IMPEDANCE BUDGET FOR CRAB CAVITY IN MEIC ELECTRON RING *

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Abstract

The Medium Energy Electron-Ion Collider (MEIC) at Jefferson Lab has been envisioned as a first stage high energy particle accelerator beyond the 12 GeV upgrade of CEBAF. The estimate of impedance budget is important from the view point of beam stability and matching with other accelerator components driving currents. The detailed study of impedance budget for electron ring has been performed by considering the current design parameters of the e-ring. A comprehensive picture of the calculations involved in this study has been illustrated in the paper.

INTRODUCTION

The high luminosity (~ 6×10^{33} cm^{-2} s^{-1}) for the future electron ion collider at Jefferson Lab known as MEIC (see Fig. 1) can be achieved by small beam sizes at the interaction point (IP) in conjunction with a large number of stored bunches having low charge per bunch (4 nC for electron, ~ 0.7 nC for proton) and high repetition rate (750 MHz) using the finite crossing angle scheme [1]-[3]. The design parameters are illustrated in Table 1. In the present design, MEIC employs the crab crossing concept [4]-[5] to avoid parasitic collisions and the minimization of synchrotron-betatron resonance near the IP [6]-[9]. In this paper, we present an estimate of impedance budget for MEIC electron ring for designing crab cavity.

A charged particle beam in its motion interacts with the external electromagnetic (EM) fields of accelerator optics as well as with the surrounding vacuum chambers through the self generated EM fields. The equations governing the motion are described by

$$\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \tag{1}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \tag{2}$$

$$m\frac{d^2x}{dt^2} = q(\mathbf{E} + v \times \mathbf{B}) \tag{3}$$

where q and m are the charge and mass of the particle moving with velocity v, the current density $\mathbf{J} = \mathbf{J}_c + \mathbf{J}_d$ is the combination of conduction $\mathbf{J}_c = \sigma\mathbf{E}$ and displacement current $\mathbf{J}_d = \epsilon_0 \partial\mathbf{E}/\partial t$.

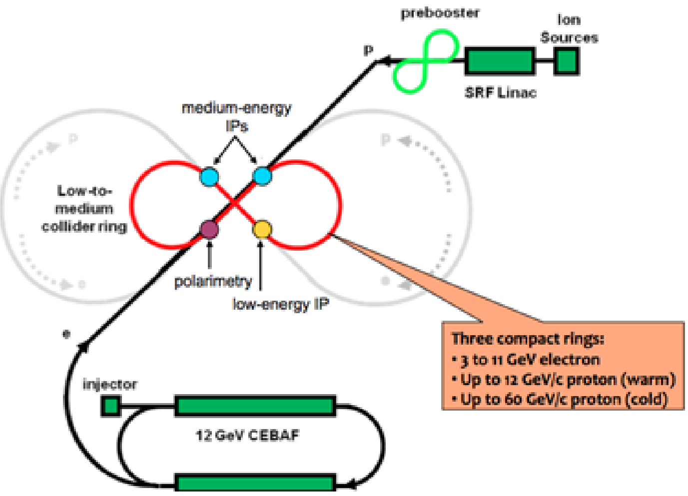

Figure 1: Conceptual layout of MEIC at JLab.

Table 1: Parameters for MEIC e-Ring Impedance Budget

Parameters	Unit	**Value**
Energy E_0	GeV	5.0
Collision frequency f_c	MHz	750
Circumference C	m	1340
Beam pipe radius r_b	mm	30
Harmonic number h		3350
Number of bunches K_B		3350
Bunch spacing s_b	cm	40
Bunch population N_e (10^{10})		2.5
Beam current I_0	A	3
Revolution Time T_0	μs	4.46
Revolution frequency f_0	kHz	224
Momentum Compaction α_c		6×10^{-4}
Synchronous Tune ν_s		4.5×10^{-2}
Cavity β_x	m	350
Cavity β_y	m	350
Radiation Integrals:		
$I_1 = \oint \frac{D_x}{\rho} ds$	m	0.76
$I_2 = \oint \frac{1}{\rho^2} ds$	m^{-1}	0.2
$I_3 = \oint \frac{1}{\rho^3} ds$	m^{-2}	4.8×10^{-3}
$I_4 = \oint \frac{D_x}{\rho}\left(\frac{1}{\rho^2} - 2k\right) ds$	m^{-1}	0.0
I_5	m^{-1}	3.62×10^{-5}

Impedance is a characteristic property of all current carrying elements and the interaction of a moving charged par-

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ticle beam with environment via impedance. This means at high current strong EM fields are generated which in turn offers high impedance to the beam and eventually leads to instabilities. The various accelerator components such as RF cavities, bellows, dielectric walls and beam pipe of finite conductivity result in scattering or trapping of beam induced fields. These fields can last for long time and interact with the following bunches causing perturbation to the energy or angle of the particle's orbit. This dynamic phenomenon is defined as collective effect which is highly disruptive to the beam and may cause heating of accelerator components by the trapped fields.

Luminosity is a key parameter for determining the performance of a collider and can be increased by increasing the beam current, which in turn increases the EM fields induced inside the structure. Consequently, the beam experiences high impedance which may lead to instability and beam loss. Following are the noteworthy points. First, sudden jump in cross-section results in the change of impedance, therefore, we should avoid it.Second, in the case of finite conductivity ($\mathbf{J} = \sigma E$) fields are trapped, which may lead to beam instability. Third, the longitudinal and transverse impedance depend strongly on the pipe radius. This means smaller beam pipes have higher cut-off frequency (c/b) and offer high impedance.

The detailed calculations of impedance budget for other components mentioned earlier have been reported in [10]. The objective of this work is to provide impedance budget for crab cavity in MEIC electron ring so that the higher-order modes can be damped to the required level. Details of the calculations follow the procedure adapted by the APS [11] for impedance budget of crab cavity.

COUPLED BUNCH INSTABILITIES GROWTH RATES

In MEIC, the high current (3 A) is supposed to achieve by distributing the total current among large number (3350) of circulating bunches. This way we can avoid single bunch instabilities and improve the beam lifetime. However, the couple bunch instability may arise due to coupling between bunches through the long range wake fields. The longitudinal and transverse coupled bunch instabilities (CBI) for equally spaced and equally populated m rigid bunches are defined as in Ref. [12]
Longitudinal Growth Rate:

$$\frac{1}{\tau_g^{\parallel}} = \frac{\alpha_c I_{tot}}{4\pi(E_b/e)\nu_s}\sum_p \omega_p Re(Z_z(\omega_p)) \qquad (4)$$

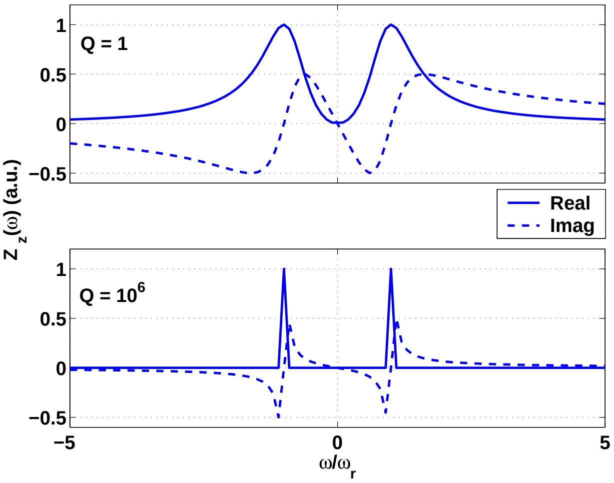

Figure 2: Illustration of longitudinal impedance for broadband (Q =1) and narrow band (Q = 10^6) RF structure.

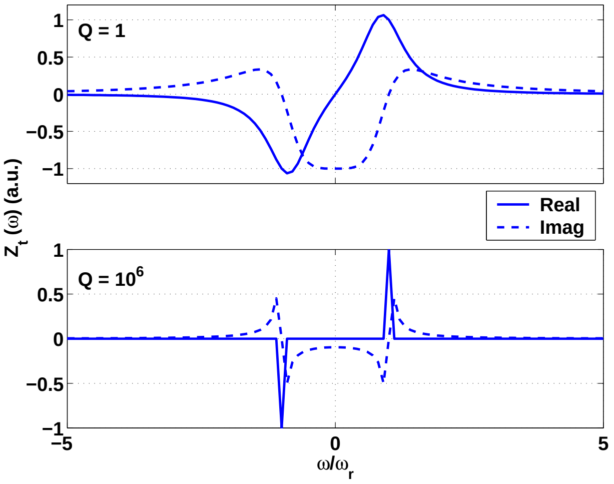

Figure 3: Illustration of transverse impedance for broadband (Q =1) and narrow band (Q = 10^6) RF structure.

where the sampling frequency ω_p for the n$^{\text{th}}$ longitudinal dipole mode is expressed in terms of the revolution frequency ω_0 and the synchrotron frequency ω_s as $\omega_p = pm\omega_0 + n\omega_0 + \omega_s$. The impedance of longitudinal higher-order modes in RF cavity of shunt impedance R_s and quality factor Q is defined as

$$Z_z(\omega) = \frac{R_s}{1 + jQ(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega})}$$

Transverse Growth Rate:

$$\frac{1}{\tau_g^{\perp}} = \frac{\omega_0 I_{tot}}{4\pi(E_b/e)}\beta_{\perp}\sum_p Re(Z_t(\omega_p)) \qquad (5)$$

where, for rigid dipole transverse oscillations $\omega_p = pm\omega_0 + n\omega_0 + \omega_\beta$ and ω_β is the betatron frequency. The transverse impedance is defined as

$$Z_t(\omega) = \frac{\omega_r}{\omega}\frac{R_t}{1 + jQ(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega})}$$

The figures and illustrate that for a given HOM resonance, width is significantly narrow than the sampling frequency of 750 MHz. So, the sum is effectively one term. We damp these longitudinal and transverse CBI by radiation damping to achieve stability. The damping integrals are defined as

$$I_1 = \oint \frac{D_x}{\rho} ds \quad (6)$$

$$I_2 = \oint \frac{1}{\rho^2} ds \quad (7)$$

$$I_3 = \oint \frac{1}{\rho^3} ds \quad (8)$$

$$I_4 = \oint \frac{D_x}{\rho} \left(\frac{1}{\rho^2} - 2k \right) ds \quad (9)$$

Damping time:

$$\tau_d^x = \frac{3T_0}{r_e \gamma^3} \frac{1}{I_2 - I_4} \quad (10)$$

$$\tau_d^y = \frac{3T_0}{r_e \gamma^3} \frac{1}{I_2} \quad (11)$$

$$\tau_d^\epsilon = \frac{3T_0}{r_e \gamma^3} \frac{1}{2I_2 + I_4} \quad (12)$$

The stability criterion is defined as

$$\tau_g > \tau_d \quad (13)$$

For MEIC, $\tau_x = 25.4$ ms, $\tau_y = \tau_x$, $\tau_\epsilon = 12.7$ ms. The longitudinal and transverse stability thresholds are given by

$$R_S f_p < \frac{2(E_b/e)\nu_s}{\alpha_c I_{tot} \tau_\epsilon} \sim 0.0197 M\Omega - GHz \quad (14)$$

$$R_t < \frac{2(E_b/e)}{f_0 I_{tot} \beta_\perp \tau_\perp} \sim 1.7 k\Omega/m \quad (15)$$

Fig. 4 shows the longitudinal impedance of MEIC electron ring. It is important to note that y-axis is in log scale. For 750 MHz, $R_S \sim 0.026$ MΩ and $R_t \sim 1.7$ kΩ/m are longitudinal and transverse impedance budget for crab cavity in MEIC electron ring. Note that the transverse impedance is independent of frequency and hence a constant for all frequencies.

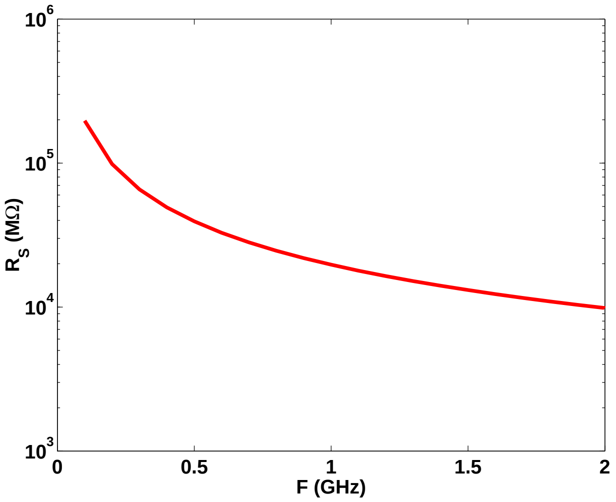

Figure 4: Longitudinal impedance for MEIC electron ring. Y-axis is in log scale.

CONCLUSION

We have presented a comprehensive picture of impedance budget calculation for the electron ring of the proposed electron-ion collider MEIC at Jefferson Lab. The present calculations will help the crab cavity design team.

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