.70	43.47	17.28	4.22	6.00	4.02	0.82	0.63	0.18	0.16
30.0	73.63	74.68	38.91	22.49	4.97	8.40	5.15	1.43	1.05	0.36	0.31
35.0	68.20	70.28	34.72	26.92	5.37	10.90	6.17	2.20	1.56	0.59	0.50
40.0	63.53	66.50	31.10	30.45	5.41	13.47	6.93	3.07	2.05	0.89	0.74
50.0	56.15	60.20	24.97	35.40	4.99	18.00	7.76	5.20	3.06	1.68	1.31

Table 4

A comparison of the phase shifts, in degrees, obtained
using Set III with the empirical phase shifts of Giamati
and Thaler at 32 MeV for the p + α system

	δ_0	δ_1^+	δ_1^-	δ_2^+	δ_2^-	δ_3^+	δ_3^-	δ_4^+	δ_4^-	δ_5^+	δ_5^-
This Calculation	71.3	72.8	37.1	24.4	5.2	9.5	5.6	1.7	1.3	0.5	0.4
Giamati and Thaler	66.5	74.5	33.8	23.8	6.5	12.8	7.1	3.6	0.3	1.5	0.9

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Figure Captions

Fig. 1: Comparison of n + α phase shifts obtained with the central potential of eq. (5) (solid lines) and the potential of ref. 6 (dashed lines). The data points shown are obtained by using the central potential given in eq. (8) of ref. 9.

Fig. 2: Phase shifts for n + α scattering obtained with Set I (dashed lines) and Set III (solid lines). The data points represent the empirical phase shifts of Morgan and Walters (ref. 10).

Fig. 3: Phase shifts for p + α scattering obtained with Set I (dashed lines) and Set III (solid lines). The data points represent the empirical phase shifts given in refs. 9, 11 and 12.

Fig. 4: Comparison of p + α differential cross sections calculated using Set I (dashed line) and Set III (solid line) with experimental data at 13.94 MeV. The experimental data are those of ref. 15.

Fig. 5: Comparison of p + α differential cross section calculated using Set III with experimental data at 24.77 MeV. The experimental data are those of ref. 16.

Fig. 6: Comparison of p + α polarizations calculated using Set I (dashed line) and Set III (solid line) with experimental data at 7.9 MeV. The experimental data are those of refs. 17 and 18.

Fig. 7: Comparison of p + α polarizations calculated using Set I (dashed line) and Set III (solid line) with experimental data at 13.94 MeV. The experimental data are those of ref. 12.

Fig. 8: Comparison of p + α   polarizations calculated using Set III
with experimental data at 23.04 and 31.96 MeV. The experimental
data are those of ref. 19.

Fig. 9: Comparison of p + α spin-rotation parameter calculated using
Set III with experimental data at 38.3 MeV. The experimental
data are those of ref. 20.

Fig.10: Phase shifts for p + α scattering with (solid lines) and
without (dashed lines) the exchange Coulomb interaction.
The crosses represent the values of the phase shifts calculated
without the exchange Coulomb interaction, but with u adjusted
to be 0.955.

Fig. 11: Differential cross sections for p + α scattering at 13.94 MeV
with (solid line) and without (dashed line) the nucleon-
nucleon spin-orbit potential.

Fig. 12: The effective central potential V_{ce} at 2 MeV, calculated with
Set III, and the direct central potential V_N in the n + α case.

Fig. 13: The effective spin-orbit potential $V_{\sigma e}$ at 2 MeV, calculated
with Set III, and the direct spin-orbit potential V_{SO} in the
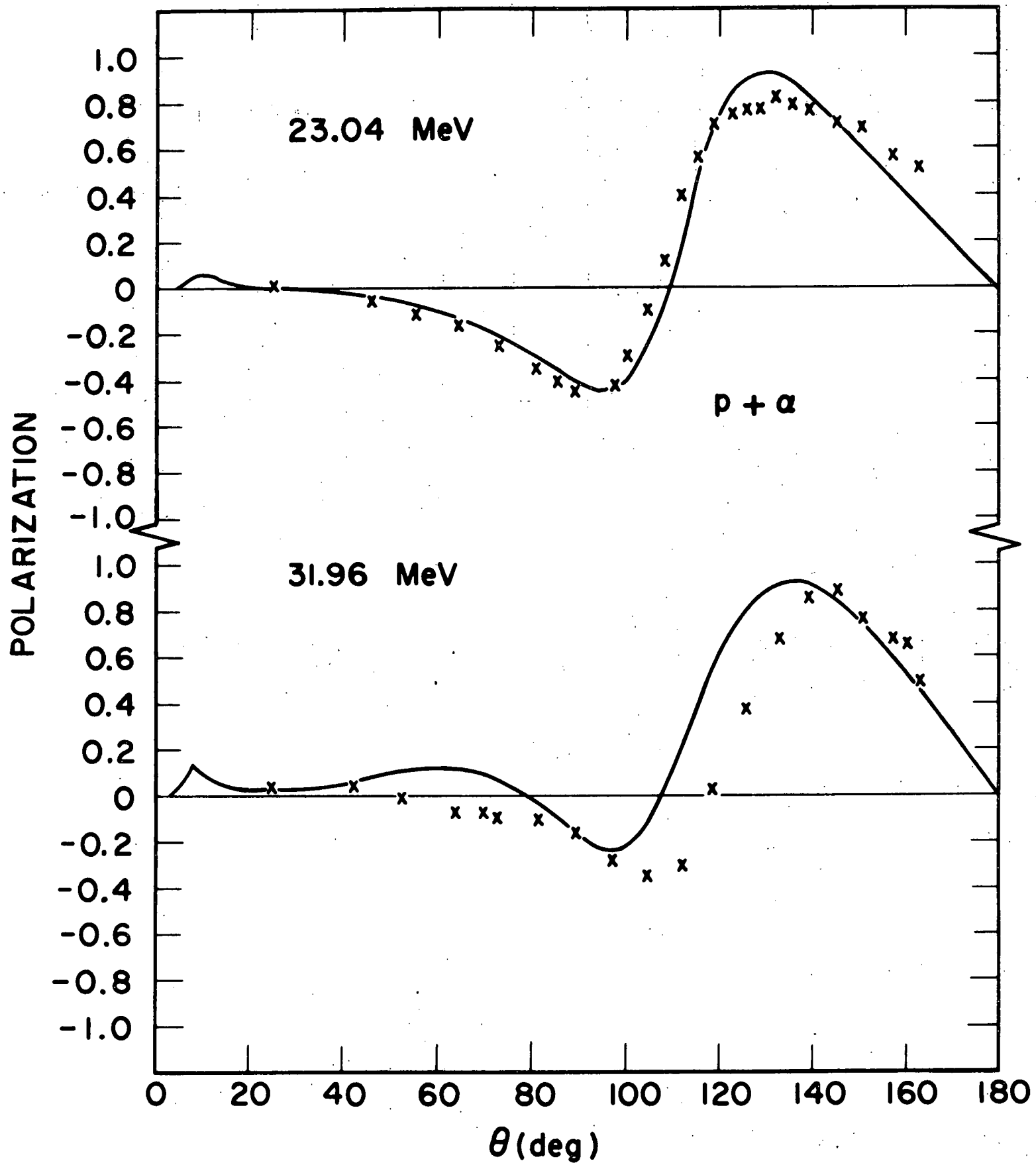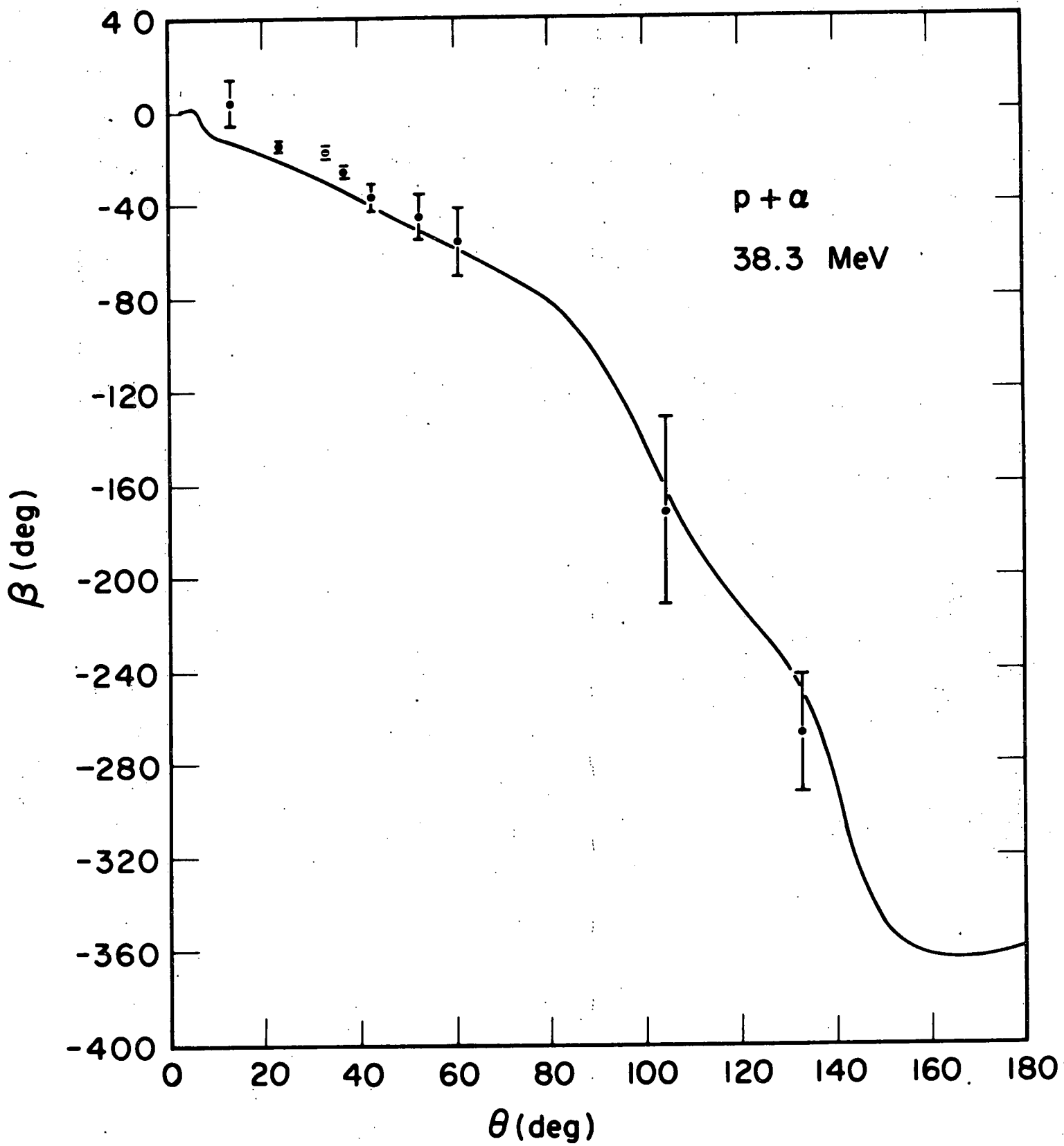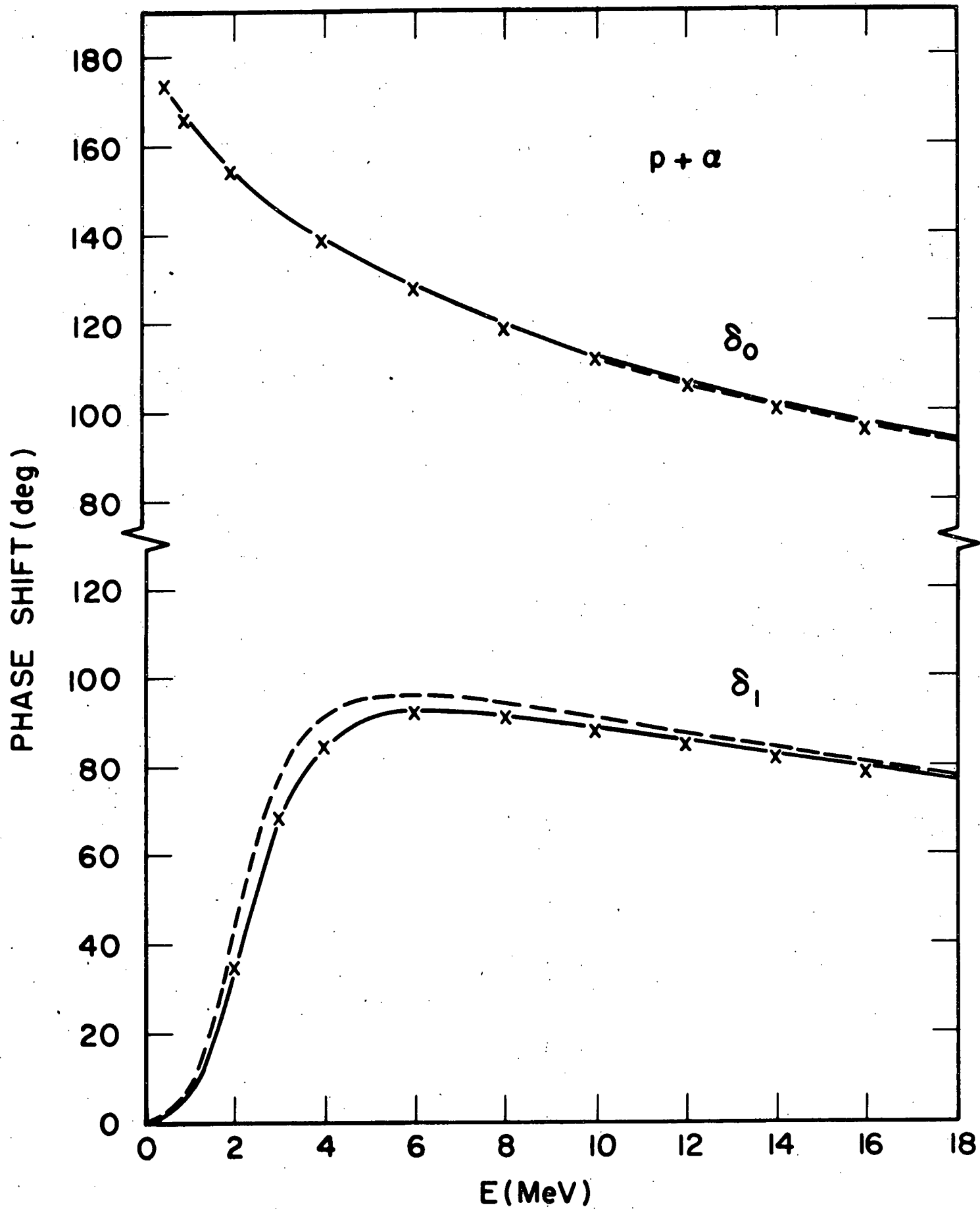n + α case.
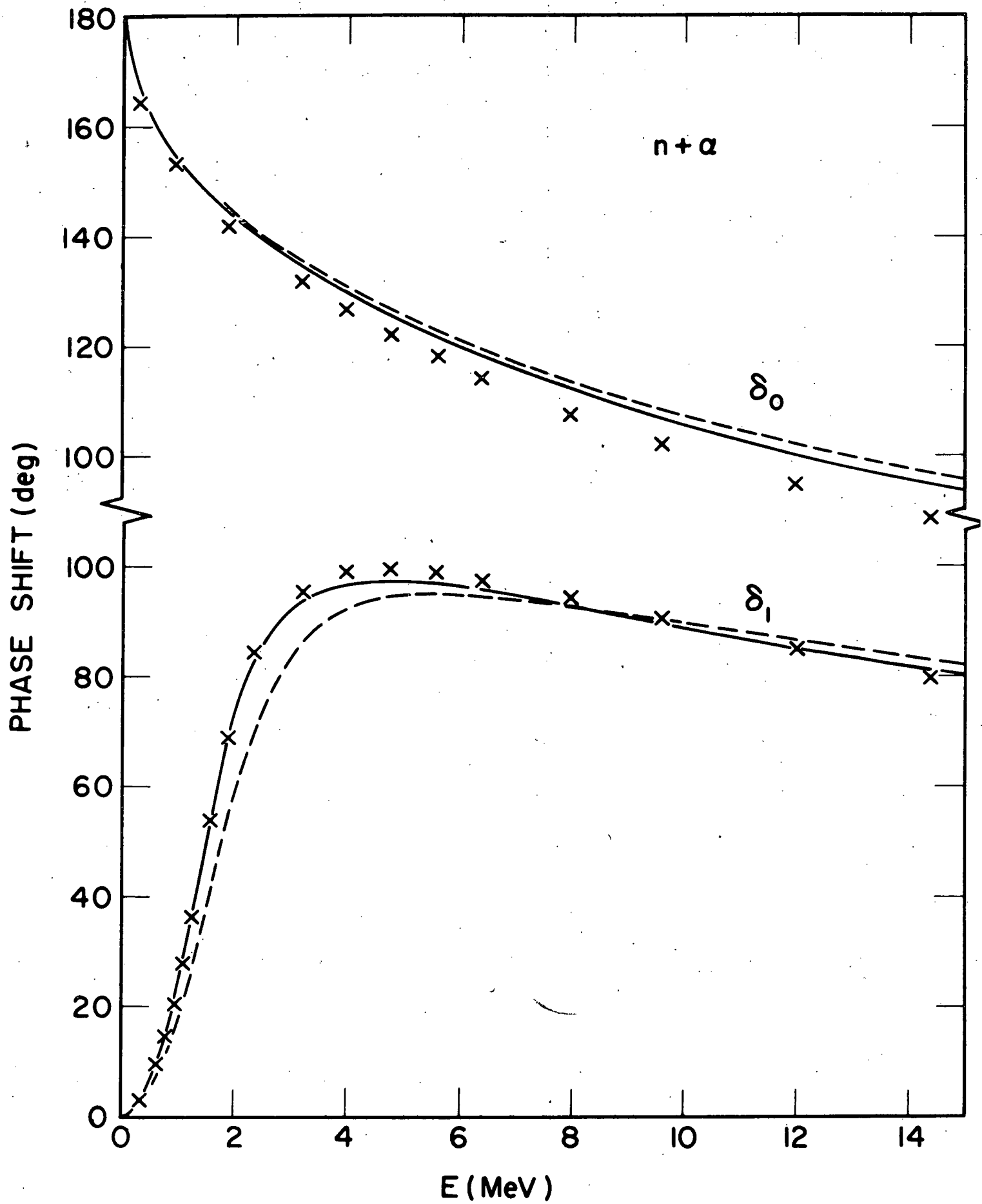
Fig. 1

Fig. 2

Fig. 3

Fig. 4

p + α

24.77 MeV
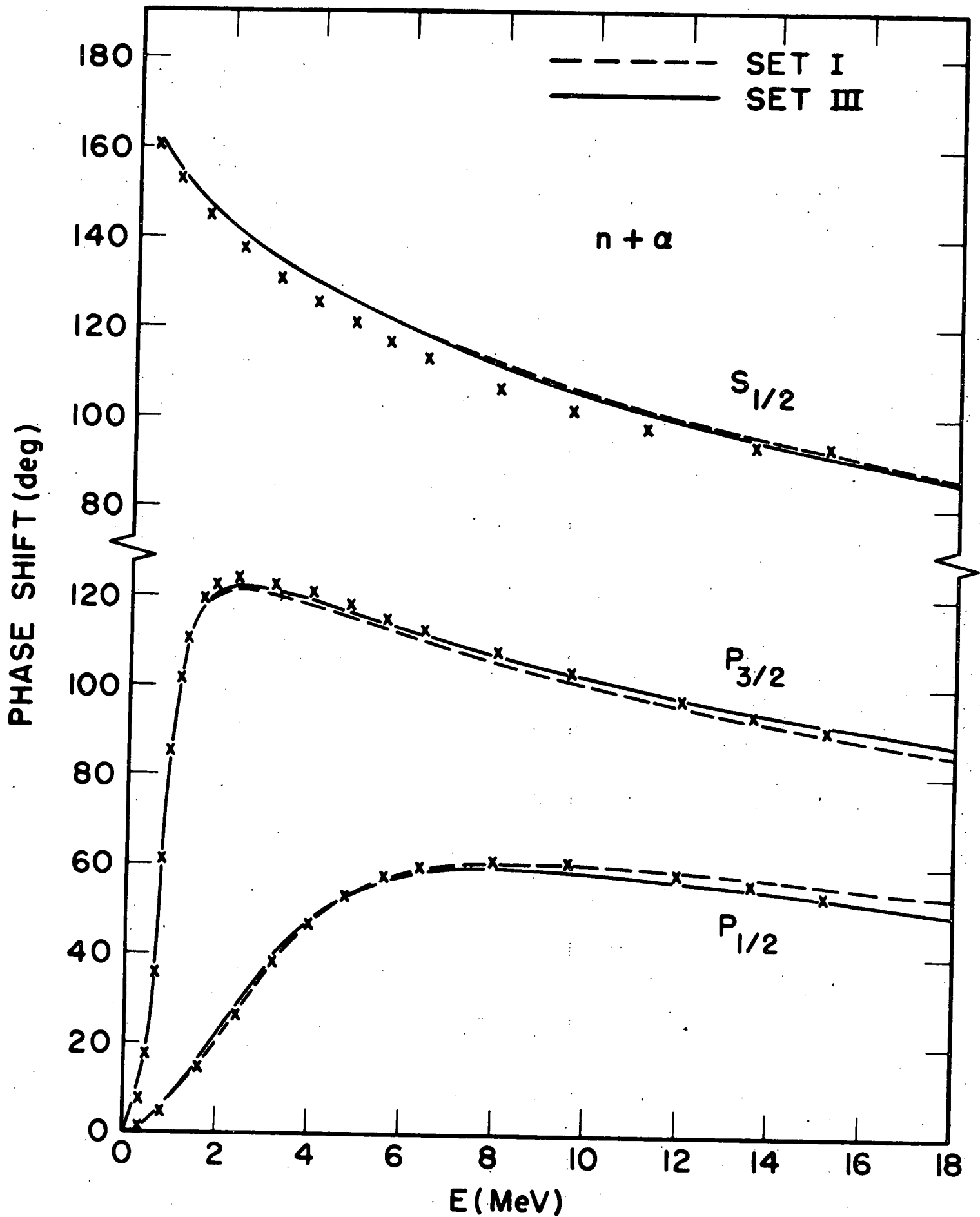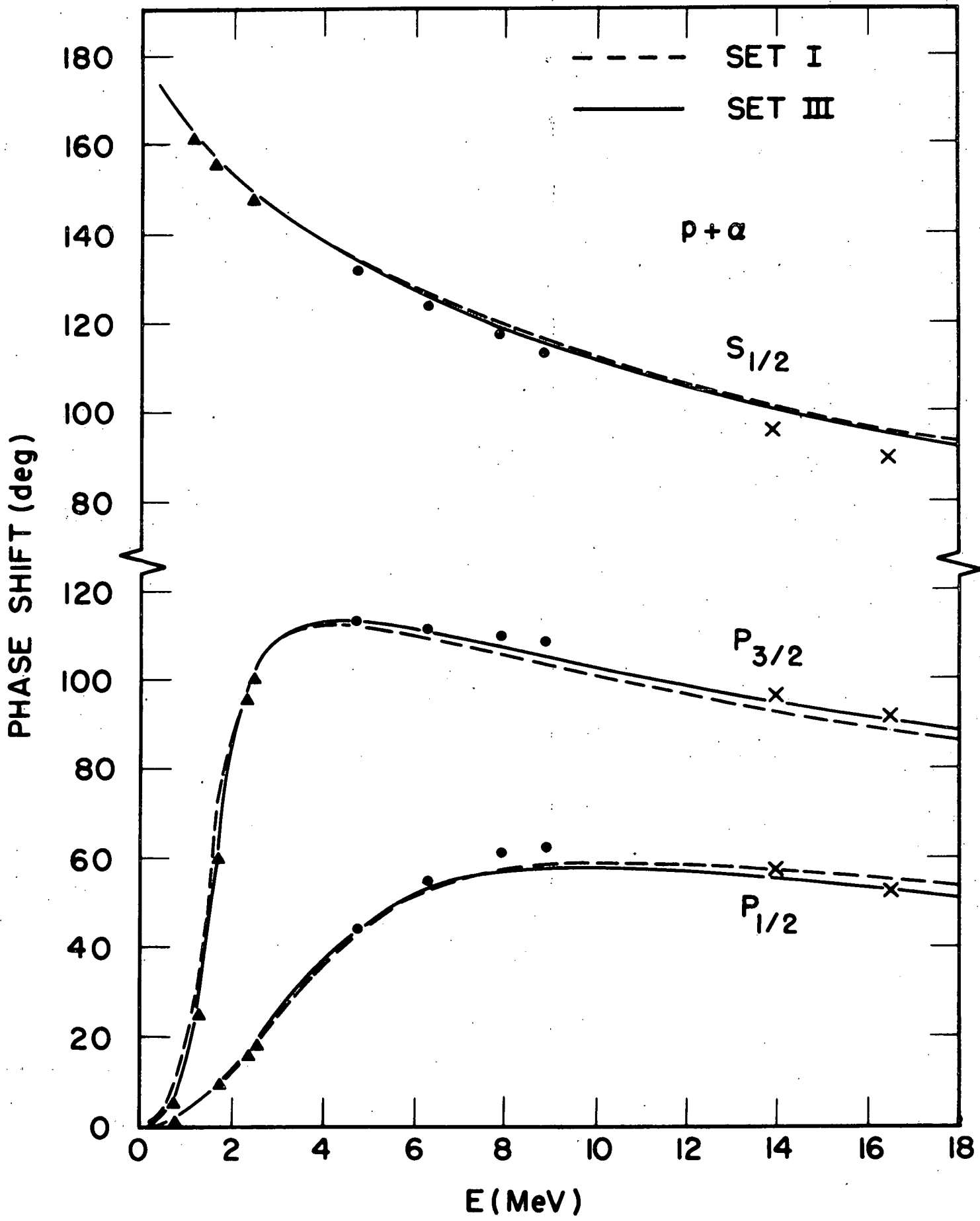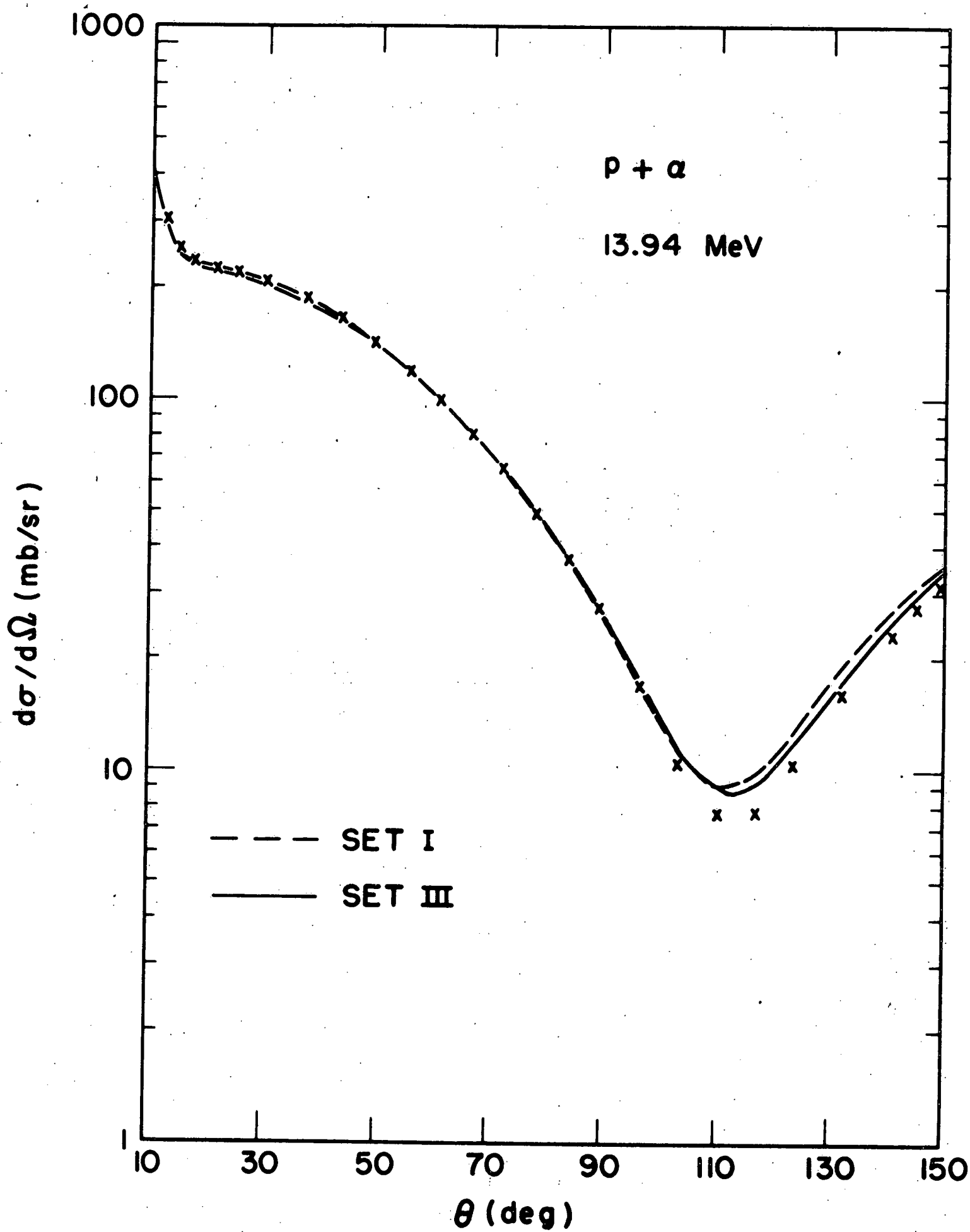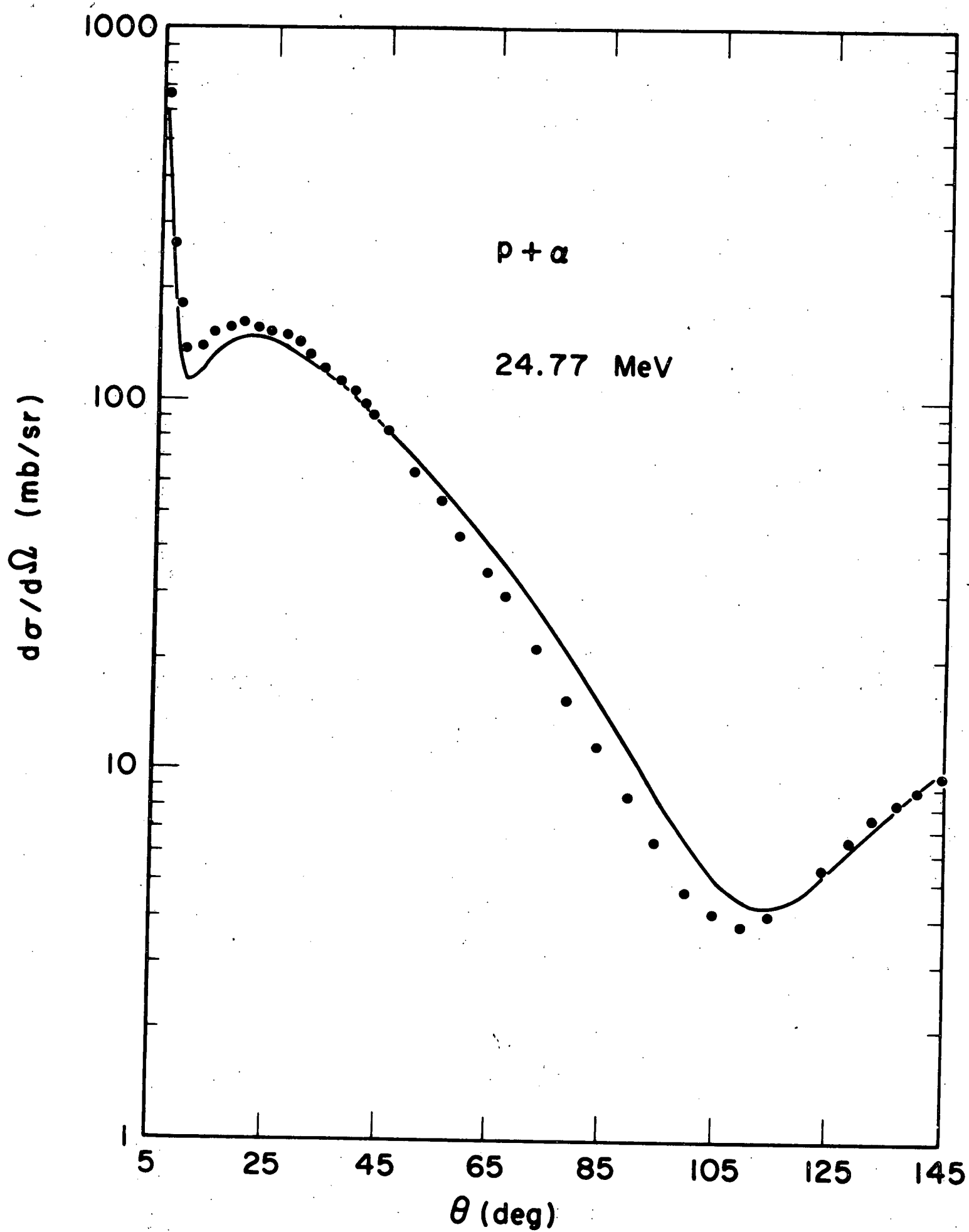
Fig. 5
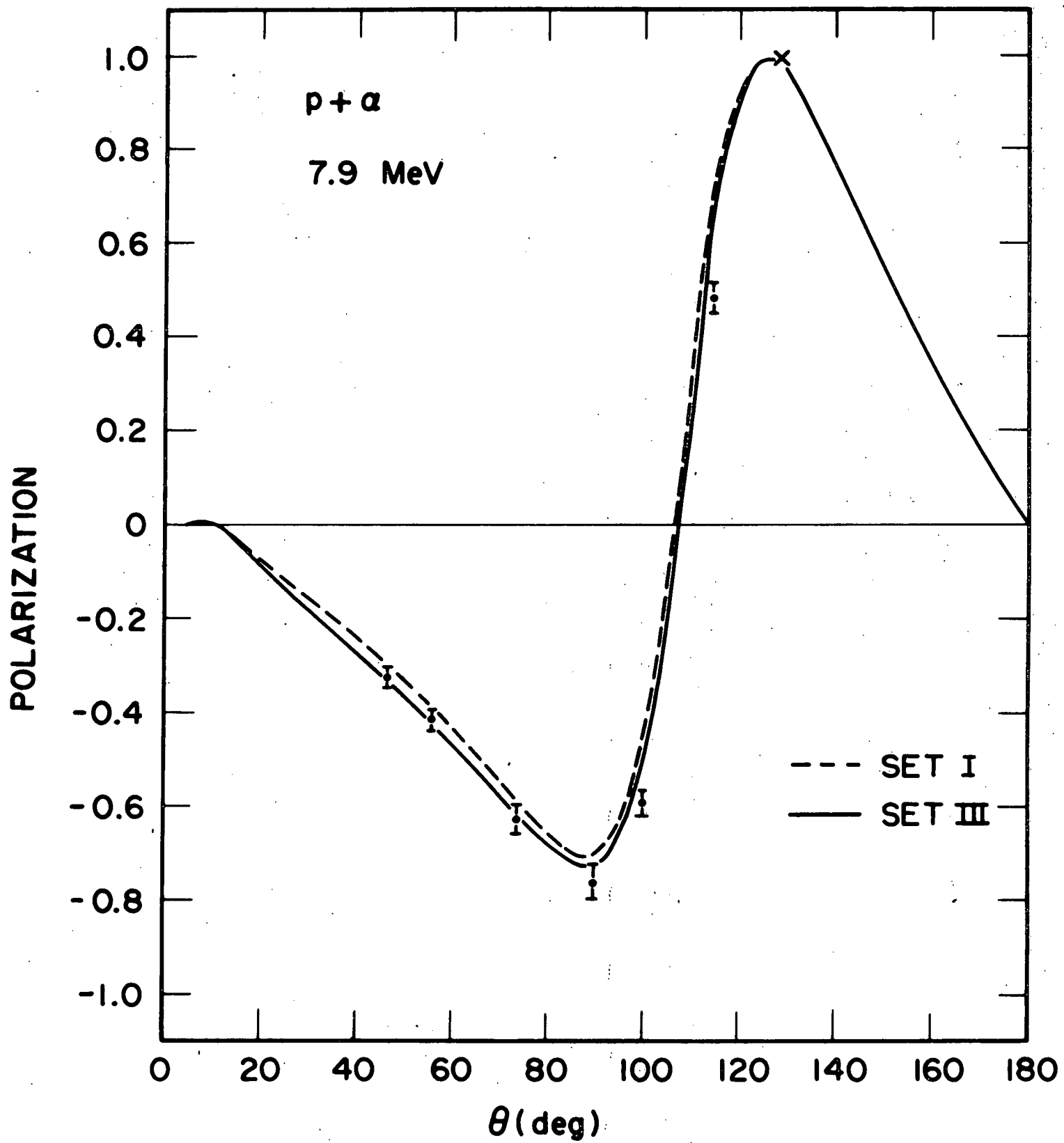
Fig. 6
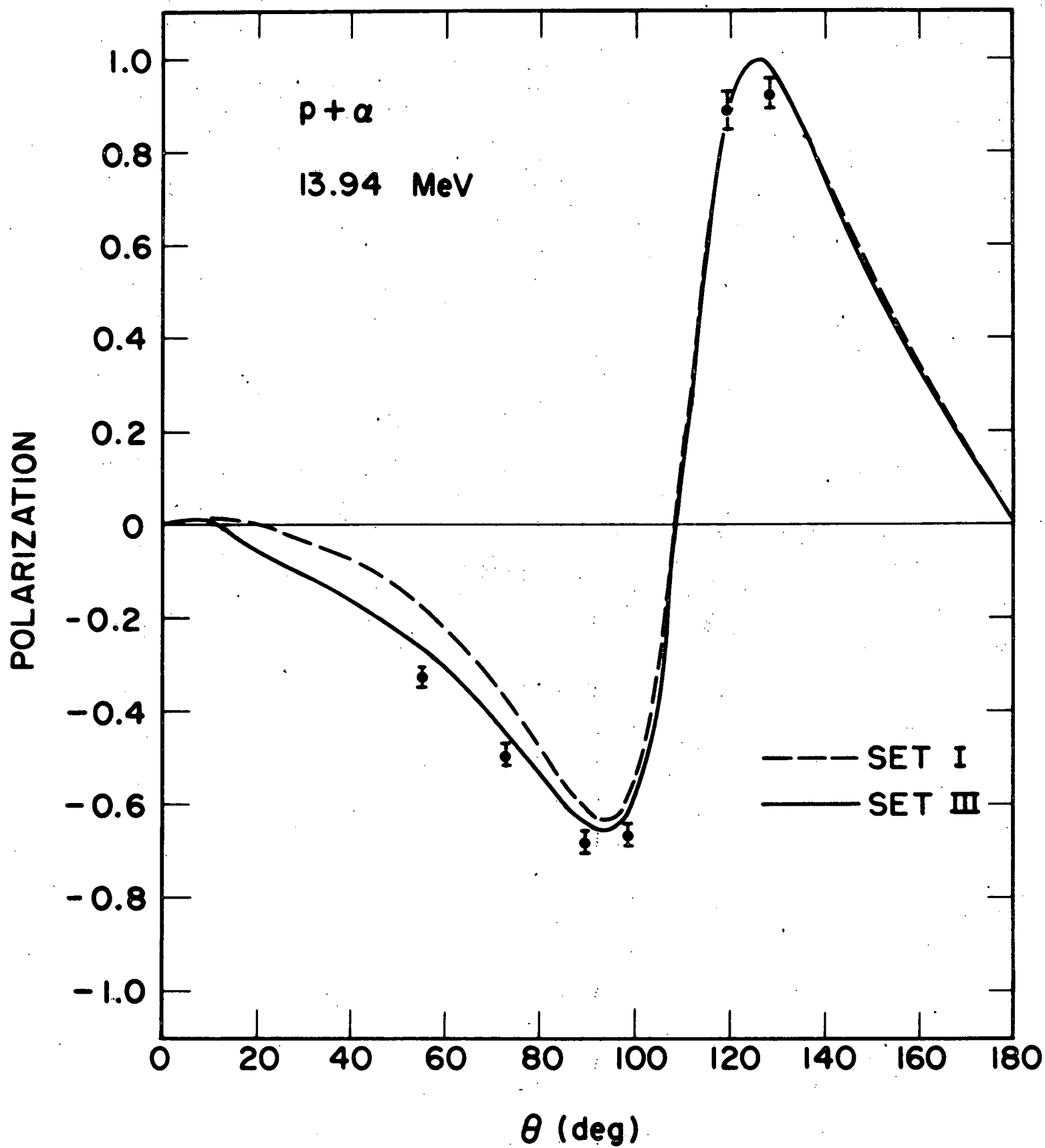
Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13