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## Using Whole-Building Electric Load Data in Continuous or Retro-Commissioning

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# Using Whole-Building Electric Load Data in Continuous or Retro-Commissioning

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## Synopsis

Whole-building electric load data can often reveal problems with building equipment or operations. In this paper, we present methods for analyzing 15-minute-interval electric load data. These methods allow building operators, energy managers, and commissioning agents to better understand a building's electricity consumption over time and to compare it to other buildings, helping them to 'ask the right questions' to discover opportunities for electricity waste elimination, energy efficiency, peak load management, and demand response. For example: Does the building use too much energy at night, or on hot days, or in the early evening? Knowing the answer to questions like these can help with retro-commissioning or continuous commissioning.

The methods discussed here can also be used to assess how building energy performance varies with time. Comparing electric load before and after fixing equipment or changing operations can help verify that the fixes have the intended effect on energy consumption.

Analysis methods discussed in this paper include: ways to graphically represent electric load data; the definition of various parameters that characterize facility electricity loads; and a regression-based electricity load model that accounts for both time of week and outdoor air temperature. The methods are illustrated by applying them to data from commercial buildings. We demonstrate the ability to recognize changes in building operation, and to quantify changes in energy performance.

Some key findings are:

- 1) Plotting time series electric load data is useful for understanding electricity consumption patterns and changes to those patterns, but results may be misleading if data from different time intervals are not weather-normalized.
- 2) Parameter plots can highlight key features of electric load data and may be easier to interpret than plots of time series data themselves.
- 3) A time-of-week indicator variable (as compared to time-of-day and day-of-week indicator variables) improves the accuracy of regression models of electric load.
- 4) A piecewise linear and continuous outdoor air temperature dependence can be derived without the use of a change-point model (which would add complexity to the modeling algorithm) or assumptions about when structural changes occur (which could introduce inaccuracy).
- 5) A model that includes time-of-week and temperature dependence can be used for weather normalization and can determine whether the building is unusually temperature-sensitive, which can indicate problems with HVAC operation.

### **About the Authors**

Phillip N. Price received B.A. degrees in physics and mathematics from Oberlin College and the Ph.D. degree in physics from the University of Kentucky, Lexington. He has worked at Lawrence Berkeley National Laboratory since 1992 and has a wide variety of professional interests, including model-measurement comparison for complicated systems. He is also active in the environmental movement, with a special interest in protecting endangered species and sensitive habitat. Dr. Price was elected a Fellow of the American Physical Society in 2003.

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## **Introduction**

“Whole-building electric load” is the total electrical power used by a building at a given moment. The load changes with time in response to changes in lighting levels; heating, ventilating, and air conditioning (HVAC) requirements; and end uses such as computers, copy machines, and so on. The curve that represents load as a function of time, called the “load shape,” can often yield useful information. Unexpectedly high night-time loads may indicate waste (such as lights that needlessly remain on when the building is unoccupied); a change in load shape may indicate an equipment or thermostat malfunction; unexpectedly high sensitivity to outdoor temperature may indicate that excessive outdoor air is being brought into the building by the HVAC system; and so on.

In this paper, we present methods for analyzing 15-minute-interval electric load data from commercial buildings. These methods allow building managers to better understand their facility's electricity consumption over time and to compare it to other buildings, helping them to 'ask the right questions' to discover opportunities for electricity waste elimination and energy efficiency. The same methods can also be used for peak load management and to assess effectiveness of demand response strategies (Mathieu et al. 2011).

The overarching theme of this paper is that a lot of useful information can be obtained from time-resolved (5-minute, 15-minute, or even 1-hour) whole-building electric load data. We begin by demonstrating the usefulness of plotting electric load data and making a few suggestions concerning graphical displays. We then define some terminology to describe load shapes, introduce several ways of describing load shapes statistically, and show a real-world example of using those statistical descriptions to identify changes in building behavior. We then discuss methods for quantifying the sensitivity of building load to outdoor air temperature, and show how those methods can be used to compare building performance from years in which summer weather was different.

## **Graphical approaches**

In this section, we first present some graphical displays of electric load data. These plots demonstrate the usefulness of comparing load shapes between buildings and analyzing a single building's load shape over time. We then present some rules of thumb for displaying load data.

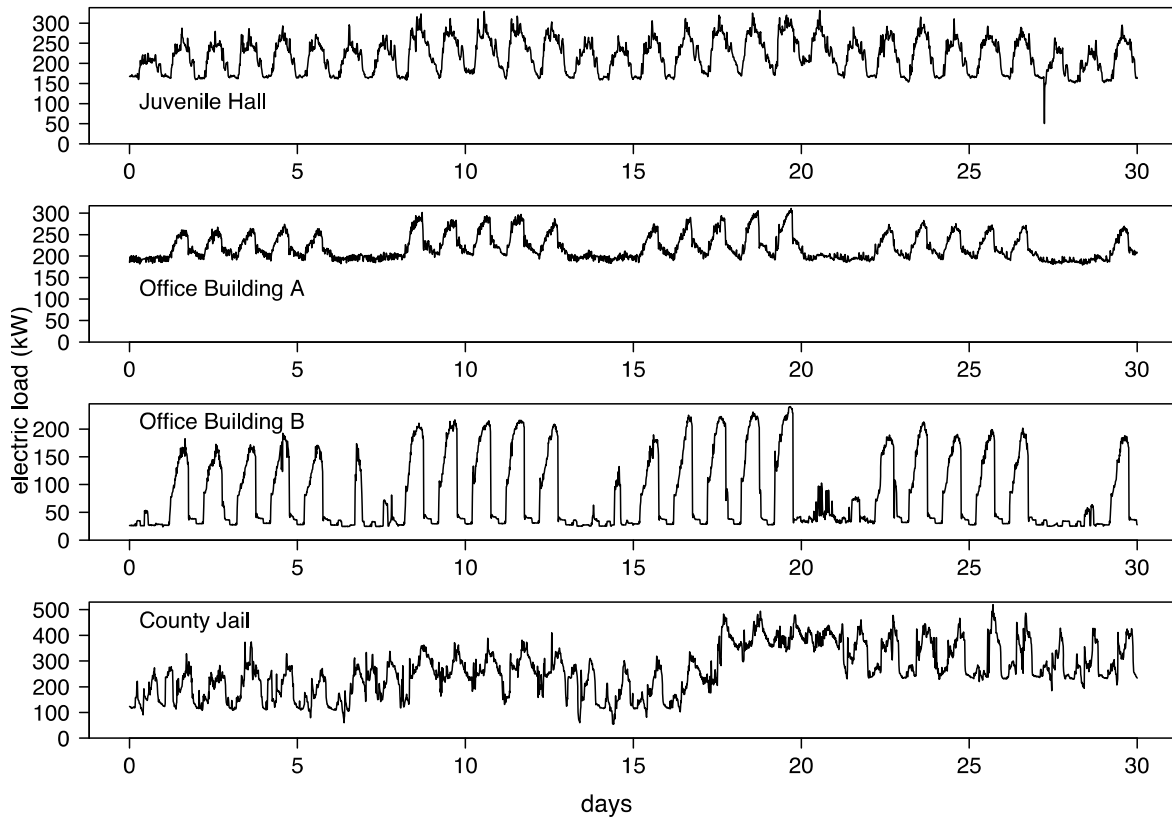
Figure 1 shows the load shape for four buildings that are in the same city in California. We have plotted 15-minute interval data over a one-month period in summer. All of the buildings have higher load during the day than at other times, but the load curves are strikingly different. Even in the two office buildings (the second and third panels), which are within a few hundred yards of each other, the load curves are different. For instance, Office Building A has a high base load, consuming 250 kW even at night and on weekends, whereas Office Building B has much lower base load and thus a much greater difference between peak and base.

The load shape of the County Jail is especially odd, looking very chaotic and showing a huge jump in the daily base load (by which we mean the minimum load for each day): for the first couple of weeks of the month, the base load is between 100-200 kW, but then it jumps to nearly 400 kW for a while before returning to a value near 250 kW that is still substantially higher than earlier in the month. By the end of the month, this building is using twice as much energy every day as it was at the beginning of the month.

Figure 1 shows that simply plotting electric load data over time helps building operators, energy managers, and commissioning agents identify anomalies, or changes in building operations, that they can investigate to discover opportunities to reduce building energy consumption. In the case of the office buildings, the commissioning agent might ask, "What are the differences in building systems, controls, operations, and end uses that lead to the differences in load shape? Is possible to change anything in Office Building A to make it run more like Office Building B?" In the

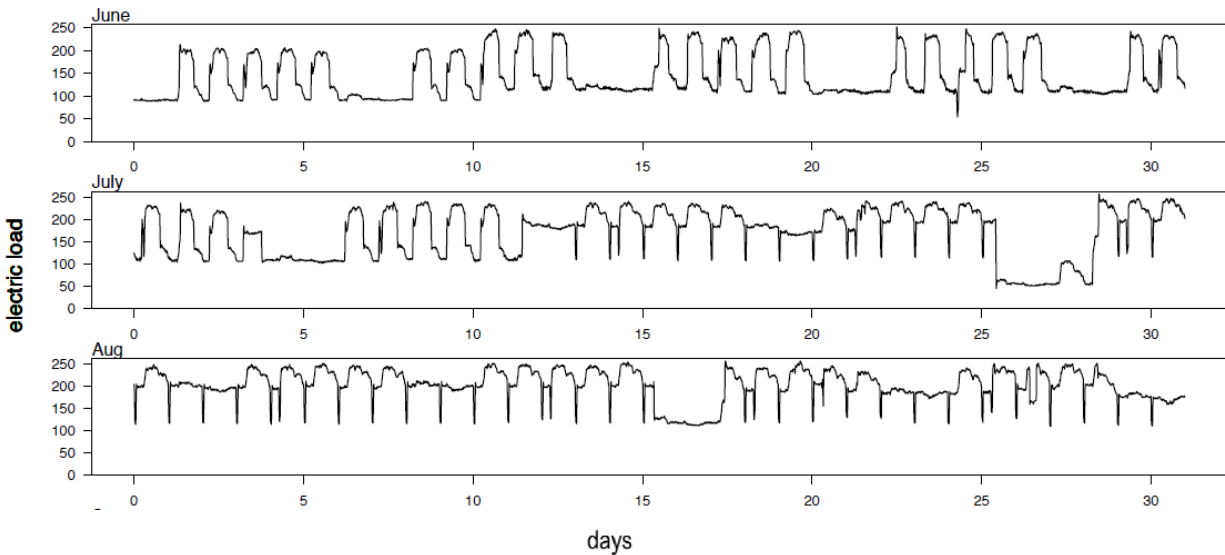
case of the jail, the commissioning agent might ask, “What changes have taken place, especially near the 18<sup>th</sup> day of the month? Is it possible to return the building to the way it was initially operating?” These questions could be dead-ends: perhaps Office Building A can not be run more like Office Building B (Building A might include a data center, for example), and perhaps the increase in energy use in the jail is necessary (because of an increase in occupancy, for instance). In general, analyzing electric load data does not allow us to identify specific problems; it simply allows us to generate a list of informed questions that help focus an evaluation of building systems, controls, operations, and end uses, making that evaluation more efficient.

**Figure 1: Load versus time for four buildings in the same city in California during June 2008.**



Sometimes plots of electric load data point to more specific problems. Figure 2 shows the load versus time for a single building during three summer months. The base load increases sometime around the 10<sup>th</sup> day in June, but generally the building’s load shape during June looks normal: high load during the day, low on nights and weekends. But after the second week in July, something goes wrong: the building no longer shuts down at night, or at least not completely. Either the building’s lights are being left on, or the HVAC system is operating around the clock, or both. Simply looking at the load shape shows that there is a problem and suggests that the nighttime HVAC and/or lighting schedules are likely culprits.

**Figure 2: Load versus time for three summer months in a building in Richmond, California.**



### ***Rules of Thumb for Graphical Displays***

While plotting electric load data is a good way to start an investigation into retro-commissioning opportunities, the usefulness of the plots is a function of how well the data is plotted; plots can differ greatly in how well they display the same information. Figure 3 is an example: this figure shows the same data as in Figure 2 but it is much harder to see what is going on. The baseline shift in early June (around day 10) is easy to see, but other features are almost completely obscured.

Here are some helpful general principles when plotting load shapes:

- 1) The y-axis should always start at zero.
- 2) Choose the plot aspect ratio so that major features have a slope of between 30 and 60 degrees up or down.
- 3) It is often useful to superimpose plots on each other (such as current week and previous week, or current week and average week). Displayed in black and white, two or three curves are often the most that can be shown without becoming visually confusing, but with the use of color this can be increased to four or five.
- 4) Plots that show a time period of a few days, up to about a week or two, are best for comparing one time period to another. Longer periods require the plot to be too compressed along the time axis.

**Figure 3: Same data as in Figure 2, shown in a single time series rather than as separate months. This plot is much harder to interpret.**

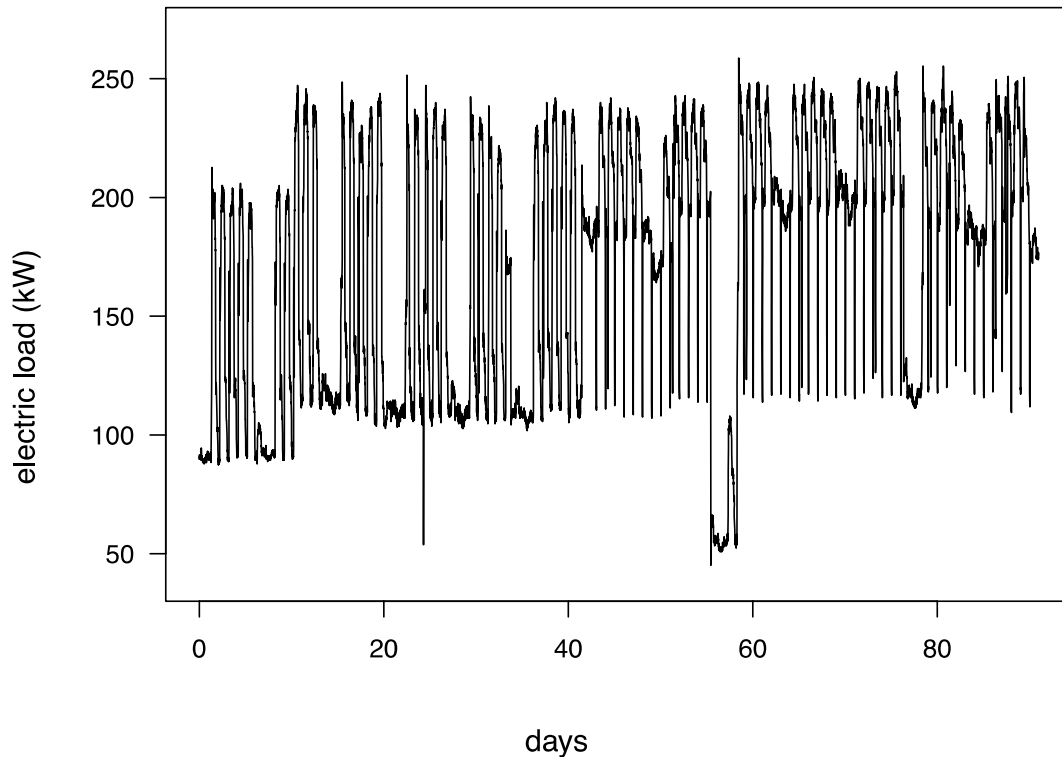


Figure 2 follows these principles, while Figure 3 violates items 1, 4, and especially item 2: the major features of interest are nearly vertical. The plot should be made much smaller from top to bottom (or, on a large computer screen, stretched out a long way from left to right), or else the number of days should be greatly reduced.

## Load Shape Characteristics

We now discuss load shape features that are shared by many commercial buildings, and define some parameters that can be used to quantify aspects of the load shape.

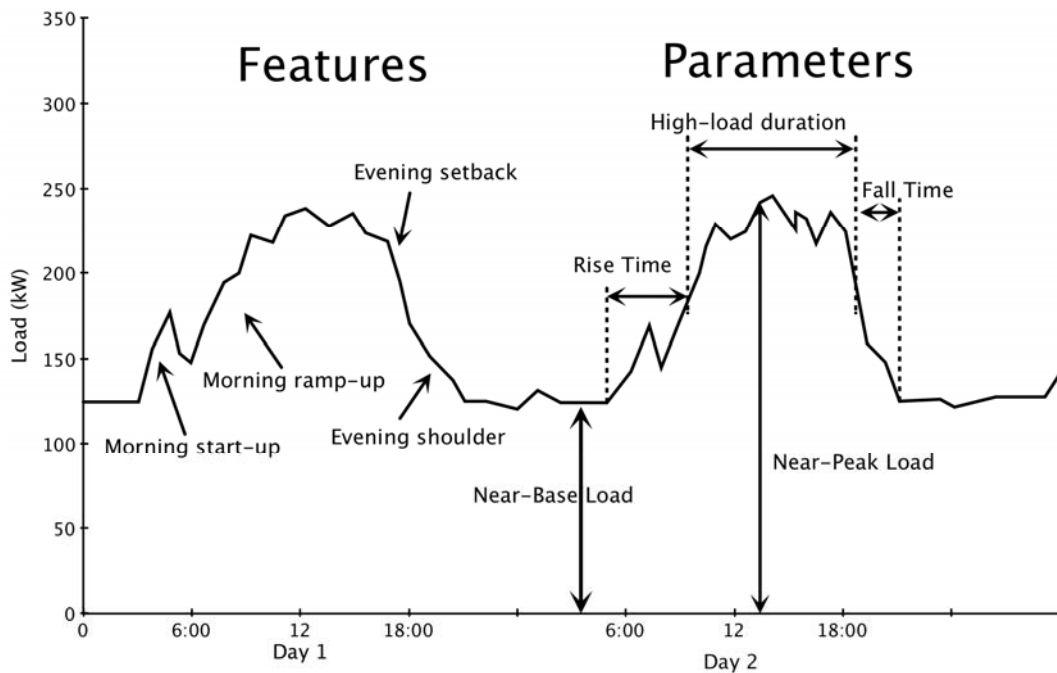
As Figure 2 shows, sometimes anomalies or faults can be seen simply by looking at a plot of a building's load as a function of time. But, as we show below, sometimes problems are not so obvious, and some statistical analysis is needed in order to flag problems. In our examples we will focus on flagging *changes* in building behavior, whether those changes are good or bad, intended or unintended. Once changes are identified a commissioning agent should be able to 'ask the right questions' and then must investigate the building's systems, controls, operations, and end uses to determine the cause of the change.

We begin by defining some terminology. Figure 4 shows a stylized load curve that is typical of many commercial buildings, as well as parameters (formally defined Table 1) that help quantify



the load shape. In this paper, we will not discuss the *morning start-up* peak, which is due to HVAC operation bringing the building back to occupied thermostat setpoints, but we note that if the start-up occurs before the *morning ramp-up* then it can probably be rescheduled to start later: there is no reason to bring the building back to occupied conditions before people have begun arriving in the morning.

**Figure 4: Features of a typical commercial building load shape, and parameters that describe the shape.**



**Table 1: Parameter definitions.**

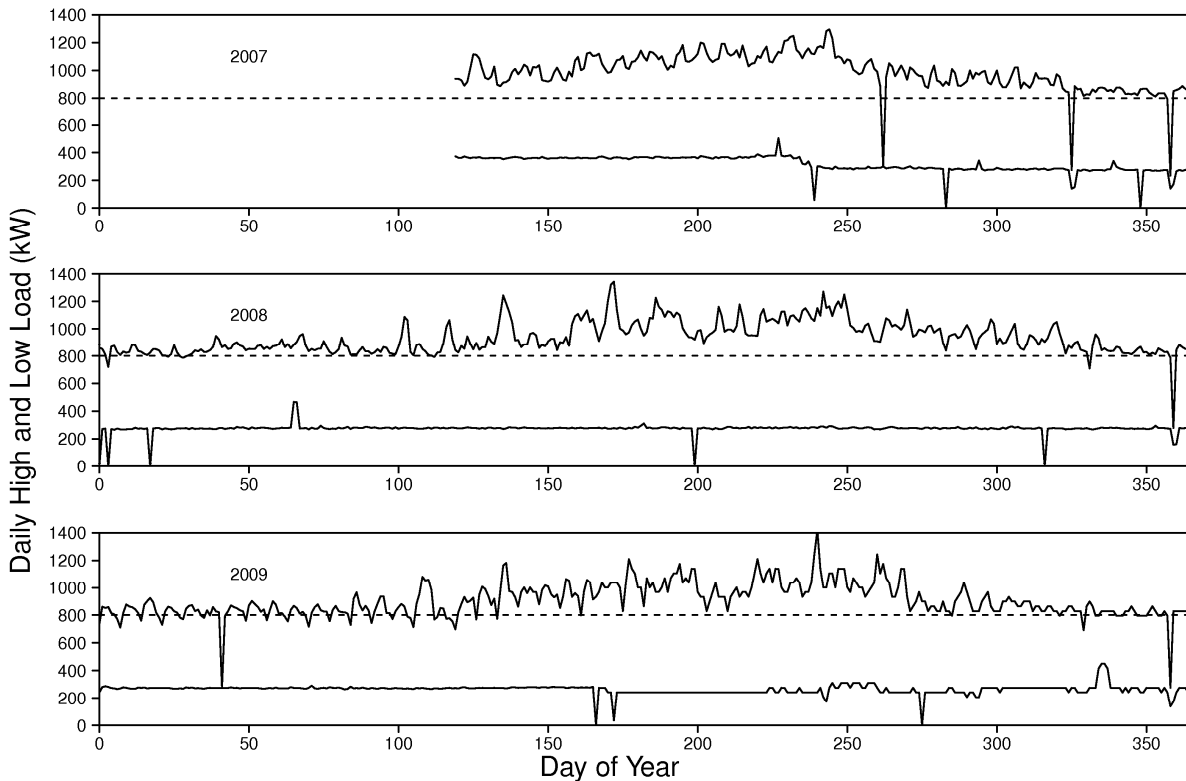
Parameter	Definition
Near-peak load	96.5 percentile of daily load
Near-base load	3.5 percentile of daily load
High-load duration	Duration for which load is closer to near-peak load than to near-base load.
Rise time	Duration for load to go from near-base load to the start of the high-load period.
Fall time	Duration for load to go from the end of the high-load period to the base load.

As indicated in Table 1, we recommend characterizing the load with three time intervals – *high-load duration*, *rise time*, and *fall time* – and the *near-peak load* and *near-base load*, which are used instead of the more-traditional peak and base load. Figure 2 shows why we suggest discarding the extremes of the data to determine the near-base and near-peak load: even after the building’s behavior changes radically, its daily minimum load returns to its normal, low level for just 15 to 30 minutes each day. This sort of behavior is not unusual: often there is a single, short-lived spike or valley in load. Although commissioning agents might be interested in why such features occur – they could indicate a minor problem with scheduling – those short abnormal intervals do not usually have major energy or comfort implications (though the peaks can result in demand charges) and, therefore, should not be used to characterize the building.

Plots of the parameters listed in Table 1 (referred to as “parameter plots”) can highlight key features of electric load data. Figure 5 shows near-peak and near-base load for a large furniture store. Attempting to look at the 15-minute load data for entire years would probably be fruitless, generating an uninterpretable forest of near-vertical lines (even more than in Figure 3), but displaying the near-base and near-peak load simplifies matters:

- 1) Christmas stands out every year, with a near-peak load about the same as the near-base load.
- 2) The peak load is obviously higher in the middle of the year than at the ends, almost certainly because of increased cooling load in the summer.
- 3) Around day 240 of 2007, and day 170 of 2009, the near-base load decreased noticeably. From an energy standpoint, even a modest decrease in base load is worth a lot, since the building uses this power so much of the time. This building, for instance, is in unoccupied mode about 12 hours a day.
- 4) A dashed line at 800 kW has been added to all of the plots to help compare across years. In 2007 and 2008, the near-peak load exceeded 800 kW almost every day; in 2009, the near-peak load was frequently below 800 kW and was almost always lower than on the corresponding days of previous years.

Judging purely from Figure 5, this furniture store seems to be doing a good job at reducing both near-base and near-peak power. (Of course one could also look at daily or weekly energy use, or many other parameters). However, weather is an important complicating factor when trying to evaluate changes in energy or power consumption, at any timescale: A building will use more energy during a hot day or week or season than during a cool one. We present a regression-based method for weather normalization in the next section.

**Figure 5: Daily near-base and near-peak load for a large furniture store.**

## Adjusting for Weather with Load Prediction Models

We are generally interested in comparing building power and energy use from one time period to the next *with weather held constant*, i.e. we would like to compare weather-normalized data. We can do this by employing load prediction models. In this section, we introduce our load prediction model and then demonstrate how it can be used to weather-normalize data.

### **Load Prediction Model**

We use linear regression models because – when constructed appropriately – they provide a good fit to load data in most buildings; their results are easy to interpret; they are easy to modify; and they present modest computational burden. Other methods for load prediction are reviewed in Price (2010) and Mathieu et al. (2011).

Divide a week into intervals (indexed by  $i$ ); for instance, if electric load data are available for 15-minute intervals, the first interval is from midnight to 12:15 on Sunday morning, the second interval is from 12:15 to 12:30, and so on. A different regression coefficient,  $a_i$ , for each period allows each time-of-week to have a different predicted load. This is an improvement on models that only include a time-of-day regression coefficient since we would expect load shapes to

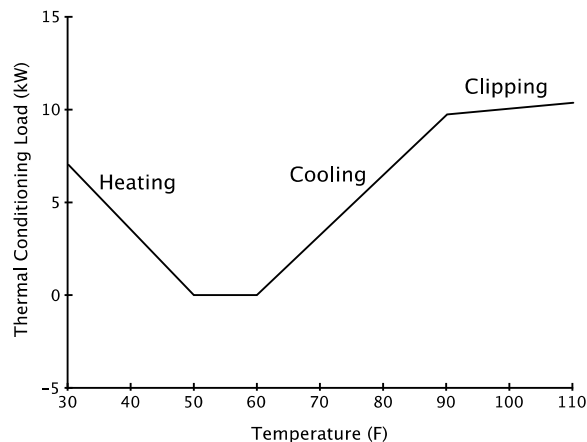
change on different days of the week. Some models include both time-of-day and day-of-week regression coefficients allowing load shapes to be shifted up or down on different days of the week; however, these models do not allow load shapes to change in all of the many ways they might on different days of the week.

We expect that there is a relationship between thermal conditioning load and outdoor air temperature like that shown in Figure 6. When the outdoor temperature is high, cooling load will increase with temperature, and when the outdoor temperature is low, heating load will increase as temperature decreases. For some range of moderate temperatures, the load may be insensitive to temperature because neither cooling nor heating is needed. When the cooling system is maxed out at high outdoor temperatures, the load will be clipped. The outdoor temperature at which cooling or heating is needed or clipping occurs can be estimated if there is enough data from a building, although this estimation adds complexity (Kissock et al. 1998). Therefore, instead of trying to find these *change-points*, divide the range of temperatures that the building experiences into  $N_T$  temperature intervals. We recommend  $N_T$  be around twice the expected number of expected change points. A different regression coefficient,  $\beta_j$ , for each temperature interval allows each interval to have a different linear relation between thermal conditioning load and outdoor air temperature,  $T$ . The temperature is broken into temperature components,  $T_{c,j}$ , computed as in Table 2, which ensure that the temperature dependence is continuous. The predicted electric load,  $\hat{L}$ , at time  $t$  is:

$$\hat{L}(t, T(t_i)) = \alpha_i + \beta_1 T_{c,1} + \dots + \beta_{N_T} T_{c,N_T} \quad (1)$$

The regression coefficients,  $\alpha_i$  and  $\beta_j$ , can be computed using historical load and temperature data, and a regression solver. A building's temperature dependence changes in different operational modes (e.g., occupied, unoccupied, start-up). Therefore, we suggest calculating a different set of regression coefficients,  $\beta_j$ , for each operational mode, or at least for occupied and unoccupied mode, the start and end of which can usually be determined by examining the building's HVAC schedules but which can be guessed, if necessary, by examining load shape plots.

**Figure 6: Thermal conditioning load versus outdoor air temperature.**

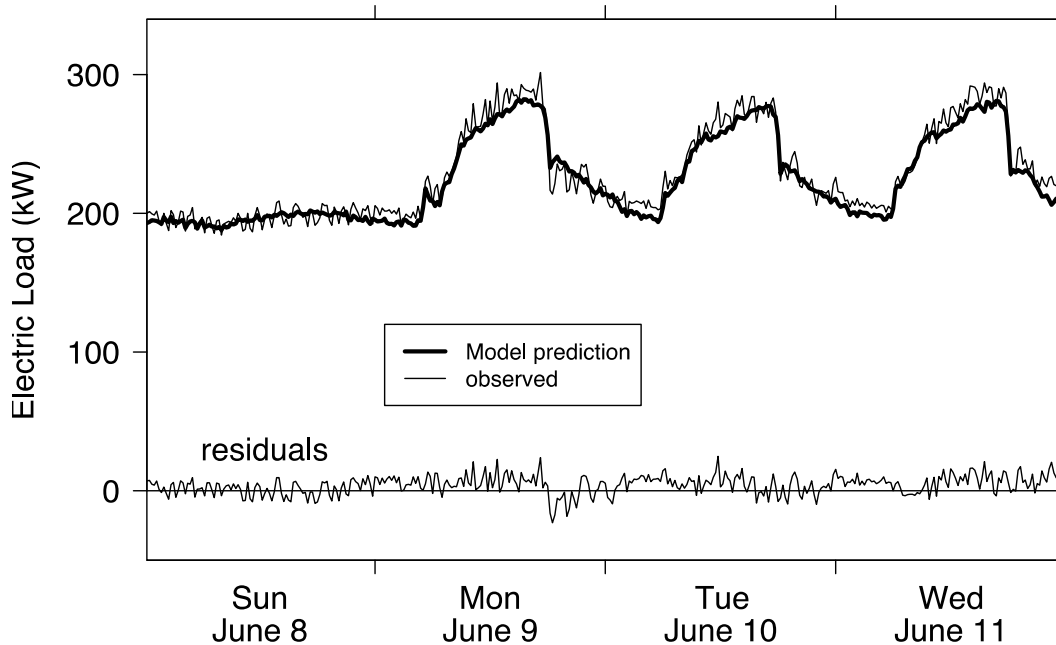


**Table 2: Computation of component temperatures, assuming that  $N_T=6$  and the boundaries between temperature intervals are 50, 60, 70, 80, and 90 °F.**

$T$ (°F)	$T_{c,1}$	$T_{c,2}$	$T_{c,3}$	$T_{c,4}$	$T_{c,5}$	$T_{c,6}$
43	43					
57	50	7				
65	50	10	5			
76	50	10	10	6		
89	50	10	10	10	9	
92	50	10	10	10	10	2
108	50	10	10	10	10	18

The regression model in Equation 1 works well for many commercial buildings. An example, which compares observed data (power measurements) to the model prediction, is shown in Figure 7. The model is unable to predict the portion of the load that varies independently of time-of-week and temperature. For example, load is also affected by occupancy, which varies from week to week.

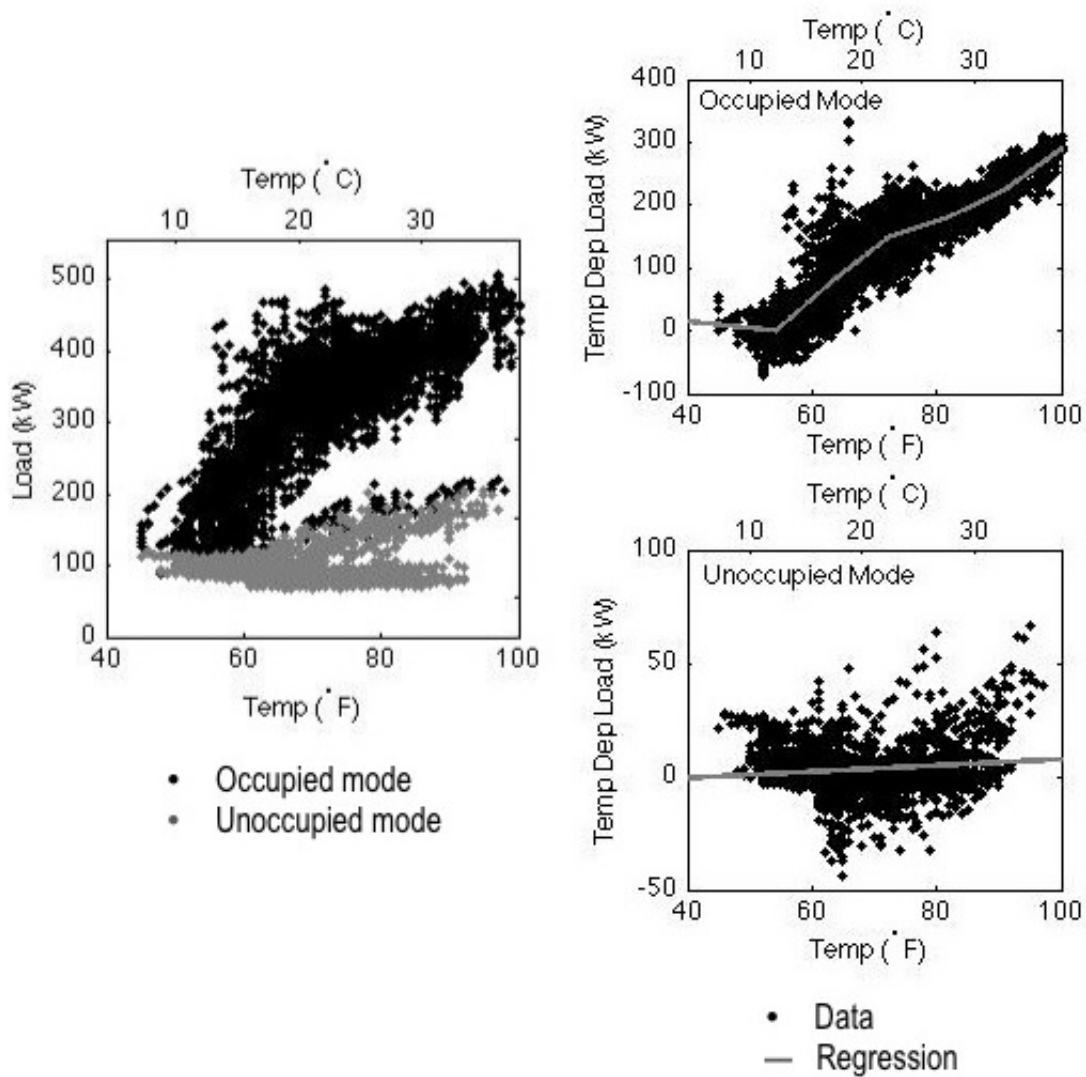
**Figure 7: Performance of the regression model for an office building.**



Load prediction models allow us to understand how a building's load changes with changes in outdoor air temperature. Too often people only inspect plots of load versus temperature to understand a building's temperature dependence. In plotting load versus temperature, time-of-day effects are not distinguished from temperature effects. Load tends to be highest in the afternoon, when temperatures also tend to be highest, but in most buildings the high load in the afternoon is not exclusively or even largely *caused* by the higher temperature.

The temperature-dependent effects can be separated from the time-dependence using the model of Equation 1: The sum of the temperature-dependent terms is an estimate of the *temperature-dependent load*. In Figure 8, we show plots of both ‘Load v. Temperature’ and ‘Temperature-dependent Load v. Temperature’ for an office building. The data points on the right-hand plots show the load after subtracting the time-of-week effect, and the gray line on each plot shows the estimated temperature-dependent load. Models are fit separately for occupied and unoccupied mode data. Six temperature intervals are used for the occupied mode regression. Only one temperature interval is used for the unoccupied mode regression.

**Figure 8: Comparing plots of ‘Load v. Temperature’ (left) and ‘Temperature-dependent Load v. Temperature’ (right) for an office building.**



The distinction between load and temperature-dependent load is important. For instance, the plot of *load* versus temperature slopes steeply upwards, even below 50 °F, but *temperature-dependent* load is flat until the temperature is above 55 °F. The load is, on average, more than 200 kW higher at 70 °F than at 50 °F, but that is mostly because the 50 °F data are usually from mornings and evenings (when load is always low) and the 70 °F data are usually from afternoons (when load is always high). In contrast, the *temperature-dependent* load is only 100 kW higher at 70 °F than at 50 °F. The temperature-dependent load is partly a function of building HVAC performance, so a large change from one year to the next might indicate a problem in the HVAC system.

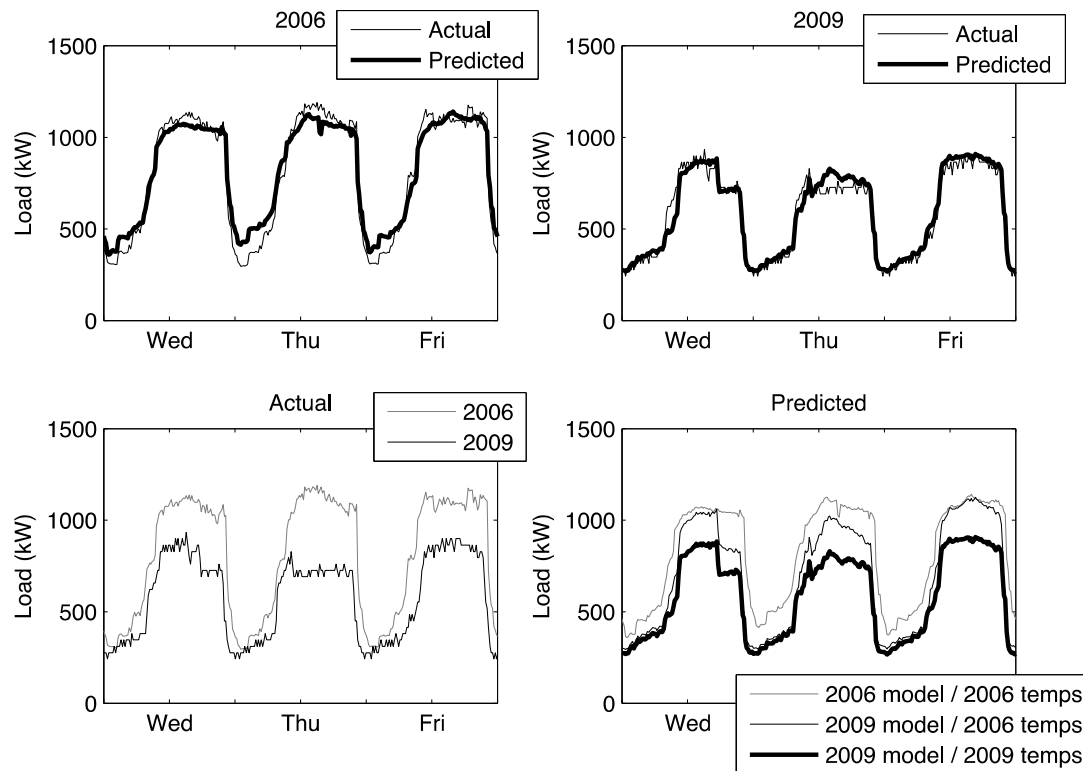
### **Weather-normalization**

The regression model described in the previous section can be used to weather-normalize data so that power usage and energy consumption comparisons can be made between different time periods. To weather-normalize data, a model is built with load and temperature data from time period A, and a second model is built with load and temperature data from time period B. Then, we predict load in time period A with temperature data from time period A, and we predict load in time period B with temperature data *from time period A* (i.e. both data sets are normalized with temperature data from time period A). Therefore, all differences in the weather-normalized data between the two time periods are due to factors other than weather.

Figure 9 shows the results of weather-normalization on data from a furniture store. To give a sense for model accuracy, the top plots compare actual and predicted load for three days in July in 2006 and 2009; these figures confirm that the models make fairly accurate predictions. The bottom left plot compares actual load data across years. From this plot, we learn that the facility used significantly less energy in 2009 than 2006. We want to know how much of the difference is due to changes in equipment, operations, and use, and how much is simply due to weather. The bottom right plot shows weather-normalized predictions. Predictions from 2006 and 2009 are shown (same as the ‘predicted’ lines in the upper plots), as well as predictions that use the 2009 model but 2006 temperatures. The point of using the 2009 model with 2006 temperatures is to see what the building load shape *would have been* in 2009, if the temperatures had been the same as in 2006. Comparing the gray and thin black lines we can see the portion of the savings that is *not* due to weather, while comparing the thin and thick black lines we see the portion of savings that *is* due to weather. Some (but not all) of the difference in daytime load is due to milder weather in 2009, while almost none of the difference in nighttime load is due to weather.

Weather normalization is especially important for benchmarking. Whether an energy manager is comparing buildings under his management with each other in areas where microclimates exist, or comparing buildings to themselves with prior years’ data, weather normalization is required in order to adjust for the weather’s impact on whole building electric demand. By benchmarking the weather-normalized peak and average demand per square-foot, for example, an energy manager can determine the building’s demand trend over several years and determine on which daily or weekly time periods his or her demand savings efforts must concentrate.

**Figure 9: Weather-normalization applied to data from a furniture store.**



## Discussion

The methods outlined in this paper for analyzing commercial building 15-minute-interval electric load data can be used by building operators, energy managers, and commissioning agents to evaluate current building performance and ensure on-going performance. These methods provide ways of visualizing and analyzing time series load data to determine not only how much energy is being used, but also when the energy is consumed and how temperature-dependent the load is. We have shown that these approaches can be used to identify major waste in buildings such as early start times or late shutdown times, excessive temperature-dependence, high base load, and so on.

With load prediction models that remove or adjust for weather dependence, a building's performance can be compared to its past performance as well as its predicted performance. These techniques allow commissioning agents to develop realistic time-differentiated demand targets as well as monitor and maintain demand savings. As deviations from predictions and targets occur, the next step is to collect more information from a building energy management and control system about the performance of individual systems and components to identify malfunctions and other issues related to building operations.



## Conclusions

Our key findings are as follows:

- 1) Plotting time series electric load data is useful for understanding electricity consumption patterns and changes to those patterns, but results may be misleading if data from different time intervals are not weather-normalized.
- 2) Plots of derived parameters such as near-base and near-peak load, and high-load duration, can highlight key features of electric load data and may be easier to interpret than plots of time series data themselves.
- 3) A time-of-week indicator variable (as distinct from time-of-day and day-of-week indicator variables) improves the accuracy of regression models of electric load.
- 4) A piecewise linear and continuous outdoor air temperature dependence can be derived without the use of a change-point model (which would add complexity to the modeling algorithm).
- 5) A model that includes time-of-week and temperature dependence can be used for weather normalization and can determine whether the building is unusually temperature-sensitive, which can indicate problems with HVAC operation.

Now that time-resolved load data are available for most commercial buildings, monitoring of whole-building energy performance should be routine. Changes – especially changes for the worse – should be investigated as part of continuous commissioning. Model results can be used to adjust for weather and quantify changes in performance, thereby allowing the effects of retro-commissioning to be evaluated.

Future directions for this research include (1) applying these methods to whole-building electric load data from more buildings, (2) quantifying the effectiveness of these methods in identifying opportunities for energy waste reductions and energy efficiency projects, and (3) creating easy-to-use tools so practitioners can use these methods both to identify energy saving opportunities and for continuous measurement and verification (M&V) of energy savings from retro-commissioning projects.

## Acknowledgement

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