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An Empirical Model for the Response of BtA Multiwires to Different Ions

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1. Introduction

BtA multiwire data for different ion species and velocities has been extracted from setup books. This data will be used to determine the response of the multwires to these different species and velocities. The presumption is that the response of a multiwire wire is proportional to the number of electrons yielded when an ion (which may enter in a partially stripped state) interacts with it. This yield is thought to have two components. The first is a consequence of those bound electrons that are freed from the incident ion and remain in the wire. The other, called the secondary emission yield, is the number of electrons knocked out of the wire as the ion interacts with it. The net result of these two components is that the charge on the wire changes. The signal is expected to be proportional to this change in the charge on the wire.

The Bethe-Bloch Equation and Induced Charge

In theory, as given by Plum (see references), for some ideal geometry, the secondary emission yield is proportional to the stopping power, *dE/dx,* which is the change in an incident particle's energy with distance traveled into the target material.[1] In this case, the target material is a multiwire wire. The Bethe-Bloch equation relates *dE/dx* to the velocity of the incident particles and its atomic number *z*:[2]

$$-\frac{dE}{dx} = K\frac{z^2}{\beta^2}\frac{Z}{A}\left[\frac{1}{2}\ln\frac{2m_e c^2\beta^2\gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta)}{2}\right] \qquad \textbf{Equation 1}$$

where K is a constant, Z and A are the atomic and mass numbers of the target material (a wire), m_e is the mass of an electron, z is the incident particle's atomic number, c is the speed of light, β is v/c, γ is the relativistic Lorentz factor, T_{max} is the maximum energy transferred to an electron during an encounter with an incident particle, I is the average ionization for the target material, and $\delta(\beta)$ is a small correction due to polarization of the target material. $\delta(\beta)$ is negligible when the kinetic energy of the incident particle does not exceed its rest mass. This is the case with the majority of the data considered here.[3] To a good approximation T_{max} can be expressed as $T_{\max} = 2m_e c^2\beta^2\gamma^2$. [4] The above formulation of the Bethe-Bloch equation neglects the so-called "shell correction" which can contribute as much as 6% to the stopping power (for the case of incident protons in the 1-100 MeV range).[5]

Using the T_{max} approximation above and neglecting the $\delta(\beta)$ correction, the Bethe-Bloch equation becomes,

[1] Plum, pg. 30
[2] Plum, pg 24
[3] Zeigler, pg. 21
[4] Zeigler, pg. 10
[5] Zeigler, pg. 11

$$-\frac{dE}{dx} \approx K\frac{z^2}{\beta^2}\frac{Z}{A}\left[\ln\frac{2m_e c^2\beta^2\gamma^2}{I} - \beta^2\right] \qquad \textbf{Equation 2}$$

which, since the multiwire properties don't change is,

$$-\frac{dE}{dx} \approx K_0\frac{z^2}{\beta^2}\left[\ln(2m_e c^2\beta^2\gamma^2) - \beta^2 - K_1\right] \qquad \textbf{Equation 3}$$

where K_0 and K_1 are constants that incorporate the wire properties and the other physical constants.

If both β and γ can be considered constant, then the equation simplifies to:

$$-\frac{dE}{dx} \approx K_2 z^2 \qquad \textbf{Equation 4}$$

where K_2 is a constant at a particular value of β.

The secondary emission yield, *Y*, from an ion incident on a multiwire wire is given by,

$$Y = K_3\frac{dE}{dx} \qquad \textbf{Equation 5}$$

where K3 is a constant.[6] The voltage associated with the resulting charge emitted from one of the multiwire wires is given by, $V = Q/C$, where Q=-Y is the charge on the wire that results from this emission and *C* is the capacitance in the integrator circuit.[7] If the ion has no bound electrons, then the charge that develops on the wire, which is proportional to the voltage given by the multiwire electronics, should be proportional to dE/dx. This resulting charge would be associated with the number of electrons emitted upon exiting the multiwire as well as entering it. If the ion did not fragment, stop in the multiwire, accept electrons, or lose a significant amount of energy then one would expect the voltage from the electronics to be,

$$V \approx K_4\frac{z^2}{C} \qquad \textbf{Equation 6}$$

There are a lot of assumptions here. Indeed for some initial conditions one expects that the ions with high z, such as Gold, would stop in the multiwire. If so, that positive charge

[6] Plum, page 30
[7] Instrumentation reference

would be deposited there and add to the total charge. In the simplest case, the nucleus of the ion remains intact as it passes through the multiwire and does not lose a significant amount of energy. Considering this case, if a number of ions with different z, but the same β are incident on the multiwire, and the above conditions hold, then the data from these ions should go as z^2. [8]

Now, if the z dependence can be determined by analyzing the data for a constant β, it can be removed from the equation by dividing it by that dependence, g(z). That is,

$$\frac{1}{g(z)}\frac{dE}{dx} = S(\beta) \approx \frac{K_0}{\beta^2}\left[\ln(2m_e c^2\beta^2\gamma^2) - \beta^2 - K_1\right] = \frac{K_0}{\beta^2}\left(f(\beta) - K_1^*\right) \qquad \textbf{Equation 7}$$

Where $K_1^* = \ln(2m_e c^2) - K_1$ and $f(\beta) = \ln(\beta^2\gamma^2) - \beta^2$. The data with the *z* dependence taken out can then be analyzed for the β dependence.[9]

Experimental data from Plum[10] for protons shows that the stopping power decreases with increasing momentum from 200 MeV/c to about 3 GeV/c where it reaches a minimum. As the momentum increases from 3 GeV/c, the stopping power slowly increases. The momentum at which the minimum occurs does not have a strong dependence on the composition of the target material.

The stopping power only relates to the secondary emission yield associated with the nucleons in the incident ion. Another component of the signal may be dependent on the electrons that are bound to the ion before it interacts with the wire. Specifically, these electrons may become attached to the wire. So, there may be a component of the multiwire charge that is linearly dependent on the initial number of electrons bound to the incident ion.

Understanding the Data

Table I shows the data collected from various runs over the past 5 years or so. The "intensity" column shows measurements from the Booster current transformer late in the acceleration cycle. These values are used as the intensities at the multiwire. However, if the transmission to the multiwire is poor, the actual intensity there will be lower. Also, in general, the intensity value associated with a set of multiwire data is not from the particular cycle during which the data was acquired from the multiwire, but was taken around the same time. What "around the same time" means is a judgement call on my part, determined on a case by case basis from the information in the setup book. In order to flag this uncertainty in the intensity at the multiwires it seems reasonable to assign a somewhat arbitrary uncertainty in that intensity of ±10%.

[8] The effect of ions that only have a glancing interaction with a wire are also neglected.

[9] Nominally, g(z) would be proportional to z^2.

[10] Plum, pgs. 25-26

As mentioned above, the voltage associated with the charge that develops on a multiwire wire is given by $V = Q/C$, where Q is the charge on the wire and C is the capacitance in the integrator circuit. This capacitance can be changed, and the voltage generated per incident ion, or the *gain* of the multiwire will change accordingly.[11] There is a capacitor that can be changed through the control system. When the Multiwire gain is set to *high* this capacitor is 180 pF, if it is in *low* gain it is 1000 pF. So, profiles that are acquired in *High* gain have a voltage signal that is 5.56 (=1000/180) times that of *low* gain.[12]

There is also an additional *x10* gain possible associated with the presence of another capacitor that is installed/removed locally and cannot be monitored through the controls system. So, there are four possible gains that the multiwire can be in: *Low, High, Low x10, and High x10*. This *x10* gain was originally intended for looking at Polarized proton profiles (which generally have the smallest signal) together with the high gain capacitor. This extra gain was more important before the OPPIS source came into operation. In any event, in looking back over data from years ago it is difficult to determine whether this *x10* gain was present or not. So, in collecting the data an educated guess generally had to be made as to whether this *x10* gain was present or not.

The effect of the capacitor on the voltage produced per incident ion is not of fundamental interest here. What is of interest is the amount of charge that develops on the wires when the beam is incident on them and how this changes with the type of particle. The voltage produced for a given capacitance when an ion is incident on a wire is representative of this since $Q=CV$. So, all the voltage Sum data is normalized to low gain. For example, if there were 10V generated for a particular multiwire data acquisition, and the hardware was in high gain, the voltage produced in low gain would be 10V/5.56=1.8V. In the following, the *'gain'* of the multiwire, or rather the response of the multiwire to a particular beam, will mean the amount of voltage produced on the multiwire in the low *"hardware gain"* setting where *C* is *1000 pF*.

In addition, there are sets of horizontal and vertical wires for each multiwire, and each measurement contains the voltage sums for both sets. The values for either plane are generally within 1 or 2% of each other. The data shown in the table is generally the average of the two, or if the profile in one plane is better than the other, (e.g.- if wires are saturating in one plane) then the sum from the other plane is taken.[13] In table I the gains of the multiwires appear in the V/Ion rows. These values are simply the multiwire Sum indicated by the BeamLineInstrument program divided by the intensity and then normalized to low gain.

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[11] Instrumentation reference (untitled)
[12] This was confirmed with Cu beam on MW060.
See **View this Shiftlog: Sun_Sep_19_2004_112605_AM**
[13] The wire voltage saturates at 10V.

	P	**PP**	**D (h=3)**	**D (h=6)**	**Si**	**Fe**	**Cu**	**Au**	**Si**	**C**	**Fe**
Atomic Number (z)	1	1	1	1	14	26	29	79	14	6	26
Charge 006	1	1	1	1	5	10	11	32	13	5	20
Charge 060	1	1	1	1	14	26	29	77	14	6	26
Electrons at MW006	0	0	0	0	9	16	18	45	1	1	6
# of cycles	5	1	3	1	1	1	4	4	1	1	1
MW006 Sum (V)	20.3	8.4	9.3	1.5	2.9	10.6	72.4	43.2	34.1	18.6	38.4
MW060 Sum (V)	11.6	3.55	N/A	0.85	3.5	7.8	56.8	157.6[1]	23.2	15.6	27.8
Intensity 006	23e12	1.3e11	1.7e11	1.1e10	3e9	25e8[3]	8e9	60e8	5.0e9	12.6e9	1.0e9
Intensity 060									5.6e9	12.3e9	1.3e9
Gain 006	L	10H	10H	10H	H	H	10L	L	10H	10H	10H
Gain 060								H			
Ions/V 006[2]	1.13e12	8.60e11	1.02e12	4.08e11	5.73e9	1.31e9	1.10e9	1.39e8	8.15e9	3.77e10	1.45e9
Ions/V 060[2]	1.98e12	2.04e12	N/A	7.2e11	5.12e9	1.78e9	1.41e9	2.17e8	1.34e10	4.38e10	2.6e9
V/Ion 006[2]	8.83e-13	1.16e-12	9.84e-13	2.45e-12	1.75e-10	7.63e-10	9.05e-10	7.20e-10	1.10e-10	2.72e-11	5.31e-10
V/Ion 060[2]	5.04e-13	4.91e-13	N/A	1.39e-12	2.10e-10	5.62e-10	7.1e-10	4.72e-9	7.45e-11	2.28e-11	3.85e-10
V/Ion[4] Norm. 060	8.87e-13	8.64e-13	N/A	2.45e-12	3.69e-10	9.89e-10	1.27e-9	8.31e-9	1.31e-10	4.01e-11	6.77e-10
v/c	0.95	0.92	0.76	0.43	0.44	0.42	0.43	0.43	0.73	0.70	0.64

Table I: Multiwire, particle, intensity, and velocity data as well as derived values.

1- The Sum given through the BeamLineInstrument program could not be used since some of the beam in different charge states did not appear on the multiwire. So, it was necessary to calculate the sum of the Au^{+77} beam by hand. This was then divided by 0.639, the percentage of beam in that charge state.
2- These values are normalized to L gain which is 1/5.56 x H gain.
3- Poor (60%) transmission between Booster and AGS
4- This "gain" is normalized to the gain of mw006 so that the response of the 2 multiwires can be compared in a meaningful way (see text)

There is a stripping foil located between MW006 and MW060. So, many of the particles studied will carry bound electrons at MW006, but will no longer carry them at MW060. As a result, one would expect that the gains at the two multiwires may differ. Protons and deuterons don't contain electrons so one might expect their gains to be the same at both multiwires. Unfortunately, looking at the table, it is clear that the gains for these species are significantly different at the two multiwires. Nevertheless, what is of interest here is the general response of a multiwire to beam (e.g.- their z^2 dependence), not small differences in the response of particular multiwires to beam. So, to account for this discrepancy, a normalized gain will be used for MW060. To arrive at this normalized gain, the V/Ion obtained for MW060 (G_{60}) for a particular case will be multiplied by the ratio of the gain for MW060 divided by the gain for MW006 for a species without electrons at both multiwires. The most logical species to choose seems to be deuterons (h=6) because it has a velocity that is similar to that of many of the other particles in the table. This fact will be useful in the analysis. So, the normalized MW060 gain will be

$$G_n = \frac{deuterons(h=6)@MW006}{deuterons(h=6)@MW060} G_{60} = \frac{2.45e-11}{1.39e-11} G_{60} \qquad \textbf{Equation 8}$$

or $1.76G_{60}$. This normalized MW060 gain, G_n, is shown in the table in an additional row called "V/Ion Norm. 060".

The wires in MW006 and MW060 are composed of the same material.[14] The wires in MW006 are spaced by 1.5 mm, whereas those in MW060 are spaced by 2.5mm. The fact that the horizontal and vertical Sums generally agree and the profiles in these two planes are generally quite different suggests that these differences in the two multiwires are not important as far as there response to a particular ion is concerned. Therefore, it will be assumed in the following that, aside from the normalization described above, the response of the multiwires is identical. This will be important for distinguishing the effect of the ion's bound electrons on the multiwire response.

2. Data Analysis

Z Dependence

As discussed above, for a constant velocity and no electrons, one might expect the multiwire gain to be a quadratic function of atomic number (z). The data in Table I for *d (h=6), Si (+5/+14), Fe (+10/+26), Cu*, and *Au* were taken all at essentially the same velocity ($\beta=0.43 \pm 0.01$).[15] With the exception of *Au*, the ions in question are fully stripped at MW060. Since *Au* has only 2 of its 79 electrons remaining at MW060, and neglecting them will simplify the analysis, there presence will be neglected.

[14] The wires are Tungsten and 3% Rhenium They are 1mm in diameter. (Dave Gassner).

[15] The numbers in parentheses are the charge states of the ion at MW006 and MW060, respectively.

The transmission from MW006 to MW060 will be assumed to be 100%, except in the case of Au^{+77} where 63.9% transmission will be used. This transmission efficiency is equal to the percentage of Au that comes out of the foil as Au^{+77}.[16]

Figure 1 shows this data together with a fit of the form $G_n = kz^2$ for the MW060 data (red-dotted line). With *k=1.34e-12 Volts/ion*, this fit describes the data reasonably well. However, if the exponent of *z*, *m*, is left as a free parameter and a linear fit of *log (z)* vs. *log* $G_n(z)$ is performed, a fit with *m=1.855* and *k=2.49e-12* results (R=0.9998). This fit is significantly better than the one for *m=2*. It is the brown curve in figure 1. Figure 2 is a close-up of the data for lower *z* where it is easier to compare the fits to the data.

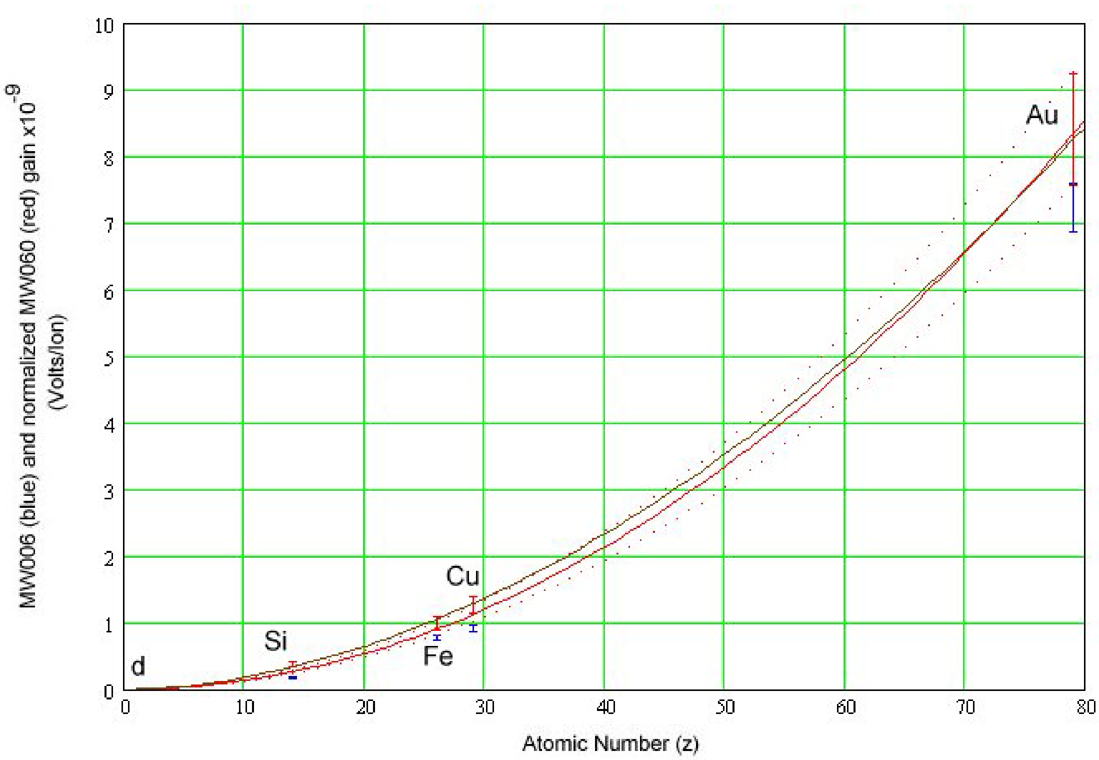

Figure 1: Data and fits for multiwire gain (*V/Ion*) vs. *z* for cases where $\beta=0.43\pm 0.01$ (d^{+1}, Si^{+14} *(charge +5 at MW006)*, Fe^{+26}, Cu^{+29}, and Au^{+77}). The error bars reflect the ±10% uncertainty in intensity. The red solid line is a fit for the G_n (MW060) data of the form $G_n = kz^2$. The z^2 fit gives *k=1.34e-12 V/ion*. The red dotted lines are fits of the same form for the high and low cases (k_{high}*=1.49e-12* and k_{low}*=1.22e-12 V/ion*). The brown curve is the $z^{1.855}$ fit with *k=2.49e-12 Volts/Ion*.

The MW006 gain (in blue) is generally lower than the (normalized) MW060 gain. If this difference is treated as though it were due to the presence of electrons on the ions at MW006, then one would infer that the electrons produce an opposite charge to that of the protons in the nucleus. This is consistent with the idea that these bound electrons add to

[16] P. Thieberger, charge state distribution for BtA Carbon foil from 4/18/03.

the charge on the wire, whereas nucleons cause electrons to be emitted from the wire thereby causing the wire to become positively charged.

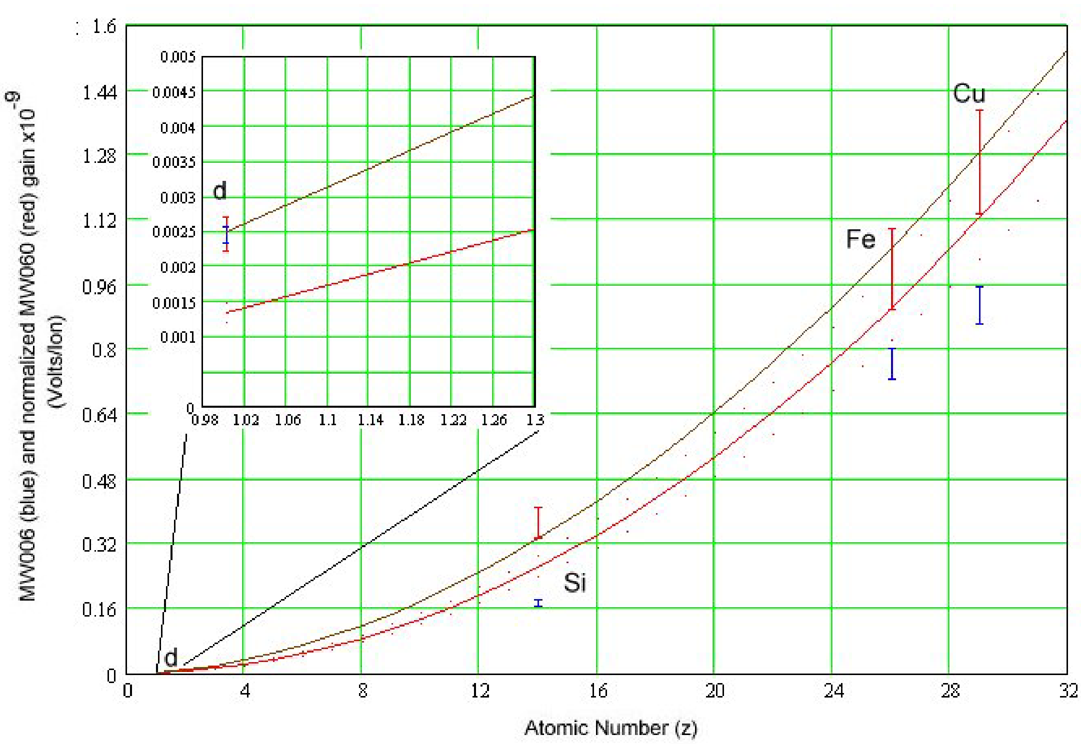

Figure 2: Data and fits for multiwire gain (*V/Ion*) vs. *z* for cases where $\beta=0.43\pm 0.01$ (d^{+1}, Si^{+14} *(charge +5 at MW006)*, Fe^{+26}, and Cu^{+29}). This is a close-up of the graph in figure 1 for lower *z* including the deuteron case. The legend is the same as it is for figure 1.

To parameterize the effect of the bound electrons, the difference between the gains at MW060 and MW006 was taken. This difference is shown in figure 3 together with a linear fit. The linear fit to the data is given by,

$$\Delta G = (2.52\cdot 10^{-11})N - 6.73\cdot 10^{-11} \qquad \textbf{Equation 9}$$

where *ΔG* is the difference in gains and *N* is the number of bound electrons (*R=0.985*). The non-zero y-intercept is likely due to errors in the data such as those due to inaccuracy in the normalization between the two multiwires and so will be discarded. This inaccuracy could simply stem from uncertainty in the intensities at the two multiwires stemming. The response of the multwires to an ion containing electrons can then be described by,

$$G(z,N) = kz^{m} + (2.52e-11)N \qquad \textbf{Equation 10}$$

where, if m=2 then k=1.34e-12, if m=1.855 then k=2.46e-12. Figure 4 shows the data and functions derived from the fits for both multiwires, using m=2.

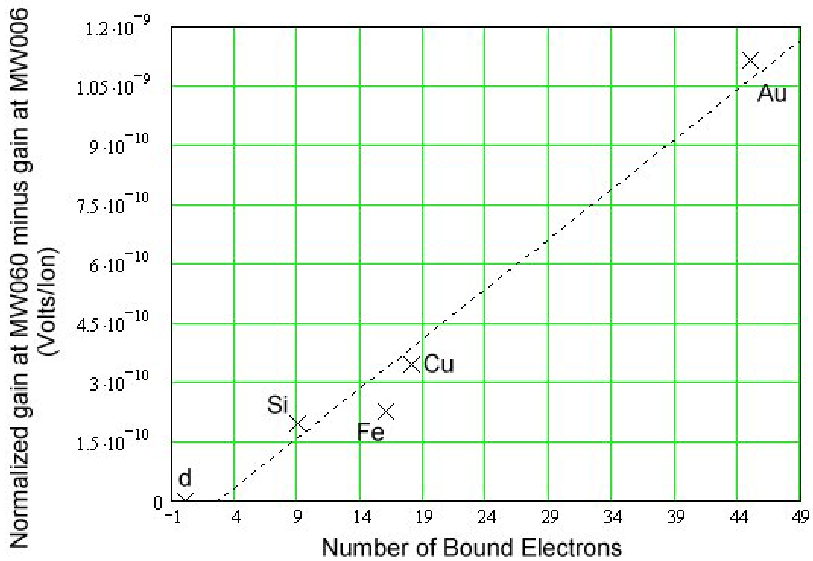

Figure 3: Normalized MW060 gain minus gain at MW006 (ΔG) versus number of bound electrons (N) for ions at $\beta=0.43$. The dotted line is a linear fit of the data, $\Delta G=(2.52e-11)N -(6.73e-11)$.

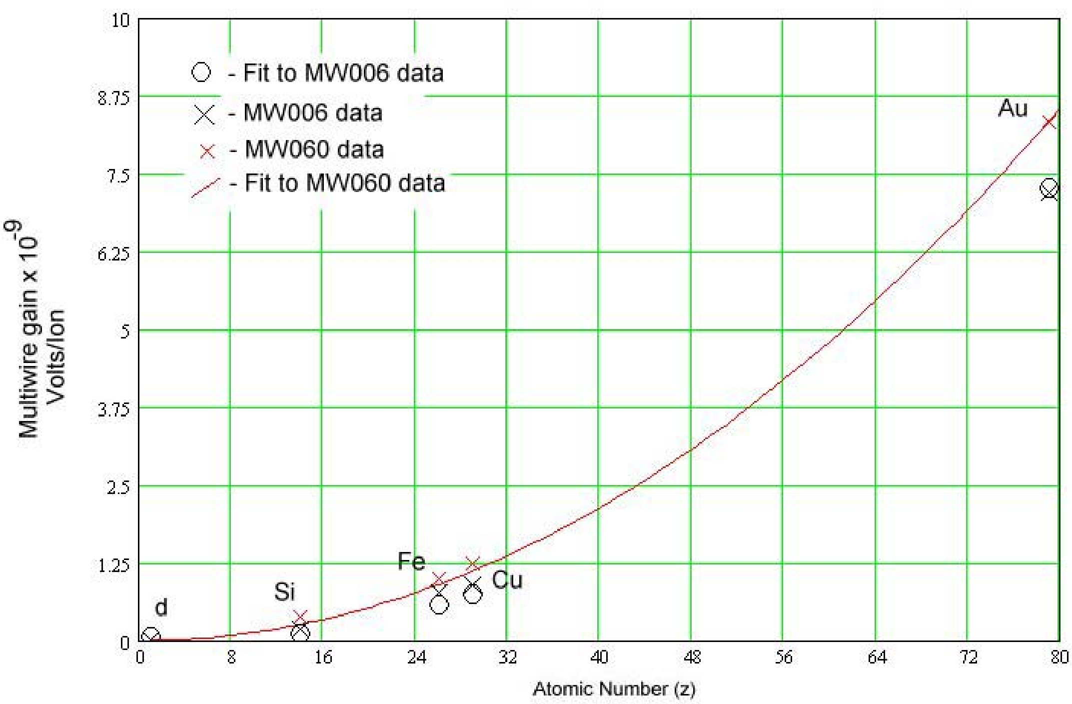

Figure 4: Data and fits for both multiwires for ions at $\beta=0.43^{\pm}0.01$ for $m=2$.

β *Dependence*

As mentioned above (eq 7), if all the data is divided by its z dependence a function *h(β)* should result. From the discussion above one would expect that *h(β)* would take the form $h(\beta) = \frac{K_0}{\beta^2}\left(f(\beta) - K_1^*\right)$, where $f(\beta) = \ln(\beta^2\gamma^2) - \beta^2$. Figure 5 shows the G_n data from Table I divided by z^2 (red points) and $z^{1.855}$(blue points). Ideally, for a given β the points would fall on top of each other. The points for *β=0.43± 0.01* are of particular interest since they are markedly closer to each other with the $z^{1.855}$ fit.

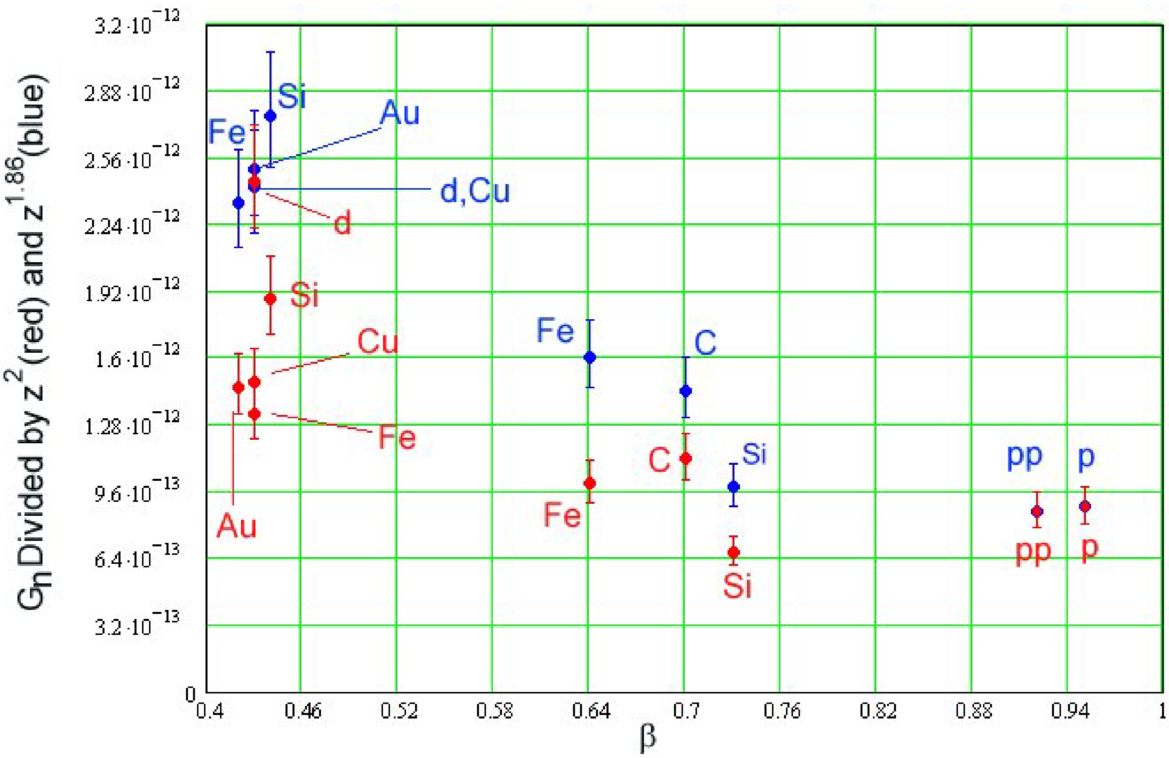

Figure 5: β dependence of MW060 gain ($h(\beta)=G_n/z^m$) for *m=1.855* (blue) and *m=2* (red) for all data in Table I.

So, although the z^2 fit seems reasonable for describing the gain as a function of *z* for β~0.43 (figs. 1 and 2), this fit does not seem adequate to describe the behavior of *h(β)*. It seems reasonable to use m=1.855 to continue with this analysis, even though that value for m does not agree with the assumptions made previously about the gain's expected z dependence. One assumption was that the nuclei remain intact during the passage through the wire, but perhaps some of the nuclei do not. Another was that the effect of ions that only make a glancing blow to the wire could be neglected. There are several reasons why the data might not agree with what would be expected from the simple model described in the introduction.

Figure 6 shows the data with the results of a linear fit of the form *log(h)* vs. *log(β)*. This yields the equation,

$$h(\beta) = \frac{7.94 \cdot 10^{-13}}{\beta^{1.35}}$$ **Equation 11**

which fits the data reasonably well. Also, shown in the graph is a curve of the form $h(\beta) = \frac{K_0}{\beta^2}\left(f(\beta) - K_1^{*}\right)$ with K_0*=1.34e-13* and K_1*=-5.12*.[17]

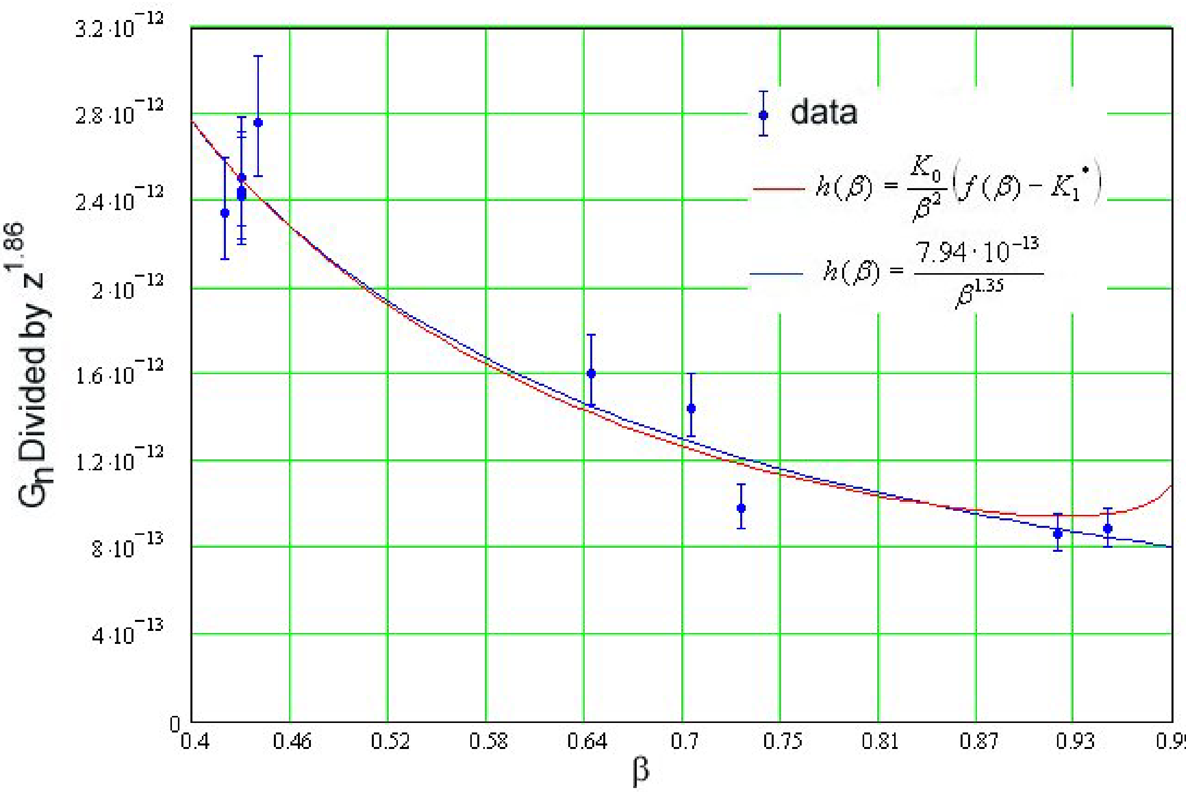

Figure 6: β vs $h(\beta)$ for MW060 G_n data in Table I.

Revisiting the Electron Response

In light of the improvement in the fit when the power of z, m, for constant β is 1.85 instead of 2 (figure 5), another look at the electron response parametrization seems in order.

Figure 7 shows the β*=0.43* data. Comparing figures 4 and 7 it is clear that the m*=1.855* fit is significantly better for the MW006 data.

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[17] This curve was arrived at by minimizing the standard deviation by succesively scanning K_0 and K_1.

3. Summary

The MW060 data shows that the *z* dependence of the multiwire data for $\beta=0.43$ can be described quite well by G α $z^{1.855}$ (see figs. 1 and 2), and not so well by the expected G α z^2 (see fig. 5). Assuming that the response of MW006 is identical to that of MW060, except for a scale factor, the effect of additional electrons can be seen as a relatively small effect on this response for the cases studied that acts in the opposite sense and is proportional to the number of bound electrons on the incident ion (figs. 3 and 4). The effect is most apparent for Au which has 45 electrons at MW006. The parameterization that describes this effect is improved if $z^{1.855}$ is used (compare figs 4 and 7).

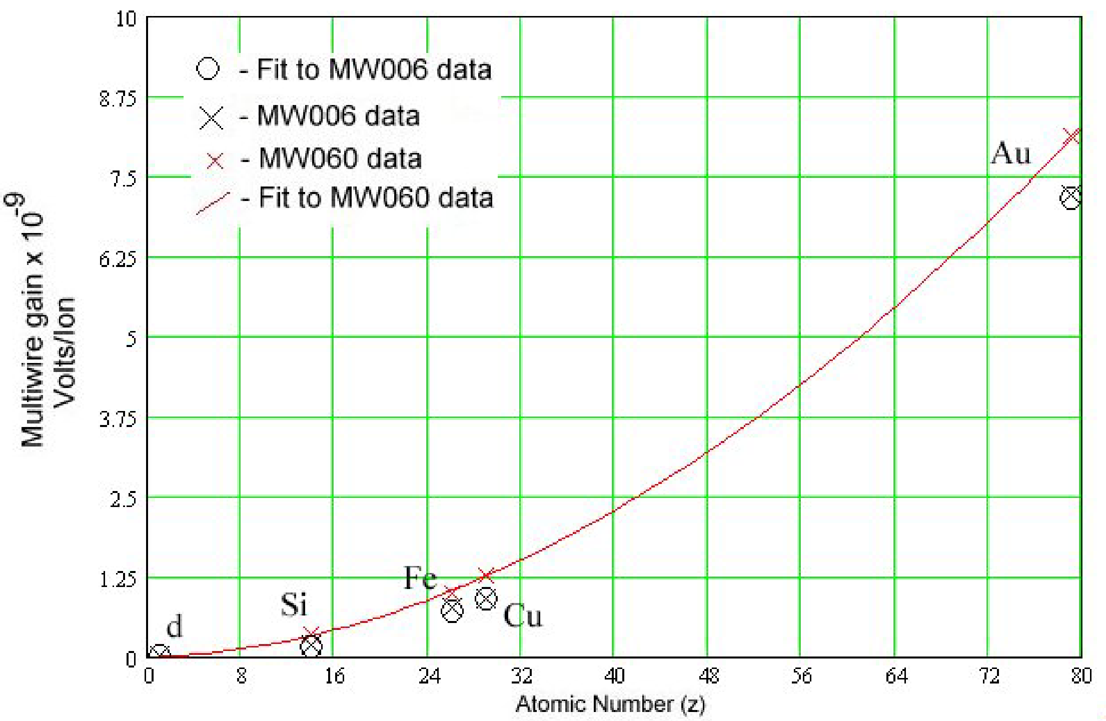

Figure 7: Data and fits for both multiwires for ions at β=0.43±0.01 for m=1.855. Compare to figure 4 where m=2.

The β dependence of the MW060 data, which is the case where there are no bound electrons, fits the functionality of the Bethe-Bloch equation dependence rather well (see figure 6). It seems reasonable to assume that the response of a multiwire to bound electrons is unrelated to the β of the incident particles, since those electrons will simply add to the charge on the wire regardless of the velocity of the incident ion. Therefore, the response of a (normalized) multiwire to incident ions can be parameterized as,

$$G(z,N,\beta) = z^{1.855}h(\beta) - K_1 N \quad \textbf{Equation 12}$$

where $h(\beta) = \frac{K_0}{\beta^2}\left(f(\beta) - K_1^*\right)$, $f(\beta) = \ln(\beta^2\gamma^2) - \beta^2$, K_0=*1.34e-11V/ion*, K_1*=*-5.12*, and K_1=*2.52e-11V/ion*. These values are appropriate for MW006, for MW060 G_{060}=*G/1.76* should be used. It should be noted that $h(\beta) = (7.94e-13)\beta^{-1.35}$ could just as well be used. In fact, it may be preferable to use it in estimating the multiwire response to a particular ion because it is simpler.

Figure 8 shows all the data using the parameterization shown in equation 12.

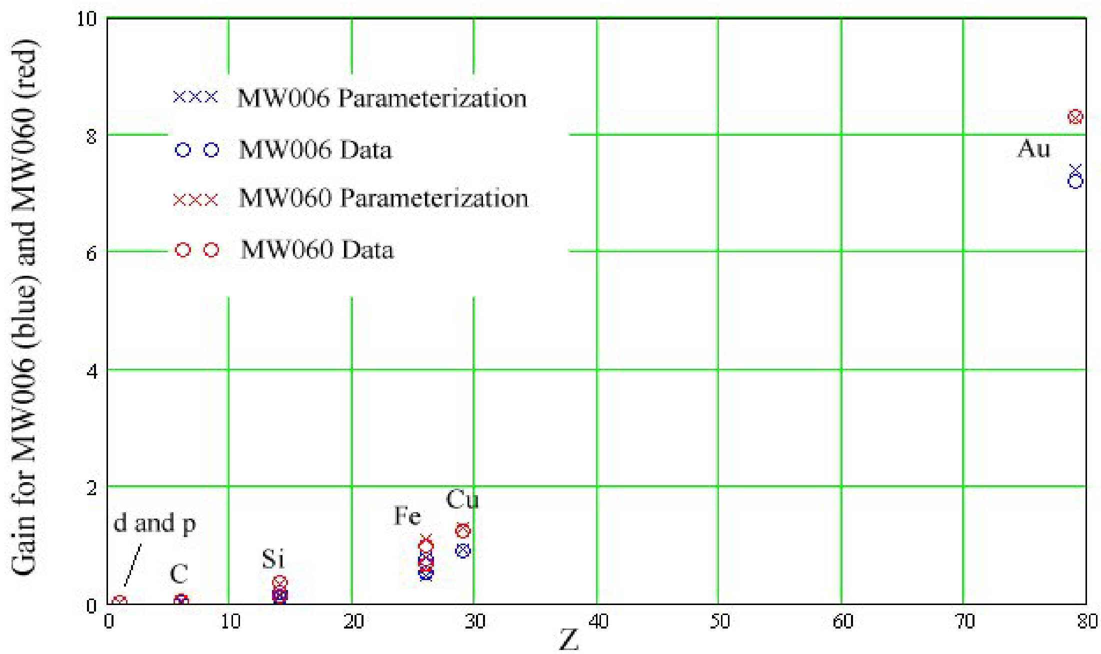

Figure 8: Gain (in units of V/ion x 10^9) vs. Atomic number for all multiwire data together with parameterization (equation 12).

Figure 9 shows a close-up that neglects Au. Figure 10 shows a close-up that neglects Au and Fe as well.

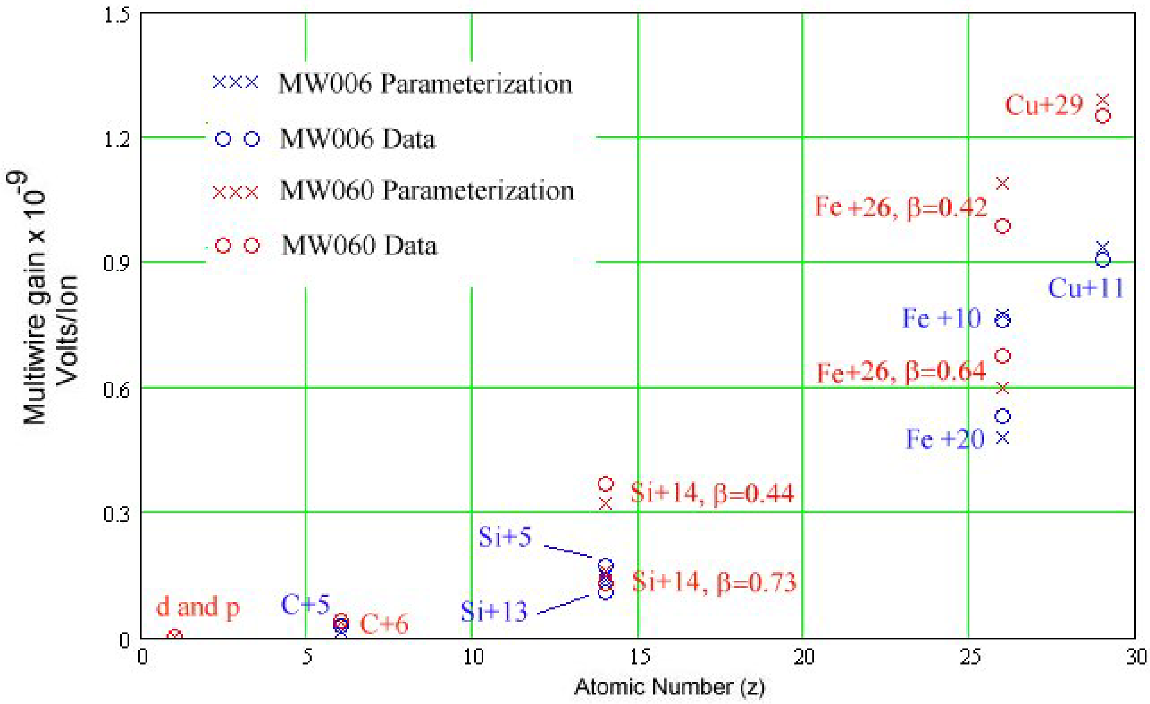

Figure 9: Gain (V/ion) vs. Atomic number for multiwire data with parameterization for ions up to Copper.

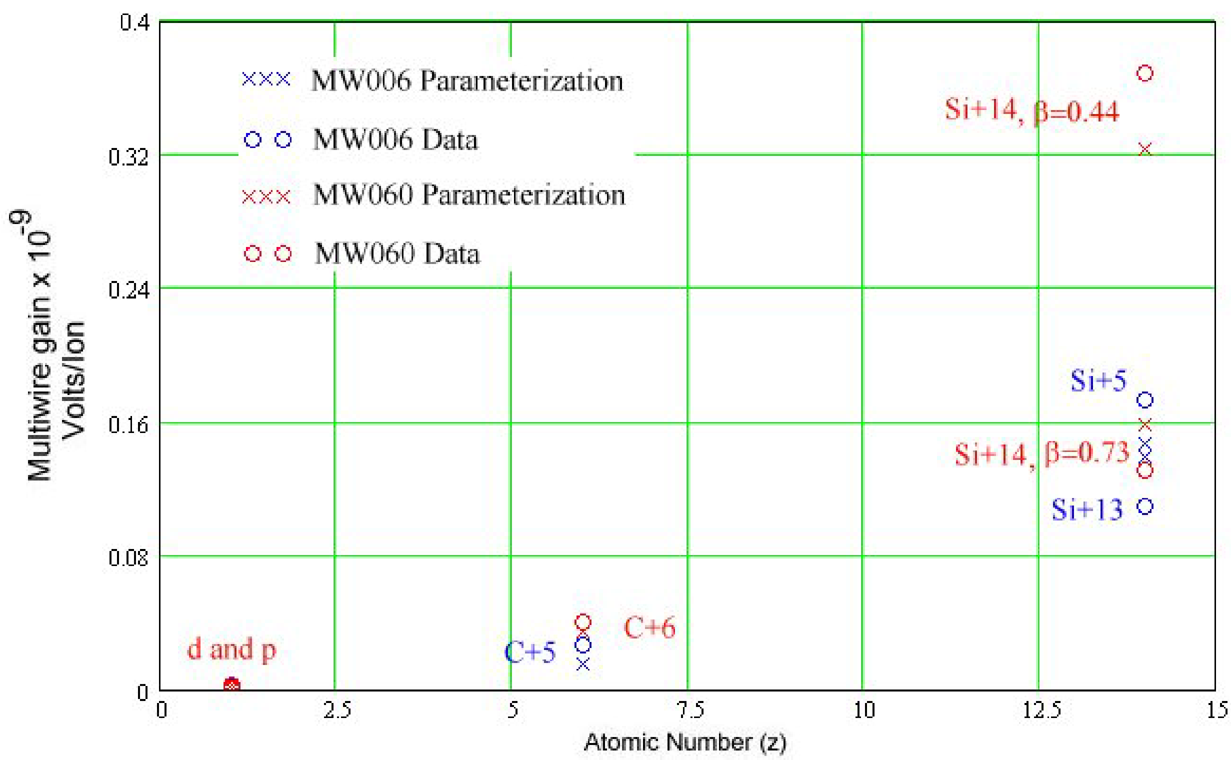

Figure 10: Gain (V/ion) vs. Atomic number for multiwire data with parameterization for ions up to Silicon.

Acknowledgements:

I would like to thank Leif Ahrens for his encouragement and guidance, and Dave Gassner for providing information about the BtA multiwire design as well as other resource material.

Data References:

Protons +1: HEP Setup Book, pg. 132.

Polarized Protons +1: data taken March 23, 2005

Deuterons +1 (h=3): Setup Book I pg 138, Setup Book II pg. 57.

Deuterons +1 (h=6): Setup Book I FY 03 pgs. 120, 125, and 129.

Si +5/14: HIP Setup Book FY 99/00, pg. 89

Fe +10/26: HIP Setup Book FY 99/00, pg. 57

Cu +11/29: Booster_AGS_fy05 elog, Feb 28, 2005, 1328-1329 hrs

Gold +32/77: Booster_AGS_fy04 Au elog, Dec 15, 2003

Si +13/14: AGS_NASA_2005 elog, July 7, 2005, 1555 hrs.

C +5/6: AGS_NASA_2005 elog, July 8, 2005, 0959 hrs.

Fe+20/26: AGS_NASA_2005 elog, June 29, 2005, 1823 hrs.

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Instrumentation Multiwire information provided by D. Gassner.