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THE EFFECTIVE SPECIFIC IMPULSE OF A PULSED ROCKET ENGINE

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(Title: Unclassified)

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THE EFFECTIVE SPECIFIC IMPULSE OF A PULSED ROCKET ENGINE

E. A. Platt and D. W. Hanner

Lawrence Radiation Laboratory, University of California
Livermore, California

May 5, 1965

NOMENCLATURE

M_s = weight of pressure vessel, tons
ρ = density of vessel material
V = volume of vessel
σ = allowable stress in vessel
p = quasi-steady initial pressure, atmospheres
q = allowable fraction of yield stress for vessel material
T_s = temperature of the pressure vessel, degrees Kelvin
T'_H = temperature of the hydrogen before detonation, degrees Kelvin
λ = thermal leakage fraction
M_H = weight of hydrogen charge, tons
Y = bomb energy, tons of high explosive equivalent
M_P = weight of propellant mixture, tons
E = specific internal energy of propellant mixture after detonation, calories per gram
χ = hydrogen fraction of propellant mixture
τ = characteristic discharge time, seconds
T = temperature of propellant after detonation, degrees Kelvin
R = radius of pressure vessel, feet
v = specific volume of propellant, cubic centimeters per gram
M_N = weight of nozzle, tons
q_N = allowable fraction of yield stress for nozzle material
r_t = radius of the throat, feet
ϵ = expansion ratio
$\tilde{T}$ = effective temperature of the nozzle, degrees Kelvin
M_{TV} = weight of throat valve, tons
m = weight of propellant in plenum, tons

t = time after detonation, seconds

γ = ratio of specific heats

a_0 = sonic velocity in throat, initial, feet per second

$\overline{I}_{sp}$ = weight average specific impulse, lbf-sec/lbm

H = specific inthalpy of propellant mixture, initial, calories per gram

η = nozzle velocity coefficient

α = ratio of coolant hydrogen to propellant

θ = time between detonations, seconds

δ = fraction of energy radiated from the motor

M = motor weight, tons

M_C = bomb weight, tons, or pounds

INTRODUCTION

If the energy of a fissioning nuclear bomb were added to rocket propellant, the enthalpy of the mixture of propellant and bomb debris could be very great. If such a mixture were expelled from a rocket motor quickly enough, the walls of the motor might be undamaged by propellant temperatures and pressures which would be impossible to contain within the walls if the rocket motor were in continuous operation. The specific impulse achieved in this manner could be greater than that of any continuous flow engine. To a certain extent, this increase in specific impulse would be obtained at the expense of motor weight and average thrust. This paper considers the first of these limitations, motor weight, and estimates the highest effective specific impulse to be expected from a nuclear-pulsed rocket motor with respect to the weight of the motor.

For each motor weight, bomb weight, and firing rate, a set of values of the other significant parameters of the engine exists which gives a maximum specific impulse. The optimums of some of these other parameters change relatively little over the range of motor weights which is of interest to us. For example, in our study it is found that the amount of hydrogen in the propellant should be between 35 and 45 percent, and that the temperature of the propellant should be close to 6000° K. It also appears from this study that nuclear devices in the range of 1 to 10 tons of high explosive equivalent are needed for missions of current interest; e.g. a 50-ton payload and a ΔV of 60,000 fps. The results given in this report, however, are independent of the details of the mission. They depend only upon the weight of the motor and upon considerations of heat effects and the usable strength of materials. The results were obtained by selecting system parameters to give optimum performance in each motor weight considered.

The rocket motor model used in this analysis is a spherical plenum emptying into a 20° conical nozzle. The throat of the nozzle is closed by a valve which is capable of quick opening. This valve is composed of mating circular sectors which pivot back against the walls of the nozzle throat to open under the pressure of the explosion. The bomb is assumed to be available in whatever yield of energy suits the engine best. Its weight is a parameter. The thermodynamics of the propellant mixture are treated by assuming the primary propellant to be cryogenic hydrogen and the bomb debris to be all carbon.

The pulse cycle begins with the valve closed. Cold hydrogen is admitted to the vessel and is allowed to reach equilibrium temperature with the vessel walls. The nuclear device is conveyed to the center of the vessel and detonates. The pressure of the propellant mixture drives the throat valve open, and in so doing pumps some additional hydrogen which is in the connected buffer cylinders to a higher pressure. This hydrogen is expended through the surfaces of the valve sectors and the surface of the nozzle throat area to prevent them from melting. Pressure in the nozzle drops quickly, and the throat valve buffer cylinders are recharged with hydrogen, which causes the throat valve to close again.

SUMMARY

We find that while the specific impulse is not strictly limited by any of the assumptions which were made, specific impulses greater than 1400 seconds would require very heavy engines for which, it is believed, desirable combinations of mission and payload do not exist. Figure 1 illustrates this trend and also the influence of the weight of the explosive charge upon the obtainable specific impulse. Figure 2 shows the limited effects on the problem of the other free parameter, the time between detonations. The longer pulsing periods benefit by increasing the radiation cooling from the nozzle and reducing the requirement for coolant hydrogen. Figure 3 illustrates the weight distribution for optimum motors. The pressure vessel is generally more

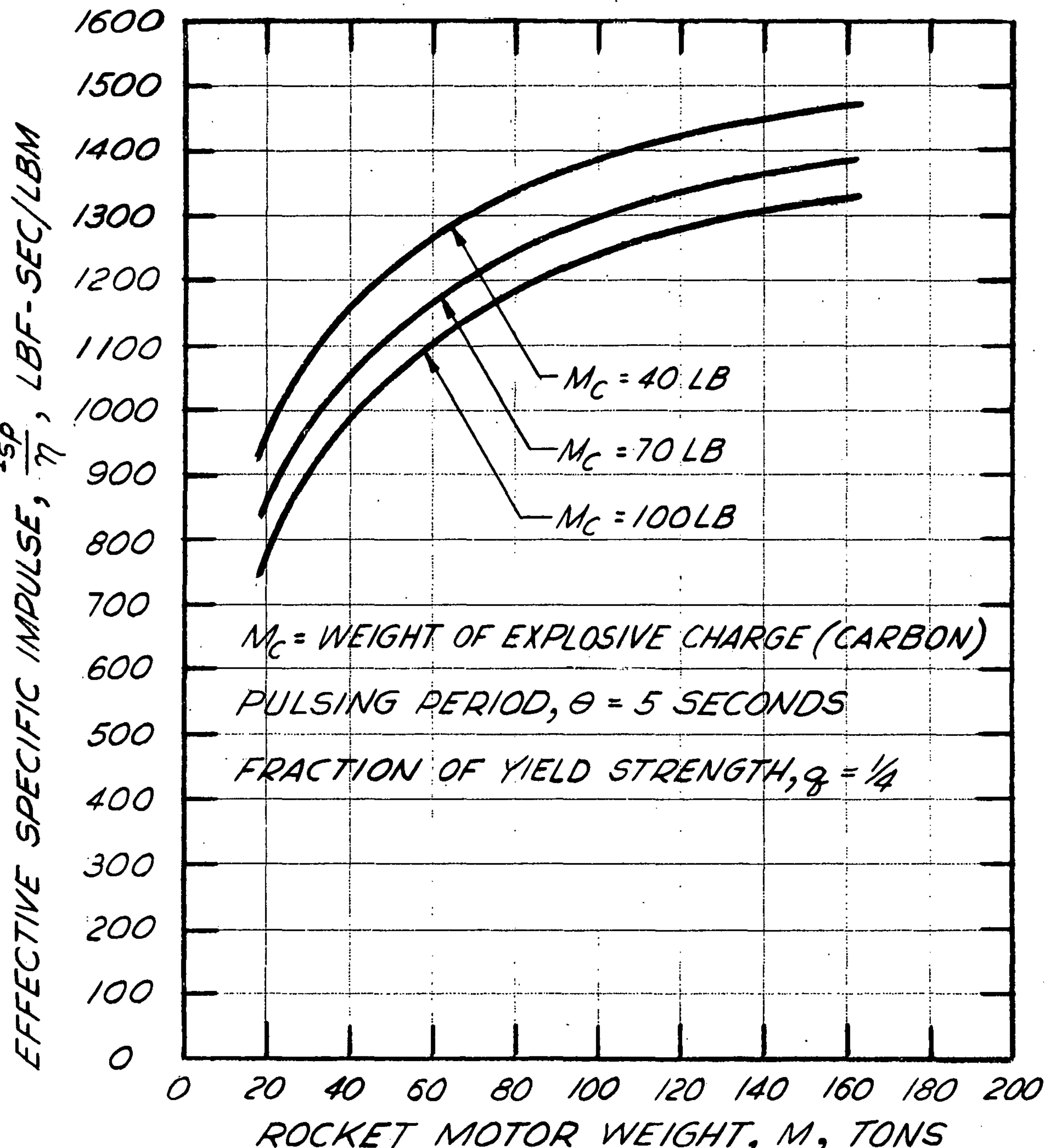

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Fig. 1. Maximum effective specific impulse vs motor weight.

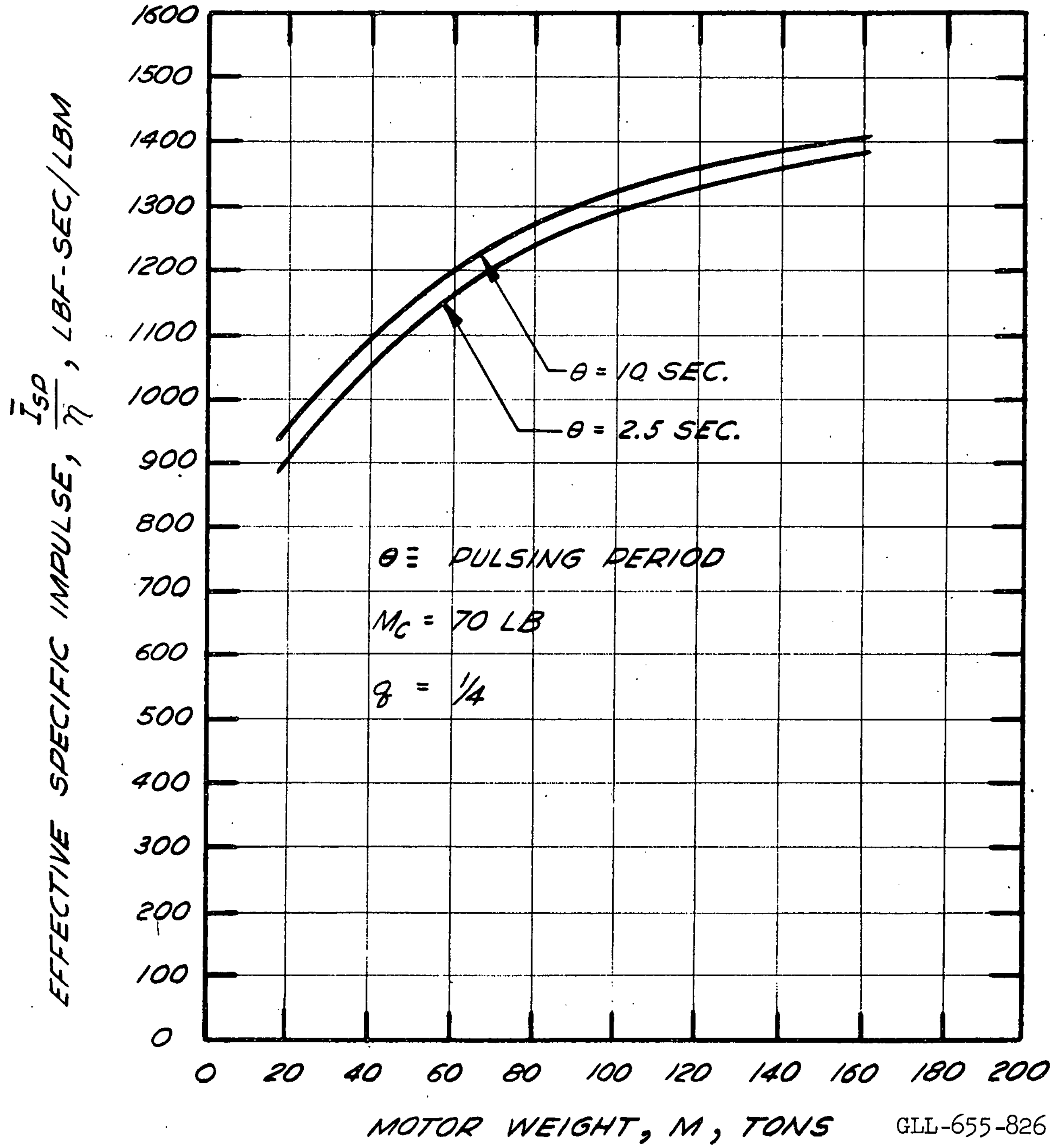

Fig. 2. Specific impulse vs motor weight for different pulsing periods.

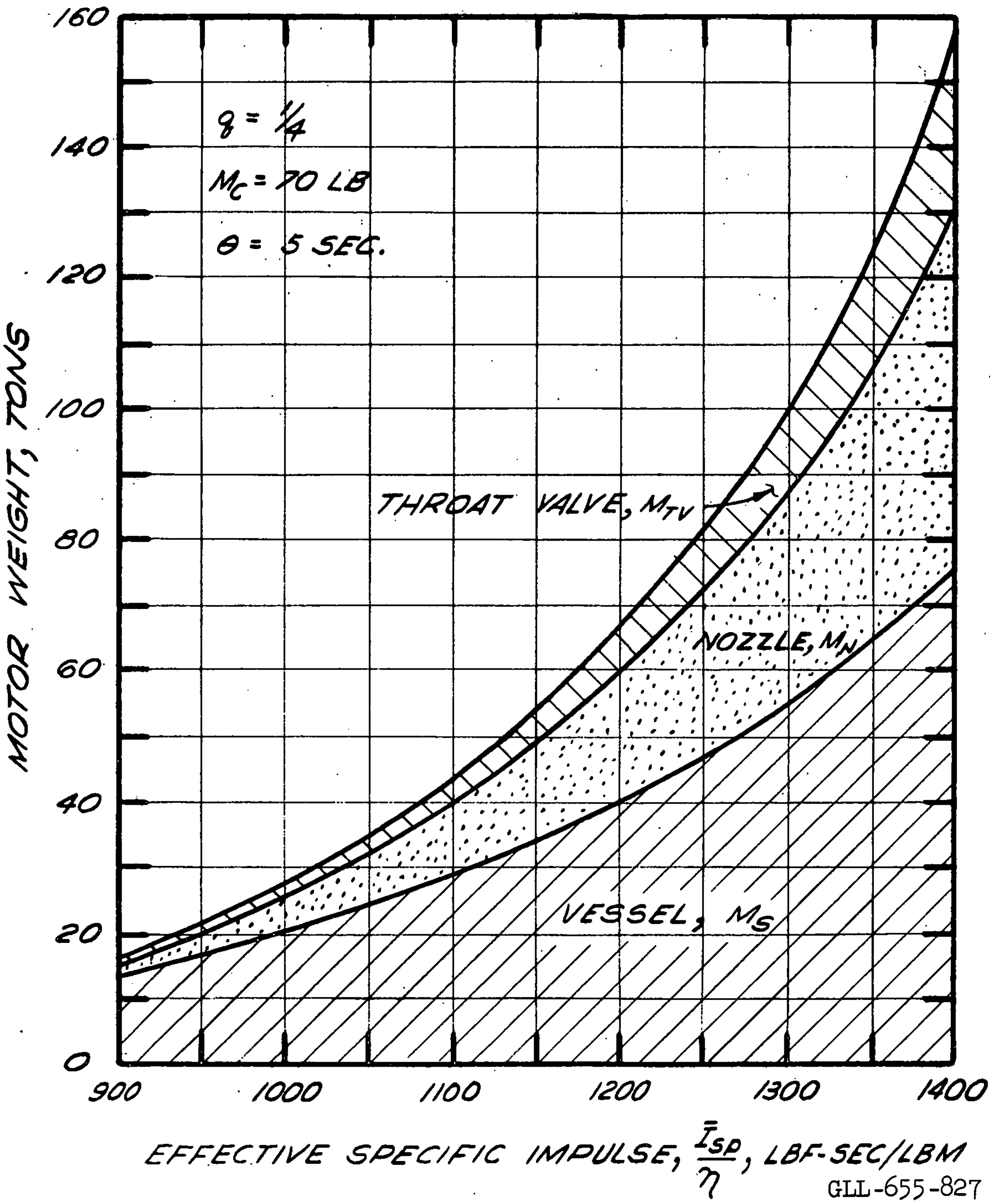

Fig. 3. Weight distribution for optimum motors vs specific impulse.

than half of the total weight, while the nozzle weight increases from 30 to 40 percent of the whole as the total weight increases. The trend in heavier motors is for the pressure vessel to increase in thickness rather than size, while the throat diameter and the nozzle becomes relatively larger. The shapes of motors of 1000 seconds and 1400 seconds specific impulse are compared in Fig. 4. The diameters of the plenums are 20 ft and 26 ft, respectively. The largest diameters of the nozzles are 55 and 175 feet, respectively. Figure 5 illustrates the way in which the thermodynamic parameters find optimum values as the motor weight changes. These are all optimized with respect to the highest obtainable specific impulse.

ANALYSIS

Motor Weight

The principal variable in this study is the weight of the rocket motor. The first problem is to relate that weight to the propellant conditions and to the properties and conditions of the motor material. For the spherical vessel, we adopt the maximum shear strain theory of yielding and write

$$M_s = \rho V\left(\frac{2}{3}\frac{\sigma}{p} - 1\right)^{-1}, \tag{1}$$

in which σ is the allowable stress. The allowable stress is assumed to be a fixed fraction, q, of the yield stress. This fraction is chosen with regard to shell dynamics, material fatigue properties and damping capacity, and motor reliability.

The yield stress depends upon the temperature of the vessel, and the temperature of the vessel is related to the maximum propellant energy in a simple way under the following assumptions:

1. The incoming hydrogen reaches equilibrium with the vessel walls.
2. A negligible amount of energy can be radiated from the vessel. After the temperature stabilizes in a few hundred pulses, the energy intercepted by the vessel from one pulse must be returned to hydrogen for the next.
3. The specific internal energy of the incoming hydrogen is negligible compared to that which it receives from the vessel. The average specific heat at constant volume of hydrogen up to feasible vessel temperatures is 2.5 calories per gram degree Kelvin.

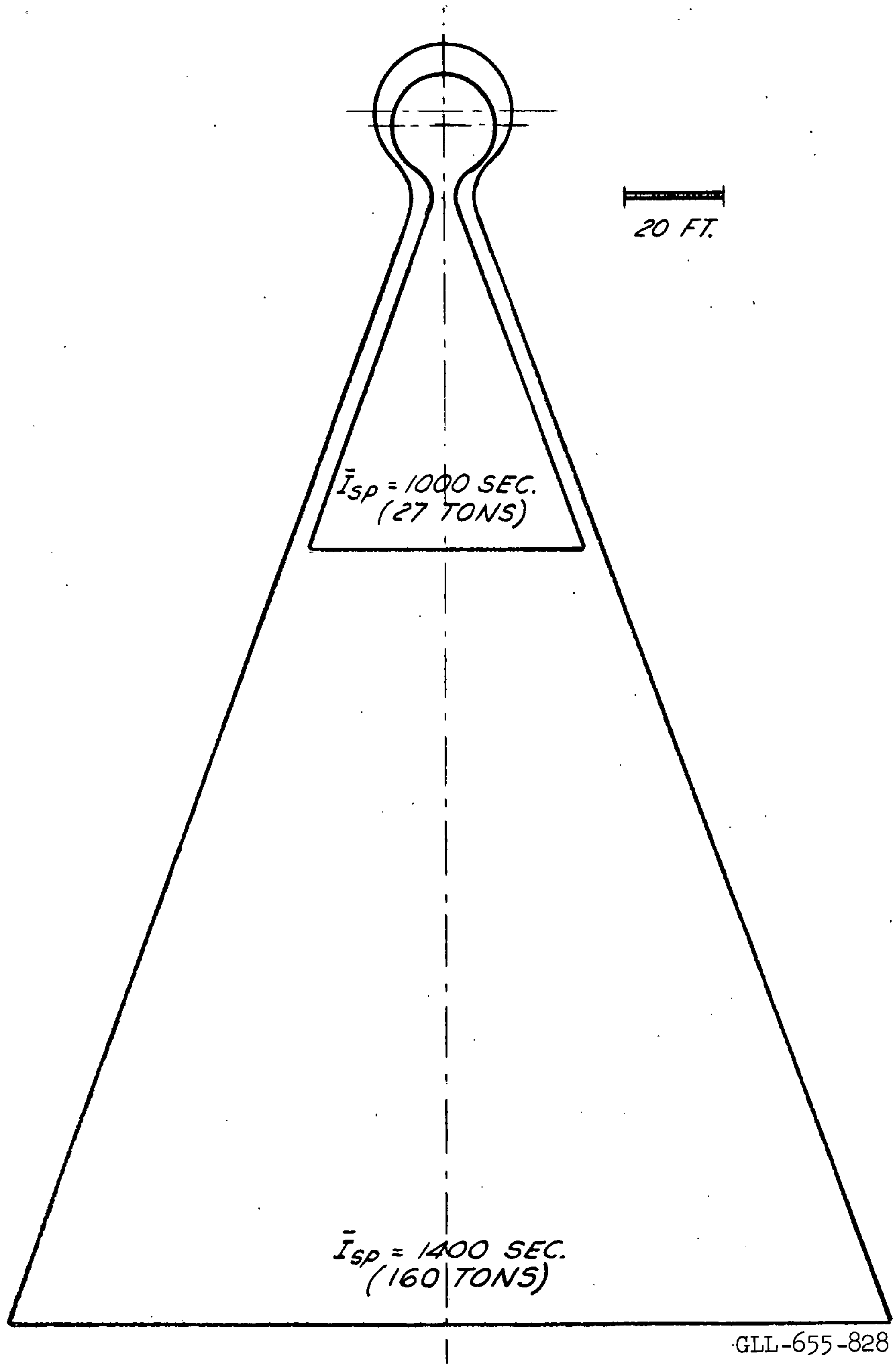

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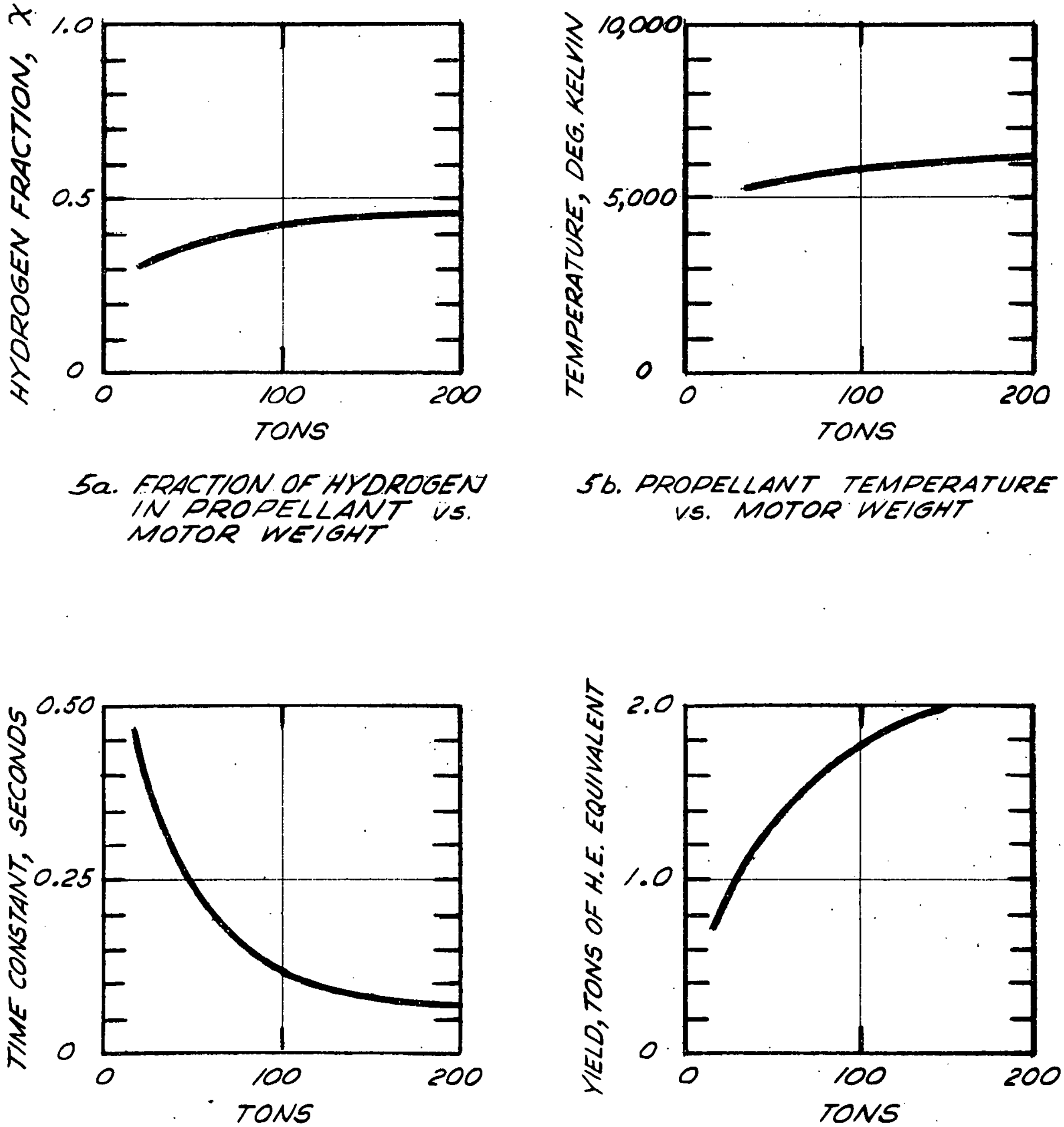
Fig. 4. Profiles of optimum motors for specific impulses of 1000 sec and 1400 sec.



5a. FRACTION OF HYDROGEN IN PROPELLANT vs. MOTOR WEIGHT

5b. PROPELLANT TEMPERATURE vs. MOTOR WEIGHT

5c. CHARACTERISTIC DISCHARGE vs. MOTOR WEIGHT

5d. BOMB YIELD vs. MOTOR WEIGHT

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Fig. 5. Parameters of optimum rocket motors vs motor weight. Bomb weight 70 lb. Pulsing period 5 sec. Fraction of yield strength, q; 1/4.

Then,

$$T_s = T'_H = \frac{\lambda \frac{Y}{M_H}}{2.5} = \frac{\lambda \frac{Y}{M_P}}{2.5 \frac{M_H}{M_P}} = \frac{\lambda E}{2.5\chi}. \tag{2}$$

The first equality in Eq. (2) is the first assumption, and the second equality depends upon the other assumptions. Consequently, the temperature of the vessel is fixed by the maximum internal energy of the propellant, the proportion of hydrogen in it, and the fraction of the energy getting into the walls.

What the thermal leakage fraction, λ, amounts to is the subject of continuing study. Some estimates have been reported.[1] For this study we adopt the estimate of Ref. 1 in the following form:

$$\lambda = 0.01 + 1.02(10)^{-16} \tau T^4 + 5.59(10)^{-8} T^{3/2} p^{1/2} + 0.128 \frac{R^2}{EM_P}. \tag{3}$$

Currently, maraging steel is preferred for the vessel. The temperature dependence of the yield strength of Grade 250 of this alloy has been applied to the analysis and the results approximated by a cubic:

$$\frac{M_s}{M_P} = 7.92(10)^{-4} \frac{pv}{q} \left[1.00 + 1.23\left(\frac{\lambda E}{1200\chi} - 1.00\right)^3\right]. \tag{4}$$

Equations (3) and (4) define a relationship between the weight of the plenum vessel, the weight of the propellant, its hydrogen fraction, and its initial conditions.

The calculation of the weight of the nozzle is based upon the analysis of Brewer and Levin.[2] Two formulae are used. Downstream from the throat to an expansion ratio at which the wall thickness is reduced to 40 mils, the partial weight is estimated as

$$M_N = 3.5 \frac{pr_t^3 \epsilon^{\frac{1}{2}}}{q_N\left(\frac{\sigma_{ty}}{\rho}\right)}. \tag{5}$$

Beyond that point the thickness is assumed constant. The nozzle material is not specified but is characterized by a yield strength-to-density ratio as a function of temperature, which is the best of several materials over the range of interest.

$$\frac{\sigma_{ty}}{\rho} = \frac{4.9(10)^8}{\tilde{T} + 2000} - 1.3(10)^5 \; \frac{\text{atm - ft}^3}{\text{ton}}. \tag{6}$$

The temperature to be used in this formula must be an average which suits the nozzle design problem.

The feasibility of a throat valve of the kind described above seems to depend upon solving the problem of protecting the surfaces of its parts from the propellant during discharge. If the transpiration of hydrogen through the walls can indeed prevent surface melting, a formula for the weight of the valve and the valve gear which has developed from the initial study is

$$M_{TV} = 2.0(10)^{-3} \, p r_t^{\,3}. \tag{7}$$

The motor weight, M, is defined as the weight of vessel, nozzle, and valve. Equations (4), (5), (6), and (7) give this weight as a function of the weight of a propellant charge, initial state of the propellant, the composition of the mixture, the radius of the nozzle throat, the temperature of the nozzle, and a characteristic time for the discharging process.

The Discharging Process

Giffen,[3] Weaving,[4] and Progelhof[5] have developed the theory of the rapid discharge of gas from a vessel, with special reference to cylindrical plenums and internal combustion engines. It is shown in Ref. 5 that the quasisteady theory gives results very close to the more rigorous wave theory if allowance is made for the velocity of approach in the vessel. It appears that for the purposes of this study, the quasisteady approximation is satisfactory. As there is no back pressure, only sonic discharge needs to be considered. Insofar as the process of discharging into the throat can be characterized by a constant ratio of specific heats, the fraction of propellant remaining in the plenum at time, t, is the following;

$$\frac{m}{M_P} = \left(1 + \frac{t}{\tau}\right)^{-\frac{2}{\gamma - 1}}. \tag{8}$$

The constant with the units of time is the characteristic discharge time referred to above. It depends upon motor proportions and upon propellant thermodynamics:

$$\tau = \frac{4}{3}\,\frac{2}{\gamma - 1}\left(\frac{\gamma + 1}{2}\right)^{\frac{1}{\gamma - 1}}\,\frac{R^3}{r_t^2\, a_0}. \tag{9}$$

Equation (8) implies a rate of discharge which is a strong function of the specific heat ratio; however, for mixtures and temperatures of interest to this study that ratio appears always less than 1:3, and the fraction of propellant remaining in the plenum after one time constant is less than 1%. In the same time the throat velocity decreases by only 50%, so that the specific impulse remains relatively high. The effect of pulse decay on specific impulse has been examined in detail, and it has been shown that the effective specific impulse, defined as the weight average impulse, is a function of γ and of the steady flow specific impulse,

$$\bar{I}_{sp_{pulsed}} = \frac{2}{\gamma + 1}\, I_{sp_{steady\ flow}}. \tag{10}$$

This result is applicable to motors with nozzles of any expansion ratio, constant γ always assumed.

A somewhat more general result than Eq. (10) has been obtained which has the effect of removing the specific heat ratio from primary consideration. It has been shown that for complete expansion, the specific impulse is very nearly equal to the root of twice the specific internal energy of the propellant. This result does not depend upon any assumptions concerning the equation of state or isentropic behavior of the propellant gas except that its internal energy must be considerable larger than pv. Since the specific impulse of a continuous flow motor with complete expansion would be equal to the root of twice the specific enthalpy, an equation comparable to (10) can be written:

$$\bar{I}_{sp_{pulsed}} = \sqrt{\frac{E}{H}}\, I_{sp_{steady\ flow}}. \tag{11}$$

While this equation has been shown valid only for complete expansion, in view of the general applicability of (10) and the similarity of (11) we assume that (11) holds for finite expansion also. Therefore, other things being equal, the weight average specific impulse scales with the root of the internal energy of the propellant. An heuristic explanation is that most of the gas is exhausted with velocities near the maximum. Such being the case, the average of the squared velocities, which is equal to twice the internal energy of the propellant, is the same as the square of the average velocity, which is the square of the average specific impulse.

Finite Expansion

At a location in the nozzle where the expansion ratio is large enough, the recombination rate of dissociation propellant should become relatively slow considering the gas velocity and should cause additional complications in calculating the velocity. A preliminary study has shown, however, that the point of freezing should be relatively near the skirt of any nozzles of interest to this investigation, at least until almost all of the propellant were expelled. Freezing near the end of the nozzle could have only a small effect on the overall enthalpy recovery. Consequently, this study assumes chemical equilibrium.

Other real gas effects must be considered, but these appear to be principally dependent upon the composition of the mixture. The impulse of various nozzles using several mixtures of hydrogen and carbon has been obtained by machine calculations which are based upon shifting equilibrium conditions. It appears that the same results, within reasonable accuracy in the range of interest, could be obtained by a perfect gas calculation if γ were the following function of the mixture hydrogen fraction:

$$\gamma = 1.105 + 0.072\chi. \tag{12}$$

Using Eq. (12) and the results of perfect gas theory as expressed in Eqs. (13) and (14) give a measure of the effect on specific impulse of using nozzles of finite length.

$$\frac{I_{sp_{\epsilon = \epsilon}}}{I_{sp_{\epsilon = \infty}}} = \eta \, \frac{1 + \frac{\gamma - 1}{2\gamma} (\zeta - 1)}{\zeta^{1/2}} \tag{13}$$

$$\epsilon = \left(\frac{\gamma - 1}{\gamma + 1} \zeta\right)^{1/2} \left[\frac{2\zeta}{(\gamma + 1)(\zeta - 1)}\right]^{\frac{1}{\gamma - 1}} . \tag{14}$$

In these equations, η is the velocity coefficient accounting for both divergence at the skirt and friction effects within the nozzle and ζ, which is fixed here by ϵ and γ, is related to the overall pressure ratio.

Parasitic Losses

Using cold propellant for transpiration cooling degrades the specific impulse. The size of this effect depends upon the relative amount of propellant used for cooling and upon the velocity of the stream into which it is injected. The limits of this effect can be foreseen. If the coolant were injected into the plenum, it would degrade the specific internal energy of the propellant by dilution, and in view of the previous discussion,

$$\left.\frac{I_{sp_{\alpha = \alpha}}}{I_{sp_{\alpha = 0}}}\right|_{plenum} = (1 + \alpha)^{-1/2}. \tag{15}$$

If, on the other hand, the injection were near the exit of the nozzle the propellant so used could do little work, and there would be only an increase in flow rate so that

$$\left.\frac{I_{sp_{\alpha = \alpha}}}{I_{sp_{\alpha = 0}}}\right|_{exit} = (1 + \alpha)^{-1}. \tag{16}$$

Calculations have been made for injection at intermediate locations, assuming coolant and propellant to be the same perfect gas and mixing to be instantaneous. The results are indicated in Fig. 6, in which the expansion ratio at the point of injection is the variable and the overall expansion ratio of the nozzle is a parameter. The function actually plotted is exact at the limits and seems to correlate other data accurately enough for our purposes. The bands cover the spread of data points. This figure illustrates that mixing losses can be partly recovered if the nozzle is long enough. For some representative motors which have been investigated, the distribution of transpiration coolant has suggested that assuming that all mixing occurs at an expansion ratio of three should give reasonable results. For convenience in computation, the situation is expressed by the following approximation:

$$\frac{I_{sp_{\alpha = \alpha}}}{I_{sp_{\alpha = 0}}} = 1 - \frac{0.9 + \epsilon^{-0.26}}{2}\,\alpha. \tag{17}$$

The fraction, α, is the ratio of coolant weight to the primary charge weight for a pulse. Kramer and Gronich[6] report the result of a computer

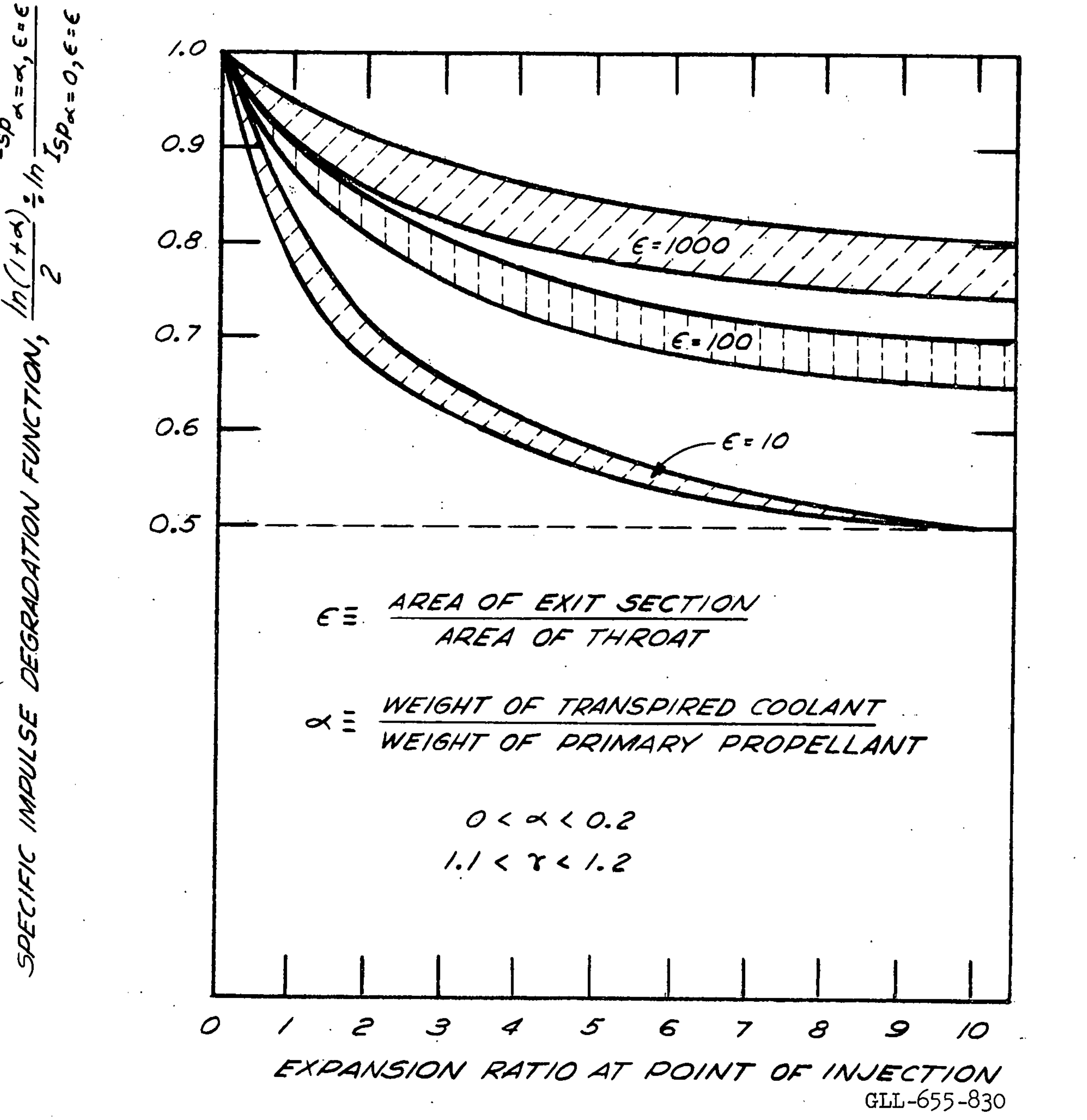

Fig. 6. Degradation of specific impulse due to transpiration cooling.

study of the transpiration cooling of a nozzle using pure hydrogen. If their results are put into our terms and approximated in analytic form, we obtain

$$\alpha \approx \chi\left(\frac{p}{300}\right)^{0.8} \tilde{T}^{-0.37}\left(3.4\frac{T}{10^4} - 0.68\right), \tag{18}$$

which is applicable to a steady flow engine. The same blowing parameter for a pulsed engine appears to be directly proportional to the characteristic time of discharge, τ, and inversely proportional to the pulsing period, θ. Specifically, for $T_0 = 6000°K$ and $P = 100$ atmospheres,

$$\alpha \approx 3.0\frac{\tau\chi}{\theta}. \tag{19}$$

The expression which has been used here to calculate the blowing parameter combines Eqs. (18) and (19):

$$\alpha = 3(10)^{-4}\chi p^{0.8}\frac{T - 2190}{\tilde{T}^{0.37}}\frac{\tau}{\theta}. \tag{20}$$

Heat losses may also degrade the specific impulse. The losses to be considered are limited to the thermal radiation from surfaces and the radiation from the nuclear reaction which penetrates the walls of the plenum. As in the case of coolant addition, the effect of heat losses on the specific impulse depends upon the position and the velocity of the effluent from which it is removed. Because of the dependence of specific impulse upon specific internal energy it appears that if the energy were lost from the pressure vessel, then

$$\left.\frac{I_{sp_{\delta = \delta}}}{I_{sp_{\delta = 0}}}\right|_{plenum} = (1 - \delta)^{1/2}. \tag{21}$$

As the station from which the heat is extracted approaches the nozzle exit, the effect of heat loss should diminish so that actually

$$\frac{I_{sp_{\delta = \delta}}}{I_{sp_{\delta = 0}}} > (1 - \delta)^{1/2}. \tag{22}$$

Considering the effect of temperature on the strength of steel, it is not likely that the vessel can be operated at a higher temperature than 800°K. The amount of heat which could be radiated from a black vessel between detonations would be approximately

$$\delta Y_{vessel} = 2.7(10)^{11} R^2 \theta \text{ ergs.} \tag{23}$$

The radius might be 10 ft and the pulse time 5 sec, in which case

$$\delta Y_{vessel} = 1.35(10)^{14} \text{ ergs.}$$

This energy would be approximately 1/3% of the yield of a 1-ton bomb. Estimating an additional 1/3% loss in penetrating radiation,

$$\frac{I_{sp_{\delta = \delta}}}{I_{sp_{\delta = 0}}} = (1 - 0.0067)^{1/2} \approx 0.997.$$

The effective temperature of the nozzle might be considerably higher than the temperature of the plenum. Using relationships and estimates which it seems unnecessary to develop here, we can write approximately,

$$\delta Y_{nozzle} \approx 4.8(10)^8 R^3 (\epsilon - 1)\left(\frac{\tilde{T}}{1000}\right)^4 \frac{\theta}{\tau}, \tag{24}$$

which assumes that the temperature of the nozzle is allowed to reach $\tilde{T}$ on the average. With R = 10 ft, ϵ = 200, $\tilde{T}$ = 1000°K, θ = 5 sec, and τ = 0.1 sec.

$$\delta Y_{nozzle} = 4.8(10)^{15} \text{ ergs}$$

or 11.5% of a 1-ton yield, therefore for this example

$$\left.\frac{I_{sp_{\delta = \delta}}}{I_{sp_{\delta = 0}}}\right|_{nozzle} > 0.94.$$

As this effect should be reasonably small, an arbitrary intermediate value has been used. With a change of variables the effect of heat loss is calculated as

$$\frac{I_{sp_{\delta = \delta}}}{I_{sp_{\delta = 0}}} = 1 - 3.4(10)^{-5} \frac{v\epsilon\theta}{E}\left(\frac{\tilde{T}}{1000}\right)^4. \tag{25}$$

The Computation Program

The effective specific impulse is obtained as a product of functions;

$$\bar{I}_{sp} = 9.34\sqrt{E}\,\frac{I_{sp_{\epsilon = \epsilon}}}{I_{sp_{\epsilon = \infty}}}\,\frac{I_{sp_{\alpha = \alpha}}}{I_{sp_{\alpha = 0}}}\,\frac{I_{sp_{\delta = \delta}}}{I_{sp_{\delta = 0}}}. \tag{26}$$

The total rocket motor weight is given by

$$M = M_s + M_N + M_{TV}. \tag{27}$$

As developed so far, Eqs. (26) and (27) together depend upon 15 variables: E, p, T, χ, τ, ϵ, $\tilde{T}$, η, θ, M_P, v, q, R, r_t, and q_N. This set can be reduced in number. The specific impulse is generally linear in the velocity losses. Also we assume that

$$q_N = q,$$

so that

$$\frac{\bar{I}_{sp}}{\eta} = \frac{\bar{I}_{sp}}{\eta}\,(E, p, T, \chi, \tau, \epsilon, \tilde{T}, \theta) \tag{26'}$$

and

$$qM = qM\,(E, p, T, \chi, \tau, \epsilon, \tilde{T}, M_P, v, R, r_t). \tag{27'}$$

The equations of state for mixtures of hydrogen and carbon exist in tabulations and on magnetic tape, so that formally,

$$E = E(p, T, \chi),$$

$$v = v(p, T, \chi),$$

and

$$R = R(p, T, \chi, M_P).$$

These data also provide the initial throat sonic velocity, so that by Eq. (9) and the above,

$$\dot{r}_t = r_t\,(\tau, p, T, \chi, M_P).$$

The propellant weight can be replaced by the weight of a bomb and the quality ratio. By definition

$$M_P = \frac{M_c}{1 - \chi}.$$

With these functions and solving (26') and (27') together,

$$\frac{\overline{I}_{sp}}{\eta} = \frac{\overline{I}_{sp}}{\eta}\,(qM, M_c, \theta, p, T, \chi, \tau, \epsilon). \tag{28}$$

In the process of calculating Eq. (28), the rocket motor weight, the bomb weight, and the pulsing period are treated as free parameters. The practical limits of the latter two are not well understood, and the specific impulse is monotonic with respect to each of them. The nuclear explosive device required should have low weight and a very reliably reproducible energy yield in the 1- to 10-ton high-explosive equivalent range. The actual design of an appropriate device has not been undertaken, and so the weight remains a matter of some conjecture. For computation we have used weights of 40, 70, and 100 pounds.

The time between detonations, while related to some of the variables of this study, is a complex problem involving external processes such as charge handling, propellant pumping, and the time to reach temperature equilibrium between propellant and vessel. In general, longer pulsing periods give higher specific impulses. However, there are overriding reasons, concerned with average acceleration, for using the shortest possible pulsing time. A time of 5 sec has been assumed, and in addition a limited amount of data have been collected for 2-1/2 sec and for 10 sec.

With the motor weight, bomb weight, and pulsing period fixed, the effective specific impulse has a maximum value with respect to the remaining parameters. The machine computation program is one of systematically varying these parameters to maximize the specific impulse.

DISCUSSION

The results which have been obtained are in terms of the nozzle efficiency and of an estimate of the usable strength of maraging steel. While it is thought that the nozzle efficiency should be high, there is uncertainty about the effect on it of the throat valve. Most of the gas would be discharged before the valve were completely open, so that if efficiency depended upon smooth flow in the throat, it would be degraded. On the other hand the nozzles favored by this study have relatively large expansion ratios and should be expected to recover some of the losses.

The selection of a working stress for the motor design is a crucial engineering problem for a pulsed motor, and it is one which is not far advanced as yet. Some of the aspects of the problem which make it complex are the fatigue life of the vessel considering the number of pulses, which in a typical mission might be four to six thousand, and the consequent ringing as modified by the damping capacity of the material; the design of the breach and the throat; the reliability of the structural materials in the presence of high temperature hydrogen; the fracture toughness of the material; and the feasibility of establishing the necessary reliability for such a motor before a mission is attempted. The fraction of the yield stress which has been generally used in this study is 1/4, and the results are reported with that fraction as base. The selection of 1/4 as the fraction is not equivalent to having a safety factor of four. The initial stress upon which the calculation is based is not the actual dynamic stress but an average based on steady conditions. In the absencé of stress concentrations the peak dynamic stress should be two or three times that of the average. This factor, among others which tend to degrade performance, must be allowed for in the selection of a proper value for "q".

The present study designates the maximum specific impulse obtainable with a given motor weight. If the cost of the vehicle were proportional to its weight, the best motor for a given mission should be nearly like one of those described here. Discrepancies arise because of the fact that the weight of storage and handling structures are not the same fraction of hydrogen as of explosive devices and also because of the independent relationship between the weight of the motor shock mounting and the discharge time constant. It has been verified by calculations that these descrepancies are small.

Nevertheless, it does appear that significantly more is involved in the cost of the vehicle than its initial weight which is obtainable from the specific impulse. The optimum engines found by the methods used here have relatively small-yield explosive devices. Relatively many of them would be required for a given mission, and since the cost of a smaller device could hardly be less than that of a larger, a serious cost disadvantage arises. Still another effect outside of the scope of this investigation is that of the low average acceleration implied by small-yield devices and fixed pulsing periods. Other things being equal, the sum of the pulsed velocity increments must be greater if the period of acceleration is lengthened.

Some shifting of the optimum toward lower specific impulse engines should be expected in a comprehensive system study. In general, it can be said that if a mission is fixed and if the weight and relative value of bombs is known along with a rational firing rate, optimum values of the other variables may be obtained. There will be an optimum motor weight. The effective specific impulse to be expected from such a motor may not be the same as that found in this study, but it can be no greater under equivalent assumptions.

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