

GPDs &
Regge
behavior

FFs

PDFs

DAs

GPDs

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Models

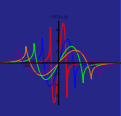
Pion GPDs

Nucleon
GPDs

Singularities of Generalized Parton Distributions

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&
Theory Center, Jefferson Lab
May 19, 2013,
CAQCD Workshop, Minneapolis



Hadrons in Terms of Quarks and Gluons

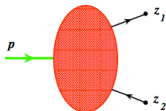
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How to relate hadronic states $|p, s\rangle$

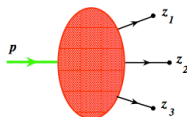
to quark and gluon fields $q(z_1), q(z_2), \dots$?

Standard way: use matrix elements

$$\langle 0 | \bar{q}_\alpha(z_1) q_\beta(z_2) | M(p), s \rangle, \quad \langle 0 | q_\alpha(z_1) q_\beta(z_2) q_\gamma(z_3) | B(p), s \rangle$$

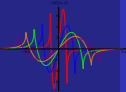


Meson-quark matrix element



Baryon-quark matrix element

- Can be interpreted as hadronic wave functions



Phenomenological Functions

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“Old” functions:

- Form Factors
- Usual Parton Densities
- Distribution Amplitudes

“New” functions:

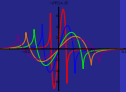
Generalized
Parton Distributions
(GPDs)

GPDs = Hybrids of

Form Factors, Parton Densities and
Distribution Amplitudes

“Old” functions

are limiting cases of “new” functions



Form Factors

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Form factors are defined through matrix elements

of electromagnetic and weak currents between hadronic states

Nucleon EM form factors:

$$\langle p', s' | J^\mu(0) | p, s \rangle = \bar{u}(p', s') \left[\gamma^\mu F_1(t) + \frac{\Delta^\nu \sigma^{\mu\nu}}{2m_N} F_2(t) \right] u(p, s)$$
$$(\Delta = p - p', t = \Delta^2)$$

- Electromagnetic current

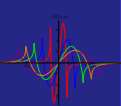
$$J^\mu(z) = \sum_{f \text{ flavor}} e_f \bar{\psi}_f(z) \gamma^\mu \psi_f(z)$$

- Helicity non-flip form factor

$$F_1(t) = \sum_f e_f F_{1f}(t)$$

- Helicity flip form factor

$$F_2(t) = \sum_f e_f F_{2f}(t)$$

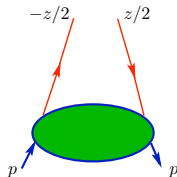


Usual Parton Densities

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Parton Densities are defined through
forward matrix elements

of quark/gluon fields separated by
lightlike distances

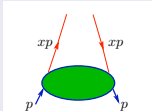


Unpolarized quarks case:

$$\langle p | \bar{\psi}_a(-z/2) \gamma^\mu \psi_a(z/2) | p \rangle \Big|_{z^2=0}$$

$$= 2p^\mu \int_0^1 [e^{-ix(pz)} f_a(x) - e^{ix(pz)} f_{\bar{a}}(x)] dx$$

Momentum space
interpretation



$f_{a(\bar{a})}(x)$ is
probability

to find a (\bar{a}) quark
with momentum xp

Local limit $z = 0$

\Rightarrow **sum rule**

$$\int_0^1 [f_a(x) - f_{\bar{a}}(x)] dx = N_a$$

for valence quark
numbers

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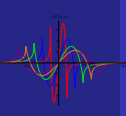
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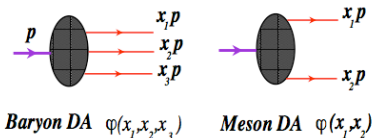
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Distribution Amplitudes

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DAs may be interpreted as

- LC wave functions integrated over transverse momentum
- Matrix elements $\langle 0 | \mathcal{O} | p \rangle$ of LC operators

For pion (π^+):

$$\begin{aligned}
 & \langle 0 | \bar{\psi}_d(-z/2) \gamma_5 \gamma^\mu \psi_u(z/2) | \pi^+(p) \rangle \Big|_{z^2=0} \\
 &= i p^\mu f_\pi \int_{-1}^1 e^{-i\alpha(pz)/2} \varphi_\pi(\alpha) d\alpha
 \end{aligned}$$

with $\alpha = x_1 - x_2$ or $x_1 = (1 + \alpha)/2$, $x_2 = (1 - \alpha)/2$

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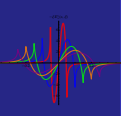
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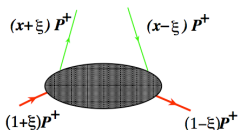
GPDs



Generalized Parton Distributions

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Momentum fractions taken wrt average momentum $P = (p + p')/2$



4 functions of x, ξ, t :

$H, E, \tilde{H}, \tilde{E}$

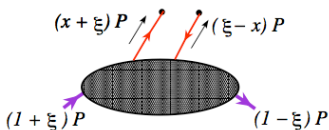
wrt hadron/parton helicity flip

$+ / +, - / +, + / -, - / -$

• Skeweness $\xi \equiv \Delta^+ / 2P^+$ is $\xi = x_{Bj} / (2 - x_{Bj})$

• **3 regions:**

- $\xi < x < 1$ \sim quark distribution
- $-1 < x < -\xi$ \sim antiquark distribution
- $-\xi < x < \xi$ \sim distribution amplitude for $N \rightarrow \bar{q}qN'$



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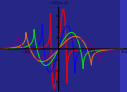
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Definition of GPDs

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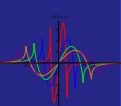
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- In scalar case, define **GPD** by

$$\begin{aligned} & \langle P + r/2 | \psi(-z/2) \psi(z/2) | P - r/2 \rangle |_{z^2=0} \\ &= \int_{-1}^1 e^{-ix(Pz)} H(x, \xi; t) dx \end{aligned}$$

- Invariant momentum transfer $t = r^2$
- Skeweness $\xi = r^+ / 2P^+$
- $r = 0 \Rightarrow$ usual (forward) distribution

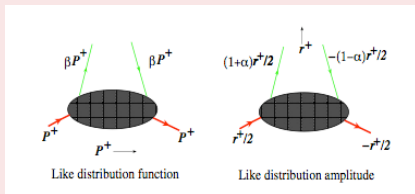
$$f(x) = H(x, \xi = 0; t = 0)$$



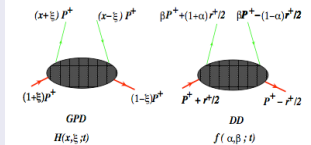
Double Distributions

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“Superposition” of P^+ and r^+ momentum fluxes

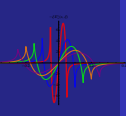


Connection with GPDs



Basic relation
between fractions

$$x = \beta + \xi\alpha$$



Parton distributions and matrix elements

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- For a scalar target, one may write

$$\begin{aligned} & \langle P + r/2 | \psi(0) \{ \overleftrightarrow{\partial}_{\mu_1} \dots \overleftrightarrow{\partial}_{\mu_n} \} \psi(0) | P - r/2 \rangle \\ & = A_{n0} \{ P_{\mu_1} \dots P_{\mu_n} \} + A_{nn} \{ r_{\mu_1} \dots r_{\mu_n} \} \\ & + \sum_{l=1}^{n-1} A_{nl} \{ P_{\mu_1} \dots P_{\mu_{n-l}} r_{\mu_{n-l+1}} \dots r_{\mu_n} \} \end{aligned}$$

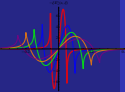
- $r = 0 \Rightarrow$ usual (forward) distribution $f(\beta)$ related to $l = 0$ moments

$$\int_{-1}^1 f(\beta) \beta^n d\beta = A_{n0} \quad (1)$$

- $P = 0 \Rightarrow$ D -term $D(\alpha)$ related to $l = n$ moments

$$\int_{-1}^1 D(\alpha) (\alpha/2)^n d\alpha = A_{nn} \quad (2)$$

- D comes with r_{μ_i} factors: it is invisible in DIS (then $r = 0$)



Definition of DDs

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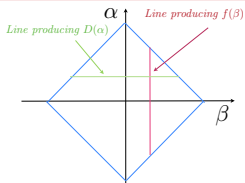
- Define **Double Distribution** (DD)

$$\frac{n!}{(n-l)! l! 2^l} \int_{\Omega} F(\beta, \alpha) \beta^{n-l} \alpha^l d\beta d\alpha = A_{nl}$$

- Support region Ω is given by rhombus $|\alpha| + |\beta| \leq 1$
- “DD parameterization” of the matrix element

$$\left\langle P - \frac{r}{2} \left| \psi(-z/2) \psi(z/2) \right| P + \frac{r}{2} \right\rangle \Big|_{z^2=0} = \int_{\Omega} F(\beta, \alpha) e^{-i\beta(Pz) - i\alpha(rz)/2} d\beta d\alpha$$

Getting PDF and D-term from DDs

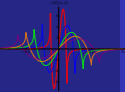


- Usual (forward) distribution

$$f(\beta) = \int_{-1+|\beta|}^{1-|\beta|} F(\beta, \alpha) d\alpha$$

- D-term

$$D(\alpha) = \int_{-1+|\alpha|}^{1-|\alpha|} F(\beta, \alpha) d\beta$$



Isolating D -term

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- Using $e^{-i\beta(Pz)} = [e^{-i\beta(Pz)} - 1] + 1$
- split DD-integral into “plus” part

$$\int_{\Omega} [F(\beta, \alpha)]_+ e^{-i\beta(Pz) - i\alpha(rz)/2} d\beta d\alpha$$

- and D -term part

$$\int_{-1}^1 D(\alpha) e^{-i\alpha(rz)/2} d\alpha$$

- with

$$[F(\beta, \alpha)]_+ = F(\beta, \alpha) - \delta(\beta) \int_{-1+|\alpha|}^{1-|\alpha|} F(\gamma, \alpha) d\gamma$$

- “Plus” “+” D representation:

$$F(\beta, \alpha) = [F(\beta, \alpha)]_+ + \delta(\beta)D(\alpha)$$

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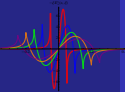
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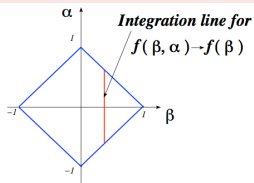
Getting GPDs from DDs

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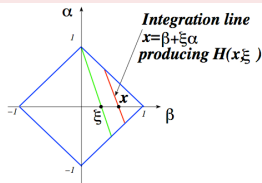
$$H(x, \xi) = \int_{\Omega} F(\beta, \alpha) \delta(x - \beta - \xi\alpha) d\beta d\alpha$$

DDs live on rhombus

$$|\alpha| + |\beta| \leq 1$$



Converting DDs into GPDs



“Munich” symmetry:

$$f_a(\beta, \alpha; t) = f_a(\beta, -\alpha; t)$$

GPDs $H(x, \xi)$ are obtained
from DDs $f(\beta, \alpha)$

by scanning DDs
at ξ -dependent angles

\Rightarrow DD-tomography

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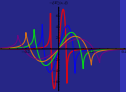


Illustration of DD \rightarrow GPD conversion

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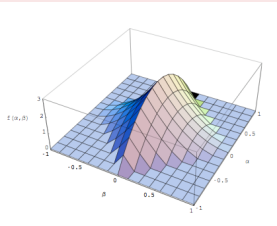
Factorized model for DDs:

(\sim usual parton density in β -direction) \otimes

(\sim distribution amplitude in α -direction)

Toy model for double distribution

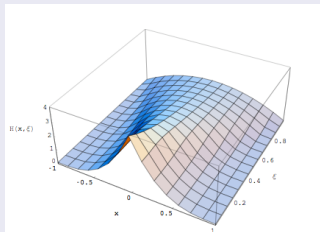
$$f(\beta, \alpha) = 3[(1 - |\beta|)^2 - \alpha^2] \theta(|\alpha| + |\beta| \leq 1)$$



● Corresponds to toy "forward" distribution

$$f(\beta) = (1 - |\beta|)^3$$

GPD $H(x, \xi)$ resulting from toy DD



- For $\xi = 0$ reduces to usual parton density
- For $\xi = 1$ has shape like meson distribution amplitude

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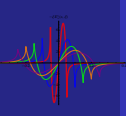
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“DD plus D” Model for GPDs

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- Factorized Ansatz for DDs:

$$F(\beta, \alpha) = f(\beta)h_\alpha(\beta, \alpha)$$

Normalization

$$\int_{-1}^1 d\alpha h(\beta, \alpha) = 1$$

Guarantees forward limit

$$\int_{-1}^1 d\alpha f(\beta, \alpha) = f(\beta)$$

- DD modeling misses terms invisible in the forward limit:
 - Meson exchange contributions
 - D-term, which can be interpreted as σ exchange
- Inclusion of D-term induces contribution confined to $|x| < \xi$ region

$$H_D(x, \xi) = \frac{1}{|\xi|} D(x/\xi)$$

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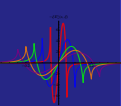
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Model for GPDs based on DDs

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- DD+D Ansatz: $F(\beta, \alpha) = f(\beta)h_a(\beta, \alpha) + \delta(\beta)D(\alpha)$
- General form of model profile

$$h(\beta, \alpha) = \frac{\Gamma(2 + 2b)}{2^{2b+1}\Gamma^2(1 + b)} \frac{[(1 - |\beta|)^2 - \alpha^2]^b}{(1 - |\beta|)^{2b+1}}$$

- Power b is parameter of the model
- $b = \infty$ gives $h(\beta, \alpha) = \delta(\alpha)$ and $H(x, \xi) = f(x) + D(x/\xi)/|\xi|$

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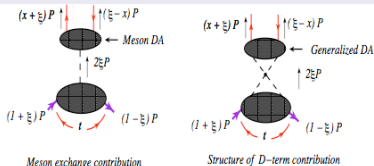
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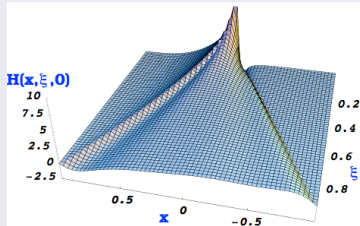
Pion GPDs

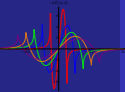
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Meson and D-term terms



DD + D-term model





Model with Regge behavior of $f(\beta)$

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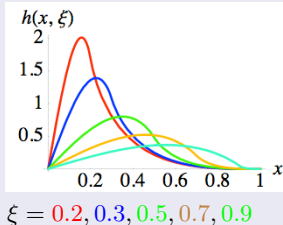
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- PDFs $f(\beta)$ are known to be singular for small β
- $f(\beta) \sim \beta^{-a}(1-\beta)^3$
- $x_+ = (x + \xi)/(1 + \xi)$
- $x_- = (x - \xi)/(1 - \xi)$
- $\sim |x - \xi|^{2-a} + \text{const}$ behavior for $x \sim \xi$

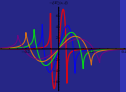
$b=1$ DD with Regge PDFs



- Model $H(x, \xi) = \int_{\Omega} d\beta f(\beta) h_b(\beta, \alpha) \delta(x - \beta - \xi\alpha)$ with $b = 1$

$$H(x, \xi)|_{|x| \geq \xi} = \frac{1}{\xi^3} \left(1 - \frac{a}{4}\right) \left\{ [(2-a)\xi(1-x)(x_+^{2-a} + x_-^{2-a}) + (\xi^2 - x)(x_+^{2-a} - x_-^{2-a})] \theta(x) - (x \rightarrow -x) \right\}$$

$$H(x, \xi)|_{|x| \leq \xi} = \frac{1}{\xi^3} \left(1 - \frac{a}{4}\right) \left\{ x_+^{2-a} [(2-a)\xi(1-x) + (\xi^2 - x)] - (x \rightarrow -x) \right\}$$



Spin-1/2 quarks: two-DD representation

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- For a (pseudo)scalar target

$$\begin{aligned} & \langle P - r/2 | \bar{\psi}(-z/2) \gamma_\mu \psi(z/2) | P + r/2 \rangle_{\text{twist}-2} \\ & = 2P_\mu f((Pz), (rz), z^2) + r_\mu g((Pz), (rz), z^2) \end{aligned}$$

- Two-DD parametrization

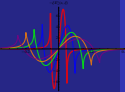
$$\begin{aligned} & z^\mu \langle P - r/2 | \bar{\psi}(-z/2) \gamma_\mu \psi(z/2) | P + r/2 \rangle_{z^2=0} \\ & = \int_{\Omega} e^{-i\beta(Pz) - i\alpha(rz)/2} \left[2(Pz)F(\beta, \alpha) + (rz)G(\beta, \alpha) \right] d\beta d\alpha \\ & = \frac{2}{i} \int_{\Omega} e^{-i\beta(Pz) - i\alpha(rz)/2} \left[\frac{\partial F(\beta, \alpha)}{\partial \beta} + \frac{\partial G(\beta, \alpha)}{\partial \alpha} \right] d\beta d\alpha \end{aligned}$$

- Not unique: invariant under transformation

$$\begin{aligned} F(\beta, \alpha) & \rightarrow F(\beta, \alpha) + \partial\chi(\beta, \alpha)/\partial\alpha, \\ G(\beta, \alpha) & \rightarrow G(\beta, \alpha) - \partial\chi(\beta, \alpha)/\partial\beta, \end{aligned}$$

- “DD+D” form corresponds to “gauge” in which one has

$$2(Pz)F_D(\beta, \alpha) + (rz)\delta(\beta)D(\alpha)$$



Spin-1/2 quarks: one-DD representation

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- **Note:** in local twist-2 operators $\bar{\psi}\{\gamma_\mu \overleftrightarrow{\partial}_{\mu_1} \dots \overleftrightarrow{\partial}_{\mu_n}\}\psi$ index μ is symmetrized with μ_i indices that produce $\beta P_{\mu_i} + \alpha r_{\mu_i}/2$
- $\Rightarrow \mu$ also produces $\beta P_\mu + \alpha r_\mu/2$, i.e.

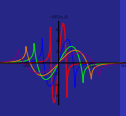
$$2(Pz)F(\beta, \alpha) + (rz)G(\beta, \alpha) = [2\beta(Pz) + \alpha(rz)]f(\beta, \alpha)$$

- Or $F(\beta, \alpha) = \beta f(\beta, \alpha)$ and $G(\beta, \alpha) = \alpha f(\beta, \alpha)$
- GPD in two-DD parametrization

$$H(x, \xi) = \int_{\Omega} [F(\beta, \alpha) + \xi G(\beta, \alpha)] \delta(x - \beta - \xi\alpha) d\beta d\alpha$$

- GPD in one-DD formulation

$$\begin{aligned} H(x, \xi) &= \int_{\Omega} (\beta + \xi\alpha) f(\beta, \alpha) \delta(x - \beta - \xi\alpha) d\beta d\alpha \\ &= x \int_{\Omega} f(\beta, \alpha) \delta(x - \beta - \xi\alpha) d\beta d\alpha \end{aligned}$$



One-DD formulation

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- D -term in the one-DD case

$$D(\alpha) = \alpha \int_{-1+|\alpha|}^{1-|\alpha|} f(\beta, \alpha) d\beta$$

- Separating D -term

$$f(\beta, \alpha) = [f(\beta, \alpha)]_+ + \delta(\beta) \frac{D(\alpha)}{\alpha} \quad (3)$$

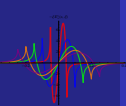
- Forward distribution

$$f(x) = \int_{-1+|x|}^{1-|x|} F(x, \alpha) d\alpha = x \int_{-1+|x|}^{1-|x|} f(x, \alpha) d\alpha$$

- Suggests factorized model

$$f(\beta, \alpha) = \frac{f(\beta)}{\beta} h(\beta, \alpha)$$

- \Rightarrow Reconstructing DDs/GPDs from $f(x)/x$:
very singular $\sim x^{-\alpha(0)-1}$ for small x !



GPDs in one-DD representation

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- “DD₊₊ + D” separation corresponds to the representation

$$H(x, \xi) \equiv H_+(x, \xi) + \text{sgn}(\xi)D(x/\xi) ,$$

- “Plus” part of GPD

$$H_+(x, \xi) \equiv \int_{\Omega} (\beta + \xi\alpha) f(\beta, \alpha) \left[\delta(x - \beta - \xi\alpha) - \delta(x - \xi\alpha) \right] d\beta d\alpha .$$

- Using $f(\beta, \alpha) = F(\beta, \alpha)/\beta$ we may rewrite

$$\begin{aligned} H_+(x, \xi) &= \int_{\Omega} F(\beta, \alpha) \delta(x - \beta - \xi\alpha) d\beta d\alpha \\ &+ \xi \int_{\Omega} \frac{\alpha F(\beta, \alpha)}{\beta} \left[\delta(x - \beta - \xi\alpha) - \delta(x - \xi\alpha) \right] d\beta d\alpha \end{aligned}$$

- GPD constructed from DD $F(\beta, \alpha)$ by “classic” formula

$$F_{DD}(x, \xi) = \int_{\Omega} F(\beta, \alpha) \delta(x - \beta - \xi\alpha) d\beta d\alpha$$

- GPD built from the “plus” part of the DD $\alpha F(\beta, \alpha)/\beta = G(\beta, \alpha)$.

$$F_+^1(x, \xi) \equiv \int_{\Omega} \left(\frac{\alpha}{\beta} F(\beta, \alpha) \right) \delta(x - \beta - \xi\alpha) d\beta d\alpha$$

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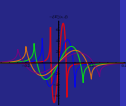
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Pion GPDs for $n = 1$ profile $\sim (1 - \beta)^2 - \alpha^2$

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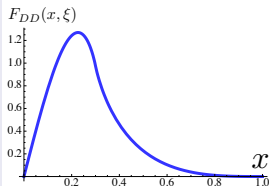
DDs

Models

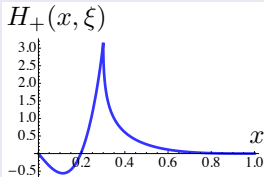
Pion GPDs

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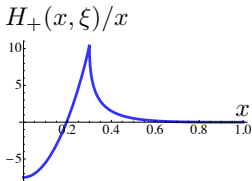
GPD $F_{DD}(x, \xi)$ for $\xi = 0.3$



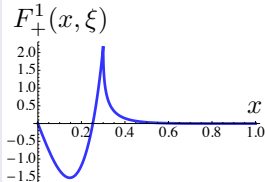
Pion GPD $H_+(x, \xi)$ for $\xi = 0.3$

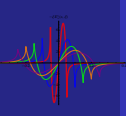


GPD $H_+(x, \xi)/x$ for $\xi = 0.3$



Function $\xi F_+^1(x, \xi)$ for $\xi = 0.3$





Definitions of Nucleon DDs and GPDs

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- In nucleon case for unpolarized target, one can parametrize

$$\begin{aligned}
 & \langle p' | \bar{\psi}(-z/2) \not{\epsilon} \psi(z/2) | p \rangle |_{\text{twist-2}} \\
 &= \int_{\Omega} e^{-i\beta(Pz) - i\alpha(rz)/2} \left[\bar{u}(p') \not{\epsilon} u(p) a(\beta, \alpha) \right. \\
 & \left. + \frac{\bar{u}(p') u(p)}{2M_N} [2\beta(Pz) + \alpha(rz)] b(\beta, \alpha) \right] d\beta d\alpha
 \end{aligned}$$

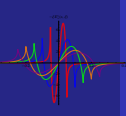
- DDs a, b correspond to $A = H + E$ and $B = -E$ of usual H and E
- A is given by simple “classic” DD representation

$$A(x, \xi) = \int_{\Omega} a(\beta, \alpha) \delta(x - \beta - \xi\alpha) d\beta d\alpha \quad (4)$$

- B is given by one-DD representation

$$B(x, \xi) = x \int_{\Omega} b(\beta, \alpha) \delta(x - \beta - \xi\alpha) d\beta d\alpha . \quad (5)$$

- Since $H = A + B$, it is given by combination of both types of DD-representation



Modeling a and b

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- In the forward limit, we have for A

$$A(x, 0) = H(x, 0) + E(x, 0) = f(x) + e(x)$$

- and for B

$$B(x, 0) = -E(x, 0) = -e(x)$$

- Suggest model representation for a

$$a(\beta, \alpha) = f(\beta, \alpha) + e(\beta, \alpha)$$

- and for b

$$b(\beta, \alpha) = -\frac{e(\beta, \alpha)}{\beta}$$

- Possible singularity of $e(\beta, \alpha)/\beta$ at $\beta = 0$, demands “ $DD_+ + D$ ”

$$b(\beta, \alpha) = -\left(\frac{e(\beta, \alpha)}{\beta}\right)_+ + \delta(\beta)\frac{D(\alpha)}{\alpha}$$

- Here $D(\alpha)$ is the D -term

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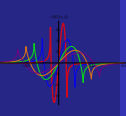
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Start modeling E and H

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- For H GPD:

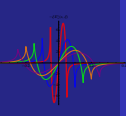
$$\begin{aligned}
 H(x, \xi) &= A(x, \xi) + B(x, \xi) \\
 &= \int_{\Omega} [f(\beta, \alpha) + e(\beta, \alpha)] \delta(x - \beta - \xi\alpha) d\beta d\alpha \\
 &\quad - x \int_{\Omega} \left[\left(\frac{e(\beta, \alpha)}{\beta} \right)_+ - \delta(\beta) \frac{D(\alpha)}{\alpha} \right] \delta(x - \beta - \xi\alpha) d\beta d\alpha \\
 &= F_{DD}(x, \xi) + E_{DD}(x, \xi) - E_+(x, \xi) + \text{sgn}(\xi) D(x/\xi),
 \end{aligned}$$

- Terms constructed using the simplest DD formula

$$\begin{aligned}
 F_{DD}(x, \xi) &= \int_{\Omega} f(\beta, \alpha) \delta(x - \beta - \xi\alpha) d\beta d\alpha \\
 E_{DD}(x, \xi) &= \int_{\Omega} e(\beta, \alpha) \delta(x - \beta - \xi\alpha) d\beta d\alpha
 \end{aligned}$$

- “Plus” part of E/x GPD:

$$\frac{E_+(x, \xi)}{x} = \int_{\Omega} \frac{e(\beta, \alpha)}{\beta} \left[\delta(x - \beta - \xi\alpha) - \delta(x - \xi\alpha) \right] d\beta d\alpha$$



Continue modeling E and H

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- Function $E_+(x, \xi)$ is similar to $H_+(x, \xi)$ of pion case

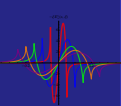
$$\begin{aligned}
 E_+(x, \xi) &= \int_{\Omega} \frac{e(\beta, \alpha)}{\beta} (\beta + \xi\alpha) [\delta(x - \beta - \xi\alpha) - \delta(x - \xi\alpha)] d\beta d\alpha \\
 &= \int_{\Omega} e(\beta, \alpha) \delta(x - \beta - \xi\alpha) d\beta d\alpha \\
 &\quad + \xi \int_{\Omega} \frac{\alpha}{\beta} e(\beta, \alpha) [\delta(x - \beta - \xi\alpha) - \delta(x - \xi\alpha)] d\beta d\alpha \\
 &= E_{DD}(x, \xi) + \xi \int_{\Omega} \left(\frac{\alpha}{\beta} e(\beta, \alpha) \right)_+ \delta(x - \beta - \xi\alpha) d\beta d\alpha \\
 &\equiv E_{DD}(x, \xi) + \xi E_+^1(x, \xi)
 \end{aligned}$$

- Important function

$$E_+^1(x, \xi) \equiv \int_{\Omega} \left(\frac{\alpha}{\beta} e(\beta, \alpha) \right)_+ \delta(x - \beta - \xi\alpha) d\beta d\alpha$$

- Modifies “DD+D” construction to

$$H(x, \xi) = F_{DD}(x, \xi) - \xi E_+^1(x, \xi) + \text{sgn}(\xi) D(x/\xi)$$



Nucleon GPDs for $n = 1$ profile $\sim (1 - \beta)^2 - \alpha^2$

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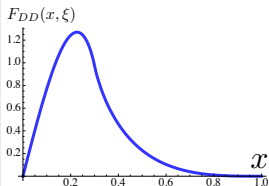
DDs

Models

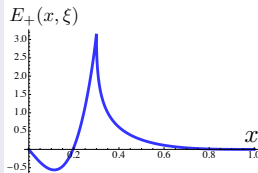
Pion GPDs

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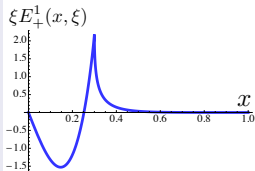
GPD $F_{DD}(x, \xi)$ for $\xi = 0.3$



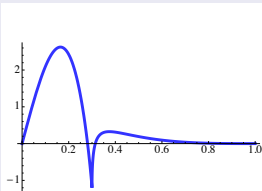
GPD $E_+(x, \xi)$ for $\xi = 0.3$

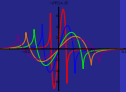


Function $\xi E_+^1(x, \xi)$ for $\xi = 0.3$



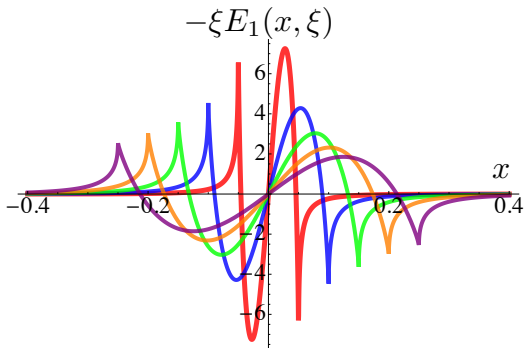
Nucleon GPD $H(x, \xi)/x$ without
 D -term for $\xi = 0.3$



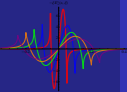


What is added on top of D term

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- “DD plus D” model is substituted by
“DD $-\xi E_+^1(x, \xi) + \text{sgn}(\xi)D(x/\xi)$ ”
- Important differences between $E_+^1(x, \xi)$ and $D(x/\xi)$:
- Support region of $E_+^1(x, \xi)$ is not restricted to $|x| \leq \xi$
- $E_+^1(x, \xi)$ does not vanish at border points $|x| = \xi$



Conclusions

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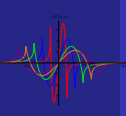
DDs

Models

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- Singular Regge behavior of usual PDFs implies singular structure of double distributions generating GPDs
- DD for E GPD reduces to $e(x)/x$ in forward limit – very strong singularity
- Formal expression for D -term diverges: need for renormalization
- Old “DD plus D” construction for GPD H is modified by extra non-monotonic term related to GPD E
- New term does not vanish at border point $x = \xi$
- New phenomenology for GPD modeling



Summary

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- 2 Usual Parton Densities
- 3 Distribution Amplitudes
- 4 Generalized Parton Distributions
- 5 Double Distributions
- 6 Models
- 7 Pion GPDs
- 8 Nucleon GPDs