AD/RHIC-AP-39

Tracking Considerations

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TRACKING CONSIDERATIONS

1. DO WE INTERPRET OUR INITIAL CONDITIONS CORRECTLY?

CONSIDER A BEAM HAVING AN ELLIPTICAL PROFILE WITH HALF AXES $\hat{X}$ AND $\hat{Y}$.

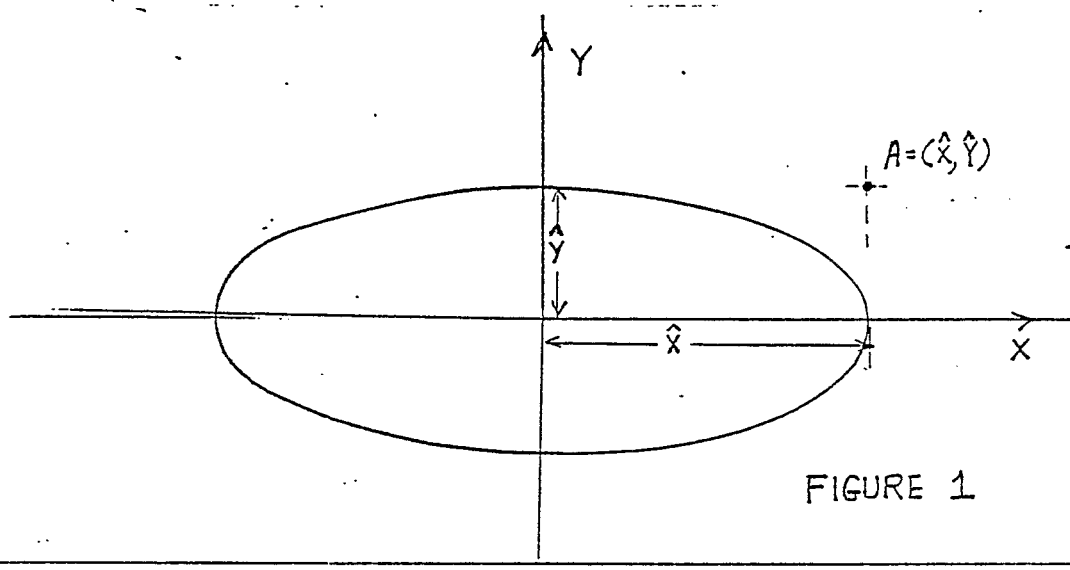

FIGURE 1

WE STATE THAT THE BEAM HAS EQUAL EMITTANCES IN THE X AND Y DIRECTIONS. USING THE NOTATION OF FIGURE 1, $\frac{\epsilon_{x_0}}{\pi} = \frac{\hat{X}^2}{\beta_x} = \frac{\epsilon_0}{\pi} = \frac{\hat{Y}^2}{\beta_y} = \frac{\epsilon_{y_0}}{\pi}$.

TRADITIONALLY WE TRACK WITH INITIAL CONDITIONS $\epsilon_{x_i} = \epsilon_{y_i}$ BUT IGNORE THE FACT THAT X_i AND Y_i ARE CORRELATED BY:

$$\frac{X_i^2}{a^2} + \frac{Y_i^2}{b^2} = 1 \quad \text{WITH } a = \hat{X} \text{ AND } b = \hat{Y}.$$

WHEN WE SELECT THE INITIAL VALUE OF x AS $X_i = \hat{X}$, THE VALUE OF y ($Y_i = \hat{Y}$) THAT SATISFIES $\epsilon_{x_i} = \epsilon_{y_i}$ GIVES AN INITIAL STARTING POINT "A" THAT DOES NOT LIE ON THE BEAM PROFILE ELLIPSE OF FIGURE 1. IN OTHER WORDS, WE ARE TRACKING A PARTICLE THAT LIES OUTSIDE OF THE BEAM AND HAS AN EMITTANCE LARGER THAN WE THINK!

2. DETERMINE X_i WHEN $\epsilon_{x_i} = \epsilon_{y_i}$,
$X_i = \hat{X} = a = (\epsilon_0 \beta_x)^{1/2} = 18\,mm$, AND $\hat{Y} = b = (\epsilon_0 \beta_y)^{1/2}$.

FOR: $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$, $\epsilon_{x_i} = \epsilon_{y_i}$, AND $\frac{X_i^2}{\beta_x} = \frac{Y_i^2}{\beta_y}$,

$$\frac{X_i^2}{a^2} + \frac{y_i^2}{b^2} = \frac{X_i^2}{a^2} + \frac{\beta_y X_i^2}{\beta_x b^2} = \frac{X_i^2}{a^2} + \frac{\beta_x X_i^2}{\beta_x a^2} = 1.$$

THUS $\frac{2X_i^2}{a^2} = 1$ AND $X_i = \frac{a}{\sqrt{2}} = 12.7\,mm$,

AND WITH $\beta_x = 50\,m$ AND $\beta_y = 8.5\,m$, $Y_i = 5.2\,mm$.

HENCE WHEN $\epsilon_{x_i} = \epsilon_{y_i}$, IT SEEMS WE DON'T NEED STABILITY TO $X_i = 18\,mm$ FOR RHIC AND THAT WE CURRENTLY SATISFY THE REQUIREMENT OF ACCOMODATING A BEAM OF $6\sigma_x$.

3. CONSIDER INITIAL TRACKING VALUES WHEN $\epsilon_{x_i} \neq \epsilon_{y_i}$.

$x_i = (\epsilon_{x_i}\beta_x)^{1/2}$, $y_i = (\epsilon_{y_i}\beta_y)^{1/2}$, AND $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$$\frac{x_i^2/\beta_x}{a^2/\beta_x} + \frac{y_i^2/\beta_y}{b^2/\beta_y} = \frac{\epsilon_{x_i}}{\epsilon_0} + \frac{\epsilon_{y_i}}{\epsilon_0} = 1$$

THUS $\epsilon_{x_i} + \epsilon_{y_i} = \epsilon_0$, AND ANY POINT ON THE BEAM ELLIPSE IS A VALID STARTING POINT FOR TRACKING.

NOTE: THE TRACKING PROGRAM RACETRACK ADJUSTS THE RELATION BETWEEN X AND Y INTERNALLY SO THAT ALL STARTING POINTS SATISFY THE CONDITION THAT $\epsilon_{x_i} + \epsilon_{y_i} = \epsilon_0$.