

TID/SNA - - 1440

KEYPUNCHED

MASTER

LIQUID ROCKET PLANT

OPERATIONAL DESCRIPTION
OF
THE DUCT ANALYSIS PROGRAM
JOB 12007

Report 8209-64-7

February 1964

DISTRIBUTION OF THIS DOCUMENT UNLIMITED



DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

TID/SNA -- 1440

MASTER

OPERATIONAL DESCRIPTION
OF
THE DUCT ANALYSIS PROGRAM
JOB 12007

Report No. 8209-64-7

Analysis By:



R R Stiger
Design Engineer
Systems Analysis Section

Approved By:



W W Madsen, Supervisor
Systems Analysis Section
NERVA Rocket Operations
Liquid Rocket Plant

Programed By:
Dick Wick, Supervisor
Grain Design Group
Engineering and Test
Data Programming
Computing Sciences

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

DISTRIBUTION OF THIS DOCUMENT UNLIMITED

TABLE OF CONTENTS

	<u>Page No.</u>
I. OBJECTIVE	1
II. SUMMARY	1
III. TECHNICAL DISCUSSION	1
A. MATHEMATIC MODEL	1
B. LOGIC	1
IV. PROGRAM OPERATION	2
A. INPUT	2
B. OUTPUT	3
APPENDIX A - ANALYTICAL MODEL	A-1
APPENDIX B - SAMPLE CASE	B-1

20

I. OBJECTIVE

The purpose of the Duct Analysis Program, IBM Job Number 12007, is to compute flow rate, pressures, and the shock location, if there is one, in a system containing a converging diverging nozzle and a long duct.

II. SUMMARY

The Duct Analysis Program, using an iterative logic scheme, calculates flow rate, pressures and temperatures, and determines if the flow is subsonic or supersonic for any duct fed by a converging,diverging nozzle.

The analysis assumes an isentropic nozzle, a constant area duct, and an adiabatic system.

III. TECHNICAL DISCUSSION

A. Mathematic Model

The mathematical development used as the analytical model for the duct analysis is attached as Appendix A.

B. Logic

The program assumes supersonic flow, and calculates the pressure at the duct exit. The calculated back pressure is compared with the input pressure; if the input back pressure is smaller, the problem is solved. If not, the program assumes a shock at the duct exit and compares pressures again. If the exit pressure is higher than the back pressure, the program is terminated; if not, the program assumes a shock at the exit of the nozzle which coincides with the duct entrance. The revised duct exit pressure is again calculated, and the program investigates three possibilities: (1) the back pressure equals the exit pressure and the system is balanced; (2) the back pressure is lower than the duct exit

pressure, **and** the shock is standing in the exit duct. The program iterates and finds the shock position that results in a system balance; and (3) the back pressure is larger than the duct exit pressure, therefore, the shock is in the nozzle or the system is subsonic. The program iterates on the shock location to balance the system. If the shock occurs at a mach number of one or less, the system is treated as being subsonic and is solved.

IV. PROGRAM OPERATION

Attached as Appendix B is a sample input and its resulting output.

A. Input

The following is a list of the input parameters:

<u>Location</u>	<u>Symbol</u>	<u>Remarks</u>
L ₀	K	Ratio of specific heats (dimensionless)
L ₁	R	Universal gas constant ($\frac{\text{in-lbm}}{\text{lbm-}^\circ\text{R}}$)
L ₂	A ₁	Inlet nozzle area (in. ²)
L ₃	A*	Throat area (in. ²)
L ₄	A ₂	Duct area (in. ²)
L ₅	L	Duct equivalent length (in.) (as used in $\frac{4}{D}$ FL/D)
L ₆	F	Friction factor of the pipe (as used in $\frac{4}{D}$ FL/D)
L ₇	D	Duct diameter (in.)
L ₈	P _i	Initial stagnation pressure (psia) (not inputted if \dot{w} is input)
L ₉	T _i	Initial stagnation temperature (°R)
L ₁₀	P _e	Static back pressure (psia)
L ₁₁	W _i	Initial \dot{w} (lb/sec). If \dot{w} is inputted, P _i is calculated; if not P _i input is used. (if flow is subsonic, \dot{w}_i may not be inputted)
L ₁₂	Dump Flag	= 1 for dump of all calculated parameters = 0 no dump

Each case must be followed by a card with "T" in column one. Columns 2-72 of this card may be used for identification. This identification will be printed for each case.

"Stacked" cases may be run; the parameters that are to be changed are inputted directly behind the previous "T" card and an additional "T" card is added.

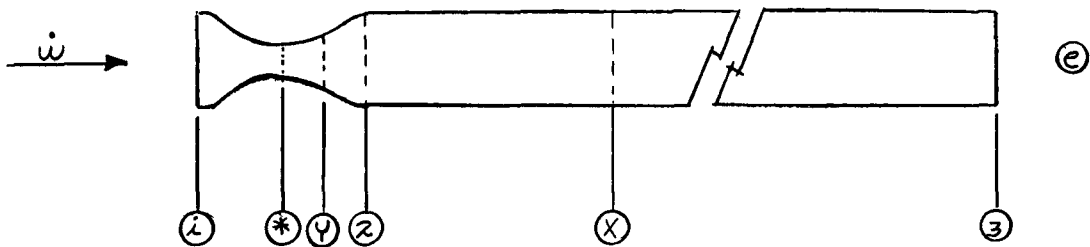
B. Output

The program output consists of the input parameters, a message, and either the flow rate or the inlet pressure which ever is applicable.

The following are messages that may be incurred as output:

<u>Message</u>	<u>Remarks</u>
Supersonic - no shock	The entire duct is supersonic and the exit pressure is greater than or equal to the back pressure.
Shock at exit of duct	The system is balanced with a shock standing at the exit of the duct.
Shock at Station 2	The system is balanced with a shock standing at the duct entrance.
Shock at A_y and M_y	The system is balanced with a shock standing in the nozzle exit cone at a Mach No. and area of M_y and A_y .
Shock at L_x and M_x	The system is balanced with a shock standing L_x equivalent inches down the duct from the nozzle exit; the Mach No. at the shock is M_x .
Flow is subsonic	The system balances subsonically. The flow rate is \dot{w} .
Impossible Physical Condition	There is an input error and the program converged on an impossible balance.

DUCT SHOCK ANALYSIS



SYSTEM SCHEMATIC

SYMBOL

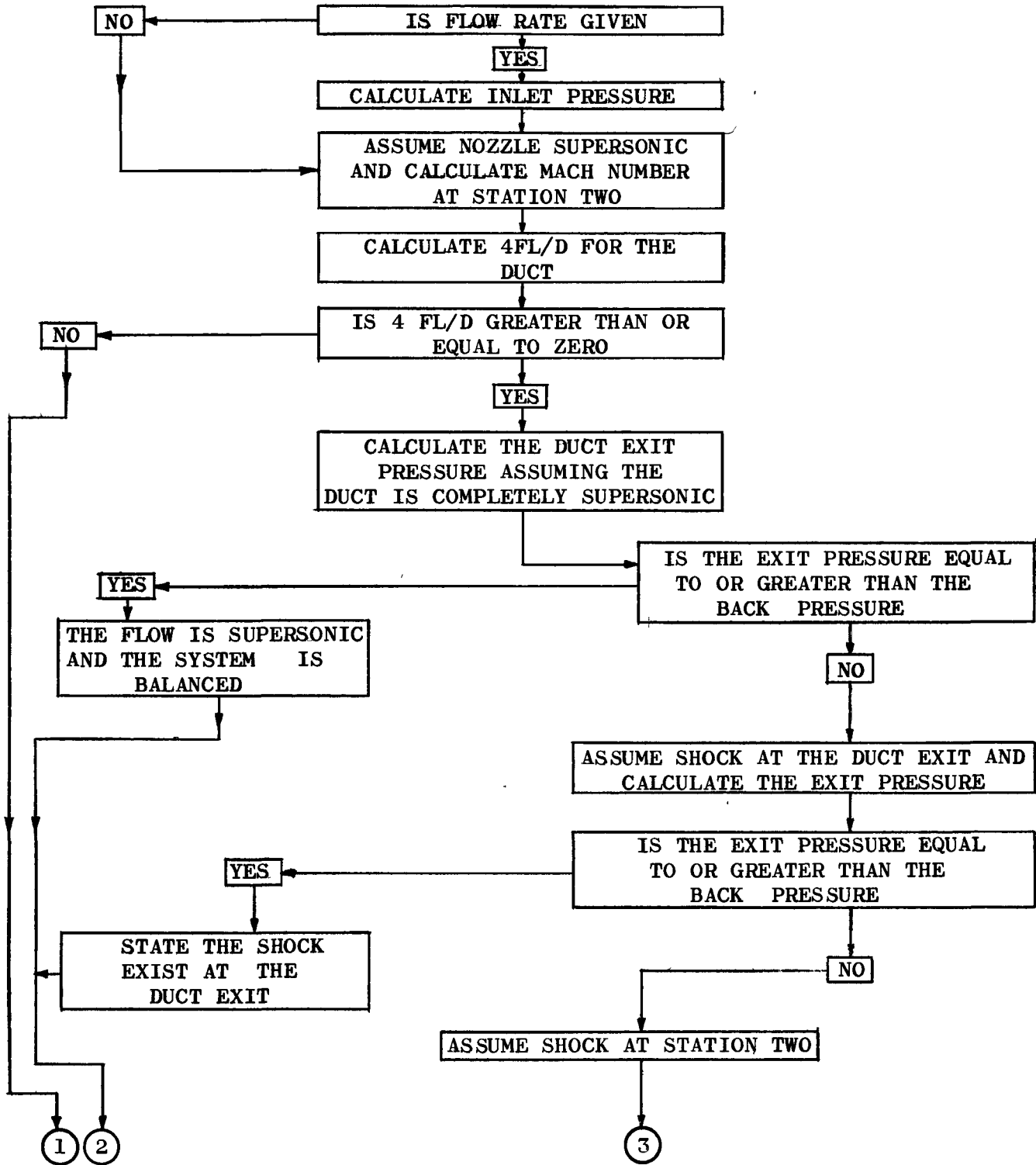
P = Pressure, psia	D = Duct diameter, in
T = Temperature, °R	M = Mach Number
A = Area, in ²	K = Ratio of specific hts.
L = Equivalent length, in	R = Gas Constant

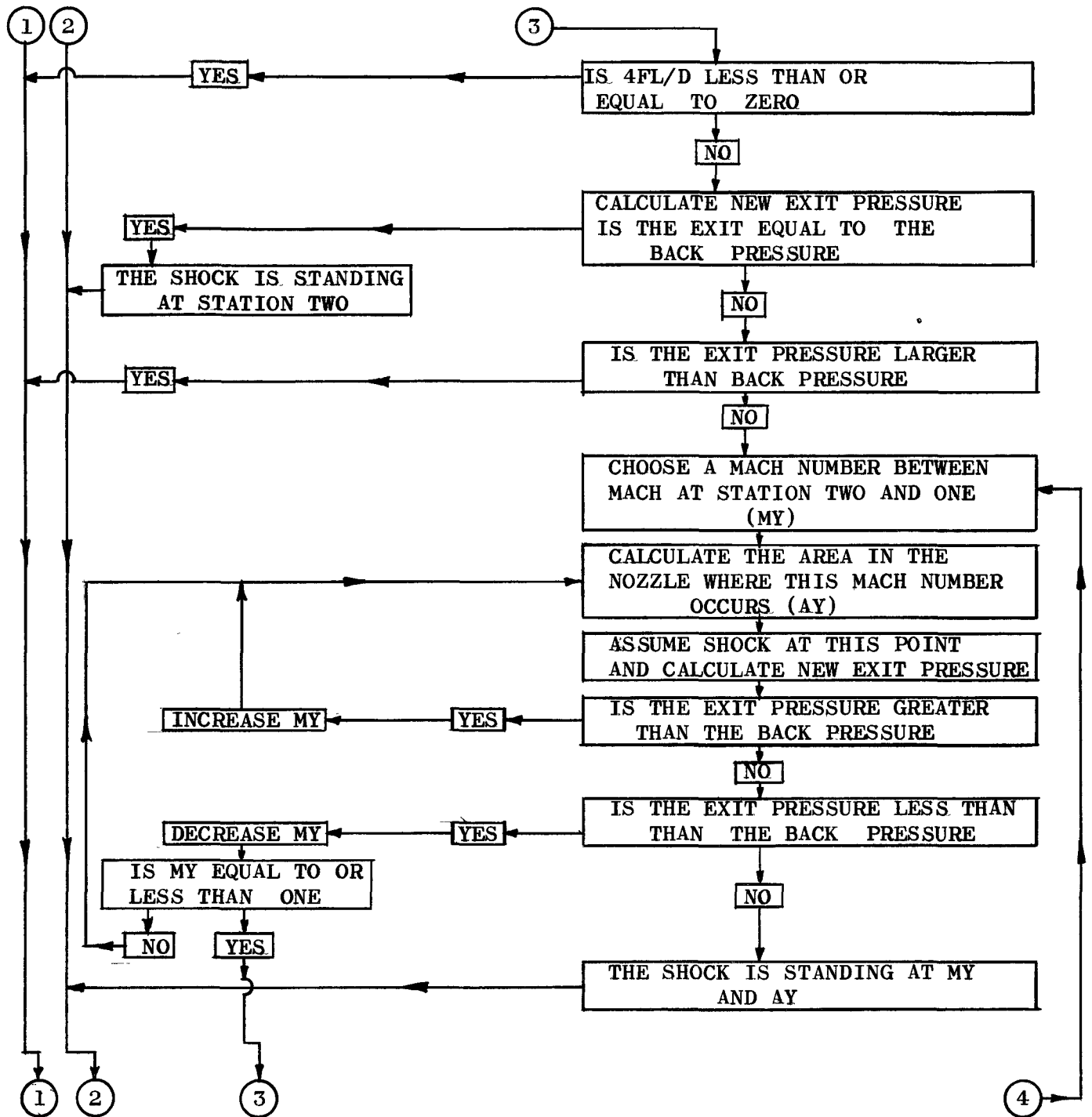
SUBSCRIPTS

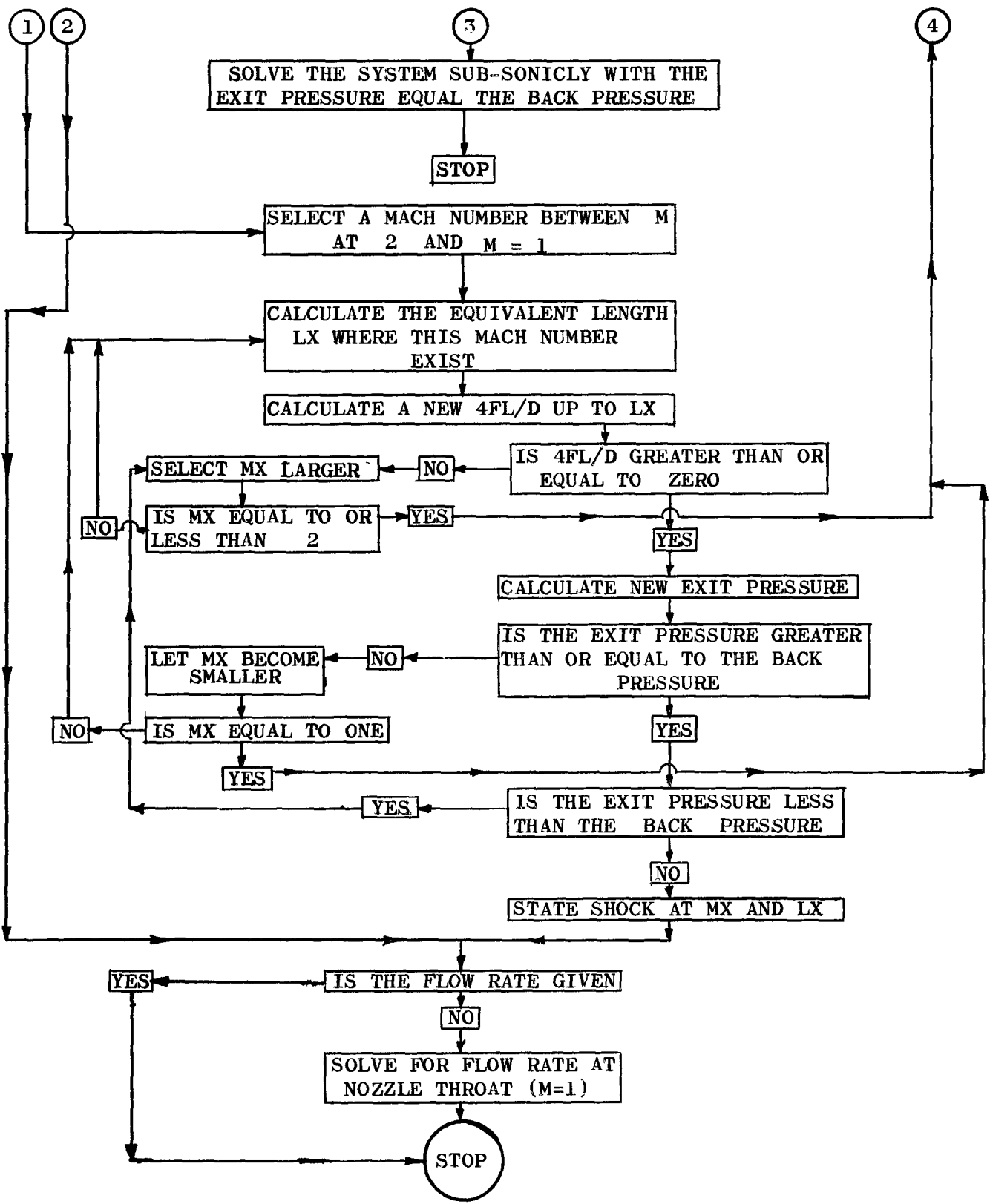
i = Initial	2 = Duct entrance
* = Throat	X = Station X
Y = Station Y	3 = Duct exit
	e = Ambient

LOGIC DIAGRAM

LONG DUCT SHOCK ANALYSIS







ANALYTICAL MODEL

1. Equations

0.) Is $W_i = 0$; yes, skip to 1.)

$$P_i^o = \frac{W_i (T_i^o)^{1/2}}{A^* \left[\frac{kgc}{R} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \right]^{1/2}}$$

1.) Calculate $M_2 > 1$ by Newton's Iteration

$$\frac{A^*}{A_2} = \frac{\left(\frac{k+1}{2} \right)^{\frac{k+1}{2(k-1)}}}{\left(1 + \frac{k-1}{2k} M_2^2 \right)^{\frac{k+1}{2(k-1)}}}$$

$$2.) \quad 4FL/D(MAX) = \left[\frac{k+1}{2k} \ln \left(\frac{\left(1 + \frac{k-1}{2} M_2^2 \right)}{1 + \frac{k-1}{2} M_2^2} \right) \right] + \frac{1 - M_2^2}{k M_2^2}$$

3.) $X = 4FL/D$

4.) $4FL/D(1) = 4FL/D(MAX) - X$

5.) a) If $4FL/D(1) > 0$ go to 6.)

b) If $4FL/D(1) < 0$ go to 13.)

6.) Solve for M_3 by Newton's Iteration:

$$M_2 > M_3 > 1$$

$$4FL/D(1) = \left[\frac{k+1}{2k} \ln \frac{\left(1 + \frac{k-1}{2} M_3^2 \right)}{1 + \left(\frac{k-1}{2} \right) M_3^2} \right] + \frac{1 - M_3^2}{k M_3^2}$$

$$7.) \quad P_2 = P_i \left(1 + \frac{k-1}{2} M_2^2 \right)^{\frac{k}{k-1}}$$

$$8.) P_i^* = P_2 M_2 \left(\frac{k+1}{2(1 + \frac{k-1}{2} M_2^2)} \right)^{-1/2}$$

$$9.) P_3 = \frac{P_i^*}{M_3} \left(\frac{k+1}{2(1 + \frac{k-1}{2} M_3^2)} \right)^{1/2}$$

10.) a) If $P_3 \geq P_e$ skip to 63.) and state No Shock

b) If $P_3 < P_e$ skip to 11.)

$$11.) P_{3a} = P_3 \left(\frac{2kM_3^2 - (k-1)}{k+1} \right)$$

12.) a) If $P_{3a} < P_e$ skip to 13.)

b) If $P_{3a} \geq P_e$ shock at exit of duct; skip to 63.)

$$13.) M_{2a} = \left[\frac{(k-1) \left(\frac{2kM_2^2 - (k-1)}{k+1} \right) + k+1}{2 \left(\frac{2kM_2^2 - (k-1)}{k+1} \right)} \right]^{1/2}$$

$$14.) 4FL/D \text{ MAX}(2) = \frac{k+1}{2k} \ln \frac{(k+1)M_{2a}^2}{2(1 + \frac{k-1}{2} M_{2a}^2)} + \frac{1 - M_{2a}^2}{k M_{2a}^2}$$

$$15.) 4FL/D(2) = 4FL/D \text{ MAX}(2) - X$$

16.) a) If $4FL/D(2) < 0$ skip to Eqn. 22.)

b) If $4FL/D(2) \geq 0$ skip to 17.)

17.) Solve for M_3' by Newton's Iteration

$$M_3' \quad 1$$

$$4FL/D(2) = \frac{k+1}{2k} \ln \frac{(k+1)M_3'^2}{2(1 + \frac{k-1}{2} M_3'^2)} + \frac{1 - M_3'^2}{M_3'^2}$$

$$18.) P_{2a} = P_2 \frac{2kM_2^2 - (k-1)}{k+1}$$

$$19.) \quad P_{2a}^* = P_{2a} M_{2a} \left[\frac{k+1}{2(1 + \frac{k-1}{2} M_{2a}^2)} \right]^{-1/2}$$

$$20.) \quad P_{3'} = \frac{P_{2a}^*}{M_{3'}} \left[\frac{k+1}{2(1 + \frac{k-1}{2} M_{3'}^2)} \right]^{1/2}$$

$$21.) \quad a) \quad \text{If } \left| \frac{P_{3'}}{P_e} - 1 \right| < .0001 \text{ Shock at Station 2; skip to 63.)}$$

$$b) \quad \text{If } P_{3'} > P_e \text{ Shock in duct skip to 50.)}$$

$$c) \quad \text{If } P_{3'} < P_e \text{ Shock in nozzle or flow is subsonic; skip to 22.e.)}$$

$$22.) \quad \text{SET } M_y = \frac{M_2 + 1}{2}$$

$$23.) \quad A_y = \frac{A^*}{M_y} \left[\frac{2(1 + \frac{k-1}{2} M_y^2)}{k+1} \right]^{\frac{k+1}{2(k-1)}}$$

$$24.) \quad P_y = P_i \left(1 + \frac{k-1}{2} M_y^2 \right)^{\frac{k+1}{2(k-1)}}$$

$$25.) \quad M_{ya} = \left[\frac{(k-1) \left(\frac{2 M_y^2 - (k-1)}{k+1} \right) + (k+1)}{2 \frac{2kM_y^2 - (k-1)}{k+1}} \right]^{1/2}$$

$$26.) \quad P_{ya} = P_y \left(\frac{2kM_y^2 - (k-1)}{k+1} \right)$$

28.) Solve for M_{n2} by Newton's Method

$$M_{n2} < 1$$

$$\frac{A_y M_{ya}}{A_2} \left(1 + \frac{k-1}{2} M_{ya}^2\right)^{-\frac{k+1}{2(k-1)}} = M_{n2} \left(1 + \frac{k-1}{2} M_{n2}^2\right)^{-\frac{k-1}{2(k-1)}}$$

$$29.) P_n^* = \frac{M_{n2} P_{ya}}{2 \left(1 + \frac{k-1}{2k} M_{n2}^2\right)^{1/2}} \frac{1 + \frac{k-1}{2} M_{ya}^2}{1 + \frac{k-1}{2} M_{n2}^2} \frac{k}{k-1}$$

$$30.) 4FL/D(MAX)(N) = \frac{k+1}{2k} \ln \frac{\left(1 + \frac{k-1}{2} M_{n2}^2\right)}{1 + \frac{k-1}{2} M_{n2}^2} + \frac{1 - M_{n2}^2}{k M_{n2}^2}$$

$$31.) 4FL/DCN) = 4FL/DMAX(N) - X$$

a) If $4FL/D(N) > 0$ skip to 32.)

b) If $MY < MAXMY$, $MAXMY = MY$

$$c) MY = \frac{MAXMY + 1}{2}$$

d) If $MY \approx 1$ skip to 35.)

e) If $MY > 1$ skip to 23.)

32.) Solve for M_{n3} using Newton's Method $M_{n3} < 1$

$$4FL/DCN) = \frac{k+1}{2k} \ln \frac{\left(1 + \frac{k-1}{2} M_{n3}^2\right)}{1 + \frac{k-1}{2} M_{n3}^2} + \frac{1 - M_{n3}^2}{k M_{n3}^2}$$

$$33.) P_{3N} = \frac{P_n^*}{M_{N3}} \left(\frac{k+1}{2 \left(1 + \frac{k-1}{2} M_{N3}^2\right)} \right)^{1/2}$$

34.) a.) If $\left(\frac{P_{3N}}{P_e} - 1\right) < .0001$ Shock at A_Y . skip to 63.)

b.) If $P_{3N} < P_E$ 1.) If $MY_{-1} < .001$ skip to 35.)

$$2.) MY_L = \frac{MY_{i-1} + 1}{2}$$

3.) If $MY_{i-1} < MAXMY$

$$\text{Set } MAXMY = MY_{i-1}$$

4.) Skip to 34.)d.)

c.) If $P_{3N} > P_E$ 1.) If $MY_{i-1} - M_2 < .001$ error

2.) If $MY_{i-1} > MINMY$

$$\text{set } MINMY = MY_{i-1}$$

$$3.) MY_i = \frac{MY_{i-1} + M_2}{2}$$

4.) Skip to 34.)d.)

d.) If $MINMY < MY_i < MAXMY$ Skip to 23.)

$$\text{If not } MY_i = \frac{MINMY + MAXMY}{2} \quad \text{skip to 23.)}$$

35.) Set $MT = .7$

$$36.) A_C^* = \frac{A \left(\frac{k-1}{2} \right) \frac{k+1}{2(k-1)} M_T}{\left(1 + \frac{k+1}{2} M_T^2 \right) \frac{k+1}{2(k-1)}}$$

37.) Solve for M_{2C} using Newton's Method $M_{2C} < 1$

$$\frac{A_C^*}{A_2} = \frac{\left(\frac{k+1}{2} \right) \frac{k+1}{2(k-1)} M_{2C}}{\left(1 + \left(\frac{k-1}{2} \right) M_{2C}^2 \right) \frac{k+1}{2(k-1)}}$$

$$38.) P_{2C} = P_{i0} \left(1 + \frac{k-1}{2} M_{2C}^2\right)^{-k/k-1}$$

$$39.) P_C^* = P_{2C} \cdot M_{2C} \frac{k-1}{2(1 + (\frac{k-1}{2})M_{2C}^2)} - 1/2$$

$$40.) 4FL/D_{MAXC} = \frac{k+1}{2} \ln \frac{(1 + \frac{k-1}{2})M_{2C}^2}{1 + \frac{k-1}{2}M_{2C}^2} + \frac{1 - M_{2C}^2}{k M_{2C}^2}$$

$$41.) 4FL/D_{(C)} = 4FL/D_{MAXC} - X$$

42.) If $4FL/D(C)$ LO error, If not continue

43.) Solve for M_{3C} using Newton's Method $M_{3C} < 1$

$$4FL/D(c) = \frac{k+1}{2} \ln \frac{(1 + \frac{k-1}{2})M_{3C}^2}{1 + \frac{k-1}{2}M_{3C}^2} + \frac{1 - M_{3C}^2}{k M_{3C}^2}$$

$$44.) P_{3C} = \frac{P_C^*}{M_{3C}} \left(\frac{k+1}{2(1 + \frac{k-1}{2} M_{3C}^2)} \right)^{1/2}$$

a.) If $\left| \frac{P_{3C}}{P_E} - 1 \right| < .0001$ skip to 45.)

b.) If $P_{3C} > P_E$ 1.) If $MT > MINMT$
 $MINMT = MT$, if not skip to 2.)

$$2.) MT = \frac{MT + 1}{2}$$

3.) Skip to 44d.)

c.) If $P_{3C} < P_e$ 1.) If $MT < MAXMT$
 $MAXMT = MT$, if not skip to 2.)

$$2.) MT = \frac{MT}{2}$$

3.) Skip to 44d.)

4.) If $MINMT < MT < MAXMT$ skip to 36.)

If not $MT = \frac{MINMT + MAXMT}{2}$ skip to 36.)

$$45.) T_{2C} = T_i \left(1 + \frac{k-1}{2} M_{2C}^2\right)^{-1}$$

$$46.) \dot{W} = \frac{P_{2C}}{T_{2C} R} \cdot M_{2C} (32.2 k RT_{2C})^{1/2} \cdot A_2$$

47.) Solve for M_{iC} by Newton's Iteration $M_{iC} < 1$

$$\frac{A^*_C}{A_1} = M_{iC} \left[\frac{2 \left(1 + \frac{k-1}{2} M_{iC}^2\right)}{k+1} \right] - \frac{k+1}{2(k-1)}$$

$$48.) P_{iC} = P_o \left(1 + \frac{k-1}{2} M_{iC}^2\right)^{-k/(k-1)}$$

49.) Print \dot{W} , P_{iC} and state flow **is** subsonic, end case

$$50.) M_X = \frac{M_2 + 1}{2}, \text{ MAXMX} = M_2, \text{ MINMX} = 1$$

$$51.) 4FL/D_{(\text{MAX}(X))} = \frac{k+1}{2k} \ln \frac{\left(1 + \frac{k-1}{2} M_X^2\right) M_X^2}{1 + \frac{k-1}{2} M_X^2} + \frac{1 - M_X^2}{k M_X^2}$$

$$52.) Y = 4FL/D_{\text{MAX}(1)} - 4FL/D_{(\text{MAX}(X))}$$

If $Y < 0$ $M_X = \frac{M_X + M_2}{2}$, if $M_X = M_2$ go to 22

If not go to 51.)

If > 0 go to 52a

$$52a) L_X = \frac{DY}{4F}$$

$$53.) M_{Xa} = \frac{(k-1) \frac{2 M_4^2 - (k-1)}{k+1} + (k+1)^{1/2}}{2k \left(\frac{2 M_4^2 - (k-1)}{k+1} \right)}$$

$$54.) P_X = \frac{P_i^*}{M_X} \left(\frac{k+1}{2 \left(1 + \frac{k-1}{2} M_{42} \right)} \right)^{1/2}$$

$$54A.) P_{Xa} = P_X \left(\frac{2 M_{42} - (k-1)}{k+1} \right)$$

$$55.) P_{Xa}^* = P_{Xa} \cdot M_{Xa} \left(\frac{k+1}{2 \left(1 + \frac{k-1}{2} M_{Xa} \right)} \right)^{-1/2}$$

$$56.) {}_{4FL/D}(\text{MAX})(Xa) = \frac{k+1}{2k} \ln \frac{\left(1 + \frac{k-1}{2} M_{Xa} \right) M_{Xa}^2}{1 + \frac{k-1}{2} M_{Xa}^2} + \frac{1 - M_{Xa}^2}{k M_{Xa}^2}$$

$$57.) Z = \frac{4F(L - L_X)}{D}$$

$$58.) {}_{4FL/D}(Xa) = {}_{4FL/D}(\text{MAX})(Xa) - Z$$

$$59.) \text{ If } {}_{4FL/D}(Xa) < 0 \text{ set } M_X = \frac{M_X + M_2}{2}, \text{ if } M_X = 1 \text{ go to 66.)}$$

a.) If $M_X = M_2$ error, if not skip to 51.)

b.) If ${}_{4FL/D}(Xa) \geq 0$ continue

60.) Solve for M_{3X} using Newton's Method $M_{3X} < 1$

$${}_{4FL/D}(Xa) = \frac{k+1}{2k} \ln \frac{\left(1 + \frac{k-1}{2} M_{3X} \right) M_{3X}^2}{1 + \frac{k-1}{2} M_{3X}^2} + \frac{1 - M_{3X}^2}{k M_{3X}^2}$$

$$61.) P_{3X} = \frac{P_{4a}^*}{M_{3X}} \left(\frac{k+1}{2(1 + \frac{k-1}{2} M_{3X}^2)} \right)^{1/2}$$

$$62.) a.) \text{ If } \left| \frac{P_{3X}}{P_E} - 1 \right| < .0001 \text{ state shock at LX skip to 63.)}$$

$$b.) \text{ If } P_{3X} > P_E \quad 1.) \text{ If } M_X > \text{MAXMX set MAXMX} = M_X$$

$$2.) M_X = \frac{M_2 + 1}{2}, \text{ if } M_X \approx 1 \text{ go to 68.)}$$

3.) Skip to 62d)

$$c.) \text{ If } P_{3X} < P_E \quad 1.) \text{ If } M_X < \text{MINMX set MINMX} = M_X$$

$$2.) M_X = \frac{M_2 + M_X}{2}, \text{ if } M_X \approx M_2 \text{ go to 22.)}$$

3.) Skip to 62d)

62A) If $\text{MINMX} < M_X < \text{MAXMX}$ skip to 51.)

If not $M_X = \frac{\text{MINMX} + \text{MAXMX}}{2}$ skip to 51.)

$$63.) T_i^* = \frac{T_o}{\left(\frac{k+1}{2} \right)}$$

$$64.) P^* = P_2 \left[\frac{k+1}{2(1 + \frac{k-1}{2})} \right]^{-k/k-1}$$

$$65.) \dot{W} = \rho^* A^* \sqrt{g_c R k T_i^*}$$

where ρ^* is function of para-hydrogen

66.) Print impossible physical condition.

LONG DUCT ANALYSIS SAMPLE CASE

GAMMA	R	AI	A*	A2	L	F	D	PIO	TIO	PE	WDOT
1.4000	9170.00	505.00	290.00	2360.00	888.00	.004	24.000	60.00	280.00	14.70	.00

SHOCK STANDING AT AY = 1531.44 AND MY = 3.232

WDOT = 145.71