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ELASTIC AND INELASTIC METHODS OF PIPING SYSTEMS ANALYSIS: A PRELIMINARY REVIEW

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A PRELIMINARY REVIEW

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SUMMARY AND CONCLUSIONS

A preliminary review of the methods used for elastic and inelastic piping system analysis was carried out. The following principle conclusions were reached:

1) Techniques for the analysis of complex piping systems operating in the high temperature creep regime should be further developed.

2) Accurate analysis of a complete pipework system in creep using the "complete shell finite element methods" is not feasible at the present. Similarly, the "reduced shell finite element method" still requires excessive computer time and also requires further investigation regarding the compatibility problems associated with the pipe bend element, particularly when applied to cases involving general loading conditions.

3) In view of 1) and 2) and the current size of proposed high temperature systems requiring the evaluation of long-term operating life (30 to 40 years), it is important to adopt a simplified analysis method. Such a method should be quick, relatively inexpensive, yet it should retain sufficient accuracy. A design procedure for such a method based on currently available techniques applied in a three-stage approach is outlined. The work required for implementation of these procedures together with desirable future developments are also briefly discussed. Other proposed simplified approximations also are reviewed in the text.

The current paper represents the second review of high-temperature piping-system analysis carried out at Brookhaven

National Laboratory (BNL). While the first review was quite general in scope in that piping-system analysis methods were only one of many topics briefly discussed, this paper (although preliminary) concentrates entirely on piping-system analysis methods.

In comparing the conclusions reached during the first review regarding the application of piping-system computer design analysis results to the American Society of Mechanical Engineers (ASME) Code Case evaluations with those arrived at from the present review, it is to be noted that the previous preliminary conclusions in Ref. 1 are indeed reinforced. The current state of development for piping-system analysis makes the design evaluation on the basis of the 1592 ASME Code Case more difficult for two reasons:

1) The stress and strain quantities necessary for the evaluation of the creep ratchetting and creep rules, and creep fatigue rules are less reliable, and
2) The Code Case rules are derived from theory, analytical methods, and service experience corresponding to stiff vessel-type structures where the strain accumulation is generally a local phenomenon, e.g., nozzel connections, etc., that does not alter the general structural response of the vessel.

In contrast, in an interconnected piping system, local strain conditions, e.g., at bends, affects the overall flexibility response of the entire structure. Correspondingly, the influence of the assumed constitutive equations on the computer stress and strain quantities is much greater in piping analysis.

As a consequence of these factors, the assumptions shown to be "conservative" in vessel analysis can become overly restrictive in piping analysis for some cases, or not conservative enough in others. Therefore, piping analysis is more sensitive to material property changes due to cyclic loading conditions and thermal history effects. Since some of these effects mentioned in Ref. 1 in this text may loom large on a theoretical level but in practice may be only of secondary importance. These uncertainties need to be evaluated by numerical examples corresponding to geometry configurations and load and thermal histories representative of LMFBR design conditions. On the basis of the studies proposed, the sensitivity of various material behavior assumptions can be evaluated and more reliable bounds for simplified methods can be established.

1. INTRODUCTION

Pressure vessels and components intended for nuclear service have been generally designed to satisfy the rules and limits contained in Section III of the ASME code. The rules require a detailed analysis of the stress and displacement conditions, and the limits correspond to failure modes of rupture, buckling, distortion, and fatigue. The evaluation of the failure modes is based on the "design-by-analysis" approach which provides a system of stress categories and methods for computation of membrane, bending, and local stresses. The corresponding limits are the primary, secondary, and peak stress limits that protect against instantaneous rupture and buckling, distortion and ratchetting, fatigue crack initiation and propagation. In the moderately high-temperature range (not exceeding 700° to $800^{\circ}F$), the material properties governing these failure modes are essentially independent of time. Thus the analysis methods and stress limits do not require explicit considerations of the service life of the structure.

This is a consistent and logical method for design evaluation and has been successfully applied to a variety of pressure-vessel and component geometries. While the principles of failure-mode analysis are quite general, the particular code rules are somewhat dependent on the structural analysis techniques that prevailed during the development of the code. The structures were generally idealized into axisymmetric shell components and the effect of geometry variations were evaluated by a procedure called discontinuity analysis. This method uses "influence coefficients" derived from closed solutions for the linear elastic shell components and links these together by a system of continuity and equilibrium equations. The results of such an analysis provide the average (membrane) stress, the linearly varying (bending) stress, and with the use of "stress concentration factors," the peak stress values These quantities are directly comparable to the corresponding allowable stress values.

Similarly for piping system, linear elastic techniques were available. The code provides "flexibility factors" and stress indices for common piping components, the use of which permits the evaluation of the stress categories directly. With the advent of newer nuclear power systems that operate at temperatures above 800^{o}F, time-dependent failure modes had to be considered as well. Code Case 1331 and Code Case 1592 provide rules for creep rupture, creep buckling, creep rachet, and creep fatigue evaluation that complement the time independent design bases of Section III of the code. The severity of the loading conditions induced by rapid thermal transients and the pronounced dependence of material properties on time eliminate the use of linear elastic theory in most cases. Finite element techniques have been extended in recent years to provide methods of analysis for vessel-type structures using elasto-plastic creep formulations for the general shell problem. With developments in present-day computer technology, these analysis methods are coming into general use and there is already a wealth of experience (generally favorable) of applications of the code cases to elevated temperature pressure-vessel designs.

Although the principles of failure-mode analysis are applicable to any geometry, the implementation of the code rules to complex structures such as large interconnected piping systems is not as advanced as those for the stiffer, vessel-type geometry. In a preliminary evaluation of the integrity of piping systems operating in the creep range[1] these problems were discussed in some detail. Some of the relevant conclusions are summarized here to provide continuity to these discussions.

"When the most advanced time-dependent inelastic (elasto-plastic-creep) analysis methods are used, the ASME Code rules for elevated temperature designs are ensuring a high quality component with a low probability for catastrophic failure. However, the application of the ASME rules to the large diameter thin-walled primary LMFBR piping system is made less rigorous by the following:

"a) The Code Rules are derived from theory, analytical methods, and service experience corresponding to stiff, essentially axisymmetric vessel-like structures under a pressure-type primary membrane loading, and radial thermal gradients. In contrast, piping systems are flexible non-axisymmetric beam-like structures under predominantly axial thermal expansion loading in combination with other loading conditions.

"b) Finite element inelastic analysis methods have advanced to the point where vessel-like structures may be analyzed in great detail while the analyses of thin-wall pipes, piping system, including non-axisymmetric bends, tees, restraints, etc., still require drastic simplifying assumptions and hence supply far less detailed results. Consequently the currently available piping analysis techniques are not yet fully compatible with the Code Rules for time-dependent strain, ratchet deformation, creep fatigue, and buckling evaluations."

Following the publication of Ref. 1, a series of papers by Hibbit, Pan and Jetter, Munson, Workman and Rodabaugh appeared discussing the piping system analysis problem. Hibbet in Ref. 2 states the following:

> "(For such cases) a technique is needed which will allow the non-linear analysis of the complete pipe run, with the objective of proving satisfactory design against load cycle fatigue. By analyzing so as to allow realistic inelastic behavior in all components, satisfactory design against low cycle fatigue may be proved.
>
> "The problem is not a difficult one in principle. The necessity of admitting shell-type behavior in pipe bends has long been recognized, and techniques for the analysis of elbows exist. Two possibilities have been available for some time, via computer programs:
>
> "a) Beam-type analysis, through the use of flexibility factors obtained from closed form or approximate solutions to the reduced shell theories....
>
> "b) Complete shell theory analysis using curved shell elements....
>
> "The advantage of the beam approach is its simplicity and cheapness. Unfortunately, because of its reliance on solutions to the pipe-bend problem, this technique is of necessity restricted to cases where such solutions exist, and this usually means mechanical (primary) loading of circular section, elastic elbows. This approach forms the basis of conventional pipeline design computer programs. The complete shell theory approach is quite the opposite. It can handle arbitrary loadings and any constitutive theory (depending on the generality of the computer program) but is not so straightforward to use and is too expensive on present-day computers to be applicable to more than a single component for a complete, cyclic, nonlinear analysis. Thus, the nonlinear analysis of a complete pipe run falls between the two approaches."

Kibbit concludes:

"The basic theory used to develop an economic nonlinear analysis capability for complete pipelines has been presented. The theory is based on appropriate kinematic coupling of constant bending sections, each of which is obtained by superposition of pure bending modes on an axisymmetric torus segment. Current applications of this model demonstrate its usefulness, but there remain fundamental objections to the model, primarily associated with the inevitable incompatibilities of using constant bending segments to model the pipebends."

Pan and Jetter in Ref. 3 discussing the FFTF closed loop systems state:

"Because of the confined space available for thermal expansion loops, most lines in the module cannot be shown to meet all the elastic stress criteria of FRA-152-Rev s, and inelastic analysis is therefore required. Due to the high cost and long computer processing time for a 3-D rigorous inelastic analysis based on presently available tools, it would be impractical to perform inelastic analysis for all critical pipelines."

In the general discussion of the problem, the autors write:

"At the current state-of-the-art, a direct 'rigorous' inelastic analysis is impossible due to economical reasons and the lack of availability of an extremely large and fast computer system. The analytical approach discussed above can be used to perform a reasonable inelastic analysis at a reasonable cost."

In this paper actual computer running times are also presented:

"A complete elastic-creep stress analysis of line 03 consisting of elbows, for 2860 hours, took approximately 23 hours of computer system time. Approximately 6 hours were used to finish the 220-hour creep analysis of pipeline 61 consisting of 12 elbows.

"The total time required for the elastic-plastic analysis of the critical elbow in pipeline 07 was approximately 2 hours. This long running time was caused by the large band width which resulted from the relatively fine mesh used in the analysis."

In the conclusion, however, they state:

"Perhaps the most significant conclusion to be drawn from these analyses is that it is feasible to perform an inelastic analysis of a complete pipeline with sufficient accuracy to follow the stress and strain history at critical points in the line."

This conclusion appears to be slightly optimistic from the economic standpoint when considering that 220 hr creep time requires 6 hr of computer time. While it may not be necessary in most cases to analyze a pipeline for the entire projected life of the plant (say 200,000 hr), it should be realized that the number of lines, critical locations, and transient sequences that need to be considered may require many multiples of the computer running times reported in Ref. 3.

On the same topic Munson et al.[(4)] comment, on piping analysis for structures below the creep range:

"An exact determination of the stresses in terms of today's calculational techniques would require detailed finite element modeling of localized regions of each component in the piping system, including coupled transient heat transfer and mechanical/thermal loadings. However, the size, number, and complexity of piping systems in a typical nulcear plant make such detailed modeling of all components impractical. Instead the overall piping stresses are calculated at discrete points (of maximum stress) using comparatively simple beam-type finite element models. Local stress concentrations due to non-uniformity in piping cross sections are accounted for through use of stress intensity factors."

In further discussions, they state:

> "The difficulty that arises using finite elements or finite difference computer programs is the number of cases that must be analyzed for a typical line of Class I piping. For example, a recirculation loop of a BWR will typically have about 15 different pipe wall sections and locations of discontinuity which must be analyzed for approximately 20 plant transients. Thus, a total of 300 analyses must be performed for just one line. The resulting cost in terms of manpower, computer run time, and data reduction time is nearly prohibitive."

Workman and Rodabaugh in the authors closure[14b] summarize the present situation concisely:

> "First, as probably apparent to those readers who have been involved in start-ups and shutdowns (sometimes crash shutdowns) of high temperature plant piping systems, the accurate prediction of the loading temperature-time history itself is a formidable problem. Second, the plastic response of materials used in piping systems is itself quite variable; even the yield strength of materials purchased to a given specification may vary be a factor of two, from 95% probability minimum, to a maximum. Accordingly, even if we had a precise analytical method, that method might in real life be useful only to an extent of indicating crude (but useful) engineering bounds. Precise analytical methods in the form of elastic-plastic finite element programs are becoming available. Unfortunately the cost of running such programs prohibits their general applications to piping systems. It seems, therefore, that the need exists for a simplified inelastic piping system analysis procedure. Obviously and this perhaps is the main theme of the discussion and closure, the limits of usefulness of such a procedure must be checked out by comparison with test data, precise analytical methods and experience with existing high temperature piping systems."

Thus all opinions seem to converge on the need for reliable but orders-of-magnitude less-costly analysis methods. Although design procedures based on linear elastic-flexibility analysis have advanced to a state where a great degree of sophistication can be achieved in computer application, only limited considerations have been given to expand these methods to approximate analysis of inelasticity effects. The aim of this report is to explore the most promising approaches for a rapid development of approximate methods. A survey of the field in broad divisions is made describing what is available in terms of the assumptions used in establishing the design conditions. Piping-oriented design procedures based on several discrete steps, each of which may employ different analysis methods, are discussed; but the review is preliminary and is only intended to point out the areas which should be investigated. Therefore a more careful in-depth review is required to assess the inelastic techniques available and their suitability for pipe system analysis.

Essentially then, this report describes the contents of a more detailed review wherein the theories and computer codes would be compared, concentrating on the assessment of piping system analysis methods in four areas:

1. Review of elastic-analysis techniques currently used, comparing the flexibility matrix approach that assumes beam-like behavior of piping segments with the complete shell-like formulations using isoparametric elements.

2. Discussion of inelastic analysis of creeping structures with the comparison of the initial stress, initial strain, and tangent-modulus methods.

3. Review of elasto-plastic creep analysis of piping components and entire piping systems with discussion of approximate methods used and proposed.

4. Discussion of special problems in piping analysis posed by the elbows (such as coupling effects of internal pressure, ovality, thickness variation, and boundary stiffness).

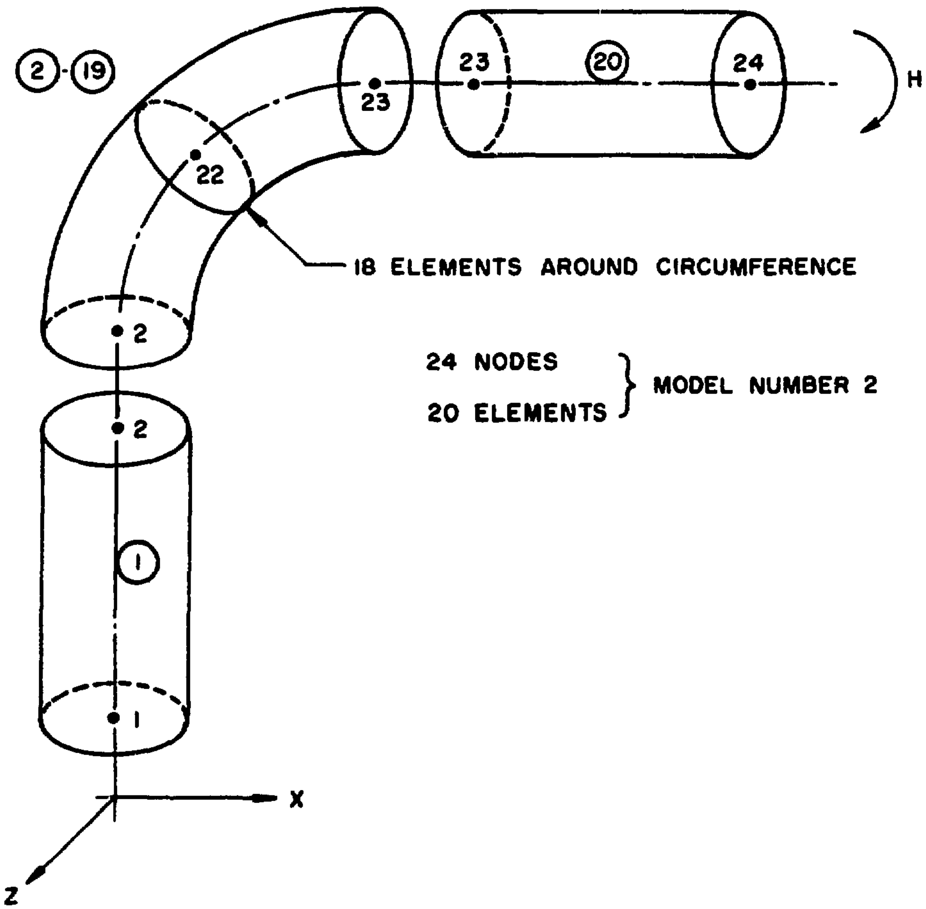
In conclusion, the outline of a series of tasks is assembled that are judged necessary to improve the state-of-the-art of piping-system analysis to a level comparable to vessel-type structures. Additionally, the effects of assumptions, approximations, errors, etc. on calculated quantities (stresses, strains, safe life) will be quantified by using typical LMFBR piping systems as benchmark problems. Because this report is concerned primarily with analysis, the techniques of fracture mechanics and fracture failure criteria are not discussed in detail, although it is an important area to be investigated (particularly for the evaluation of the "leak before failure" safety criteria for piping systems).

2. LINEAR ELASTIC ANALYSIS METHODS FOR PIPING ANALYSIS

Calculations of stresses in piping systems has been the subject of considerable attention in the past. It is considered a routine problem as there are several approaches available using graphical, manual, and analytical techniques to evaluate the linear elastic behavior of a piping system. The basic methods available for such an analysis will be discussed here.

Probably the most well known analysis technique is the so-called "matrix method" derived from classical solutions[5] for indeterminant space frames. The solution methods are based on the formation of the flexibility or stiffness matrices. Either approach is eminently suitable for computer applications and several computer programs that efficiently provide flexibility analysis for various types of piping systems are readily available. Reviews of these methods are given in Refs. 6, 7, and 8, the latter presenting extensive comments on nuclear piping applications. Matrix solution techniques applied to flexibility analyses are similar to the developments of matrix formulation for continuum mechanics problems. The application of these approaches to complex structures has reached a degree of sophistication culminating in highly versatile methods of analysis known as the "finite element methods."

In the finite element method a complex structure is replaced by an array of basic shapes, and in principle, any of the larger finite element systems such as EPACA, MARC, STAR, ANSYS, NASTRAN, etc., (see Ref. 9 or Ref. 10 for a recent summary) which use doubly curved thin-shell finite elements could deal with virtually any piping network. Such approaches have been used to analyze simple pipe systems and components in Refs. 3, 12, and



REDUCED SHELL FINITE ELEMENT IDEALIZATION OF A PIPING ELBOW WITH TANGENT STRAIGHTS

Fig. 1

17. This complete shell finite-element method is widely accepted in pressure vessel evaluation either for the analysis of details such as nozzles or for entire shell structures. For piping analysis, however, the large number of components prohibits its efficient use. In order to overcome this difficulty, a simple finite element model has been suggested called the "reduced shell finite element method."[2,11] This has been implemented in the MARC program[3,13]; a typical model from Ref. 13 is shown in Figure 1. This is a promising approach as the "reduced shell finite element method" enjoys the advantages of the "complete shell finite element method" and the simple "flexibility (matrix) method." However, it suffers from a disadvantage due to the incompatibility of constant bending segments and the resultant exclusion of out-of-plane curvature change effects.

2.1 A Comparison of Assumptions

The pipe stress problem is particularly suited to computer solution, and the scope of such present day computer programs described above is large. However, the analytical techniques on which they are based are all approximate to a greater or lesser extent.

The "flexibility (matrix) method" is founded on the primary assumption that plane cross-sections remain plain after bending; the implication being that a straight pipe can be treated using elementary beam theory and for this reason it is often called a "beam-type" method. In the method curved pipe elements are represented by equivalent straight pipe segments modified by suitable "flexibility factors." That is, if the flexibility matrices are derived from energy considerations (e.g., Ref. 6), the strain (or complementary) energy of a curved pipe under applied loading is equivalent to that of a similar straight pipe under the same loadings multiplied by appropriate "flexibility factors" which, by definition, shall depend on material as well as geometric parameters.

The "complete shell finite element method" utilizes either an assemblage of small flat-plate elements with sufficient degress of freedom to adequately describe the deformation of curved three-dimensional "isoparametric" shell elements in particular, double-curved elements for bends and single curved "shell of revolution" type elements for straights. This is the most versatile method and since it contains no assumptions other than those made in the usual finite element derivations, it provides the best mathematical, and usually the best geometric representation of pipe systems.

The "reduced shell finite element method" uses either a number of three-dimensional simple beam elements or a shell of revolution element together with a number of beam elements to model straight sections of pipe. Such elements embody a "plane section remains plane" type assumption similar to the "flexibility method." Bends are treated as a number of so-called "Von Karman Pipe Bend Elements" [2] which are basically "shell of revolution" elements with extra degrees of freedom to cope with in-plane or out-of-plane bending, tied together with simple beam elements. The pipe-bend element is a pure-bending element, that is a constant bending moment is assumed to act circumferentially over the element. This assumption also is similar to the implicit pure-bending assumption of "flexibility methods." The "flexibility method" however can allow for warping, pressure, etc. by suitable redefinition of the flexibility factors; whereas the current "reduced shell finite element method" needs further development in this area as discussed by Hibbit and Pan and Jetter.[2,3]

2.2 A Comparison of Elastic Analysis Methods

The basic criterion for the "best" method of elastic pipework analysis is that it should be versatile and simple to use at a reasonable computer cost. The "flexibility method" amply satisfies this criterion in being able to cope with a variety of loading conditions (pressure, deadweight, thermal, etc.) and with any component geometry such as elbows, reducers, and tee joints. Through the use of appropriate "flexibility" factors, it may also be extended to deal with problems of stability.[18] The most serious disadvantage of the "flexibility method" is the need for auxiliary calculations for the detailed stress evaluations of the components, particularly those represented by modified flexibility factors. For complex components, the generation of these flexibility factors may require separate complete shell finite element analysis which pose problems in matching boundary conditions. However it should be noted that for the most important geometry and loading conditions, the flexibility factors are mostly available.

The application of finite element techniques to pipework is still in the development stage. In particular, the complete shell finite element method, although by far the best and the most versatile system model, is still difficult and too costly to use in spite of large efforts in progress to improve and make this method and make it generally available.[17] The systems which may be analyzed at the present are limited to relatively simple configurations but in all fairness, this is a reflection on the current digital-computer cost rather than on the method itself.

The reduced shell finite element method represents an excellent combination of the best characteristics of the flexibility and the complete shell analysis methods. It requires further development work, however, to eliminate the theoretical objections and to reduce some of the computer costs as discussed previously (see Ref. 3).

In conclusion, the best elastic analysis method at this time is the flexibility analysis, particularly when all flexibility factors are available. However, for some cases analyzed by this method, it will be necessary to apply additional complete shell methods (using the boundary forces calculated from flexibility methods as impact) to calculate stresses and strains where local effects are expected to be excessive, (e.g., in bends, tees, or anchor intersections, etc.). The foregoing discussions are summarized schematically in Table I for easy reference.

Table I

Matrix (Beam Analysis Method)

Manual-Graphical Methods

Refs. (5) and (7)

Flexibility or Stiffness Matrix Methods

Refs. (5)-(8), (12), (69), (70)

Advantages	Disadvantages
Versatile Inexpensive	Requires Flexibility Factor and Stress Indices

Finite Element Analysis

Complete Shell Element

Refs. (9)-(11), (17), (2), (36), (42), and others

Advantages	Disadvantages
General but costly	System is size limited by computer storage and running time

Reduced Shell Element Analysis

Refs. (11), (14a), and others

Advantages	Disadvantages
General but fairly costly	System size limited, need more development for bend elements

3. INELASTIC ANALYSIS TECHNIQUES FOR STRUCTURES

This section is concerned with the computational techniques required if inelastic behavior is introduced into structural calculations. The majority of recent reviews (e.g., Refs. 41-46) tend to be repetitive and restrictive in their treatment of the problem in as much as they discuss only the finite element methods.

3.1 Basic Algorithms for Plasticity and Creep

Creep plasticity calculations have been carried out on specific components for many years and a number of papers have been written on the use of numerical techniques for obtaining solutions to complex problems. Three basic algorithms have evolved for plastic analysis:

(a) The "initial strain method" developed by A. A. Illyushin [19-20];

(b) the "initial stress method" by O. C. Zienkiewicz [21];

(c) the "tangent modulus method" by P. V. Marcal [22]; and others.

All of these methods are basically "incremental procedures" in the sense that an elastic problem is solved, then inelasticity is incorporated by incrementing some quantities (i.e., developing an initial strain, an initial stress, or alternative "elastic material parameters," respectively), and then recalculating the modified elastic problem.

The "initial strain" algorithm was adopted for creep analysis by A. Mendelson [24] and P. S. Kuratov and V. I. Rozenblium. [25] It is fundamentally also an incremental procedure, similar to that used for plastic behavior, repeated over successive time steps. A different algorithm was suggested by H. Poritsky and F. A. Fend [23] (see also Ref. 28 which is simpler to use). At the beginning of a time interval, the values of all fundamental quantities are known and their rates (in time) are found on solving an equivalent elastic treating creep-strain as an initial-strain, i.e., the values of the fundamental quantities at the end of the interval are found assuming constant creep-rate over the time interval. Various improvements to these algorithms for particular application to finite-element methods have been suggested, notably by Z. P. Bazant[26] and W. C. Carpenter and P.A.T. Gill.[27] The "initial-strain" type algorithms (also called "exact methods"[28,29]) cope successfully, on the whole, with time-dependent structural behavior at the expense of increased computing time.

Alternatively, there are the approximate methods based on equation-of-state such as superposition of states proposed for forward-creep under constant prescribed boundary forces by D. L. Marriott and F. A. Leckie,[30] but found applicable also in the relaxation situation where constant boundary displacements are prescribed by J. Spence and J. Hult.[31] The method can be extended also to time-dependent boundary conditions (to be published by J. Spence). Attempts to apply classical energy methods of structural mechanics to the creep situation have resulted in the direct (Galerkin) methods of L. M. Kachanov[34,35,57] and the energy deformation bounding methods of A. R. S. Ponder, J. B. Martin, and F. A. Leckie (see Refs. 15 and 38 for recent review).

It is worth noting that the "initial strain" algorithms have been extended to incorporate more general material behavior, viz. viscoplasticity by O. C. Zienciewicz and I. C. Cormeau.[39] Other fundamental approaches to the solution problems in inelastic material behavior are currently being formulated by the authors (to be published by Boyle) such that the preceding "algorithms" are systematically introduced as special cases of a more general treatment.

3.2 Comparison of Inelastic Techniques: Advantages and Disadvantages

The stability and convergence of these inelastic methods is often still an open question. In the case of time-dependent plastic behavior it has been established through experience with practical problems that the "tangent modulus" method has better convergence than "initial stress," which in turn exhibits better convergence than "initial strain" although it depends somewhat on the nature of the problem.[41,42,46] However, the "tangent modulus" is more difficult and time-consuming to compute, while "initial stress" is applicable to only certain constitutive laws (that is also true to some extent of "tangent modulus"). Some information on convergence properties (and a description of alternative methods) is available in Ref. 95.

The case of creep deformation is more complex since time is the important parameter. A rough summary of the general situation is provided in Table II for the more important creep-analysis method based on the "initial strain" algorithm. It is supposed that the active variables occupy N storage locations and that for each time-step a number of equivalent elastic problems should be solved. The table indicates the order of the error and the equivalence of the methods to the classical techniques for initial-value problems. While, the best method will be dependent on the problem, it is fairly obvious that the improved (Runge-kutta-Gill) algorithm is in general the most efficient. (There are other similar methods, see Ref. 97.) Although the "initial strain" algorithm is the easiest to apply (and most widely used in finite element creep codes), it has few advantages. Indeed it may exhibit numerical instability at large times due to the growth of parasitic errors (primarily

Table II

EFFICIENCY OF CREEP ALGORITHMS

Algorithm	Method	Number of storage locations required	Number of equiv. elastic problems	Order of error	Ultimate stability and convergence
Algorithm 1 (23) (30)	Euler	2 x N	1	$(\Delta t)^2$	Poor
Algorithm 2 (24) (29)	Incremental Picard Iteration	2 or 3 x N	I	$\frac{1}{2}(\Delta t)^{I+1}$	Bad
Improved algorithm (27)	Classical Runge-Kutta 4	5 x N	4	$(\Delta t)^5$	Fair
Improved algorithm (60),(96)	Runge-Kutta-Gill	3 x N	4	$(\Delta t)^5$	Good

N = number of storage locations for basic variables

Δt = magnitude of time step

I = number of iterations (usually one)

because of the accumulation of rounding errors, etc., due to the large number of small time-increments which must be used with the current methods of varying the time steps), even though the methods are stable in the strict sense. This is a problem which the "initial stress" algorithm (also widely used) multiplies as an example of the potential instability given by Rashid in Ref. 36. Some of the stability criteria (see Ref. 40) overlook this behavior. The Runge-Kutta-Gill procedure minimizes parasitic error growth and, furthermore, it is just as easy to use with a variable time stepsize. However, the best method for a particular class of problems such as piping geometries and LMFBR loading conditions still needed to be established.

Superposition of states in its simplest form for constant applied conditions assumes that the total deformation of a structure can be approximated by a pure (equivalent) elastic part found by ignoring creep, added to a pure creep part found by ignoring elasticity. It is found to be fairly accurate for simple structures.[28,30,31] Its usefulness and accuracy when applied to common piping components will be discussed in Sections 4.2 and 4.4.

Direct (Galerkin) type methods approximate some quantity of interest, say the stress, by a finite sum of chosen space functions multiplied by coefficients which are a function of time and are to be determined. In fact, this is the embodiment of finite element creep analysis! There exist simplified methods (primarily due to L. M. Kachanov[34]) which assume simple forms for the time coefficients. Such simplifications are successful if a good a priori knowledge of the time behavior is available. Again particular applications to piping analysis will be discussed later.

Finally the bounding methods of F. A. Leckie *et al.* may be used to estimate conservative (often over-conservative) bounds on the deformations or energy of a creeping structure using known solutions. They are unfortunately not in a form which is suitable for general piping system analysis (where relaxation is prevalent) but may be used with success for particular components.

A schematic overview of the preceding discussions of the current inelastic techniques is summarized in Table III.

Table III

Elasto-Plastic Analysis

1. Initial Strain
2. Initial Stress
3. Tangent Modulus

Refs. (19), (20), (21), (22), (42)-(47)

Elastic-Creep Analysis

Algorithm 1
Algorithm 2
Improved Algorithm

Refs. (23)-(28)

Elastic-Plastic Creep Analysis

Exact Methods	Approximate Methods
Viscoplasticity	Superposition of State Direct Solution Bounding Methods
Refs. (36), (39), (40), (45), (47)	Refs. (30)-(31), (34)-(35), (38)

4. APPLICATION OF INELASTIC ANALYSIS METHODS TO PIPING COMPONENTS AND SYSTEMS

The technique for elasto-plastic creep analysis of piping systems are available, but specific applications have not yet appeared. Although there are a number of papers on the plastic analysis of piping components, particularly elbows, there has not been much attention paid to the plastic behavior of complete systems. The pipe bend in pure bending has been considered by P. V. Marcal(53) and J. A. Blomfield(54) principally to describe the progression of the plastic region and deal with the nature of limit loads. Bends with attached straights were examined for plastic collapse by R. M. Mello and V. S. Griffin(13) using the "reduced shell finite element method" with good results, and by A. Hoffman et al.using the complete shell finite element method.(98) A different approach to the limit analysis of bends utilizing elastic and steady creep solutions was taken by J. Spence and G. E. Findlay.(55) A short discussion by C. R. Calladine(56) points out the usefulness of the reference stress approach linking the plastic collapse load to the steady-creep deformation of simple structures. These analyses have not been applied to piping analysis but nevertheless are useful in the study of particular components.

4.1 Creep Applications for Straight and Curved Pipe

The straight pipe as a cylindrical shell has been treated by many investigators for both creep and plasticity with a variety of loadings [an overview can be gained from the review of W. Olszak and A. Sawczuk(93)] who recently used the complete shell finite element method with full-time dependent and stationary-creep analysis. However, in piping analysis, the straight sections are treated as long beams, and the inelastic method usually becomes particularly simple. The problem will be dealt with in detail in later sections.

The flexibility in a piping system, however, is usually concentrated at the curved pipe sections. The steady (stationary or "pure") creep of bends, neglecting structural stress redistribution, was first explored by L. M. Kachanov[34, 57]. Independent and more detailed analyses were performed by J. Spence in a series of publications[58-60] that showed the potential for greatly increased flexibility. Recently the combined elastic-creep behavior of a curved pipe, with structural stress redistribution included, has become available. This analysis, using the improved time integration algorithms (carried out by Spence but still unpublished), has treated two distinct cases--forward creep under constant applied bending moment (the results bearing out those of Spence when the steady state is achieved), and relaxation under constant prescribed end rotation. Some results from these analyses are shown in Figures 2 and 3. Similar results were generated for pipe runs in Refs. 11 and 13.

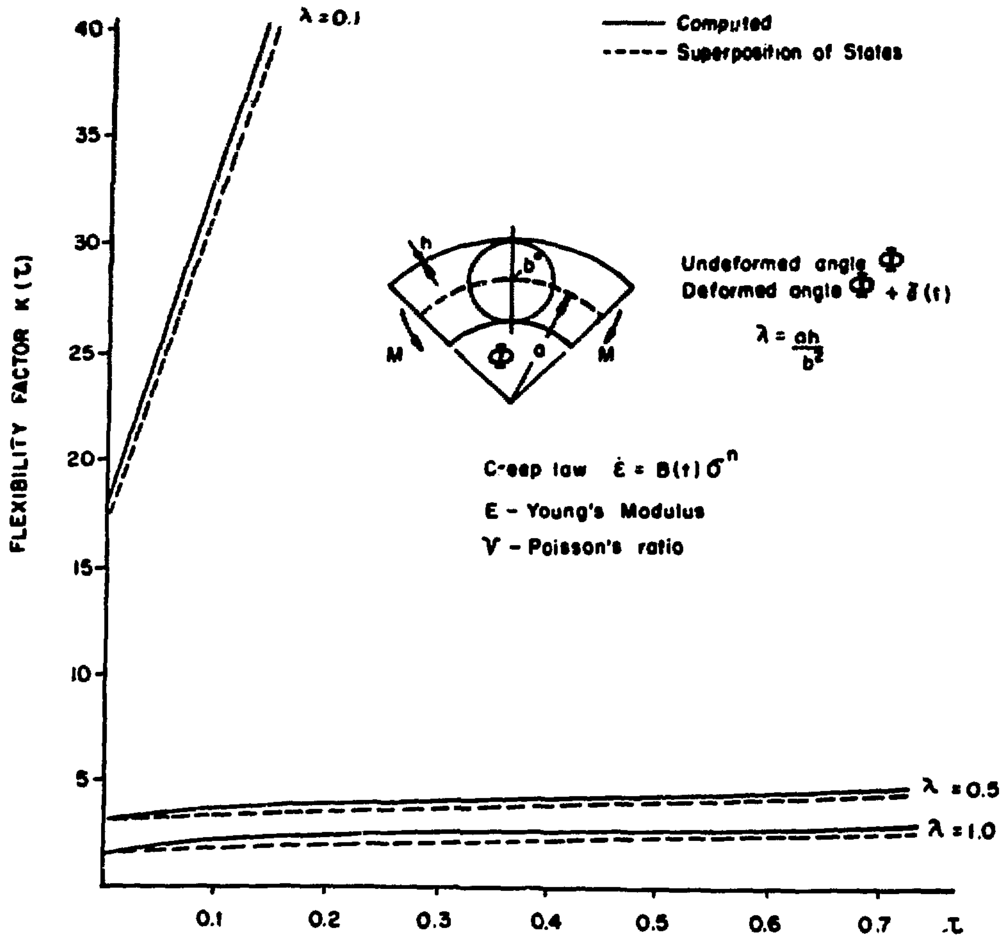
4.2 Simplified Piping System Analysis Using Superposition of States

The early analysis of E. L. Robinson[49] of simple systems unwittingly used superposition of states and included only straight pipes. Despite this, it brought out several points which were important for future developments. W. Gorczynski[50] followed the same approach. Later J. Spence[51] included the effect of bends through the use of creep flexibility factors.[61] A similar though more general method was presented by E. C. Rodabaugh and G. H. Workman.[52] In order to render the problem practical using information then available, the authors decoupled the effects of combined loading. For example, it was assumed that the total

CREEP OF A CURVED PIPE UNDER CONSTANT APPLIED IN-PLANE BENDING MOMENT M. COMPARISON WITH SUPERPOSITION OF STATES

$$K(\tau) = \frac{\text{end rotation } \vartheta(t) \text{ of curved pipe under } M}{\text{end rotation of similar elastic straight under } M}$$

M = applied in-plane bending moment

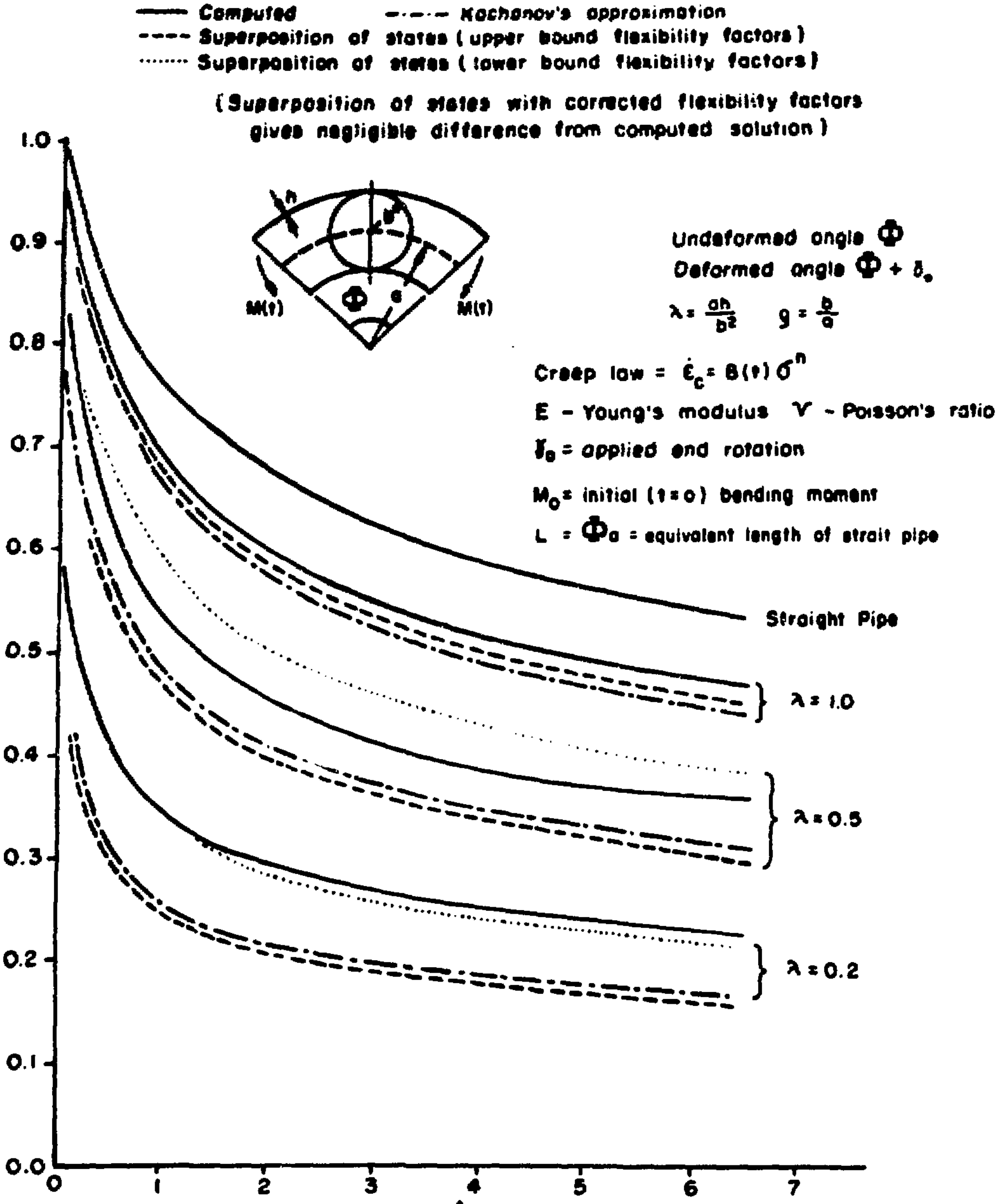


t = time, $\tau = E\sigma_0^{\,n-1}\int_0^t E(t)\,dt$, $\sigma_0 = \frac{M}{\pi h b^2}$

Case $n = 3$, $\nu = 0.33$ $e = 0.1$

Fig. 2

RELAXATION OF BENDING MOMENT M(t) IN A CURVED PIPE WITH CONSTANT APPLIED END ROTATION · COMPARISON OF APPROXIMATE METHODS



t = time, $\tau = E\sigma_o^{n-1}\int_0^t B(t)\,dt$ $\sigma_o = \frac{M_o}{\pi h b^2}$

Case n = 5, ν = 0.33 ρ = 0.1

Fig. 3

creep strain was the algebraic sum of the creep strains due to the individual loads taken separately. They also used the same flexibility factors to describe out-of-plane as in-plane bending on the curved sections. While these assumptions are acceptable in elastic calculations they may seriously underestimate the creep deformation. Some of the problems are discussed in Ref. 92. Finally, Kachanaov's simple method[34] for relaxation has been used on some simple configurations in relaxation[47,48] and particularly tc assess creep damage through rupture. The basis of the method is that all stresses in the system will exhibit relaxation in the same manner being merely of the same shape as the initial elastic stress field multiplied by a function of time. For this reason, it is unsuitable for the analysis of more complex piping systems.

4.3 Piping System Analysis Using Simplified Finite Element Method

The accurate analysis of a complete pipework system in creep using the "complete shell finite element method" is not feasible at the present as discussed earlier. Consequently several investigators have turned to the "reduced shell finite element method" for analysi of pipe runs. In particular, the Marc Code[2] has been used by Pan and Jetter[3] for cyclic creep analysis of an FFTF system. Although such an approach is fairly accurate and versatile, it still suffers from the serious disadvantage of excessive computer time and from incompatibility problems associated with the pipe bend element, particularly the general loading cases (see discussions in INTRODUCTION, pp. 3 to 6). An alternate to simple finite element type analysis has been suggested by G. H. Workman and E. C. Rodabaugh[14a] whose intent was to account for the inelastic behavior in an "initial strain" sense by solving a succession of elastic flexibility problems. This method forms the basis for a computer program called PIREX2 not yet released (February 1975). The analysis, however, required further clarification as pointed out in Refs. 14a and 14b.

4.4 The Need for a Simplified Analysis

It is obvious from the foregoing brief review that very little quantitative information is provided to aid the designer in the choice of a suitable method for the analysis of piping systems operating at high temperatures. While the use of "complete shell finite element method" is not feasible, the "reduced shell" method is a great improvement but still very costly for large systems analysis. On the other hand, superposition of states allows a simple "flexibility" method that still can account fairly well for the important features of the creep response, but other approximate methods can also be developed. The size of piping configurations alone necessitates a simple technique and appears to eliminate even "reduced shell finite element method" as a practical method for overall analysis. Furthermore, the requirement of evaluation of long operating life of piping systems conclusively points to the need for simplified analysis.

A "flexibility" time method using simple superposition of states is relatively cheap and easy to apply. Although it may not accurately predict the magnitude of field variables, it is nevertheless correctly highlighting the overall trend. In order to appreciate the significance of superposition of states, it is useful to discuss its implication with reference to a simple example: the forward creep, and relaxation of a curved pipe alone. The examples will indicate the differences in procedures.

(i) First consider the case of a curved pipe acted on by a pure in-plane bending moment M applied constantly in time (Figure 2). Let $\gamma(t)$ be the end-rotation of the pipe varying in time due to combined elasticity and creep with stress redistribution. Superposition of states in this case implies that this total end-rotation can be approximated by a pure elastic part $[\gamma_c(t)]$ found by ignoring creep, added to a pure creep part $[\gamma_c(t)]$, found by ignoring elasticity. However, since the applied load is constant, $\gamma_e(t) = \gamma_c(o) = \gamma_o$, the initial (elastic) end-rotation is known. If a Norton-type power law of creep is assumed for simplicity ($\dot{\varepsilon}_c = B(t)\sigma^n$), it is convenient to define an alternative time scale $\tau = E\sigma_o^{n-1}\int_o^t B(t)dt$ where E is Young's modulus and σ_o the maximum fibre stress in an equivalent elastic straight pipe; and define a factor $K(\tau)$ such that $K(\tau) = \gamma(t)/\gamma_{os}$ where γ_{os} is the end-rotation of an equivalent elastic straight pipe. Then superposition of states reduces to

$$K(\tau) \simeq K_o + \tau\dot{K}(\infty)$$

where K_o is identified as the classical elastic flexibility factor and $\dot{K}(\infty)$ is a scalar multiple of the "steady creep" flexibility factor of Ref. 60. Results of this approximation are shown in Figure 2 compared to an exact time integration method on a linear shell model of the curved pipe.

(ii) Secondly, consider a curved pipe constrained to a fixed end-rotation γ_o. Superposition of states implies that the total end-rotation is found by adding a pure elastic part, found by ignoring creep, to a pure creep part, found by ignoring elasticity. With reference to Figure 3 and using a Norton type power law, it may be shown[60] that the pure elastic part is

$$\gamma_e(t) = \frac{M(t)}{\pi h b^2} \frac{K_e}{\rho} \cdot \frac{\Phi}{E}$$

and the pure creep part is

$$\frac{d}{dt} \gamma_c(t) = \left(\frac{M(t)}{\pi h b^2}\right)^n \frac{K_c}{\rho} B(t) \left(\frac{\pi}{D_o}\right)^n \dot{\Phi}$$

where $D_o = 4\int_o^{\pi/2} (\sin\phi)^{1+1/n} d\phi$ and $M(t)$ is the resultant in-plane bending moment. Thus

$$\frac{d\gamma_o}{dt} = \frac{d\gamma_e}{dt} + \frac{d\gamma_c}{dt} \quad .$$

However, $\frac{d}{dt} \gamma_o = 0$ and consequently their results

$$\frac{d}{d\tau} M(\tau) + \alpha^{n-1} M^n = 0 \qquad \text{Eq. (1)}$$

where $\tau = E\sigma_o^{n-1}\int_o^t B(t)dt$, $\sigma_o = M(o)/\pi h b^2$,

$M(\tau) = M(t)/M(o)$.

Initially $M(o) = 1$ and Eq. (1) become

$$M(\tau) = \{1 + (n-1)\alpha^{n-1}\tau\}^{-1/n-1} \qquad \text{Eq. (2)}$$

where

$$\alpha^{n-1} = \frac{K_c}{K_e} \left(\frac{\pi}{D_o}\right)^n$$

K_e being the elastic flexibility factor, K_c the "steady" creep flexibility factor.[60] Results of Eq. (2) compared with an exact analysis using linear shell theory and the improved time integration technique mentioned previously are shown in Figure 3 for upper- and lower-bound creep-flexibility factors. In fact, even if more accurate creep flexibility factors are used, there is no significant difference between Eq. (2) and the exact analysis.

In conclusion, it must be remembered that when the integrity of an important piping system is in question, it would seem dangerous to choose any one method over another in the final analysis--accurate auxiliary checks must also be an important consideration.

4.5 Proposals for a Procedure Based on Current Methods

Bearing the above comments in mind the following ideal situation is visualized:

(i) Analysis of complete systems using a Stage I (global system analysis method) based on a flexibility approach with superposition of state or similar simplified creep theory. Such a method should be quick, cheap, yet retain proficient accuracy.

(ii) On the basis of the results of (i) isolate and analyze the critical locations using a Stage II (partial system analysis) method; namely the "reduced shell finite element method" with exact time integration. Both methods described here are given in Appendix I.

(iii) On the basis of results of (i) and (ii) isolate components of interest and re-analyze using Stage III (component analysis) method. It is possible at this stage to include more detailed material behavior and in-depth evaluation of damage for the possibility of rupture and fracture. Stage III would involve "complete shell finite element" methods. Such an incremental approach would eliminate the need for analysis of whole systems using finite element methods. An inexpensive, quick flexibility method can be used to focus interest on the critical areas.

Although current knowledge is sufficient in principle to cope with the more usual configurations and behavior, the requirements of design of a complete general procedure has yet to be satisfactorily sound. Several facts remain to be clarified.

(a) Suitable elastic- and creep-flexibility factors should be generated to cope with the general case of combined loading and constraints.

(b) A good curved pipe finite element needs to be developed to cope with torsion, etc., and the non-linear effects of internal and external inertia. Some work is in progress.[12]

(c) More accurate methods for exact time integration need to be incorporated into the finite element codes. The suggested design procedure was based on currently developed analysis methods. The situation would be improved if a simple flexibility-type method existed which did not rely on superposition of state.

5. SPECIAL PROBLEMS IN ELASTIC AND INELASTIC PIPING ANALYSIS

The effective use of the proposed primary method requires a number of factors so that complex bend components can be treated using beam analysis. The development of suitable "flexibility factors" is fairly straight-forward and many publications are available which describe the problem adequately. The majority of these publications, however, deal with ideal shapes neglecting load interaction and ovalization, thickness variations, etc. commonly encountered in pipe bends.

5.1 The Effect of Internal Pressure

The effect of internal pressure on the deformation of pipe bend in linearly elastic behavior has been studied by S. H. Crandall and N. C. Dahl[61,62] using shell theory and by E. C. Rodabaugh and H. H. George[63] and others (e.g., Ref. 101) using energy methods. The flexibility analysis which these authors produced were improved by J. A. Blomfield and C. E. Turner[64] and W. G. Dodge and S. E. Moore[65].

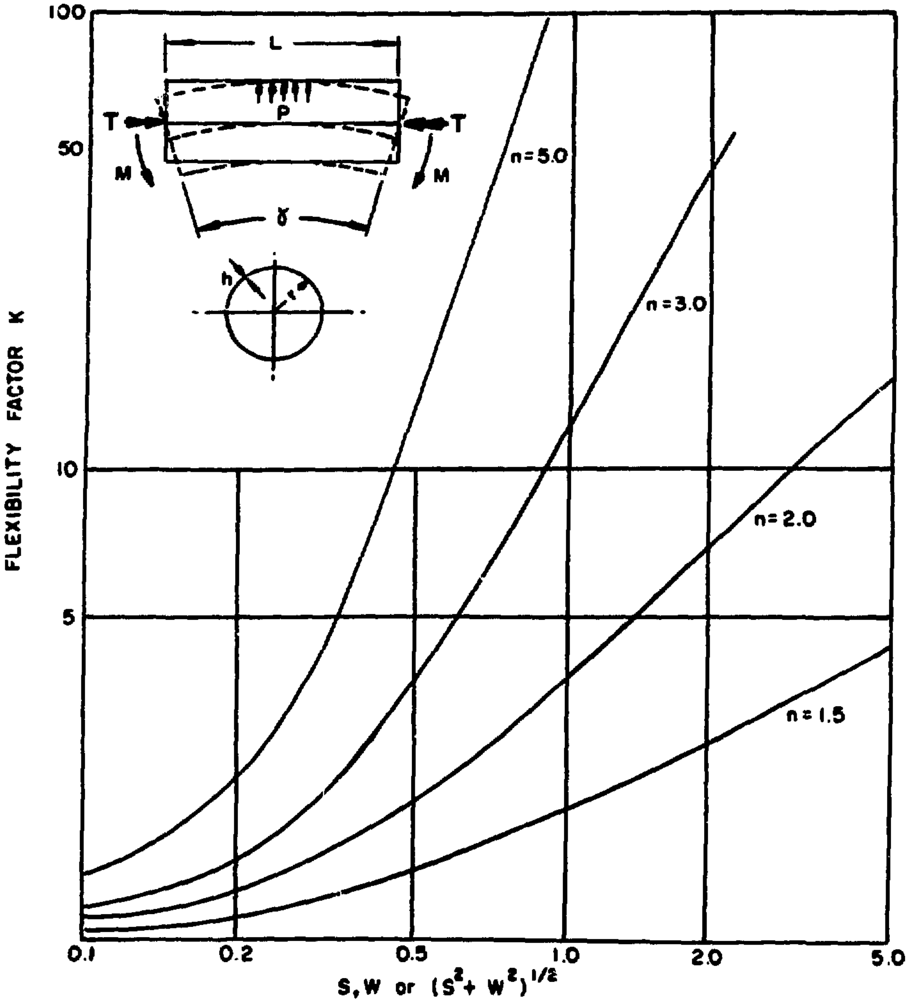
In steady creep it has been shown by J. Spence[66,67] that internal pressure has a potentially large effect on the flexibility of a straight pipe; flexibility factors were produced to account for this effect (some results are shown in Figure 4). Structural stress redistribution was investigated by Boyle and Spence (not yet published) for the relaxation situation; a typical example is given in Figure 5. The results indicate that constant pressure speeds up the redistribution of stresses and the accumulation of strain.

The effect of constant internal pressure on the creep behavior of curved pipe has not yet been treated; some information is available for a simplified situation. Finally, another loading case is the external pressure; this problem does not seem to have been documented in any detail.

INCREASE INFLEXIBILITY OF A STRAIGHT PIPE IN STEADY CREEP UNDER CONSTANT COMBINED LOADING

$$K = \frac{\text{end rotation } \delta \text{ of pipe under combined loading}}{\text{end rotation of pipe under bending alone}}$$

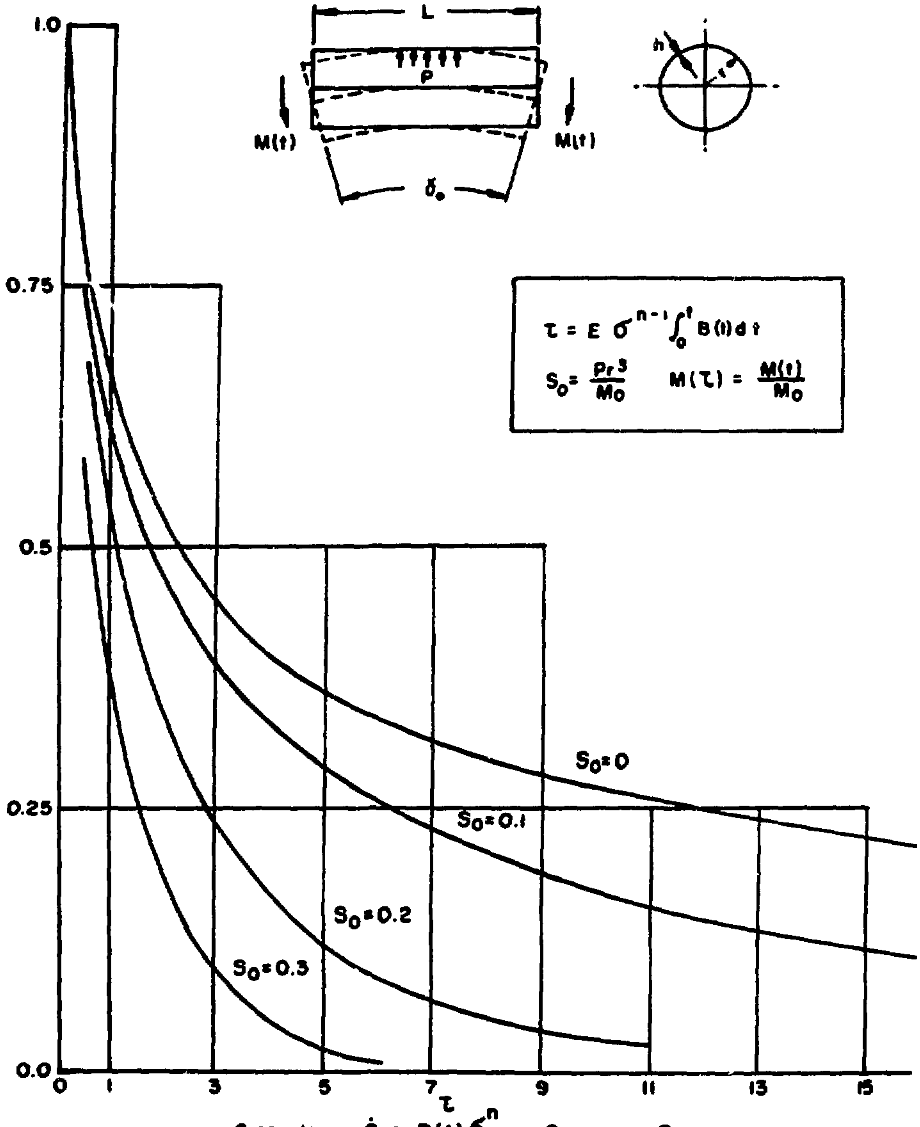
Creep law: $\dot{\varepsilon}_c = B\,\sigma^n$



$S = \frac{Pr^3}{M}$ $W = \frac{T}{\pi M}$

Fig. 4

RELAXATION OF BENDING MOMENT M(t) IN A STRAIGHT PIPE UNDER INTERNAL PRESSURE P, WITH CONSTANT APPLIED CURVATURE δ_0/L



Creep law : $\dot{\varepsilon}_c = B(t)\sigma^n$: Case $n = 3$

t = time M_0 = initial (t = o) bending moment

$$\sigma_0 = \frac{M_0}{\pi h r^2}$$

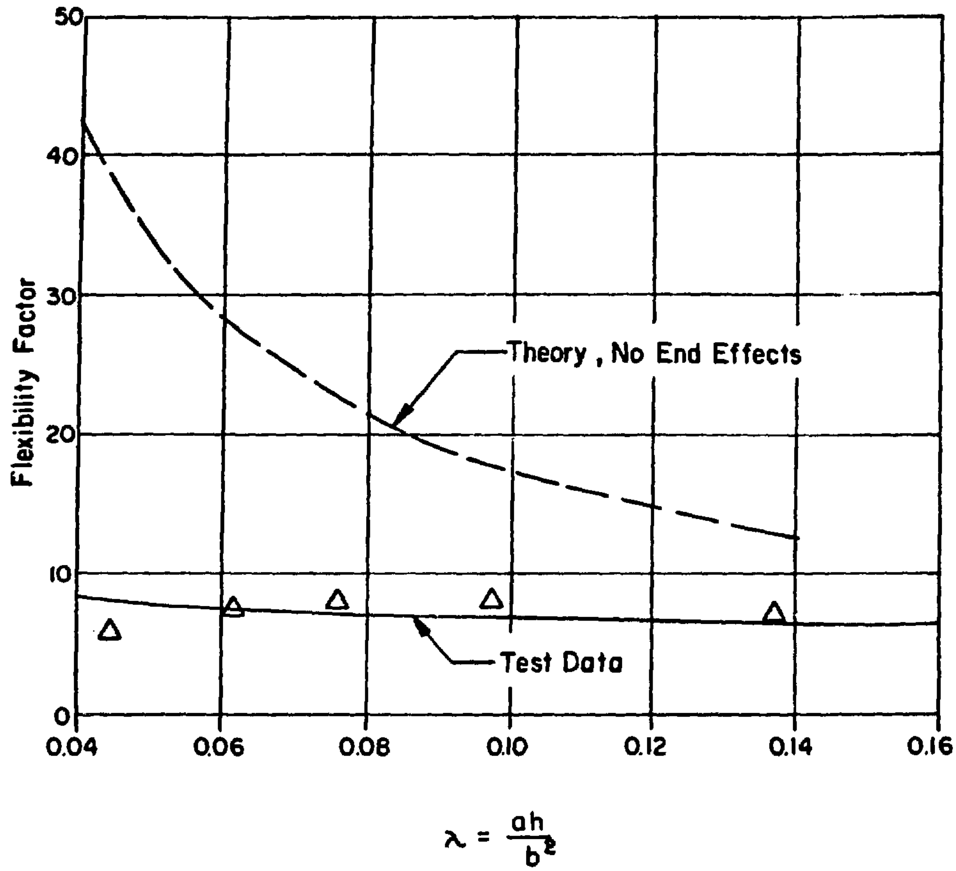
Fig. 5

5.2 Out of Roundness of Bends

Curved pipes are produced to a certain manufacturing tolerance so that the cross sections were not perfectly circular, tending towards ellipticity. Refined flexibility factors to account for noncircularity were produced by R. A. Clark et al.[69] using shell theory and by G. E. Findlay and J. Spence[70] using energy methods. These analyses were restricted to elastic behavior. However, J. Spence[71,72] extended his studies of creep analysis[60] to account for out-of-roundness. A commonly used method in the manufacture of bends also can result in nonconstant thickness. Some unpublished work indicates the thickness variation is less important than the cross-sectional shape. No results are available on the interactive effects with internal pressure.

5.3 End Constraints on Bends

The individual components of piping systems have to be attached to each other in some way. In particular, pipe bends are often linked to straight sections by straight tangent pipes or rigid flanges and it is to be expected that these should reduce flexibility. Reviews of these conditions have been presented in Ref. 12, and variations in the flexibility factors for elasticity have been given in Refs. 74, 12, and others.[99] Some information is also available in Refs. 75 to 77. although design factors were not taken into account. To illustrate the magnitude of the end effects, Figure 6 shows theoretical and test results for a 90° bend from Ref. 12. Figure 7 indicates the dramatic effect of rigid flanges on the flexibility of various bends. The influence of flanges on the creep behavior has not yet been considered.



END EFFECTS PRODUCED BY RIGID FLANGE ON BOTH ENDS OF 90° ELBOWS

Figure 6

THEORETICAL FLEXIBILITY FACTORS FOR IN-PLANE BENDING OF AN ELASTIC CURVED PIPE WITH ATTACHED FLANGES

$$K_{\Phi} = \frac{\text{end rotation } \gamma \text{ of curved pipe under M}}{\text{end rotation of similar straight under M}}$$

M = applied in-plane moment

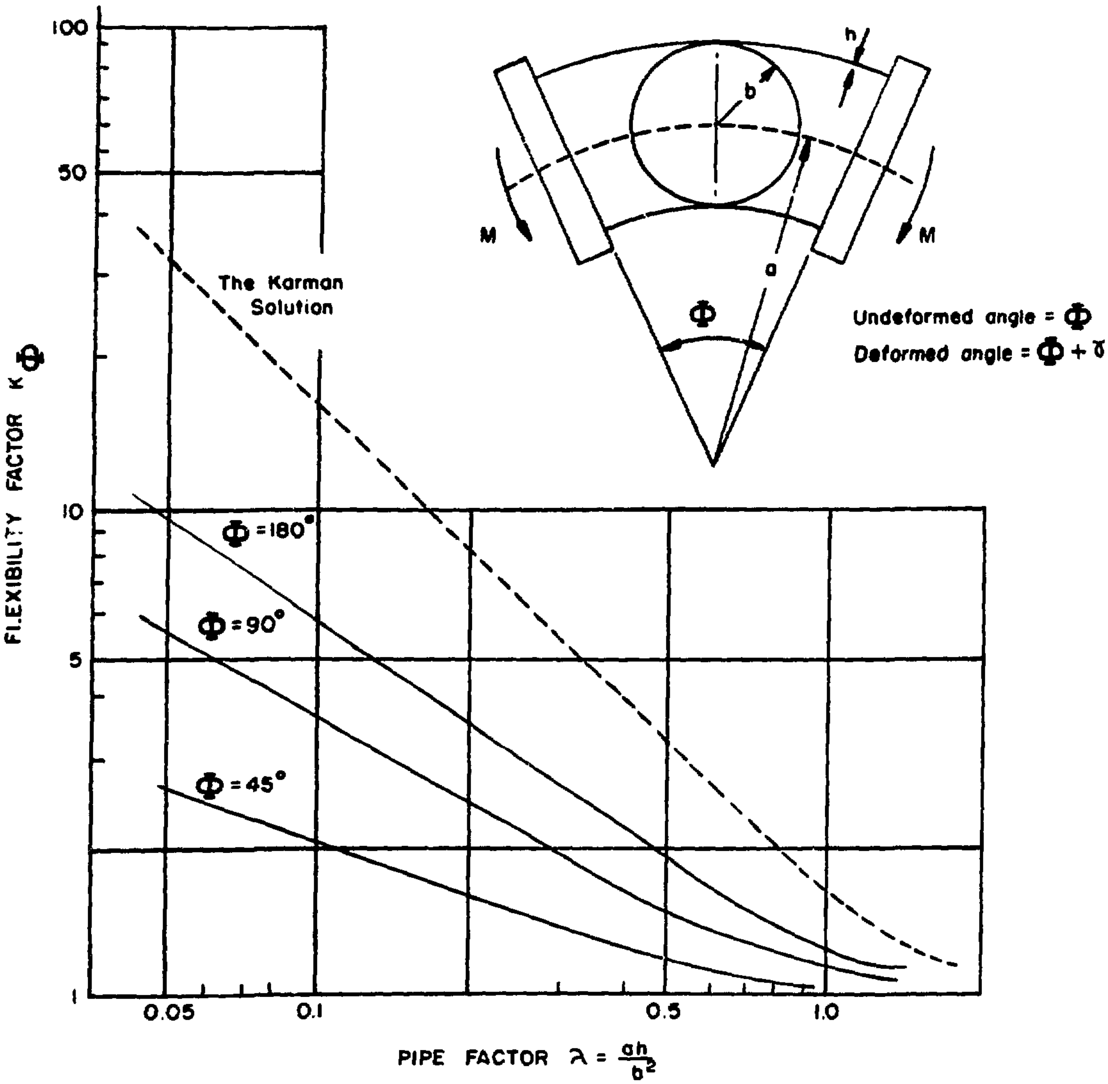


Fig. 7

5.4 Coupled Effects

Most of the work available consider pure in-plane bending, although some of the authors (Spence and Boyle) are involved in some work for out-of-plane in creep. Also, combined effects of pressure and noncircular cross-section still have need to be determined. Other topics have not been developed in detail; in particular, the effect of out-of-plane bending and torsion, common in piping, is a matter open to question.[77,78]

5.5 Additional Information Required for Complex Loading History and Cyclic Material Behavior

Virtually all of the current investigations into the time-dependent inelastic behavior of structures assume a particularly simple form of constitutive model--usually either "time-hardening" or "strain-hardening" assumptions. A number of large finite element codes do not use any definite law in favor of using directly tabulated data from constant stress uniaxial tests, and interpolating[46] from "isochronous" curves. For more complex loading histories--for example, cyclic loading--such models are insufficient and do not even predict the correct response. The point is discussed well by E. Krempl.[84] Nevertheless some analyses do exist with the particular application to piping components which cover (cyclic) ratchetting (either thermal, load, or combined plastic/creep).[3,14a,86,89,94]

For the general problem of time-dependent boundary conditions combined with complex material behavior, e.g., creep, plasticity, visco-plasticity, fatique, rupture, etc., under variable environmental conditions, a more suitable constitutive theory is needed. The work of Krempl,[32] Pugh et. al.,[33] and Rashid[36] give a good overview of the theoretical and computational problems. Corum et al.[37] give guidance for practical solutions of the numerical problems.

Many of the mathematical models which describe the aging of a material with memory are too complicated to be used in complex structural calculations. A review of such models can be found.(35,83,86) What is required is a constitutive theory with sufficient freedom to describe the material response but which is also fairly simple in its application to piping analysis. Such a model may be that of "internal state variables" but detailed discussions of this problem are outside of the scope of this brief review.

It is to be noted that the combined effects of creep and plasticity have not been discussed. The subject is very much open to question and there are differences in opinion as to how it should be handled. Corum et al.(37) in discussing combined effects of plasticity and creep state:

> "Prior creep deformations influence subsequent plastic behavior, and prior plastic deformations influence subsequent creep behavior. The extent of these interactions is discussed here on the basis of a very few test results for type 304 stainless steel.
>
> "Very little conclusive information is available regarding the influence of prior creep strain accumulation on subsequent cyclic plasticity. One reason for this is that fully annealed stainless steels often have a low yield strength relative to their creep resistance. Thus it is difficult to introduce significant creep strain without first accumulating relatively large plastic strains, especially at temperatures of 1100°F and below.

They then conclude:

> "It is generally observed that large prior plastic strains of several percent in the form of cold work can improve the creep resistance of stainless steels in the temperature range of 1000 to 1300°F. The optimum improvement depends on the specific alloy, the creep stress, and the creep temperature. Likewise, data indicate that <u>large</u> plastic strains incurred at elevated temperatures can

reduce subsequent creep rates. Relaxation tests performed after strain cycling over large strain ranges show this effect.

"It is not necessarily true, however, that small plastic strains should produce similar creep-hardening effects; in fact, the data that exist suggest that perhaps the opposite is true. That is, small plastic strains slightly accelerate creep.

"Thus, in conclusion, it would seem that for the relatively small plastic strains that will be induced in an actual component by the mechanical and thermal loadings, little effect on the creep rate should be anticipated. The inelastic analysis guidelines of Chapter 3 recommend that the effects of prior creep on subsequent plasticity be approximately accounted for, [by using the 10th cycle cyclic strain-strain curve for the instantaneous plastic response (see p. 70 (37))] but that the effects of prior plasticity on subsequent creep be ignored. Needless to say, much more experimental work is needed in this area to yield quantitative relationships."

In our opinion (i.e., the authors of this present paper), if instantaneous plasticity is taken into account only in the initial behavior (before creep has started), the problem is relatively straightforward and proceeds as mentioned in the previous discussions from this initial elastic-plastic state. Difficulties occur when the combined effects are expected to occur throughout the deformation history. However, the incorporation of the combined response into existing finite element codes is relatively straightforward. Usually the inelastic behavior is separated into an instantaneous plastic part and a time-dependent creep part,[41,42,46, or 100] and the computations proceed incrementally allowing creep and instantaneous plasticity to take place alternately in steps of time and load. On the other hand, better viscoplastic models may also be utilized.[39] Similarly, the technique of superposition of state can be readily extended to such a combined response. However, since the superposition concept has only recently been identified in its present form, further investigations are needed to assess its acceptibility. It is expected that the bounding theorms of S. A. Leckie _et al._ can be used to determine its reliability in specific applications.

It should be pointed out, however, that is not the applicability of any of the above techniques of analysis which is in doubt, but the material model itself. Indeed the separation of the inelastic response into an instantaneous plastic part and a time-dependent creep part may not be acceptable. A more suitable constitutive theory may be required. It is sufficient to say that without further detailed investigations little more can be said definitely about combined effects at present.

6. CONCLUSIONS

The analysis of complex piping systems operating at high temperature in the creep regime is not well developed. This report has attempted a preliminary survey to identify the factors which must be taken into consideration in the stress analysis.

The main conclusion is that further work will be necessary to bring piping design methods and associated computer techniques up to the standards already obtainable in other areas of pressure vessel technology. Particular proposals are made and a possible design procedure is outlined which could be implemented using existing information coupled with some development work.

The current state of development for piping systems analysis makes the design evaluations on the basis of the ASME Code Case 1592 more difficult for two reasons:

(1) The stress and strain quantities necessary for the evaluation of the creep ratchetting and creep fatigue rules are less reliable, and

(2) In a stiff vessel-type structure where the strain accumulation is generally a local phenomenon (e.g., nozzel connections) that does not alter the structural response of distant areas of the vessel.

In contrast, local strain conditions in an interconnected piping system (e.g., at bends) affect the flexibility conditions that influence the overall response of the entire structure. Correspondingly, the influence of the assumed constitutive equations on the computed stress and strain quantities is much greater in piping analysis. A consequence of this is that the assumptions that can be shown to be "conservative" in a vessel analysis can become overly restrictive in piping analysis in some cases or not conservative enough in others. Therefore, piping analysis is more sensitive to material property changes due to cycle-loading conditions and thermal-history effects. As some of these effects may loom large on a theoretical level but may be only a secondary effect in a practical case, these uncertainties need to be evaluated by numerical examples corresponding to geometry configurations, load, and thermal histories representative of LMFBR design conditions. On the basis of such studies, the sensitivity of the results of the various material behavior assumptions can be evaluated and more reliable--possibly simpler--design procedures may be developed. This report could provide a convenient starting place for an in-depth review of these problems.

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Appendix I

OUTLINE OF STAGE I FLEXIBILITY ANALYSIS METHOD

Consider for the sake of clarity a simple two anchor single run. Let one anchor be fixed and let displacements and rotations, denoted by the six-dimensional vector δ_o be given to the other anchor in order to close the loop. Initially the system will deform elastically resulting in restraining forces and moments, denoted by F_o. Using standard elastic flexibility matrix methods the forces and displacements are related by

$$\delta_o = [K]F_o \qquad (1)$$

where [K] is the "flexibility matrix." Thus $F_o = [K]^{-1}\delta_o$. Let the system now expand thermally due to creep such that, with the same applied displacement δ_o, the restraining forces shall vary in time, $F(t)$. Superposition of states here demands that the total displacement is made up of an elastic part, $\delta_e(t)$, ignoring creep, added to a creep part $\delta_c(t)$, ignoring elasticity, such that

$$\delta_o = \delta_e(t) + \delta_c(t) \quad . \qquad (2)$$

In a similar manner to the initial behavior

$$\delta_c(t) = [K]F(t) \qquad (3)$$

and, if a time hardening type creep law is assumed for convenience,

$$\frac{d}{dt}\,\delta_c(t) = k[t,F(t)] \qquad (4)$$

where $k(.,.)$ is a nonlinear vector valued function, derived, for example, from energy.

Using Eqs. (2), (3), and (4) and remembering that $\frac{d}{dt}\,\delta_o = 0$ there results

$$[K]\,\frac{dF}{dt} + k[t,F(t)] = 0 \qquad (5)$$

which is a system of initial value problems with $F(0) = F_o$. This problem may be solved using standard methods.

OUTLINE OF STAGE II METHOD

Consider the application of the "reduced shell finite element method," for example, to the analysis of a loop. Let $\{\sigma\}$ and $\{\varepsilon_c\}$ represent vectors of nodal stresses and inelastic creep strains, respectively, and suppose for convenience that the creep law is of a time-hardening type, i.e.,

$$\frac{d}{dt}\{\varepsilon_c\} = \{N(t,\{\sigma\})\} \tag{6}$$

where $\{N(.,.)\}$ is a vector valued function. Then it can be shown, that the redistribution of stresses is governed by a finite system of ordinary differential equations

$$\frac{d}{dt}\{\sigma\} = [R]\{N(t,\{\sigma\})\} + [P]\frac{d}{dt}\{F(t)\} \tag{7}$$

and the evolution of inelastic strain governed by

$$\frac{d}{dt}\{\varepsilon_c\} = \{N(t,[R]\{\varepsilon_c\} + [P]\{F(t)\})\} \tag{8}$$

where $\{F(t)\}$ is a vector representing applied loadings and boundary conditions, and $[R]$, $[P]$ are matrices which are uniquely defined for a particular system.

The initial value problems, Eqs. (7) and (8), may now be solved using one of the improved algorithms mentioned previously assuming that the initial profiles (initial elastic behavior) are known.

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