EPICYCLIC TWIN-HELIX MAGNETIC STRUCTURE FOR PARAMETRIC-RESONANCE IONIZATION COOLING*

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Abstract

Parametric-resonance Ionization Cooling (PIC) [1-3] is envisioned as the final 6D cooling stage of a high-luminosity muon collider. Implementing PIC imposes stringent constraints on the cooling channel's magnetic optics design. This paper presents a linear optics solution compatible with PIC. Our solution consists of a superposition of two opposite-helicity equal-period and equal-strength helical dipole harmonics and a straight normal quadrupole. We demonstrate that such a system can be adjusted to meet all of the PIC linear optics requirements while retaining large acceptance.

INTRODUCTION

Combining muon ionization cooling with parametric resonant dynamics should allow much smaller final transverse muon beam sizes than ionization cooling alone [1-3]. Thus, high luminosity would be achieved in a collider with fewer muons. Parametric ionization cooling is accomplished by inducing a parametric resonance in a muon cooling channel. The beam is then naturally focused with a period of the channel's free oscillations. Absorber plates for ionization cooling together with energy-restoring rf cavities are placed at the beam focal points. At the absorbers, ionization cooling maintains the angular spread constant while the parametric resonance causes a strong reduction of the beam spot size. This resonant cooling scheme should provide equilibrium transverse emittances that are at least an order of magnitude smaller than those achievable with conventional ionization cooling.

TWIN-HELIX CHANNEL

Correlated Optics Condition

To ensure the beam's simultaneous focusing in both horizontal and vertical planes, the horizontal oscillations' period λ_x must be equal to or be a low-integer multiple of the vertical oscillations' period λ_y. The PIC scheme also requires alternating dispersion D such that D is

- small at the beam focal points to minimize energy straggling in the absorber,
- non-zero at the absorber for emittance exchange,
- relatively large between the focal points to allow for aberration correction to keep the beam size small at the absorbers.

Given the above dispersion requirements, it is clear that λ_x and λ_y must also be low integer multiples of the dispersion period λ_D. Note that λ_x and λ_D should not be equal to avoid an unwanted resonance. Thus, the cooling channel's optics must have correlated values of λ_x, λ_y and λ_D:

$$\lambda_x = n\lambda_y = m\lambda_D\,, \tag{1}$$

e.g. $\lambda_x = 2\lambda_y = 4\lambda_D$ or $\lambda_x = 2\lambda_y = 2\lambda_D$.

Orbital Dynamics

The PIC dynamics is very sensitive to magnetic fringe fields. One approach to finding a practical fringe-field-free solution is to use helical harmonics [4, 5]:

$$\begin{aligned} B_\varphi^n &= b_\varphi^n(|k|,\rho)\cos(n[\varphi - kz + \varphi_0^n]), \\ B_\rho^n &= b_\rho^n(|k|,\rho)\sin(n[\varphi - kz + \varphi_0^n]), \\ B_z^n &= \mathrm{sgn}(k)\, b_z^n(|k|,\rho)\cos(n[\varphi - kz + \varphi_0^n]), \end{aligned} \tag{2}$$

where B_φ, B_ρ, and B_z are the azimuthal, radial, and longitudinal helical magnetic field components, respectively, n is the harmonic number (e.g. $n = 1$ is the dipole harmonic), and $k = 2\pi / \lambda_h$ is the helix wave number while λ_h is the helix period.

One extensively-studied system based on helical field is the Helical Cooling Channel (HCC) [4]. However, the HCC is not suitable for PIC because it has constant dispersion magnitude. It was suggested [2] that alternating dispersion could be created by superimposing the HCC with an opposite-helicity helical dipole field with a commensurate characteristic period. However, with this approach, the periodic orbit solution is somewhat complicated and producing sufficiently large acceptance seems problematic.

Here we study a somewhat different configuration of magnetic fields, however, retaining the principle of Ref. [2] of creating an alternating dispersion by superimposing two helical-dipole fields with commensurate periods. We use a superposition of two equal-strength helical dipole harmonics with equal periods and opposite helicities ($k_1 = -k_2$) as a basis for our PIC channel design. Analogously to how combining two circularly-polarized waves produces a linearly-polarized one, the magnetic field in the mid-(horizontal)-plane of this configuration is transverse to the plane. This means that the periodic orbit is flat and lies in the mid-plane. The horizontal and vertical motions are uncoupled. This is a

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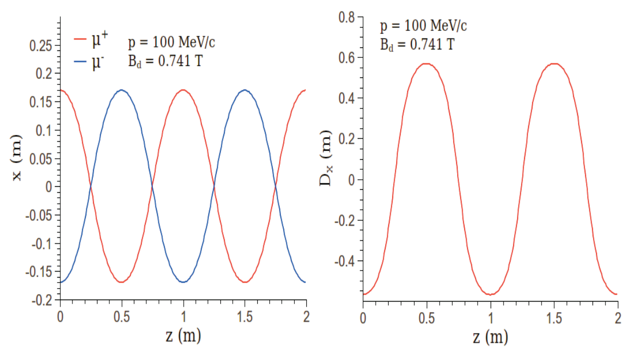
more conventional orbital dynamics problem than the one with a 3D reference orbit and coupled transverse motion.

Figure 1(a) shows an example of the periodic orbit solutions for 100 MeV/c μ^+ and μ^- in a twin-helix channel with 1 m period and 0.741 T magnetic field strength of each helical dipole harmonic. The periodic orbit was determined numerically by locating the fixed point in the phase space. For this procedure, one only needed to consider the x-x' horizontal phase space and the procedure was further simplified by selecting a longitudinal position where x' was zero. Figure 1(b) shows the dispersion as a function of the longitudinal position for the μ^+ solution shown in Fig. 1(a). Note that the dispersion has oscillatory behaviour required for PIC. Note also that the dispersion period is equal to the helix period, i.e. $\lambda_D = \lambda_h$.

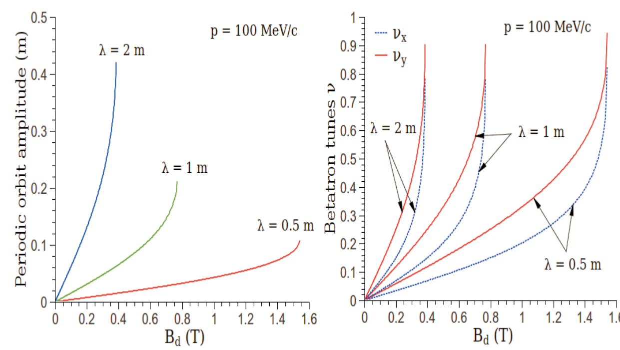


(a) (b)

Figure 1: (a) Periodic orbits of 100 MeV/c μ^+ and μ^- in a twin-helix channel with 1 m period and 0.741 T magnetic field strength of each helical dipole harmonic.
(b) Dispersion behavior as a function of the longitudinal position for the μ^+ solution in Fig. 1(a).

The transverse motion in a twin-helix consisting of two helical dipole harmonics only is stable around the periodic orbit in both dimensions as long as the helical dipole strength does not exceed a certain limiting value. Figures 2(a) and 2(b) show the periodic orbit's amplitude and the betatron tunes, respectively, vs. the helical dipole strength B_d for three different values of the helix period. In the calculations shown Figs. 2(a) and 2(b), the strength B_d was changed in small steps. On each step, the new periodic orbit was obtained as described above using the previous step's solution as the initial guess. The betatron tunes were extracted from a single-period linear transfer matrix, which was obtained numerically in terms of canonical coordinates. The canonical coordinates were calculated using an analytic expression for the helical magnetic field vector potential [5].

Since the dispersion period is determined by the helix period $\lambda_D = \lambda_h$, the correlated optics condition of Eq. (1) imposes the following conditions on the betatron tunes: $\nu_x = \lambda_h/\lambda_x = \lambda_h/(m\lambda_D) = 1/m$ and $\nu_y = \lambda_h/\lambda_y = \lambda_h/(m\lambda_D/n) = n/m$, e.g. $\nu_x = 0.5$, $\nu_y = 1$ or $\nu_x = 0.25$, $\nu_y = 0.5$. Examining Fig. 2(b) shows that it is not possible to satisfy these conditions by adjusting λ_h and B_d. Thus, we introduced a straight normal quadrupole to redistribute focusing between the horizontal and vertical dimensions. One subtlety is that, in addition to changing the focusing properties of the lattice, the quadrupole also changes the periodic reference orbit. The helical dipole strength B_d and the quadrupole gradient $\partial B_y/\partial x$ were iteratively adjusted until, at $B_d = 1.303$ T and $\partial B_y/\partial x = 1.153$ T/m, we achieved the correlated optics condition with $\nu_x = 0.25$ and $\nu_y = 0.5$. Having $\nu_x = 0.5$ and $\nu_y = 1$ would be more beneficial by allowing shorter spacing between the absorbers but it was not possible to adjust these tune values because of a strong parametric resonance at $\nu_y = 1$. We also attempted tuning the correlated optics condition by using a helical quadrupole pair instead of a straight quadrupole but that configuration did not seem compatible with the correlated optics requirements. Figure 3 shows the dependence of the betatron tunes on B_d at $\partial B_y/\partial x = 1.153$ T/m. The crossing lines indicate the correlated optics condition. Note that the straight quadrupole introduces an asymmetry into the magnetic field so that the correlated optics condition is satisfied for one muon charge only.



(a) (b)

Figure 2: Periodic orbit's amplitude (a) and horizontal and vertical betatron tunes (b) vs. the helical dipole strength.

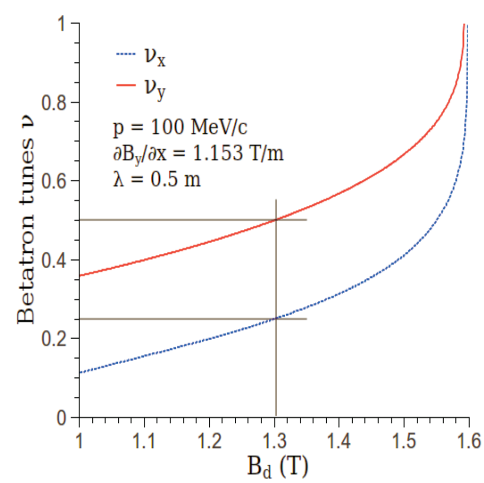


Figure 3: Horizontal and vertical betatron tunes vs. the helical dipole strength B_d at $\partial B_y/\partial x = 1.153$ T/m.

We next studied the dependence of the periodic orbit and of the betatron tunes on the muon momentum. We found that, with correlated optics, the dispersion amplitude $D_{x\ max} = p\ \partial x_{max}/\partial p$ was 0.098 m and the horizontal and vertical chromaticities were $\xi_x = p\ \partial \nu_x/\partial p = -0.646$ and $\xi_y = -0.798$, respectively. The correlated optics parameters scale with the muon momentum and helix period in the following way:

$$B_d \propto p/\lambda, \quad \partial B_y/\partial x \propto p/\lambda^2, \\ x_{\max}, D_x \propto \lambda, \quad \xi_x, \xi_y \propto \text{const.} \tag{3}$$

G4beamline simulation

To estimate the dynamic aperture of the correlated optics channel, we tracked 10^5 100 MeV/c muons through 100 periods of the channel using the GEANT-based G4beamline program. The initial muon beam was monochromatic, parallel and uniformly distributed within a 5 cm × 5 cm square. Figures 4(a) and 4(b) show the initial and final transverse muon distributions, respectively. The initial positions of the particles that were not lost after 100 periods are shown in Fig. 4(a) in blue. Figures 4(a) and 4(b) suggest a very large dynamic aperture and therefore high transmission efficiency of the twin-helix channel.

Possible Practical Implementation

In practice, the required magnetic field configuration can be obtained by installing two separate helical dipoles producing equal field strengths within the channel's aperture [5] and then superimposing a straight quadrupole. The helical dipoles should be aligned along the symmetry axis and should have the same spatial periods and opposite helicities. Alternatively, the desired magnetic fields can be obtained by combining two series of tilted current loops with their currents varied along the symmetry axis.

Next steps

We will next consider compensation of chromatic and spherical aberrations. It has been demonstrated in [3] that chromatic terms can be compensated using two sextupole harmonics $n_s(s) = n_{s1} \sin(ks) + n_{s2} \sin(2ks)$ where k is the helix wave number. The spherical aberrations can be compensated [3] using three octupole harmonics $n_o(s) = n_{o1} + n_{o2} \cos(ks) + n_{o3} \cos(2ks)$. Thus, we will next study the non-linear optics effects of these sextupole and octupole parameters.

PIC requires x-y coupling for equalization of the cooling decrements [3]. Coupling can be introduced in the twin-helix channel by inducing an x-y coupling resonance or by slightly offsetting the spatial period of one of the helical harmonics in the harmonics pair.

CONCLUSION

We designed a muon beam "twin-helix" channel with linear optics suitable for parametric ionization cooling. The channel is a superposition of two opposite-helicity equal-period equal-strength helical dipole harmonics and a straight normal quadrupole. We demonstrated that such a system can be adjusted to meet all of the PIC linear optics requirements. Our tracking simulations suggest that the twin-helix channel has very large dynamics aperture. There is a straightforward practical implementation of such a channel.

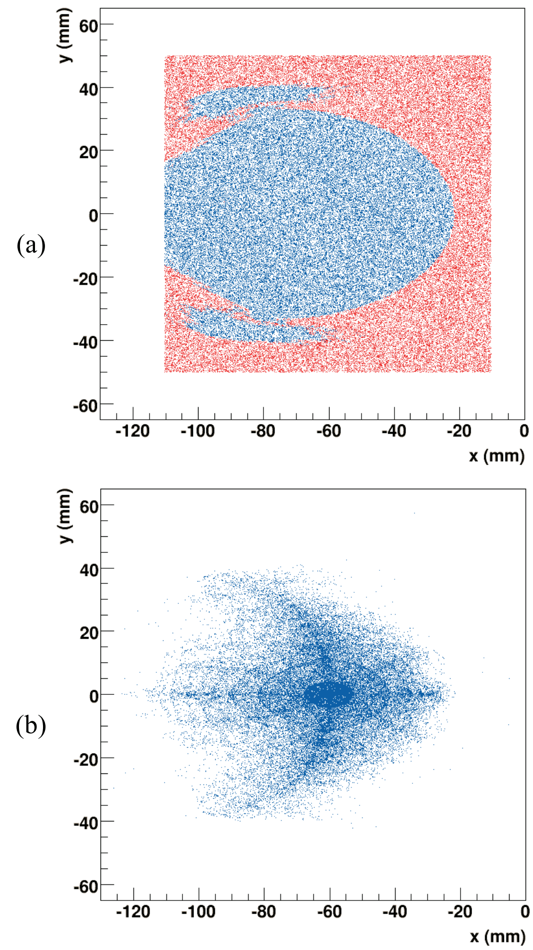


Figure 4: Initial (a) and final (b) transverse muon beam distributions in a G4beamline tracking study.

REFERENCES

[1] Y.S. Derbenev and R.P. Johnson, *Parametric-resonance Ionization Cooling and Reverse Emittance Exchange for Muon Colliders*, Proc. COOL05, AIP. Conf. Proc. 821 (2006), p. 420.

[2] Y.S. Derbenev and R.P. Johnson, *Advances in Parametric-resonance Ionization Cooling*, Proc. EPAC08, Genoa, Italy (2008), pp. 2838-2840; A. Afanasev, Y.S. Derbenev and R.P. Johnson, *Aberration-free Muon Transport Line for Extreme Ionization Cooling: a Study of Epicyclic Helical Channel*, *ibid.*, p. WEPP147.

[3] Y.S. Derbenev and R.P. Johnson, *Parametric-resonance ionization cooling*, in preparation.

[4] Y.S. Derbenev and R.P. Johnson, Phys. Rev. ST Accel. Beams 8, 041002 (2005).

[5] T. Tominaka *et al.*, Nucl. Instrum. Meth. A 459, 398 (2001).