Muon simulation at the Daya Bay site

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Abstract

With a pretty good-resolution mountain profile, we simulated the underground muon background at the Daya Bay site. To get the sea-level muon flux parameterization, a modification to the standard Gaisser's formula [1] was introduced according to the world muon data. MUSIC code [2] was used to transport muon through the mountain rock.

To deploy the simulation, first we generate a statistic sample of sea-level muon events according to the sea-level muon flux distribution formula; then calculate the slant depth of muon passing through the mountain using an interpolation method based on the digitized data of the mountain; finally transport muons through rock to get underground muon sample, from which we can get results of muon flux, mean energy, energy distribution and angular distribution.

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1 Digitization of mountain profile of the Daya Bay site

Since we have a 1:5000 topographic map of the Daya Bay area, we can use it to generate a relevant 3D mountain profile with fairly good resolution. From internet, a kind of data digitizer software named WinDIG is downloaded to digitize the contour map to get x-y-z coordinates (x-y demonstrates the location, and z is the altitude.) of sample points. The total digitized area is 3-kilometer from west to east and 4-kilometer from south to north with total sample points 0.15million. To save time, at the area far away from the proposed detector site there are less sample points digitized. Fig. 1 shows a 3D profile of the mountain generated by ROOT package using the sample points.

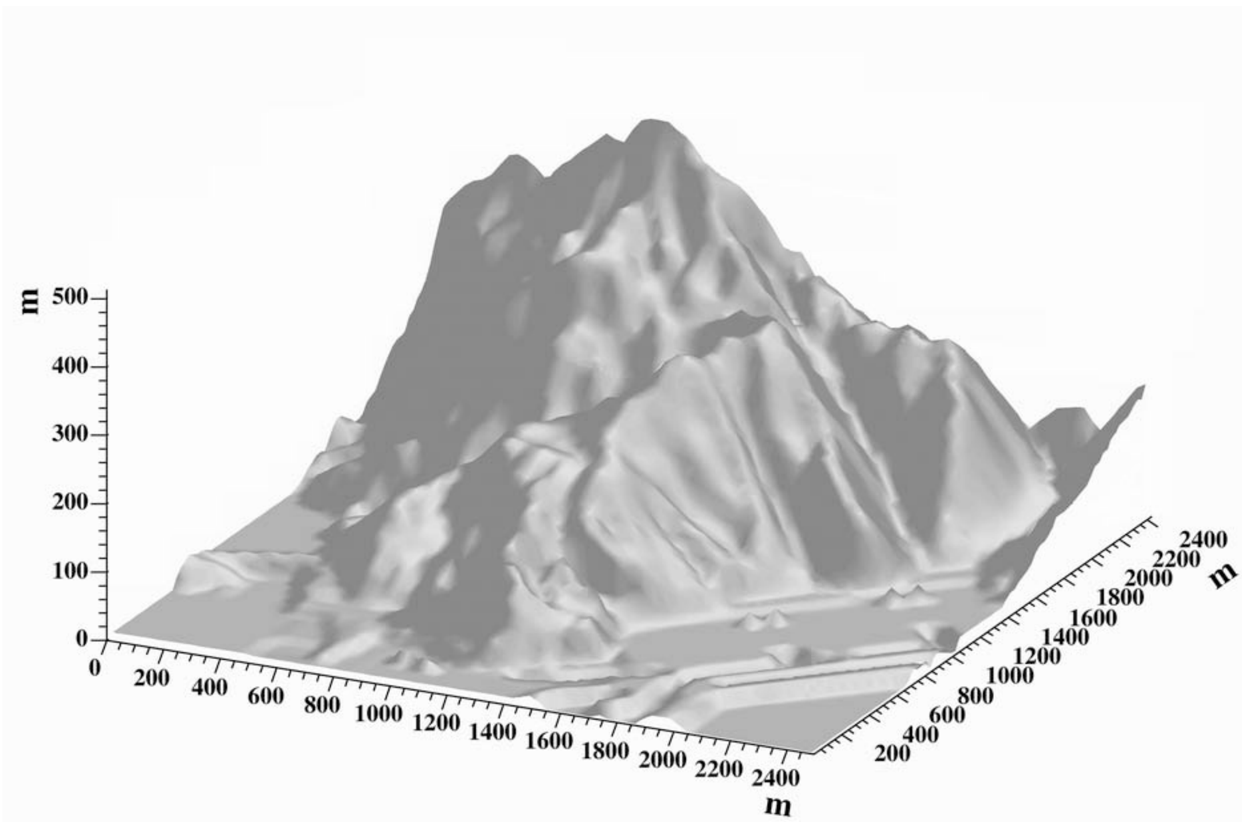

Figure 1: *3D profile of the Daya Bay area generated by ROOT.*

2 A parameterization of the sea-level muon flux

Muon flux at sea-level usually can be described by the standard Gaisser's formula [1]:

$$\frac{dI}{dE_\mu d\cos\theta} = 0.14(\frac{E_\mu}{GeV})^{-2.7}\left[\frac{1}{1+\frac{1.1E_\mu\cos\theta}{115GeV}}+\frac{0.054}{1+\frac{1.1E_\mu\cos\theta}{850GeV}}\right] \quad (1)$$

In this formula, θ is the polar angle, E_μ is the energy. There are two conditions neglected in this formula, which are the muon decay and the curvature of the earth. To obey the second condition, $\theta < 70°$ is needed. Due to the first reason, the standard Gaisser's formula cannot describe the experimental results at low energy well, we modified the formula by adding a term to the standard formula and doing fit with world muon data to get the parameters. Function 2 shows the form of this modification.

$$\frac{dI}{dE_\mu d\cos\theta} = 0.14\left(\frac{E_\mu}{GeV}(1+\frac{3.64GeV}{E_\mu[\cos\theta^*]^{1.29}})\right)^{-2.7}\left[\frac{1}{1+\frac{1.1E_\mu\cos\theta^*}{115GeV}}+\frac{0.054}{1+\frac{1.1E_\mu\cos\theta^*}{850GeV}}\right] \quad (2)$$

where

$$\cos\theta^* = \sqrt{\frac{(\cos\theta)^2+P_1^2+P_2(\cos\theta)^{P_3}+P_4(\cos\theta)^{P_5}}{1+P_1^2+P_2+P_4}} \quad (3)$$

is given by Chirkin [3]. In this paper he gave a series of parameters ($P_1 = 0.102573,\ P_2 = -0.068287,\ P_3 = 0.958633,\ P_4 = 0.0407253,\ P_5 = 0.817285$) using the CORSIKA simulation package and supposing the depth of the atmosphere as 114.8 g/cm^2 and 19.3 km. According to references [4][5], when taking the curvature of the earth into consideration, the difference between the observed zenith angle on the ground and the zenith angle at muon production at the top of the atmosphere will give reason to this modification. Fig. 2 demonstrates this relation.

About the term added to the standard formula, it can give better expression at low energy. while the energy goes high this term is negligible. To get the constant *3.64* and the index *1.29*, we fit the formula with the world muon experimental data. In mathematical view of point, that formula is much better than the standard formula, which can be found in Fig. 3.

In reference [11], some early experimental results of vertical muon intensity with different depth of material that muon transport through in standard rock can be found. According

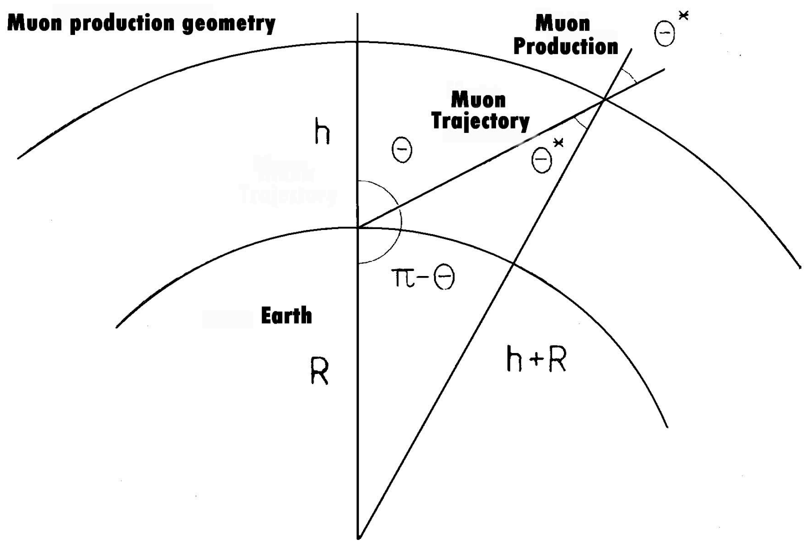

Figure 2: *Relation of the observed zenith angle of muons to the zenith angle at production at the top of the atmosphere. R is the radius of the earth. [4][5]*

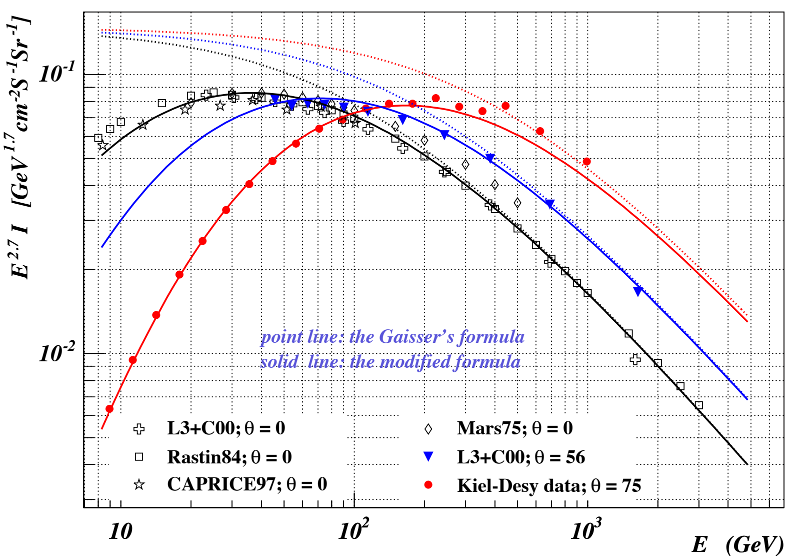

Figure 3: *The best-fit result to the experimental data. From this figure, the modified formula could fairly match the experimental data in different zenith angles with energy higher than several tens GeV . The data are quoted from [6] [7] [8] [9] [10].*

to the paper, depth of these entire data except the latest result with the lowest overburden is the depth below top of the atmosphere. So the consistency between the simulated data and the experimental data should be better than that Fig. 4 tells.

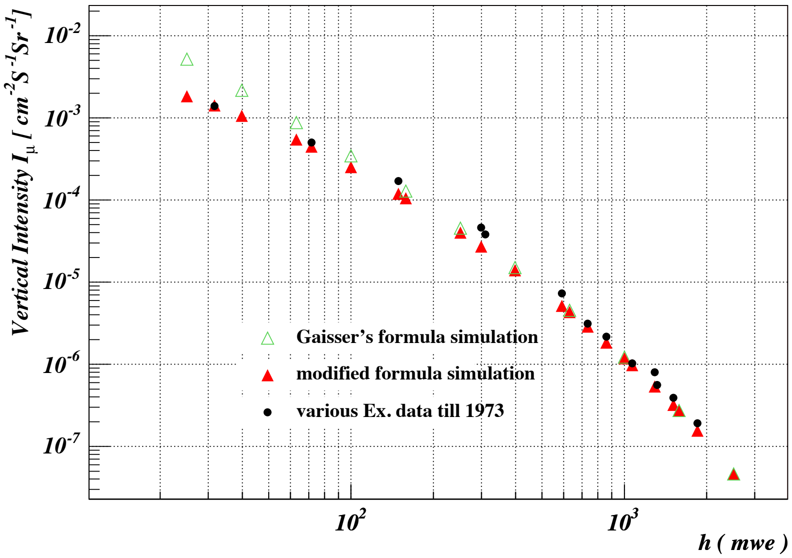

Figure 4: *Average vertical muon intensity versus depth in standard rock. Black points are experimental data from reference [11]; red solid triangles stand for the simulated results using the modified formula; green hollow triangles demonstrate the simulated results using the standard Gaisser's formula.*

3 A random variable generator

According to the simulation method, we need random variable generator to generate sea-level muon events according to the sea-level muon flux distribution formula. Next algorithm named **discrete approaching** is used:
1. Divide the energy range and the range of angle into equal bins;
2. Calculate the integral of each bin according to the formula;
3. Add the bins one by one together to get a 1D cumulative probability distribution series;
4. Generate a evenly random number in(0, 1), compare this number with the above series (need normalization) to find a right bin with nearest bin boundary;
5. Use another random number in (0, 1) to pick up a (E, theta) in this right bin.
Using double-precision variables to calculate the integral of each bin, this algorithm can

generate correct random numbers according to a function like the standard Gaisser's formula; Fig. 5 shows the consistency between the distribution of the generated variables and the formula.

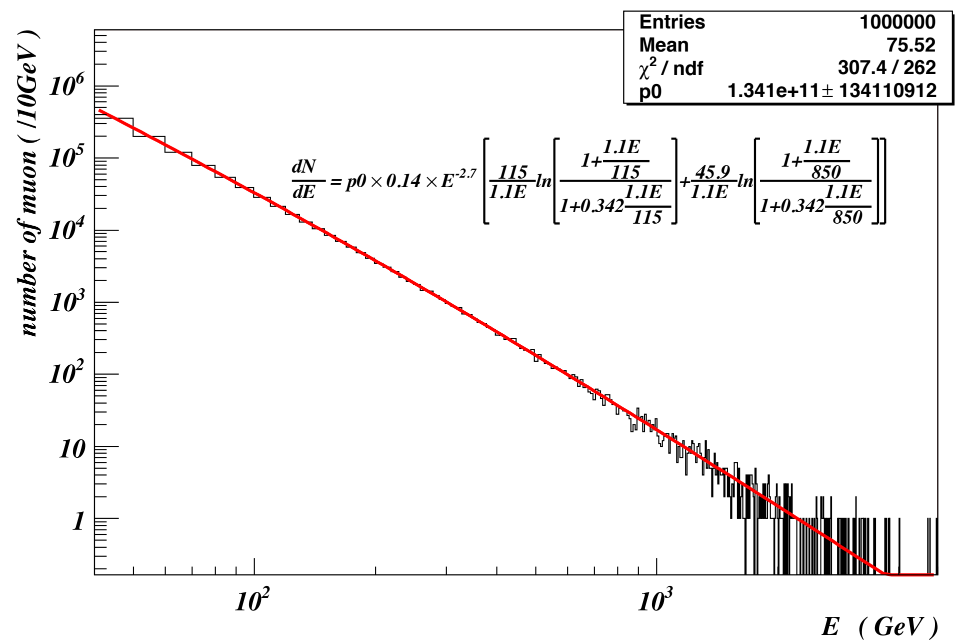

Figure 5: *Distribution of muon energy generated with 2D random numbers based on the standard Gaisser's formula. Red line is the fit of the distribution using the exact integral distribution function.*

4 A method to do interpolation

The basic idea of this interpolation can be described as that: supposing two points and their function values, if they are closely located their values are expected to be close, if they are distant it is probable that their values are quite different. In estimation of a value at a certain point, we should put more weight on function values of near sample points than those of distant points. We got the digitized data of the mountain in xyz coordinate system. But when calculating the slant depth through the mountain, it is easier to put the proposed underground site at the origin of the coordinate system and use polar coordinate system. We think about a kind of **weighted average** to deal with this problem. A formula like this is used:

$$R(\theta_0, \phi_0) \;=\; \frac{\sum \omega_i(\delta\theta) r_i(\theta, \phi)}{\sum \omega_i(\delta\theta)} \tag{4}$$

where, R and r are the distance from the underground site to the surface. $\delta\theta$ is the inclination of the direction (θ, ϕ) to the direction (θ_0, ϕ_0) ; $\omega(\delta\theta)$ is the weight function of the sample points; $\sum$ stand for the sum of the sample points in adjacent region. In the polar coordinate system there can be more than one r values in certain direction (θ, ϕ). By using a scanning algorithm, we can conveniently find the point of intersection:

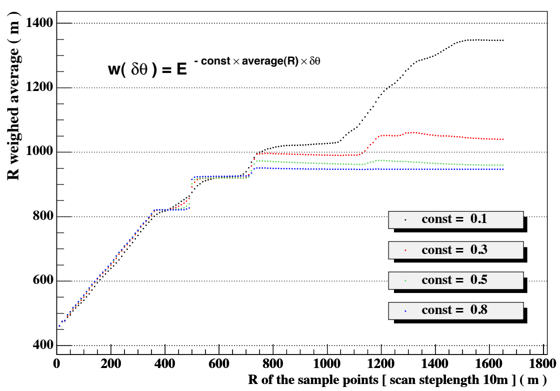

Figure 6: *Calculate weighted average with different group of sample points; several plateau emerges.*

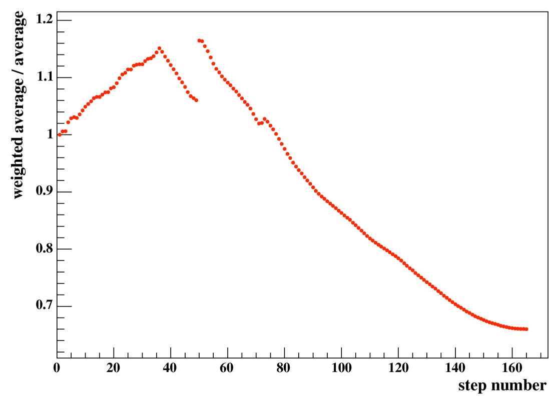

Figure 7: *Given const=0.8; there could be a point of intersection in each PEAK location.*

1, the sample points in adjacent region can be divided to a series of sub-groups according to the r *value of the sample points;*
2, calculate weighted average with different group of sample points when $r < R$;
3, increasing r *step by step, scanning along the* r *direction to find points of intersection, one can get Fig 6.*
4, divide the weighted average by the average of r *step by step. the plot Fig 7 shows the result.*

If we choose the weight function such as that shown in Fig 6, using the above algorithm, the intersections of the muon track with the mountain profile can be obtained. However, there could be fake intersections found through this method. One needs to search around this intersection to make sure that there are sample points around, which means a real intersection. Fig 8 shows the comparison of the result using ROOT interpolation method in the xyz coordinate system with that deploying above weighted average algorithm in the

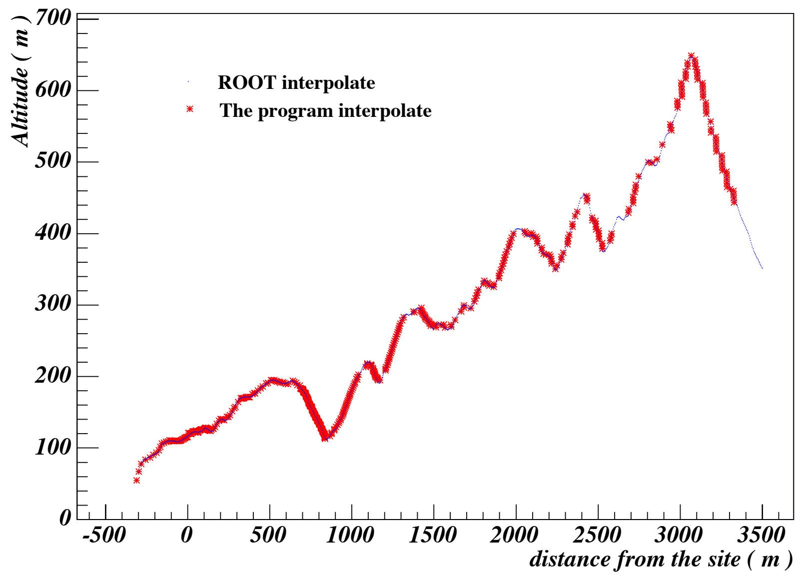

Figure 8: *Comparison of 2D mountain profile from the Daya Bay near site with azimuthal angle of* 115°.

polar coordinate system. We can see fairly good result through those two methods, but the weighted average method can save much CPU time.

5 Results of underground muon simulation

The lower edge of sampling energy of the surface muon is determined by the formula in reference [12]:

$$E_\circ^{min} = \epsilon \left(e^{\frac{X}{\xi}} - 1 \right) \tag{5}$$

where X is determined by the shortest track-length for muons in rock, $\xi \approx 2.5 \times 10^5 \; g/cm^2$ $\epsilon \sim 500 \; GeV$. The next results is calculated under the condition that the range of muon zenith angle is chosen from 0° to 75°. For the zenith angle greater than 75°, because of geomagnetic field effect [13], situation is much more complex to parameterize the muon spectrum. However, further calculation shows about 5% more in muon flux and about 5% higher in muon average energy when extrapolating the modified Gaisser's formula to full zenith angle range.

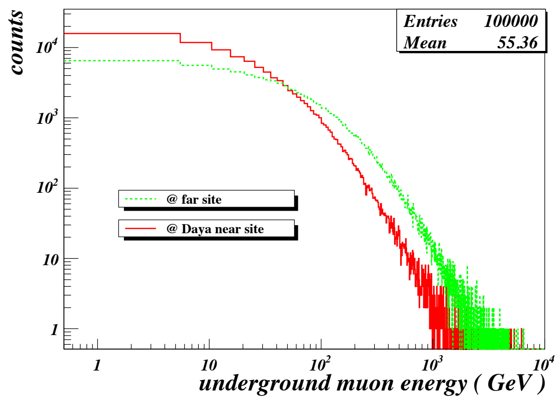

Figure 9: E_μ *distribution.*

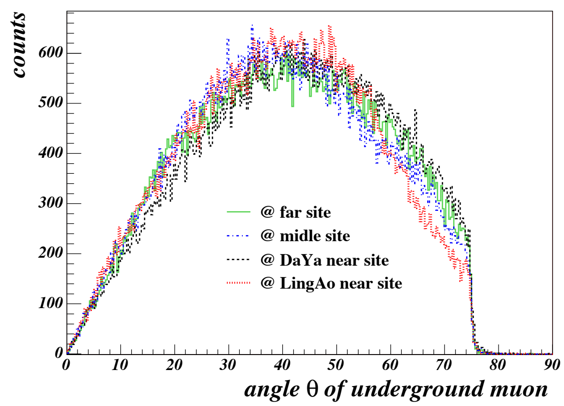

Figure 10: θ *distribution.*

For the final determined detector sites, we did the simulation and obtained the results shown in table 1. Fig 9 shows the energy distribution of underground muons at the Daya near site and at the far site. Fig 10 presents the θ distribution of the incoming muons at four sites, and the muon ϕ angle distribution is shown in fig 11.

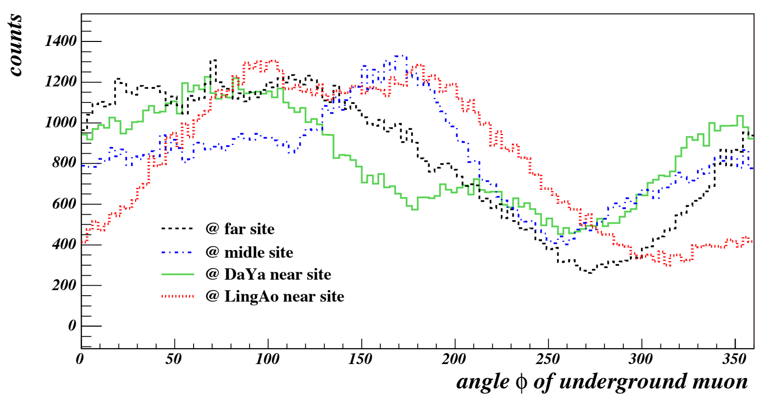

Figure 11: ϕ *distribution.*

We also did the same simulations at various locations through out the Daya bay area to show the real impact of the mountain profile to the muon flux, which was an input to the sensitivity study. Fig 12, 13, 14, 15 show a part of those results. In those plots, there is three numbers at every location. They are altitude (m), muon Flux (Hz/m^2) and average energy (GeV) respectively. The background picture is the scattering-point contour-line map of the relevant area. Stars in the plots stand for the location of the reactor cores, and the red line is the perpendicular bisector of the corresponding group of reactor cores.

Table 1: Results of underground muon simulation. Altitude in this table is given by the original map which was used to do mountain digitization.

sites	altitude (m)	muon flux (Hz/m^2)	average energy (GeV)
Daya Bay near	98	1.2	55.3
LingAo near	112	0.70	61.4
Middle	208	0.17	98.3
Far	356	0.041	140.3

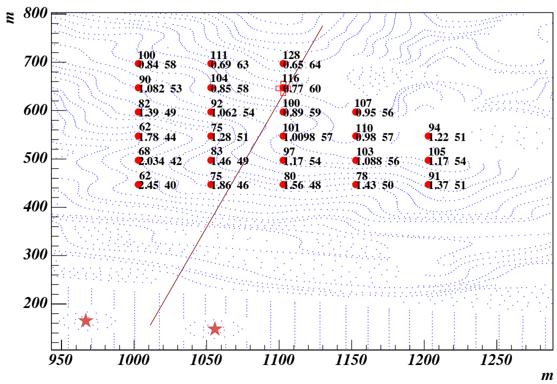

Figure 12: *Daya Bay near site scenario.*

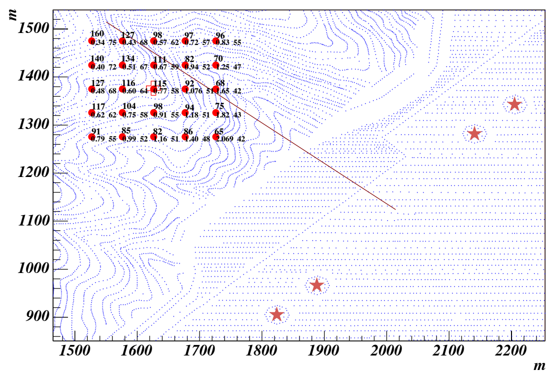

Figure 13: *LingAo near site scenario.*

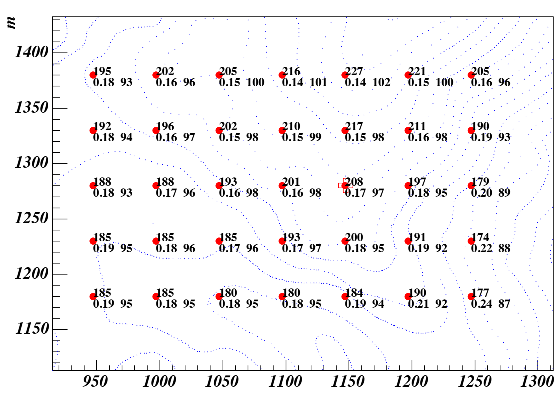

Figure 14: *Middle site scenario.*

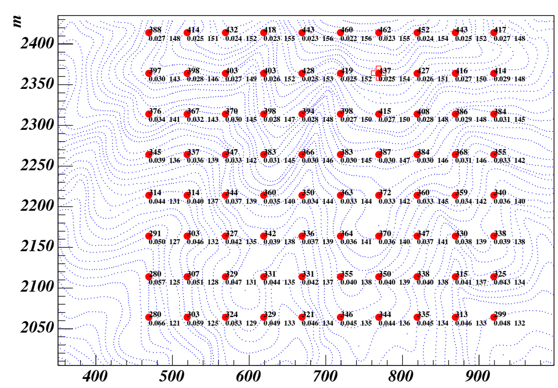

Figure 15: *Far site scenario.*

6 Summary

This note describes the method we used to simulate the underground muon at the Daya Bay site. With the real mountain profile, we obtained muon flux and the average energy at various underground locations in the Daya Bay area, which will do good to the baseline design and detector design.

Acknowledgements

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