INL/EXT-11-23600

# Integration of Reliability with Mechanistic Thermalhydraulics: Report on Approach and Test Problem Results

***Completion of Level 2 Milestone M2L11IN07040204 – "Report on Approach and Test Problem Results"***

J. S. Schroeder
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July 2011

Idaho National Laboratory

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**Prepared for the**
**U.S. Department of Energy**
**Office of Nuclear Energy**
**Under DOE Idaho Operations Office**
**Contract DE-AC07-05ID14517**

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# TABLE OF CONTENTS

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# 1 Introduction

## 1.1 Background

The Risk-Informed Safety Margin Characterization (RISMC) pathway of the Light Water Reactor Sustainability Program is developing simulation-based methods and tools for analyzing safety margin from a modern perspective. [1]

There are multiple definitions of "margin." One class of definitions defines margin in terms of the distance between a point estimate of a given performance parameter (such as peak clad temperature), and a point-value acceptance criterion defined for that parameter (such as 2200$^{o}$ F). The present perspective on margin is that it relates to the *probability* of failure, and not just the distance between a nominal operating point and a criterion. In this work, margin is characterized through a probabilistic analysis of the "loads" imposed on systems, structures, and components, and their "capacity" to resist those loads without failing. Given the probabilistic load and capacity spectra, one can assess the probability that load exceeds capacity, leading to component failure.

Within the project, we refer to a plot of these probabilistic spectra as "the logo." Refer to Figure 1 for a notional illustration. The implications of referring to "the logo" are (1) RISMC is focused on being able to analyze loads and spectra probabilistically, and (2) calling it "the logo" tacitly acknowledges that it is a highly simplified picture: meaningful analysis of a given component failure mode may require development of probabilistic spectra for multiple physical parameters, and in many practical cases, "load" and "capacity" will not vary independently.

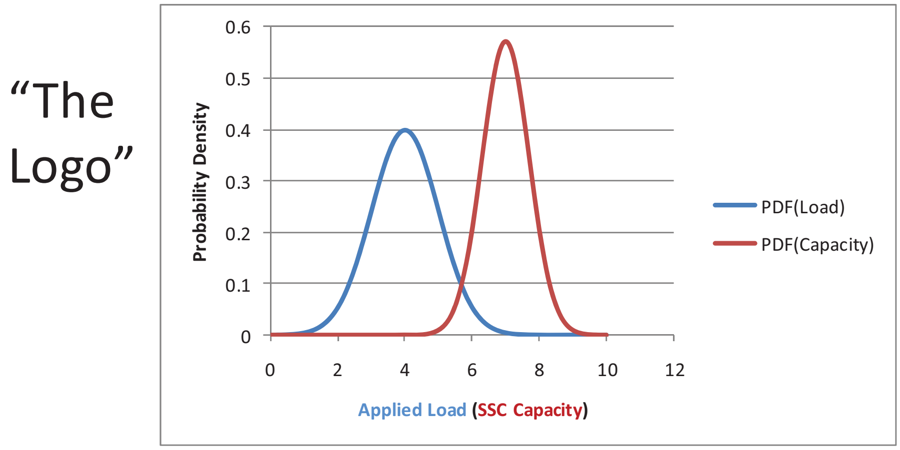


Figure 1. "The Logo:" just a position of a probabilistic load spectrum and a probabilistic capacity spectrum.

Refer to Figure 2, which shows how work is organized within the RISMC pathway. The portion in the upper right labeled "Technical Framework for Margin Management" is where the safety case is formulated in terms of "margins," analyzed in the manner stated above. The simulation tool for analyzing margin, R7, is indicated in the blue portion in the middle of the figure. This diagram shows R7 as an integration of the purely "Mechanistic Simulation of Phenomenology" and the "Generation / Quantification of Scenarios." That integration is the subject of this report.

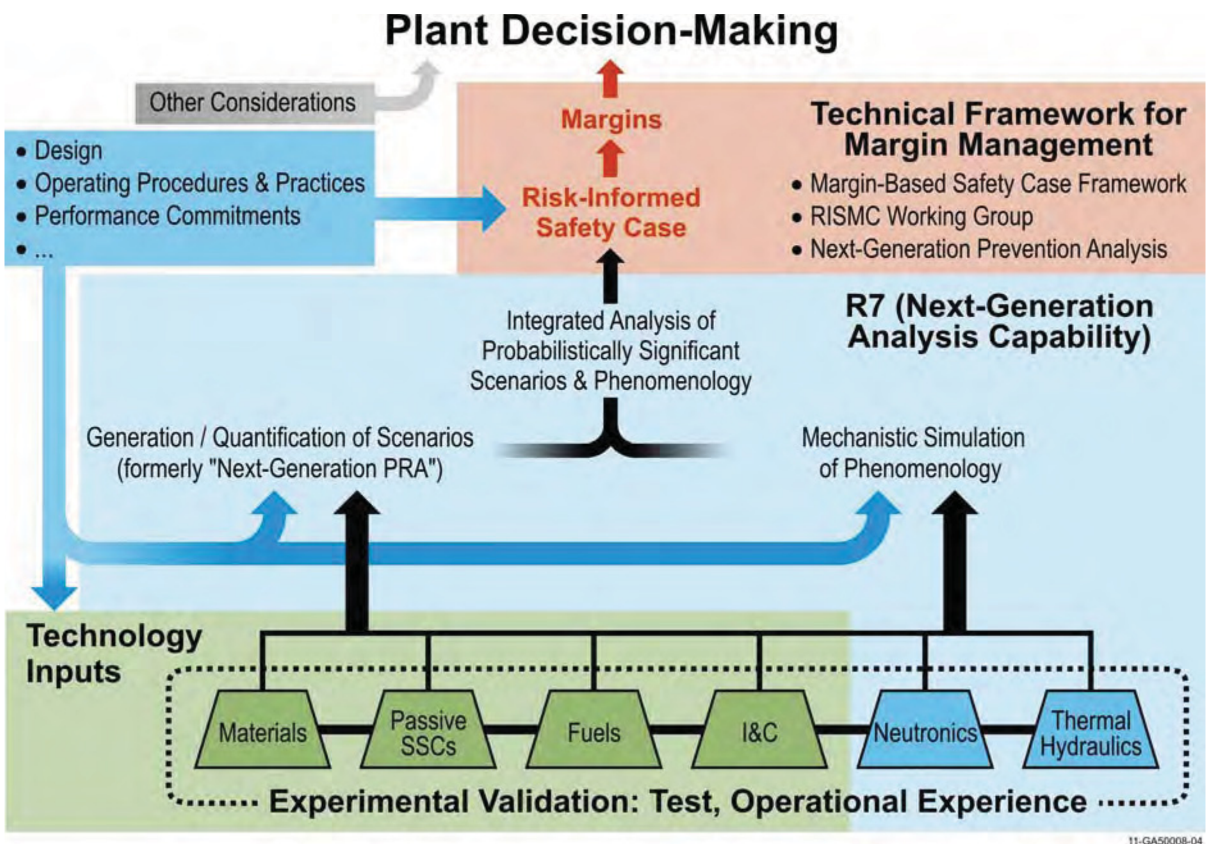

Figure 2. The Risk-Informed Safety Margin Characterization (RISMC) Pathway.

## 1.2 The Problem addressed in This Task

A completely mechanistic simulation of the behavior of a system unambiguously predicts that system's future evolution from physical laws. The solver in R7 is well equipped to do precisely that. However, in order to "do the logo," we need not only to model the physics, but also to include variations in component behavior. Unfortunately, in practice, we do not know when a given component will fail, although we know something about the distribution of failure times for a nominally homogeneous population of components; correspondingly, in reliability analysis, component failure is generally treated with models that are aleatory in nature (rather than mechanistic). Moreover, although we do not know when a component will fail, theory and practice show that certain stressors acting on certain components will change their time of failure; this limits the applicability of a simple approach based on presuming a component failure time *a priori* in a given simulation, without regard to time-history-specific stressors.

The problem treated in this report is how to deal with the latter considerations within a simulation code that is based on a physics solver.

## 1.3 Summary

Material provided in the attachments shows how to put aleatory degrees of freedom into the simulation to reflect component reliability, allowing for time-history-specific stressors on the components, without changing the way in which the solver works. Attachment 1 summarizes the conceptual background; Attachments 2 and 3 are actually part of the living documentation of R7 itself. This demonstration accomplishes the purpose of this task, and correspondingly satisfies the milestone. It presently appears that this way of addressing aleatory models in R7 can be applied more widely than just to component reliability. Suitably generalized, it might be applied to human performance as well.

The basic idea is described in the paper "Heartbeat Model for Component Failure Time in Simulation of Plant Behavior" [Ref. 2], furnished here as Appendix 1. The idea is based on an analogy between a homogeneous population of components and a homogeneous population of people. Note that "homogeneous" does not mean "absolutely identical," which would imply that all components in the designated population fail at the same instant, and all people in their designated population die at the same instant. A homogeneous population of light bulbs will fail at different times, even if they are all constantly "on" at the same voltage, and an analogous idea pertains to the people. Many years ago, it was popularly imagined that each human is born with a particular quota of heartbeats, and when these are consumed, that human dies, unless some other cause of death has occurred first. Within this model, a lifestyle that causes an average increase in heart rate will lead to a shorter life; but at identical stress levels imposed on a population of people, there will still be a variation in their life spans. Similarly, a homogeneous population of components will have different failure times, each of which could be shortened if certain kinds of wear and tear were to accumulate more rapidly, owing to application of different stressors to the components (e.g., different voltages to the light bulbs).

This "heartbeat" idea is not currently taken seriously for people, but serves as a metaphor for the "cumulative damage model of component failure." A given component will fail when its cumulative damage (wear and tear) exceeds that component's threshold. The probability density function of component failure time is then seen as a distribution over cumulative damage thresholds, assuming a constant nominal rate of damage accumulation.

In the present implementation, when a component failure mode (implying an aleatory degree of freedom) is spawned into the simulation of a given time history, random sampling is done to determine that component's destiny (the value of the cumulative damage *threshold* at which that component will fail), and the evolution of the system is completely mechanistic thereafter. This is like drawing a component at random from the homogeneous population. As the simulation progresses in time, it tracks the accumulated "damage" (the wear and tear) to that component, based on current values of the stressors, and compares the current level of damage with the component-specific, time-history-specific failure threshold. This calculation can easily reflect scenario-specific variations in the rate of damage accumulation. A suitable collection of time histories will appropriately sample the range of this component's reliability behavior in the context of the actual stressors experienced in that environment.

The above idea can be applied either to active components or to phenomena such as crack growth. The practical point is to keep the aleatory contribution out of the loop of the physics solver. An R7-compatible model of crack growth is described in Reference 4 and has been implemented in R7.

As an integral part of R7 development, code documentation is developed concurrently with source code [3]. Accordingly, the R7 implementation of the above idea for active components and for passive components is documented as part of R7's overall documentation, and the relevant chapters of Ref. 3 (chapter 24 and Chapter 16 respectively) are provided here as Appendices 2 and 3. Appendix 2 is based on Reference 2; Appendix 3 is based on work documented in Reference 4.

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# APPENDIX 1

## Heartbeat Model for Component Failure Time in Simulation of Plant Behavior

INL/CON-10-20039
PREPRINT

# Heartbeat Model for Component Failure Time in Simulation of Plant Behavior

## ANS PSA 2011

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March 2011

*ANS PSA 2011 International Topical Meeting on Probabilistic Safety Assessment and Analysis*
Wilmington, NC, March 13-17, 2011, on CD-ROM, American Nuclear Society, LaGrange Park, IL (2011)

# HEARTBEAT MODEL FOR COMPONENT FAILURE TIME IN SIMULATION OF PLANT BEHAVIOR

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## ABSTRACT

As part of the Department of Energy's "Light Water Reactor Sustainability Program" (LWRSP), we are developing a methodology and associated tools for risk-informed characterization of safety margin that can be used to support decision-making about plant life extension beyond the first license renewal. Beginning with the traditional discussion of "margin" in terms of a "load" (a physical challenge to system or component function) and a "capacity" (the capability of that system or component to accommodate the challenge), we are developing the capability to characterize realistic probabilistic load and capacity spectra, reflecting both aleatory and epistemic uncertainty in system behavior. This way of thinking about margin comports with work done in the last 10 years. However, current capabilities to model in this way are limited: it is currently possible, but difficult, to validly simulate enough time histories to support quantification in realistic problems, and the treatment of environmental influences on reliability is relatively artificial in many existing applications. The INL is working on a next-generation safety analysis capability (widely referred to as "R7") that will enable a much better integration of reliability- and phenomenology-related aspects of margin. In this paper, we show how to implement cumulative damage ("heartbeat") models for component reliability that lend themselves naturally to being included as part of the phenomenology simulation. Implementation of this modeling approach relies on the way in which the phenomenology simulation implements dynamic time step management. Within this approach, component failures influence the phenomenology, and the phenomenology influences the component failures.

*Key Words*: Safety margin, reliability, phenomenology, cumulative damage, R7

## 1 INTRODUCTION

Coupling reliability and phenomenology in simulation of complex systems has been a research topic for many years. It has long been known how to assess complex system reliability efficiently by simulating time histories, provided that certain simplifying assumptions are made about component reliability; and it has long been possible to simulate phenomenological (e.g., thermal/hydraulic, or T/H) behavior, typically with component success and failure status dictated by the simulation user in an input deck for the phenomenological simulator (or perhaps modeled in the simulator's "control" system). However, simulating reliability and phenomenology together is more difficult, especially if the plant's physical state influences component behavior. Recent decades have seen "dynamic probabilistic risk assessment (PRA)" [1, 2, 3] efforts to improve on this, but we do not know of existing T/H simulators that address time-history-specific phenomenological influences on component reliability within the simulation, except in a highly simplified way (e.g., extremely coarse sampling).

Currently, the INL is developing a "Next-Generation Systems Analysis Code," commonly called R7 [4, 5]. R7 simulates the behavior of reactor systems, and moreover does so in a way that supports realistic decisionmaking: namely, by analyzing probabilistically-significant classes of scenarios that are chosen through the sampling of key aleatory and epistemic variables. The development of R7 so far includes certain aleatory variables, and even some coupling between component behavior and the physical plant state. This paper presents and illustrates a class of component reliability models suitable for incorporation into R7, in which the component's aleatory behavior is coupled to the plant's physical state in a natural way.

Section 2.1 compares two approaches to simulation of reliability time histories, and suggests that one of these two approaches comports with R7's dynamic approach to time step control. Section 2.2 then presents and illustrates a class of component reliability models that fit naturally into this simulation approach. These models are not in themselves new, but they lend themselves to implementation in R7. Section 2.3 generalizes the discussion from its implicit focus on active components (such as pumps) to passive components (such as pipes and vessels). Finally, Section 3 offers interim conclusions.

## 2 RELIABILITY MODELING

### 2.1 Event-Driven Simulation Versus "Exhaustive" Simulation

Suppose that we wish to estimate the reliability of a system whose complexity makes it impractical to compute reliability directly (for example, by evaluating a closed-form expression for reliability in terms of component reliability models). One common way of estimating complex system reliability is to simulate a large number of time histories, sampling over different instantiations of component failure time; if the sampling is done appropriately, then the reliability can be calculated in terms of the number of time histories in which the system succeeded, and the total number of time histories simulated.

One way of simulating a time history is the following.

(1) initialize the component states,

(2) increment the system clock by a small time step $dt$,

(3) for each component,

sample a random number and compare it with $\lambda * dt$ to determine whether that component fails in the current time step; if so, propagate the effects of that state change through the system configuration;

(4) return to step (2);

continue iterating until the entire mission has been analyzed, tracking reliability, availability, and performance-related metrics as appropriate throughout the current time history.

Another way of developing time histories is the following. Instead of determining each component's failure time by marching along the time axis one $dt$ at a time, and waiting for a random number generator to decide which time step will yield a transition, sample once per component to determine its failure time directly from its failure time distribution. This is illustrated in Figure 1; the random-number sampling process for this component furnished the random number 0.7, which yields a failure time of about 51. Carrying out a similar process for all of the aleatory degrees of freedom, and knowing all of the scheduled events (like testing) *a priori*, one immediately determines all of the state transition times in this time history; one can then immediately assess the reliability / availability / performance (RAP) metrics in this time history, and move on to the next time history.

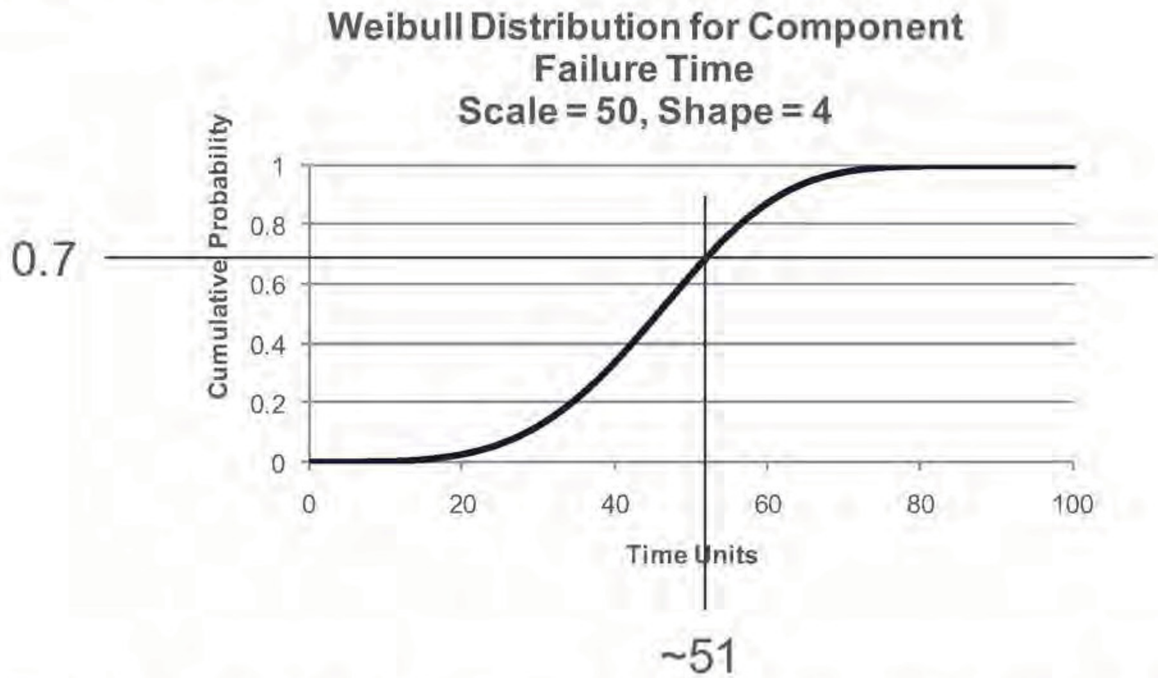


**Figure 1. Sampling from the cumulative failure time distribution to determine a time-history-specific component failure time.**

This latter approach (most commonly called "event-driven" simulation) has been used for generations to assess RAP metrics for very large-scale, very complex systems (an early example is furnished in [6]). It is much faster than evaluating RAP metrics over a long mission time by generating stochastic events in each small time interval *dt*. But in event-driven simulation, dependencies among components, or between component behavior and current physical state, are either not reflected, or are modeled rather selectively, because each component's failure time distribution is written down *a priori*, and does not reflect time-history-specific developments that might influence the component's behavior in that time history. "Exhaustive" simulation, which generates state transitions (or not) in each time step *dt*, "knows" everything about past history and current state, and can model a broad range of influences on the stochastic or deterministic state transitions based on this knowledge, but this modeling flexibility comes at a high price in terms of execution time.

## 2.2 Cumulative Damage Models for Component Reliability

The literature of accelerated life testing relates component reliability characteristics to the imposition of stressors, such as elevated temperature or mechanical loading. The illustration presented here is based on Weibull distributions, which are widely used in reliability modeling, and which lend themselves to interpretation in terms of a "cumulative damage" idea, but others can be used. See [7] among many other references on this general topic.

The general idea of the cumulative damage model is that damage to a component accumulates over time as a function of the applied stresses, and that a given component fails when its particular damage threshold is exceeded. In this picture, a cumulative distribution of failure time is interpreted as the distribution over a component population of the times at which those components' damage thresholds will be reached. At nominal conditions (constant stressors), the reliability is written as follows, assuming a damage rate that is constant in time:

$$R(t,\eta,\beta) = e^{-(\frac{t}{\eta})^{\beta}},$$

where

$R$ is the reliability as a function of time,

$t$ is time,

$\eta$ is the scale factor,

$\beta$ is the shape factor.

The cumulative distribution in Figure 1 above is

$$F(t,\eta,\beta) = 1 - R(t,\eta,\beta),$$

with shape and scale parameters as given in the figure.

In order to understand what this formulation is saying, suppose that the shape parameter $\beta >> 1$. Then when $t < \eta$, $(t/\eta)^\wedge\beta$ is a very small number, $R \sim 1$ and $F \sim 0$. The equivalent vernacular statement is that it typically takes about $\eta$ time units for damage to reach the failure threshold; the damage necessary to fail the item simply has not accumulated when $t << \eta$. However, when $t >> \eta$, the situation is reversed; the probability that the item is failed now approaches unity. The transition to failure mostly occurs around $t \sim \eta$, when sufficient damage has accumulated. The character of the shift in probability is determined by the value of $\beta$; the larger the value of $\beta$, the more narrowly the transition is focused near $t = \eta$. In a nutshell, as implied by the appearance of $t/\eta$ in the formula above, $\eta$ sets the time scale.

The cumulative damage models work by modifying $\eta$. If stressors change such that damage accumulates at twice the normal rate, then $\eta$ is halved. Moreover, at least in simple cases, they assume that the current rate of damage accumulation is determined by the current stressors, and not (for example) by history.

### 2.2.1 Arrhenius Model

The Arrhenius model is a widely used idea, according to which an underlying "reaction rate" is governed by an activation energy $E_A$ and the temperature T, thus:

$$rate \propto Ae^{-\frac{E_A}{kT}},$$

where

$A$ is a proportionality constant,

$k$ is the Boltzmann constant.

For present purposes, we associate damage rate with reaction rate. Within this formulation, then, the damage rate at a temperature $T_{current}$ differs from the damage rate at the nominal temperature $T_{nom}$ by a factor of

$$e^{\frac{E_A}{k}\left(\frac{1}{T_{nom}}-\frac{1}{T_{current}}\right)}.$$

Figure 2 compares Weibull distributions whose underlying parameters are the same except that they correspond to different temperatures, and thus have different scale parameters given by the Arrhenius model.

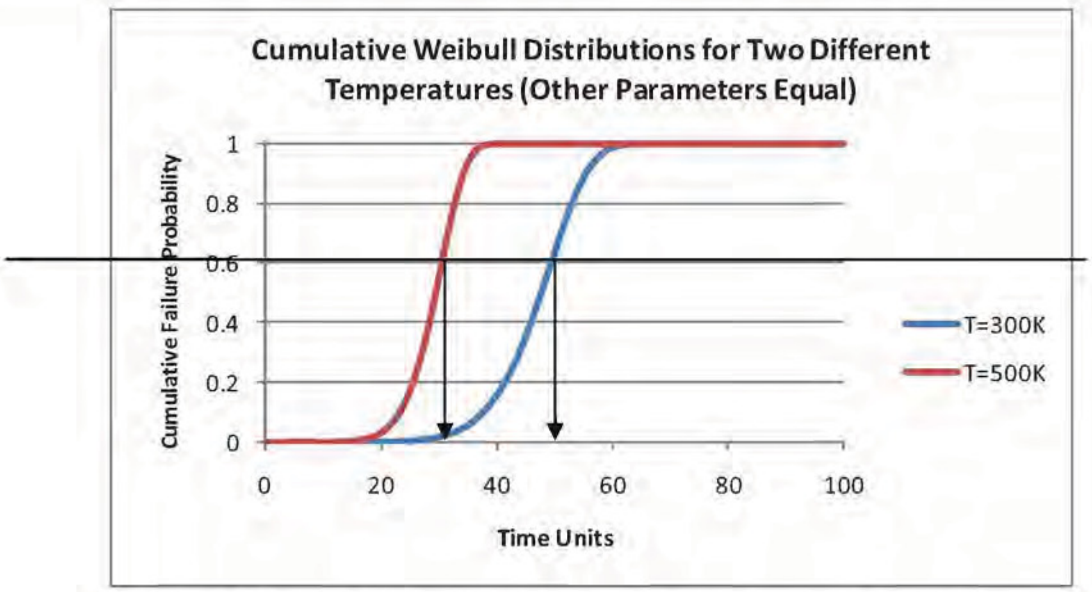


**Figure 2. Weibull distributions of failure time for two different temperatures.**

Figure 2 illustrates the point that applying the Figure 1 process to determine a component's failure time in a given time history will yield a failure time that is earlier if the component is operating at a higher temperature. (The horizontal line in Figure 2 corresponds to a sampled random number of about 0.6, which intercepts the T=300K curve at around 50 time units, but intercepts the T=500K curve at around 30 time units.)

Consider now the case where a component is subjected to varying temperature. The upper plot in Figure 3 shows an illustrative temperature history in which T=300K at time zero, rises to 500K and remains there between about $t$ = 20and $t$ = 32, and then goes back down to 300K as of about $t$ = 57. The lower plot is a plot of cumulative damage as a function of time; at constant temperature, corresponding to constant damage rate, the damage accumulation lies on a straight line (the purple line in the plot). If temperature increases, the rate of damage accumulation increases; the green line corresponds to the temperature history, and damage accumulates fastest where temperature is highest.

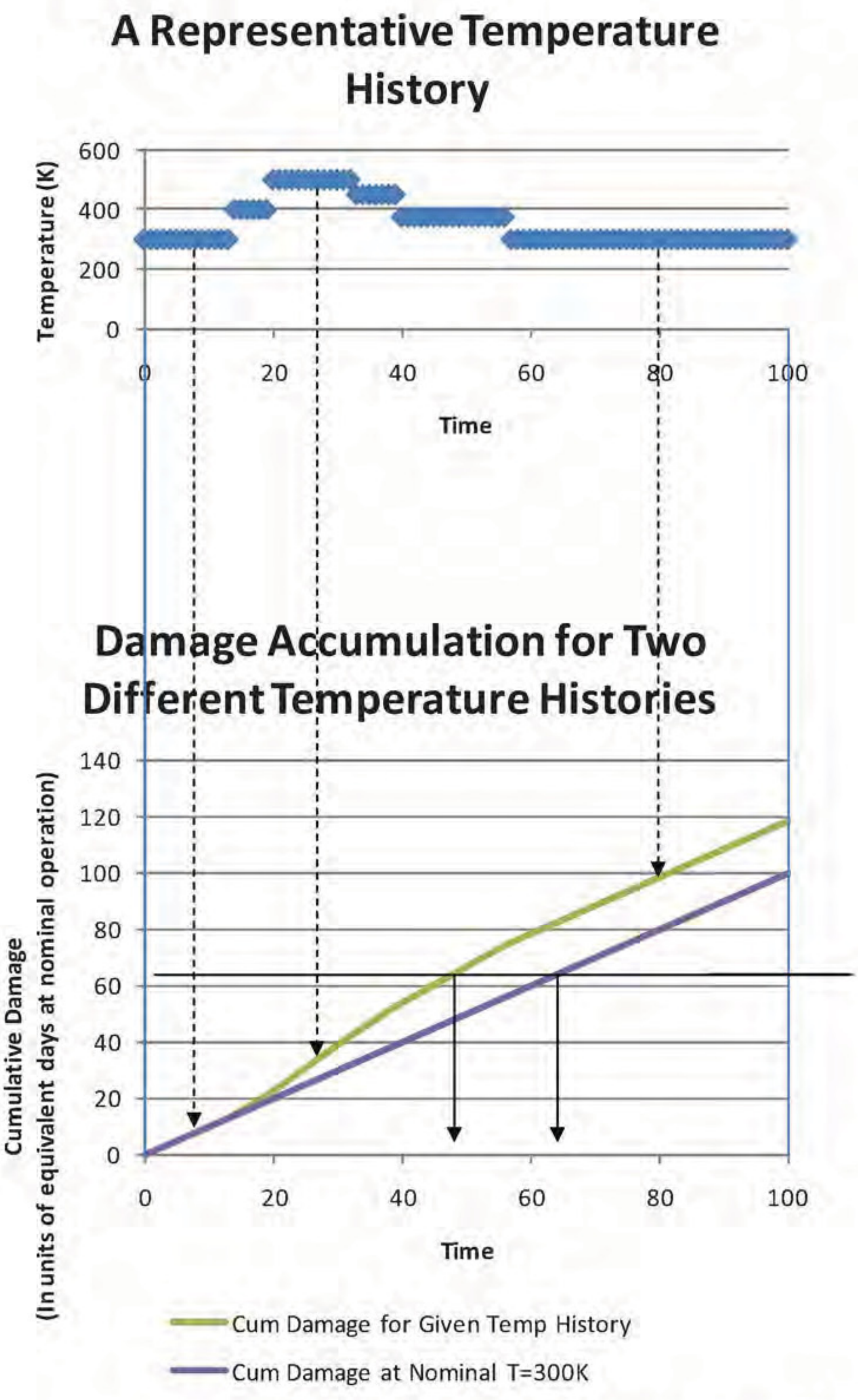


**Figure 3. Rate of Damage Accumulation Varying with Temperature.**

## 2.3 Time-Step Control and Compatibility of Cumulative Damage Models with R7

The numerics of R7 are well beyond the scope of this summary, but it is important to understand that when variables are changing rapidly, the time step is small. This is suggested notionally in Figure 4 below.

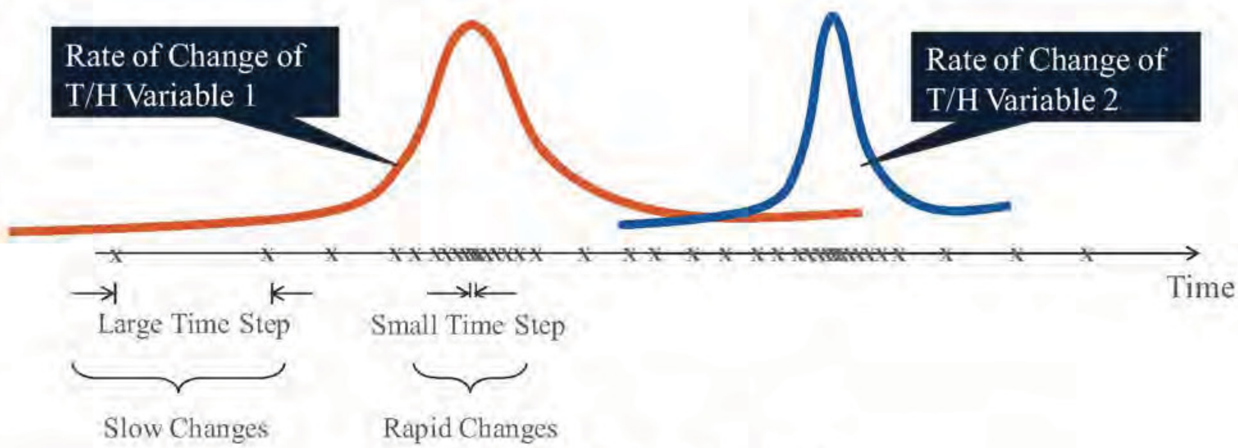


**Figure 4. Adaptive Time Stepping in R7.**

Implementation of cumulative damage models in R7 means that besides monitoring rates of change of T/H variables, as in Figure 4, R7 needs also to be aware of whether component state transitions are occurring in the near future, so that the time step can be modified appropriately. Referring back to Figure 3, for example, suppose that in a particular time history initiated at $t = 0$, a particular failure mode's aleatory damage threshold is determined to be 0.62, and the implied timing information is conveyed to R7's time step controller. That timing estimate is predicated on nominal conditions; in the case of the temperature history shown, it will become clear shortly after $t = 20$ that the damage threshold will be reached sooner than the time sampled at the beginning of the time history, and R7 will need to shrink the time step accordingly.

This leads to a notional criterion for R7 compatibility for models of aleatory component failure modes:

Based on the current plant state, the component model should be able to project a component failure time well enough to appropriately inform R7's time step control.

The simple cumulative damage models described above satisfy this criterion. In those models, when a component lifetime is spawned within a simulated time history, a single aleatory degree of freedom is sampled to initialize an estimate of failure time, and this failure time is modified with causal models as the plant state evolves.

Note that the cumulative damage models satisfy this criterion because after their aleatory aspects are sampled at the initiation of the models, they are causal in nature: the prognostication of failure time will not have a large random component that might mislead the time step control.

## 2.4 Models of Passive Component Failure

The discussion carried out above was not focused on any particular component type. In this subsection, we focus on passive components, and on degradation of those components due to factors such as mechanical cycling, thermal stress, neutron fluence, and perhaps even water chemistry.

As an example, consider the following model of passive component failure due to crack growth. In each time history, at $t = 0$, the component model is seeded with a crack distribution. This is the aleatory part of the model. Thereafter, the crack grows "mechanistically," i.e., as a result of certain mechanisms linking particular phenomenological stressors (mechanical stress, cycling, …) to crack growth. Within each time step, "damage" is accumulated, and a prognostication of component failure is developed and fed back to R7 time step control.

Even though crack growth can accelerate as catastrophic failure is approached, this, too, satisfies the R7 compatibility criterion. It is simply necessary to code the model in such a way that R7 sees the catastrophic failure coming.

## 3 SUMMARY

The class of models discussed in this paper can be applied within R7 in order to achieve full two-way coupling between component failure modes and scenario physics, as suggested in Figure 5.

This discussion would not appear relevant to evaluation of scenarios spanning only a few hours (e.g., from "initiating event" to cold shutdown). However, it currently appears that R7 has the potential to examine not only such traditionally-analyzed accident scenarios, but also very long stretches of operating time, and to do so in a way that allows for characterizing the spectrum of operating stresses to which a given component may be subjected over long periods.

In order to be R7-compatible, a component unreliability model must fit into R7's time step control paradigm. This requires models to provide a current estimate of failure time so that R7 can determine how long its time step can be without introducing additional inaccuracy.

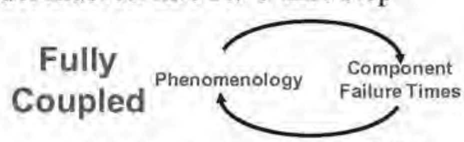

Figure 5. Full Coupling Between Phenomenology and Component Behavior.

A class of R7-compatible component reliability models was discussed above. In these "cumulative damage" models, an aleatory degree of freedom is sampled at component birth, and used, together with all the rest of the physics and component status information in the simulation, to project and update the component failure time. Depending on the component, damage (wear, total heartbeats, …) accumulates based on calendar time, or run time, or run time influenced by time-history-specific scenario physics.

Some of the parameters in some of the models in this class are not known very well at present. For now, this limits the predictive applications of this capability. However, the capability is expected to prove to be very useful for characterizing the potential implications of emergent results from materials behavior research, or cascading failure situations as well.

## 4 ACKNOWLEDGMENTS

This work was performed under Contract No. DE-AC07-051D14517.

## 5 DISCLAIMER



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# APPENDIX 2

## "Class Pipe_Pui_FSM," Chapter 16 of "R7 Documentation", describing the R7 implementation of a model of stress corrosion cracking in a pipe

# Chapter 16

# Class Pipe_Pui_FSM

COMPONENT Pipe_Pui_FSM is one of two R7 components that use an internal finite state machine (FSM) to implement stochastic calculations in the context of an R7 simulation. The other component, Pump_Pui_FSM, uses the FSM to implement a simple two-state reliability model. The Pipe_Pui_FSM component implements a multi-state physics model of passive component aging. The multi-state physics model was provided by Unwin *et al.* [ea11b] and features a cumulative damage (or heartbeat) formulation for pipe crack formation and growth.

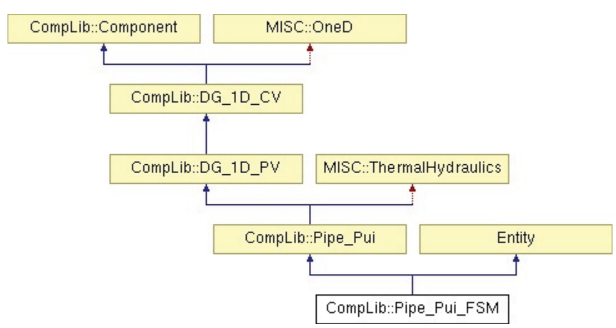


**Fig. 16.1 :** Inheritance tree for Class Pipe_Pui_FSM.

Pipe_Pui_FSM is derived from Pipe_Pui as shown in Figure 16.1. Pipe_Pui handles the pipe thermal-hydraulic state representation. Pipe_Pui_FSM extends Pipe_Pui, adding stochastic behavior to the component. The FSM functionality is incorporated by also deriving from the Entity base class as shown in the figure. The Entity class provides basic FSM functionality; component-specific behavior is produced by coding state-specific behavior into separate state objects. In this case, the pipe stochastic behavior is encoded in `PipeOwnedStates.C`.

### Background

Coupling reliability and phenomenology has been a research topic for generations. It has long been known how to assess complex system reliability efficiently by simulation of time histories, provided that certain simplifying assumptions are made about component reliability; and it has long been possible to simulate phenomenological (e.g., T/H) behavior, one time history at a time, typically with component success and failure status dictated by the simulation user in an input deck for the phenomenological simulator. However, doing the two together is more difficult. Recent decades have seen "dynamic PRA" efforts to improve on this, but we do not know of existing T/H simulators that address time-history-specific phenomenological influences on component reliability within the simulation, except in a highly simplified way (e.g., extremely coarse sampling).

Two classes of simulation-based approaches to reliability / availability analysis have been widely used. They are variously called "event-driven simulation" and "discrete-event simulation." It seems that different individuals use these names differently. Here, we will call one approach "destiny-based" and the other approach "painstaking." The traditional destiny-based approach is one in which the failure time of each component is determined *a priori* by sampling from the appropriate cumulative failure time distribution. The destiny approach has the great advantage of being much faster than approaches based on painstakingly integrating along the time axis, methodically assessing evolutions and transitions within each small $dt$. The painstaking method can model all sorts of interactions explicitly; traditionally, people have not been able to do that very well within the destiny approach, which works extremely well when everything is uncoupled and the phenomenology can be simplified.

The problem we are solving here is to unify the analyses of reliability and plant phenomenology with no more simplification than R7 already has. The present idea is to proceed by incorporating the destiny approach into R7, and couple it to phenomenology. This is distinct from addressing failure of components for essentially mechanistic reasons (such as pump failure due to loss of suction head); it is about capturing phenomenological influences on essentially aleatory failure modes.

### The Heartbeat Model and the Cumulative Damage Model

It has been suggested by some that each of us is born with a heart that is capable of a certain maximum (individually unique) number of heartbeats, and that we die when that personal number of heartbeats is consumed (if we have not died earlier as a result of other causes). Within this model, an individual's lifetime is lengthened if the average pulse rate is lowered, as may be achievable by conditioning exercise. (The pulse rate may increase during the exercise period itself, but decrease the rest of the time, incurring a net benefit.) On the other hand, chronic stress would reduce lifetime if it increased the average pulse rate. Therefore, if one were updating predictions of an individuals time of death, given knowledge of his total heartbeat endowment and his life history, one would track consumption of heartbeats to date, and model expected future consumption in terms of varying heart rates induced by life-style-induced stress. This has obvious analogs in analysis of component lifetime.

Although the heartbeat model was probably originally formulated only as a metaphor, it turns out that there is a literature of cumulative damage reliability models that is closely analogous to it. Think of an electronic component whose lifetime ends when a certain amount of damage is accumulated. Suppose further that the failure time distribution is Weibull, with a scale parameter $\eta$ and a shape parameter $\beta$, thus

$$F(t,\eta,\beta) = 1 - R(t,\eta,\beta) \tag{16.1a}$$

$$R(t,\eta,\beta) = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{16.1b}$$

where

- $F$ is the component unreliability, the probability that the item is not working at time $t$ (given that it was new and working at $t = 0$),
- $R$ is the component reliability, the probability that the item is still working at time $t$,
- $\eta$ is the Weibull distribution scale parameter,
- $\beta$ is the Weibull distribution shape parameter.

The Weibull distribution is widely discussed because its form allows it to represent a broad range of failure-time behaviors, including the simple exponential (constant-failure-rate) model, which corresponds to $\beta = 1$.

In order to understand what this formulation is saying, suppose that the shape parameter $\beta >> 1$. Then when $t < \eta, (t/\eta)^\beta$ is a very small number, $R \approx 1$ and $F \approx 0$. The equivalent vernacular statement is that it typically takes about $\eta$ time units for damage to reach the failure threshold; the damage necessary to fail the item simply has not accumulated when $t << \eta$. However, when $t >> \eta$, the situation is reversed; the probability that the item is failed now approaches unity. The transition from good to failed mostly occurs around where $t \approx \eta$, when sufficient damage has accumulated. The character of the shift in probability is determined by the value of $\beta$; the larger the value of $\beta$ , the more narrowly the transition is focused near $t = \eta$. In a nutshell, as implied by the appearance of $(t/\eta)$ in the formula above, $\eta$ sets the characteristic time scale.

The literature of the cumulative damage topic also contains models that relate stressor magnitude to $\eta$. The Arrhenius model, for example, relates temperature to aging rate. This is like elevating the pulse rate in a heartbeat model. Formulae are given below. The underlying intuition is that whatever form of damage it is that leads to failure, that damage accumulates at a rate that is influenced by the environment.

The above discussion tacitly assumes that even if the environment influences the damage rate, the rate is constant in time. In discussions of accelerated aging, the treatment generalizes to consider aging rates that are piecewise constant in time, and simply accumulate the damage as a function of time by applying the above idea within each interval of constancy, and summing damage straightforwardly as a function of time.

The spirit of the present treatment is to retain the cumulative damage idea, at least initially, while applying it to situations in which the rate of damage accumulation varies. Within an R7 time step, we will initially take the damage rate to be constant, noting that the R7 time step is adaptive, and automatically becomes small when it needs to be small for reasons of accuracy (potentially including a high rate of change in the damage rate). From this point of view, the only new thing we will be doing within R7 from a heartbeat point of view is using the basic idea to generate time histories within a powerful simulation environment.

### Tracking the Damage

As stated above, the parameter that fixes the characteristic time scale is $\eta$. The present framework operates by adjusting $\eta$ as a function of component stressors, and thereby speeding up or slowing down the heartbeat (damage) clock relative to the real time clock. It will be shown below how $\eta$ changes as a function of stressors; but generally, within this class of $\eta$-based approaches, within an R7 time step $dt$, we accumulate component damage of

$$dt\,\frac{\eta_n}{\eta_c} \tag{16.2}$$

where $\eta_n$ is the sampled aleatory characteristic lifetime conditional on nominal stress, and $\eta_c$ is the adjusted characteristic lifetime according to the current stress. Thus, for example, if

$$\eta_c \sim \frac{1}{2}\,\eta_n \tag{16.3}$$

an hour of clock time at this stress level consumes two hours' worth of heartbeats.

### Scaling the Damage Rate: Inverse Power Law

The inverse power law can be used to relate some generic stress, $V$, to characteristic lifetime, $\eta$. The scale parameter, $\eta$, of the Weibull distribution can then be expressed as

$$\eta(V) = \frac{1}{KV^n}\ \ . \tag{16.4}$$

For changing $V$ the rate of damage accumulation also changes; the characteristic lifetime $\eta$ goes down as $V$ increases.

Within an R7 time step, we therefore accumulate damage as

$$dt\,\frac{\eta_n}{\eta_c} = dt\,\frac{K{V^n}_c}{K{V^n}_n} = dt\,\left(\frac{V_c}{V_n}\right)^n\ \ . \tag{16.5}$$

### Scaling the Damage Rate: Arrhenius Model

The Arrhenius lifestress model is probably the most common lifestress relationship mentioned on the web and in accelerated life testing literature. It has been widely used when the stimulus or acceleration variable (or stress) is thermal (i.e. temperature). It is derived from the Arrhenius reaction rate equation proposed by the Swedish chemist Svandte Arrhenius in 1887. The Arrhenius reaction rate equation is

$$R(T) = Ae^{-\frac{E_A}{KT}} \tag{16.6}$$

where

$R$ is the speed of reaction,

$A$ is an unknown nonthermal constant,

$E_A$ is the activation energy (eV),

$K$ is the Boltzman's constant ($8.617 \times 10^{-5} eV K^{-1}$),

$T$ is the absolute temperature (K).

The Arrhenius life-stress model is formulated by assuming that life is proportional to the inverse reaction rate of the process, thus the Arrhenius life-stress relationship is given by

$$L(V) = Ce^{-\frac{B}{V}} \tag{16.7}$$

and the Arrhenius-Weibull model probability distribution function (PDF) is obtained by setting

$$\eta = L(V) = Ce^{-\frac{B}{V}} \quad . \tag{16.8}$$

We will, of course, still accumulate damage within an interval $dt$ according to

$$dt \, \frac{\eta_n}{\eta_c} \quad . \tag{16.9}$$

Substituting for $\eta$ gives a damage contribution within $dt$ of

$$dt \, \frac{\eta_n}{\eta_c} = dt \, \frac{Ce^{\frac{B}{T_n}}}{Ce^{\frac{B}{T_c}}} = dt \; e^{B\left(\frac{1}{T_n} - \frac{1}{T_c}\right)} \tag{16.10}$$

in which we have allowed $B$ to absorb the activation energy and the Boltzmann constant $K$.

## 16.1 Input options

Inputs for components of type `Pipe_Pui_FSM` are specified in

“INPUT/DefineComponents/ComponentName.inp”

files, where “ComponentName” is a unique name of the component, as defined in the file “INPUT/ListOfComponents.inp”. To declare the component “ComponentName” to be of the type Pipe_Pui_FSM, one must set

```
[Define]
  type = Pipe_Pui_FSM
[]
```

Currently input for components of type Pipe_Pui_FSM is identical to components of type Pipe_Pui. Parameters of the pipe cracking model are set in the ParseInputData() method, and in the constructors of the pipe state objects, and not read from the input file.

## 16.2 Governing Equations

The cumulative damage model for this component is implemented in the following way. A particular simulation trial is initialized by sampling the time to the first state transition. The sampled state transition time is referred to as the transition destiny, or $t_d$. Transitions destinies may currently be described in two ways; those not subject to cumulative damage, and those that are.

State transitions that are not subject to cumulative damage are characterized using an exponential distribution with the following probability distribution function (PDF) and cumulative distribution function (CDF)

$$f(t) = \frac{1}{\mu} e^{\left(-\frac{t}{\mu}\right)} \tag{16.11a}$$

$$F(t) = 1 - e^{\left(-\frac{t}{\mu}\right)} \tag{16.11b}$$

where $\mu$ is the mean time to a state transition event. State transition destinies, $t_d$, are then obtained by picking a uniform random variate, $U(0, 1)$, setting it equal to $F(t)$ and solving equation (16.11b) for $t$, or

$$t_d = F^{-1}(U) = -\mu \ln (1 - U) \,. \tag{16.12}$$

The R7 implementation of equation (16.12) uses random number generation routines provided by Galassi *et al.* [ea11a].

State transitions that are subject to cumulative damage are characterized using a Weibull distribution with PDF and CDF

$$f(t) = \frac{b}{a^b} t^{b-1} e^{\left[-\left(\frac{t}{a}\right)^b\right]} \tag{16.13a}$$

$$F(t) = 1 - e^{\left[-\left(\frac{t}{a}\right)^b\right]} \tag{16.13b}$$

where $a$ is a scale factor and $b$ is a shaping factor. As in the exponential case, transition destinies, $t_d$, are then obtained by picking a uniform random variate, $U(0,1)$, setting it equal to $F(t)$ and solving Eq. (16.13b) for $t$, or

$$t_d = F^{-1}(U) = a\left[-\ln(1-U)\right]^{\frac{1}{b}} . \tag{16.14}$$

After a transition destiny is obtained, cumulative damage will be evaluated as the simulation develops. This is accomplished by relating a component stressor or stressors to the Weibull scale factor, $a$, as discussed in the background sections above.

The cumulative damage calculation works by adjusting $a$ as a function of the stressors, and thereby speeding up or slowing down the damage rate relative to the simulation clock. In each R7 simulation time step the R7 solver queries FSM components for the maximum allowable timestep. This request from the R7 solver triggers an evaluation of accumulated damage by the FSM. Damage within the current timestep is evaluated as

$$dt \frac{a_n}{a_c} \tag{16.15}$$

where

$dt$ = R7 time step,

$a_n$ = scale factor evaluated at nominal stressor level,

$a_c$ = scale factor evaluated at current stressor level.

Damage is then accumulated in each R7 time step until the total damage is equal to or greater than the sampled transition destiny, or

$$\sum_{i=0}^{N} dt_i \frac{a_{ni}}{a_{ci}} \geq t_d \tag{16.16}$$

where the current timestep is $N + 1$. If the condition in Eq. (16.16) is satisfied in the $N + 1$ timestep, the state machine will set the new state to the state specified as the transition target state. If Eq. (16.16) is not satisfied in the current timestep, the state-machine calculates a maximum timestep such that Eq. (16.16) will not be true during the next time step, given conditions in the current timestep persist through the end of the next time step. A safety factor is then applied so that if component stressors change more than expected during the next timestep, accumulated damage will not over-shoot the transition destiny significantly. The estimated maximum timestep is returned to the R7 solver, which plans the next solution timestep accordingly.

Lastly, it is possible to specify multiple transitions out of a given component state into one or more target states. The state-machine keeps track of all such transitions and acts on the first scheduled to occur in either the current or next timestep.

The multi-state physics model for this component is completely specified when all ten state transition destinies are specified. The following sections provide the equations used determine the transition destinies for each state transition in the model.

### 16.2.1 Micro Crack Initiation

The duration of the initial state, before transition to the micro crack state, is specified as follows. The Weibull model is used to relate the cumulative probability of crack initiation by time $t$ to component stressors by setting the Weibull scale factor to

$$\eta(\sigma, T) = A\sigma^n e^{\left(\frac{Q}{RT}\right)} \tag{16.17}$$

where

$A$ = a fitting parameter,

$\sigma$ = the explicit stress dependence,

$n$ = a stress exponent factor,

$Q$ = the activation energy (eV),

$T$ = the absolute temperature dependence (Kelvin),

$R$ = the universal gas constant.

The cumulative damage (heartbeat) formulation for the crack initiation time is then

$$dt\frac{\eta(\sigma_n, T_n)}{\eta(\sigma_c, T_c)} = dt\frac{A{\sigma_n}^n e^{\left(\frac{Q}{RT_n}\right)}}{A{\sigma_c}^n e^{\left(\frac{Q}{RT_c}\right)}} = dt\frac{{\sigma_n}^n}{{\sigma_c}^n} e^{\left(\frac{Q}{RT_n} - \frac{Q}{RT_c}\right)} \tag{16.18}$$

where the subscripts $n$ and $c$ on $\eta$, $\sigma$, and $T$ refer to nominal or reference conditions, and to the current conditions in the current time step.

### 16.2.2 Microcrack to Radial Macrocrack Transition

Following crack initiation, the transition to a radial macrocrack is represented with a response surface of the form

$$\ln \eta = 25,248T^{-1} - 0.12P - 36.4 \tag{16.19}$$

where

$\eta$ = characteristic time in the microcrack state, before macrocrack

$T$ = material temperature in degree K,

$P$ = material stress loading (pressure) in MPa

The cumulative distribution function for the distribution of transition times is

$$F(t) = 1 - e^{\left[-\left(\frac{t}{\eta}\right)^{\gamma}\right]} \tag{16.20}$$

which gives the radial macrocrack transition destiny as

$$t_d = F^{-1}(U) = \eta \left[ -\ln\left(1 - U\right) \right]^{\frac{1}{\gamma}} , \tag{16.21}$$

and heartbeat formulation for this transition as

$$dt \frac{\eta(P_n, T_n)}{\eta(P_c, T_c)} = dt \frac{e^{\left(25{,}248 T_n{}^{-1} - 0.12 P_n - 36.4\right)}}{e^{\left(25{,}248 T_c{}^{-1} - 0.12 P_c - 36.4\right)}} \quad . \tag{16.22}$$

## 16.2.3 Radial Macrocrack to Leak Transition

The rate at which radial macrocracks become leaks is based on continuing the growth of a microcrack past the size at which it is considered a radial macrocrack, to the size at which it is considered a leak. Hence the governing equations are identical to those in the preceding section. The transition destiny determined for the transition between microcrack and radial microcrack is therefore reused with a multiplier of approximately $1.3$ to determine the transition destiny from radial macrocrack to leak. The same cumulative damage formulation also applies.

## 16.2.4 Microcrack to Circumferential Macrocrack Transition

The microcrack transition to circumferential macrocrack is also modeled with a response surface that is nearly the same as in the preceding section, or

$$\ln \eta = 25,262 T^{-1} - 0.12 P - 37 \tag{16.23}$$

where

$\eta$ = characteristic time in the microcrack state, before macrocrack

$T$ = material temperature in degree K,

$P$ = material stress loading (pressure) in MPa

The cumulative distribution function for the distribution of transition times is

$$F(t) = 1 - \exp\left[ -\left(\frac{t}{\eta}\right)^{\gamma} \right] \tag{16.24}$$

which gives the circumferential macrocrack transition destiny as

$$t_d = F^{-1}(U) = \eta \left[ -\ln\left(1 - U\right) \right]^{\frac{1}{\gamma}} , \tag{16.25}$$

and heartbeat formulation for the transition as

$$dt\frac{\eta(P_n,T_n)}{\eta(P_c,T_c)} = dt\frac{e^{\left(25{,}262T_n{}^{-1}-0.12P_n-37\right)}}{e^{\left(25{,}262T_c{}^{-1}-0.12P_c-37\right)}} \tag{16.26}$$

### 16.2.5 Leak or Circumferential Macrocrack to Rupture Transition

The rate at which leaks or circumferential cracks transition to rupture is not presently formulated as dependent on simulation variables. The cumulative distribution function for time to rupture is

$$F(t) = 1 - \exp\left(-\phi t\right) \tag{16.27}$$

where $\phi$ is the rupture rate. The rupture time transition destiny is then

$$t_d = F^{-1}(U) = -\frac{1}{\phi}\ln\left(1-U\right) \tag{16.28}$$

### 16.2.6 Repair Transitions

Detection and repair of cracks is not dependent on simulation variables. The cumulative distribution function for the time to crack detection and repair is

$$F(t) = 1 - \exp\left(-\omega t\right) \tag{16.29}$$

where $\omega$ is the detection and repair rate. The repair time transition destiny is then

$$t_d = F^{-1}(U) = -\frac{1}{\omega}\ln\left(1-U\right) \tag{16.30}$$

## 16.3 Interfaces

The `Pipe_Pui_FSM` interface is the same as `Pipe_Pui`. It can interface with the following components:

**`Pipe_Pui:`** see Chapter 37.

**`Elbow_Pui:`** see Chapter 37.

## 16.4 Verification & Validation

Verification of the `Pipe_Pui_FSM` component comprises the following test cases:

1. Two-State Heartbeat Model Test. This is a simple two-state test of the heartbeat model. The failure times are Weibull-distributed and sensitive to temperature-related heartbeat stress. The PWR60 model is simulated until the pipe component fails. This test was performed with an earlier implementation of a heartbeat model coded in component `Pump_Pui_HB`. It has not yet been redone in the context of `Pipe_Pui_FSM`. The results are included here to demonstrate the impact of equation (16.10)

2. Multi-State Pipe Cracking Heartbeat Model Test. This test is a six-state test of the heartbeat model and corresponds to full implementation of the multi-state physics model of passive component aging.

The PWR60 model transient consists of an initial period at hot-standby, heat up to full power conditions, and finally cool down to hot-standby. The transient is initially scheduled for a one year duration. The model includes one `Pipe_Pui_FSM` component; the surge line. At the time these tests were performed the R7 pressurizer model was not functioning well so the pressurizer was replaced with a constant pressure boundary condition. The result is a transient of relatively constant pressure, but with annual variations in temperature. The layout of the PWR60 model is shown in Figure 16.2

### 16.4.1 Test Case 1

#### Problem Formulation

This test case verifies that the cumulative damage calculation for the Heartbeat model is working correctly. The pump failure times for this test are input as Weibull-distributed with with parameters $\eta = 1000.$ and $\beta = 3.00$. The characteristic lifetime of the pump varies as

$$\eta(V) = Ce^{-\frac{B}{V}} \tag{16.31}$$

where

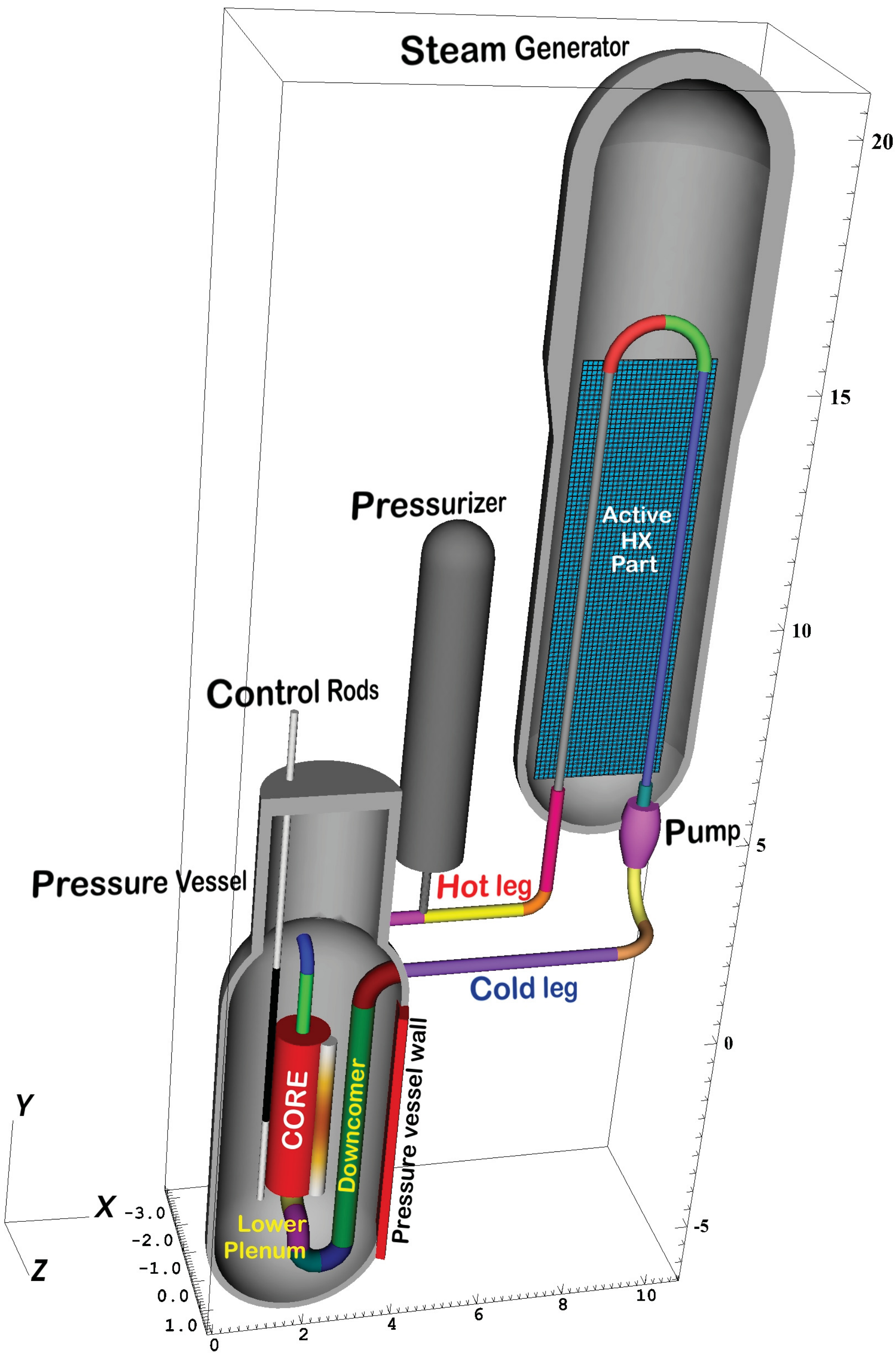


**Fig. 16.2 :** Layout of "PWR60" simulation model.

$C$ is a constant of the Arrhenius-Weibull model. Value set to $15.0$ for this test case.

$B$ is a constant of the Arrhenius-Weibull model. Value set to $1260.0$ for this test case.

$V$ is the reference value for the stressor. In this case the stressor is the average fluid temperature of the pump component, $300.$K.

For this test case the fluid temperature in the pump remains constant at a value close to $400.$K. With the test case fluid temperatures $100.$K greater than the reference temperature, the pump failure time distribution is expected to shift to the left of the input distribution. The new characteristic lifetime is given by (16.31) with $V = 400.$K. The expected pump failure times for this test case should therefore be Weibull-distributed with scale parameter $\eta = 350.$ and shape parameter $\beta = 3.00$.

### Results

Figure 16.3 shows the pump failure time density distribution plotted against the expected densities for the input distribution, and for the left-shifted distribution expected as a result of the fluid temperature through the pipe being $100.$K higher than the reference temperature. The results show good agreement, indicating that pipe failures are occurring as expected based on the model inputs.

## 16.4.2 Test Case 2

### Problem Formulation

The multi-state problem involves a full-scale simulation of the PWR60 transient for prolonged periods.

### Results

Verification results for Test Case 2 have, for the following reasons, not yet been generated:

1. The parameters specified by [ea11b] require very long transients (on the order of 50 years) to have a reasonable expectation of seeing transitions between the various crack states.

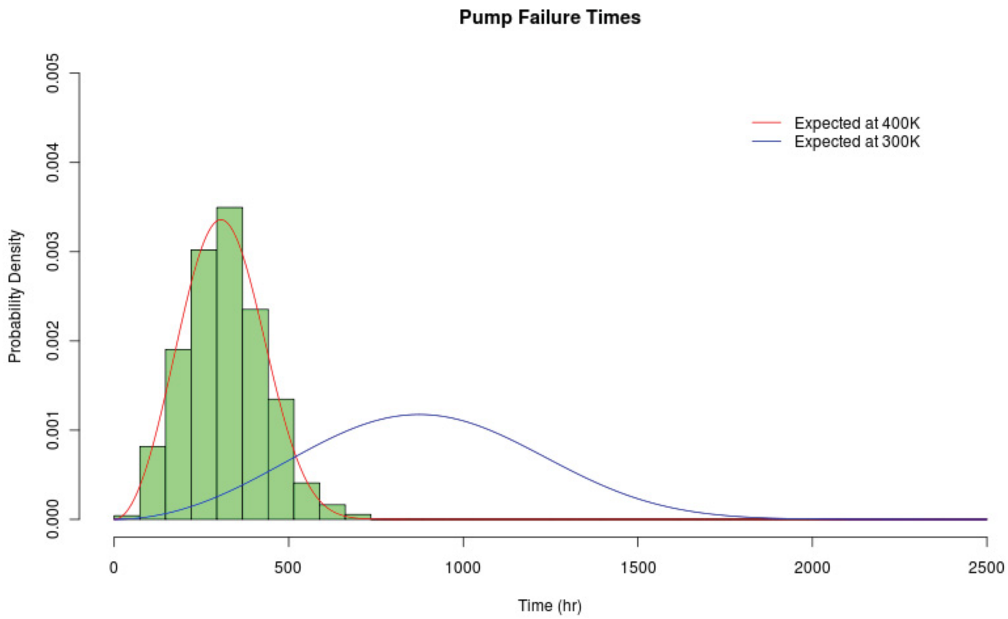


**Fig. 16.3 :** Pipe failure time density. Solution with 1000 simulation runs.

2. Because of the previous item, a large number of simulation trials would have to be performed to achieve reasonable variance in the simulation output.

3. The discovery and repair of cracking modes is specified by the current model as random through out the simulated transient. The reality is that discovery and repair of cracks may only occur during periods of shutdown operation. Leaks may be discovered at any time, but only repaired during shutdowns. Capturing these nuances is not easily achievable with the current model.

4. There is, at present, no solution that includes the temperature and stress dependence that exists in this model. So there is no result to verify the simulation solution against.

Because of the above issues no attempt was made to make the simulation runs required for verification and validation of the full model. The model was verified to some extent using accelerated transition rates and closely observing code execution to verify state transitions where occurring as programmed.

# Chapter 24

# Class Pump_Pui_FSM

COMPONENT Pump_Pui_FSM is one of two R7 components that that use an internal finite state machine (FSM) to implement reliability calculations in the context of an R7 simulation. The other component, Pipe_Pui_FSM, uses the FSM to implement a physics-based pipe cracking and cumulative damage model sometimes referred to as a heartbeat model. The reliabilty calculation for Pump_-Pui_FSM uses the same FSM algorithm as Pipe_Pui_FSM. However, the pump reliability model is a simplified two-state model that was developed first, to allow benchmarking a relatively simple model before the more complicated pipe cracking model was developed.

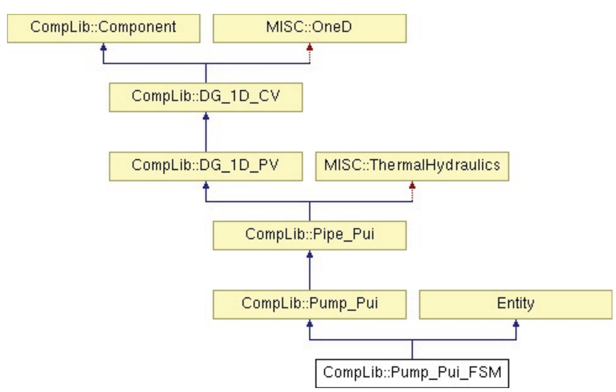


**Fig. 24.1** : Inheritance tree for Class Pump_Pui_FSM.

Pump_Pui_FSM is derived from Pump_Pui as shown in Figure 24.1. Pump_-Pui handles the pump physical state representation. Pump_Pui_FSM extends Pump_Pui by adding reliability elements to the model. The FSM functionality is incorporated by also deriving from the `Entity` abstract base class as shown in the figure. The `Entity` class provides basic FSM functionality; component specific behavior is produced by coding state-specific behavior into separate state objects. In this case, the pump reliability behavior is encoded in `PumpOwnedStates.C`. The two states modeled are ”running” and ”failed”.

## 24.1 Input options

Inputs for components of type `Pump_Pui_FSM` are specified in

“`INPUT/DefineComponents/ComponentName.inp`”

files, where “`ComponentName`” is a unique name of the component, as defined in the file “`INPUT/ListOfComponents.inp`”. To declare the component “`ComponentName`” to be of the type `Pump_Pui_FSM`, one must set

```
[Define]
  type = Pump_Pui_FSM
[]
```

Currently input for components of type `Pump_Pui_FSM` is identical to components of type `Pump_Pui`. Parameters of the reliability model are currently set in the constructors of the pump state objects, and not read from the input file. Only two parameters must be input to change the model behavior: 1) the pump failure rate, and 2) the pump repair rate.

## 24.2 Governing Equations

The governing equations comprise both a physics model and a reliability model. The physics model is decribed in Section 22.2. The reliability model for `Pump_Pui_FSM` is a two-state Markov model that incorporates both pump failure and repair transitions. The operable state for the pump is state 0, and the failed state is state 1. The transition rate from state 0 to state 1 is $\lambda$ and the repair rate from state 1 back to state 0 is $\mu$. The Markov state equations are

$$\begin{bmatrix} \dot{P}_0 \\ \dot{P}_1 \end{bmatrix} = \begin{bmatrix} -\lambda & \mu \\ \lambda & -\mu \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \end{bmatrix} \tag{24.1}$$

with boundary conditions

$$\begin{bmatrix} P_0(t=0) \\ P_1(t=0) \end{bmatrix} = \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix}. \tag{24.2}$$

The solution of the above equations can be shown [Reference] to be

$$P_0(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t} \tag{24.3a}$$

$$P_1(t) = \frac{\lambda}{\lambda+\mu}\left[1 - e^{-(\lambda+\mu)t}\right] \tag{24.3b}$$

where $P_0(t)$ is the component availability and $P_1(t)$ is the component unavailability.

## 24.3 Interfaces

The `Pump_Pui_FSM` interface is the same as `Pump_Pui`. It can interface with the following components:

**`Pipe_Pui:`** see Chapter 37.

**`Elbow_Pui:`** see Chapter 37.

## 24.4 Verification & Validation

A "PWR60" model transient was simulated for 1000 trials with transition rate $\lambda = 1.0 \times 10^{-3}$ hr$^{-1}$ and repair rate $\mu = 1.0 \times 10^{-1}$ hr$^{-1}$. The pump availability, $P_0(t)$, is plotted against equation (24.3a) in Figure 24.2. The pump unavailability, $P_1(t)$, is plotted against equation (24.3b) in Figure 24.3

Figures 24.2 and 24.3 demonstrate reasonable agreement between the simulation output and the expected state probabilities.

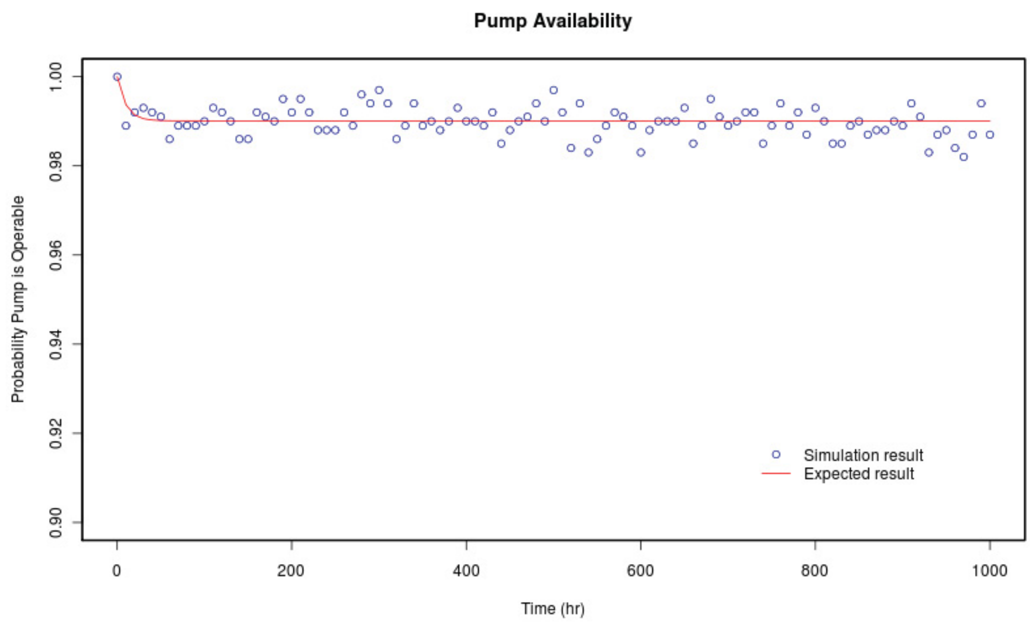


**Fig. 24.2 :** Pump availability as a function of time. Solution with 1000 simulation runs.

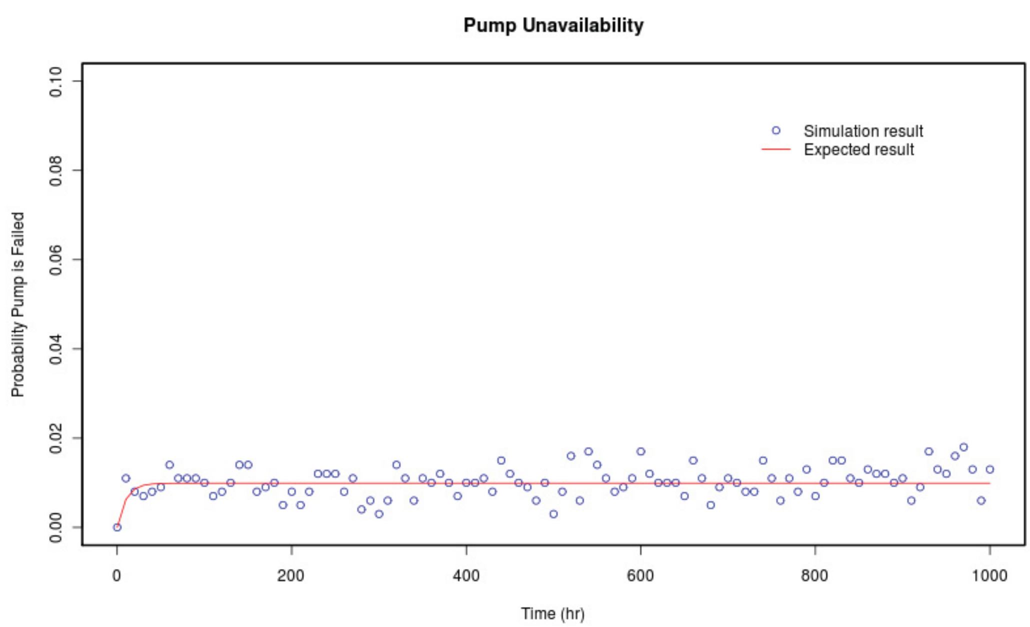


**Fig. 24.3 :** Pump unavailability as a function of time. Solution with 1000 simulation runs.

# Chapter 25

# Pump Control Components

COMPONENTS of the PumpControl family are designed to control or specify pump's (class Pump, Section 22) power history, Figure 25.1.

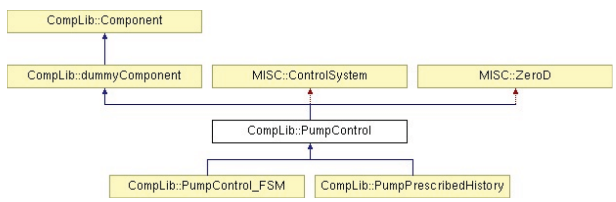


**Fig. 25.1 :** Inheritance tree for Class PumpControl.

## 25.1 Class PumpPrescribedHistory

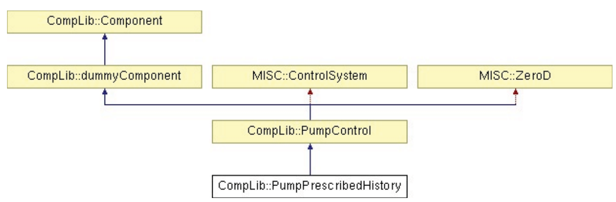


**Fig. 25.2 :** Inheritance tree for Class PumpPrescribedHistory.

The class PumpPrescribedHistory (Figure 25.2) is the simplest controlling component for pumps. This component is designed to set pump's power

# APPENDIX 3

## “Class Pump_Pui_FSM,” Chapter 24 of “R7 Documentation“, describing the R7 implementation of a model of pump unreliability

# Chapter 24

# Class Pump_Pui_FSM

COMPONENT Pump_Pui_FSM is one of two R7 components that that use an internal finite state machine (FSM) to implement reliability calculations in the context of an R7 simulation. The other component, Pipe_Pui_FSM, uses the FSM to implement a physics-based pipe cracking and cumulative damage model sometimes referred to as a heartbeat model. The reliabilty calculation for Pump_Pui_FSM uses the same FSM algorithm as Pipe_Pui_FSM. However, the pump reliability model is a simplified two-state model that was developed first, to allow benchmarking a relatively simple model before the more complicated pipe cracking model was developed.

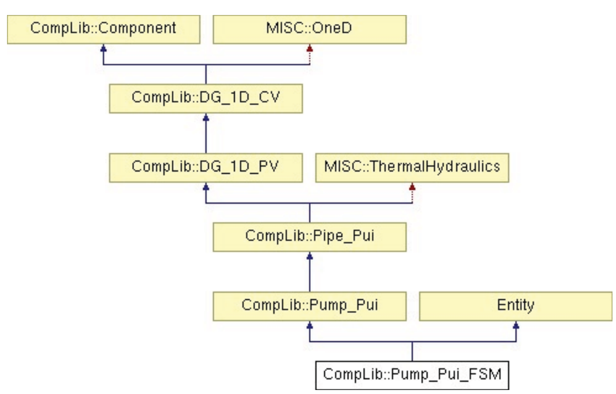


**Fig. 24.1** : Inheritance tree for Class Pump_Pui_FSM.

Pump_Pui_FSM is derived from Pump_Pui as shown in Figure 24.1. Pump_Pui handles the pump physical state representation. Pump_Pui_FSM extends Pump_Pui by adding reliability elements to the model. The FSM functionality is incorporated by also deriving from the `Entity` abstract base class as shown in the figure. The `Entity` class provides basic FSM functionality; component specific behavior is produced by coding state-specific behavior into separate state objects. In this case, the pump reliability behavior is encoded in `PumpOwnedStates.C`. The two states modeled are ”running” and ”failed”.

## 24.1 Input options

Inputs for components of type `Pump_Pui_FSM` are specified in

“`INPUT/DefineComponents/ComponentName.inp`”

files, where “`ComponentName`” is a unique name of the component, as defined in the file “`INPUT/ListOfComponents.inp`”. To declare the component “`ComponentName`” to be of the type `Pump_Pui_FSM`, one must set

```
[Define]
  type = Pump_Pui_FSM
[]
```

Currently input for components of type `Pump_Pui_FSM` is identical to components of type `Pump_Pui`. Parameters of the reliability model are currently set in the constructors of the pump state objects, and not read from the input file. Only two parameters must be input to change the model behavior: 1) the pump failure rate, and 2) the pump repair rate.

## 24.2 Governing Equations

The governing equations comprise both a physics model and a reliability model. The physics model is decribed in Section 22.2. The reliability model for `Pump_Pui_FSM` is a two-state Markov model that incorporates both pump failure and repair transitions. The operable state for the pump is state 0, and the failed state is state 1. The transition rate from state 0 to state 1 is $\lambda$ and the repair rate from state 1 back to state 0 is $\mu$. The Markov state equations are

$$\begin{bmatrix} \dot{P}_0 \\ \dot{P}_1 \end{bmatrix} = \begin{bmatrix} -\lambda & \mu \\ \lambda & -\mu \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \end{bmatrix} \tag{24.1}$$

with boundary conditions

$$\begin{bmatrix} P_0(t=0) \\ P_1(t=0) \end{bmatrix} = \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix}. \tag{24.2}$$

The solution of the above equations can be shown [Reference] to be

$$P_0(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} \tag{24.3a}$$

$$P_1(t) = \frac{\lambda}{\lambda+\mu} \left[1 - e^{-(\lambda+\mu)t}\right] \tag{24.3b}$$

where $P_0(t)$ is the component availability and $P_1(t)$ is the component unavailability.

## 24.3 Interfaces

The `Pump_Pui_FSM` interface is the same as `Pump_Pui`. It can interface with the following components:

**`Pipe_Pui`:** see Chapter 37.

**`Elbow_Pui`:** see Chapter 37.

## 24.4 Verification & Validation

A "PWR60" model transient was simulated for 1000 trials with transition rate $\lambda = 1.0 \times 10^{-3}$ hr$^{-1}$ and repair rate $\mu = 1.0 \times 10^{-1}$ hr$^{-1}$. The pump availability, $P_0(t)$, is plotted against equation (24.3a) in Figure 24.2. The pump unavailability, $P_1(t)$, is plotted against equation (24.3b) in Figure 24.3

Figures 24.2 and 24.3 demonstrate reasonable agreement between the simulation output and the expected state probabilities.

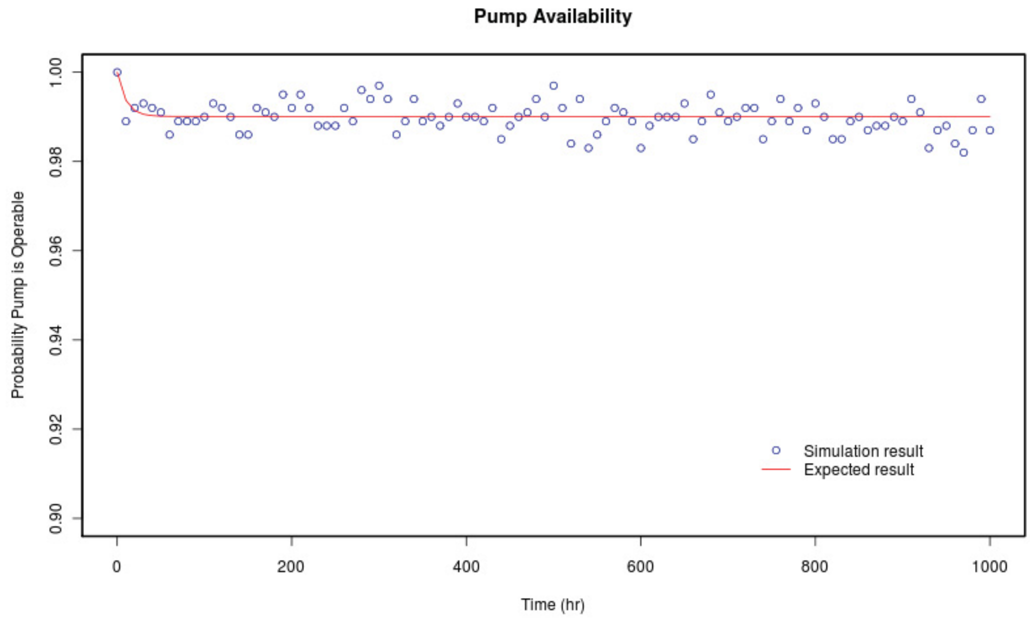


**Fig. 24.2** : Pump availability as a function of time. Solution with 1000 simulation runs.

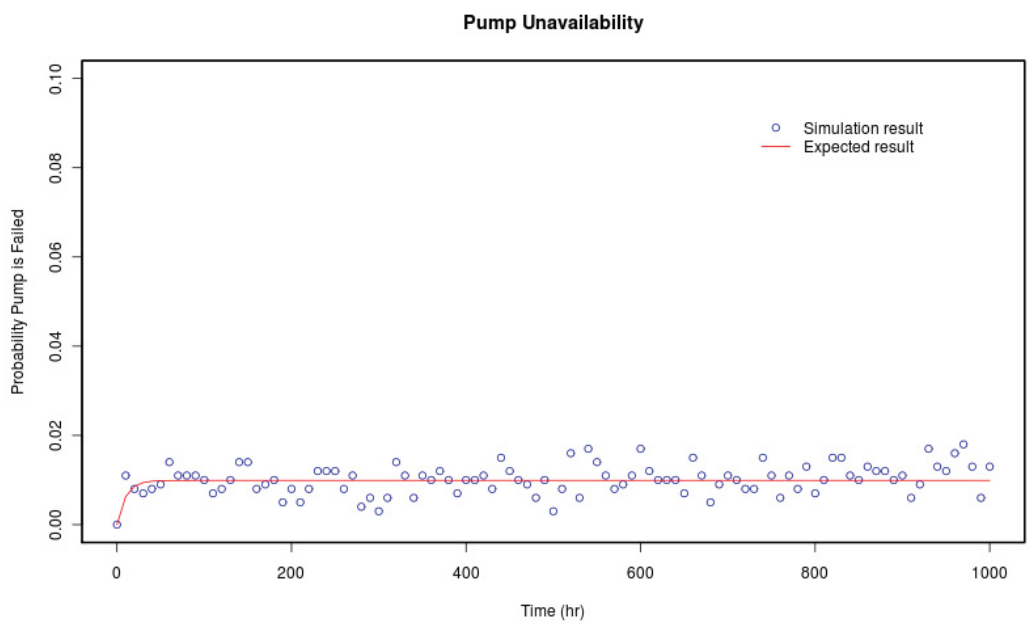


**Fig. 24.3** : Pump unavailability as a function of time. Solution with 1000 simulation runs.

# Chapter 25

# Pump Control Components

COMPONENTS of the `PumpControl` family are designed to control or specify pump's (class `Pump`, Section 22) power history, Figure 25.1.

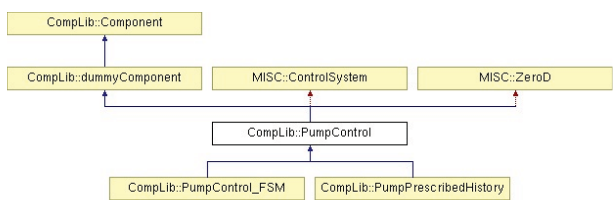


**Fig. 25.1 :** Inheritance tree for Class `PumpControl`.

## 25.1 Class `PumpPrescribedHistory`

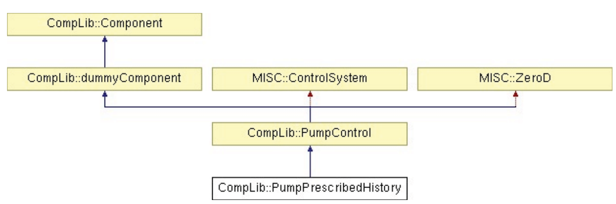


**Fig. 25.2 :** Inheritance tree for Class `PumpPrescribedHistory`.

The class `PumpPrescribedHistory` (Figure 25.2) is the simplest controlling component for pumps. This component is designed to set pump's power