FERMILAB-CONF-11-048-APC

SIMULATIONS OF THE TAPERED GUGGENHEIM 6D COOLING CHANNEL FOR THE MUON COLLIDER[*]

P. Snopok[†], Illinois Institute of Technology, Chicago, IL, USA
G. Hanson, University of California, Riverside, CA, USA
R. Palmer, Brookhaven National Laboratory, Upton, NY, USA

Abstract

Recent progress in six-dimensional (6D) cooling simulations for the Muon Collider based on the RFOFO ring layout is presented. In order to improve the performance of the cooling channel a tapering scheme is studied that implies changing the parameters such as cell length, magnetic field strength, RF frequency, and the amount of the absorbing material along the cooling channel. This approach allows us to keep the cooling rates high throughout the process. The results of the simulations carried out in G4beamline are presented.

RFOFO-BASED COOLING LATTICES

In a Muon Collider design the muon beam 6D phase space volume must be reduced by several orders of magnitude in order to be able to further accelerate it. Ionization cooling is currently the only feasible option for cooling the beam within the muon lifetime ($\tau_0 = 2.19\ \mu$s). The RFOFO ring [1] is one of the feasible options along with other designs. The layout of the RFOFO ring is shown in Fig. 1. The RFOFO ring provides a significant reduction in the 6D emittance in a small number of turns with a relatively low particle loss factor. 6D cooling is achieved by employing the concept of emittance exchange. When a dispersive beam passes through a wedge absorber in such a way that higher momentum particles pass through more material, both the longitudinal and the transverse emittances are reduced. However, the design of the injection and extraction channels and kickers is very challenging for the RFOFO, and the ring could not be used as is because the bunch train is too long to fit in the ring. Both problems are mitigated in the RFOFO helix, also known as the Guggenheim channel. In addition, using the helix solves another important issue, namely, the overheating in the absorbers. The Guggenheim performance is comparable to the original RFOFO ring. To further improve the performance of the Guggenheim lattice, it is proposed to use a tapered scheme, in which parameters of the structure change from turn to turn based on the emittance reduction rate and particle transmission. Five consecutive turns of the untapered helix are shown in Fig. 2. This channel was simulated up to 15 turns (495 m) [2], and the results of the simulation will be used as a reference in assessing the tapered channel performance.

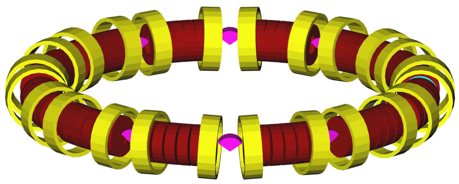

Figure 1: RFOFO ring layout. Yellow—tilted magnetic coils with alternating currents to provide necessary bending and focusing and generate dispersion, purple—wedge absorbers for cooling and emittance exchange, brown—RF cavities for restoring the longitudinal component of the momentum.

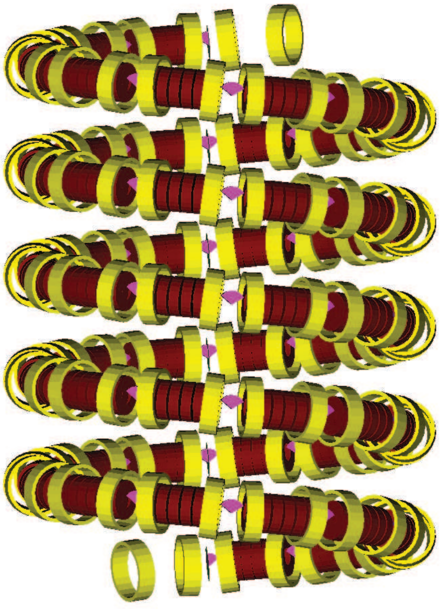

Figure 2: 5-turn slice of the Guggenheim helix. Simulated up to 15 turns (495 m). Used as a reference in assessing the tapered lattice performance.

TAPERED GUGGENHEIM

The main issue with the Guggenheim channel of fixed radius and RF frequency is its gradual loss of cooling efficiency. To keep cooling rates high throughout the cooling process, it was proposed to use a tapered channel in which various parameters change to avoid reaching the equilib-

---

[*] Work is supported by the U.S. Department of Energy.

[†] psnopok@iit.edu

rium emittance. This way the same emittance reduction can be obtained in a much shorter channel, as will be shown below.

One such channel was proposed previously in [3], and simulated in ICOOL [4] using certain approximations. It was not simulated as a helix or a set of individual rings, but as a linear channel. Hence, there was no bending field to generate dispersion crucial for the emittance exchange mechanism to work. Thus, emittance exchange was simulated via a linear matrix of the form

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1+\delta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\delta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\delta \end{pmatrix},$$

where δ is the desired emittance exchange factor.

Arguably, the linear channel simulation is easier and faster than the full ring or helix simulation that requires tilting and displacing the coils to generate necessary dipole fields, using solid wedge absorbers for emittance exchange, and tweaking individual parameters of the lattice elements to avoid overlap in the ring. However, such a simulation is required to assure the tapered helix performs as expected. The study was performed using the g4beamline [5] code.

The first 10 stages of the proposed tapered channel are shown in Fig. 3, where one can see the radius, the number of identical cells per turn, and the size of the coils change from stage to stage. For a list of parameters (excluding the coil geometry), please refer to Table 1. “Length RF” is the total length of RF per cell. It is assumed there are 4 cavities per cell. The RF gradient is 15.48 MV/m for all stages, the accelerating phase is 44° for the first five stages, and 34° afterwards. B_0 is the magnetic field strength on the reference circle running through the centers of RF cavities and absorbers. “Bend field” is the bending field required to keep muons on a circular orbit; it also provides dispersion. “Coil tilt” is the coil tilt angle required to produce the necessary bending field. “Coil displ.” is the displacement of the coils with respect to the reference circle that minimizes vertical excursion of the orbit. Starting with stage 6 there are two set of coils per cell, hence, there are two numbers. Eleven stages were simulated starting with the same beam that was used for the untapered Guggenheim simulations.

Individual rings corresponding to each stage are simulated rather than the multi-turn helix. It was shown before [2] that the performance of the helical lattice is very similar to that of individual rings assuming magnetic shielding between layers. The results of the g4beamline simulation are summarized in Figures 4–7. Both simulations use the same beam with parameters as described in [2]. The tapered channel is two times shorter than the original 201.25 MHz untapered Guggenheim when the two transmissions are the same (50%). At the same time, the final 6D emittance is 11 mm^3 for the tapered case as compared to 48 mm^3 for the untapered scenario, which is quite an improvement. There are two places where the longitudinal emittance in the tapered case demonstrates some growth, which is caused by the fact that the RF frequency changes from stage to stage and there are no special matching sections between stages.

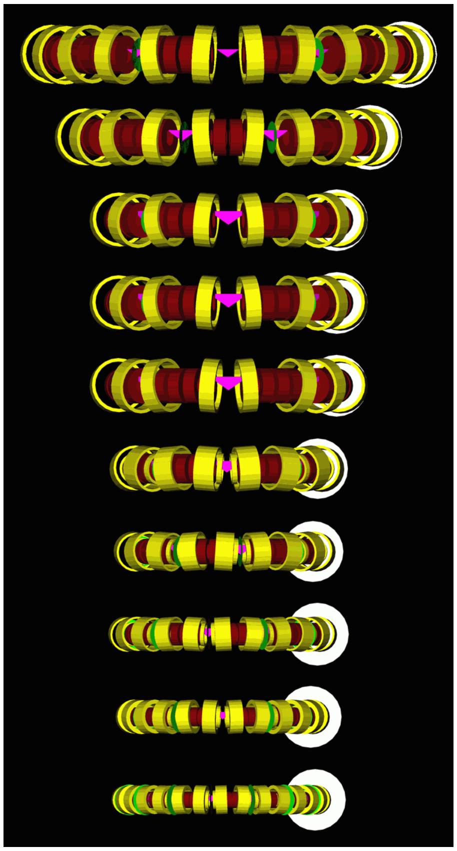

Figure 3: Geometry of the first 10 stages of the tapered Guggenheim.

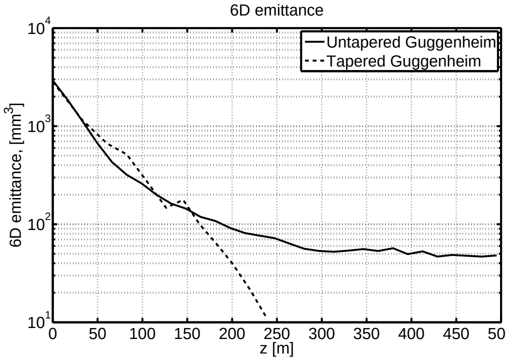

Figure 4: 6D emittance reduction in the untapered Guggenheim (solid) and the tapered Guggenheim (dashed).

SUMMARY

The study shows that tapering is a viable concept that allows to cut the length of the cooling and achieve a lower 6D emittance than the untapered scheme at the same time.

Table 1: Lattice parameters of the tapered Guggenheim stages.

Stage number	Cell length [m]	Number of cells	RF freq. [MHz]	Length RF [m]	Length abs. [mm]	B_0 [T]	Bend field [T]	Coil tilt [deg]	Coil displ. [mm]
1	2.75	12	201.25	1.88	226	2.33	0.129	3.72	100
2	2.75	10	201.25	1.88	326	2.52	0.152	4.17	119
3	2.75	8	201.25	1.88	426	2.69	0.190	4.98	148
4	2.75	8	201.25	1.88	426	2.72	0.190	4.95	146
5	2.75	8	201.25	1.88	426	2.75	0.190	4.75	146
6	2.36	8	235.00	1.61	366	3.09	0.222	4.47	44/143
7	2.02	9	274.00	1.38	314	3.60	0.230	3.96	37/110
8	1.73	11	319.00	1.18	268	4.19	0.220	3.22	26/78
9	1.49	12	373.00	1.02	230	4.90	0.234	2.62	26/69
10	1.28	15	435.00	0.87	198	5.72	0.218	2.08	16/48
11	1.09	17	507.00	0.75	169	6.68	0.226	1.84	12/36.5

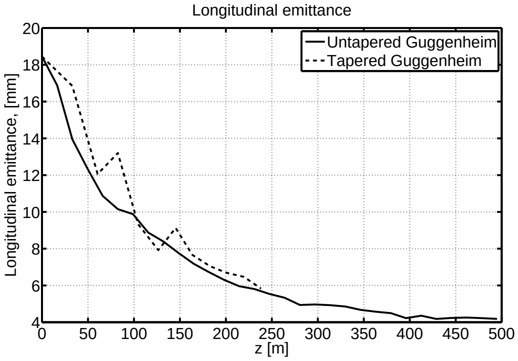

Figure 5: Longitudinal emittance reduction in the untapered Guggenheim (solid) and the tapered Guggenheim (dashed).

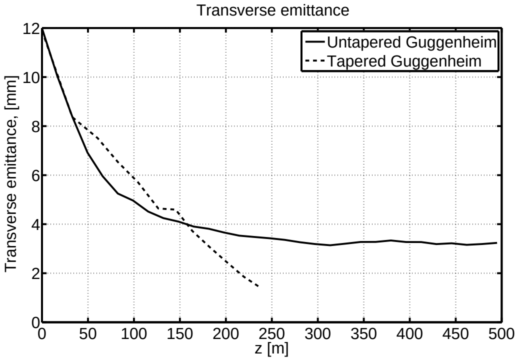

Figure 6: Transverse emittance reduction in the untapered Guggenheim (solid) and the tapered Guggenheim (dashed).

Direct comparison to the corresponding ICOOL simulation was not performed, since the goal was to compare with the original untapered Guggenheim helix. The next step would be to simulate a proper helical channel rather than the set of rings, although it is expected that the performance will be similar. Another important improvement to implement is a smaller number of different RF frequencies, ideally, 201.25, 402.5 and 805 MHz only.

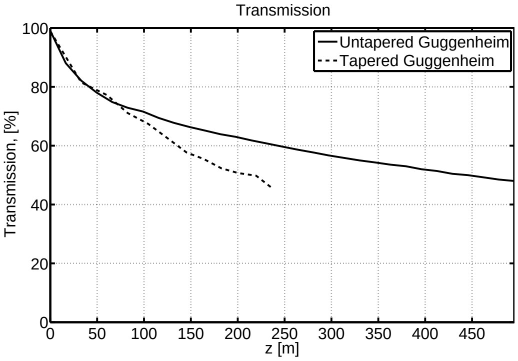

Figure 7: Particle transmission in the untapered Guggenheim (solid) and the tapered Guggenheim (dashed).

ACKNOWLEDGMENTS

The authors would like to thank Dr. Richard Fernow from BNL for many fruitful discussions and thoughtful comments.

REFERENCES

[1] R. Palmer et al., *Phys. Rev. ST Accel. Beams* **8**, 061003, 2005.

[2] P. Snopok, G. Hanson and A. Klier, *IJMPA* **24(5)**, 987–998, 2009.

[3] R. Palmer, Towards End-to-end 6D cooling Simulation, presentation at the IDS-NF 5th Plenary Meeting, Fermilab, April 2010.

[4] ICOOL code, `https://pubweb.bnl.gov/~fernow/icool/readme.html`.

[5] G4beamline code, `http://muonsinc.com`.