Final Report

Toward the Theory of Turbulence in Magnetized Plasmas

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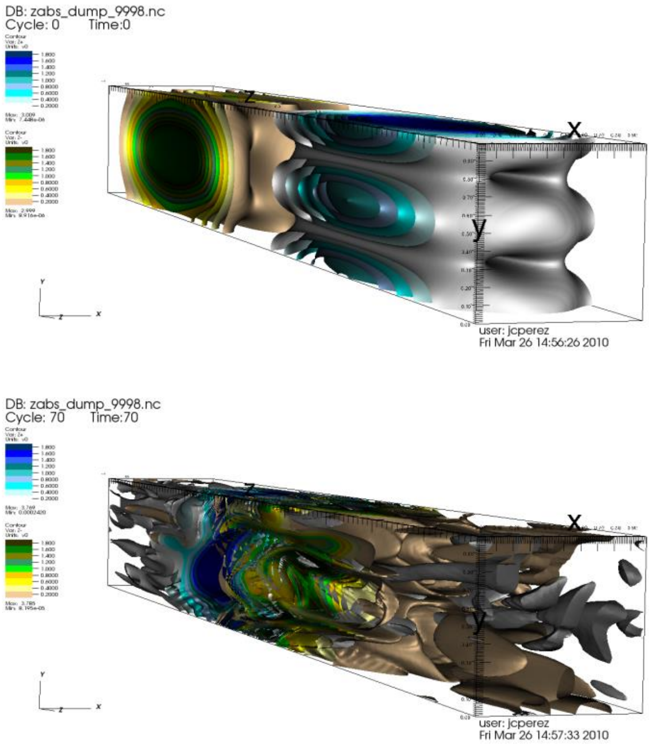
The goal of the project was to develop a theory of turbulence in magnetized plasmas at large scales, that is, scales larger than the characteristic plasma microscales (ion gyroscale, ion inertial scale, etc.). At such scales, the plasma can be reasonable well described by magnetohydrodynamic equations. In the presence of a background magnetic field, ideal MHD possesses exact nonlinear solutions - Alfven waves propagating up and down along the magnetic field lines. Interactions of counter-propagating waves redistribute energy over scales and, when the dissipation is not significant, give rise to a turbulent cascade by which energy is transferred to progressively smaller scales.



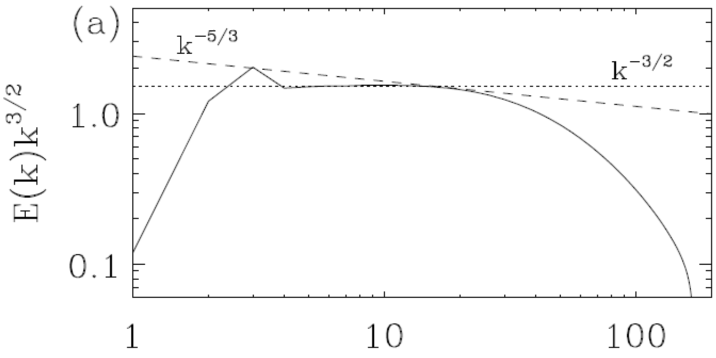
Two counter-propagating large-scale wave packets collide and create very fine structures during a single interaction. The upper panel shows wave packets before the collision, the lower panel – during the collision.

Initially, the wave packets have the energy contained in large-scale modes. During the interaction, the energy gets redistributed over a broad range of small scales.

Collisions of counter-propagating Alfven packets govern the turbulent cascade of energy toward small scales.

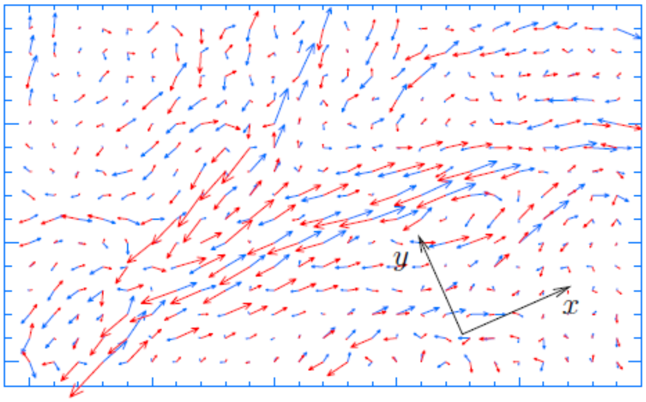
It has been established that such an energy cascade is intrinsically anisotropic, in that it predominantly supplies energy to the modes with mostly field-perpendicular wave numbers. The resulting energy spectrum of MHD turbulence, and the structure of the fluctuations were studied both analytically and numerically.

A new parallel numerical code was developed for simulating reduced MHD equations driven by an external force. The numerical setting was proposed, where the spectral properties of the force could be varied in order to simulate either strong or weak turbulent regimes. The strong turbulence regime is relevant for astrophysical applications, such as interstellar medium turbulence, while the weak turbulence regime is relevant, for example, for laboratory devices (MST, LAPD) and it may be present in solar wind turbulence.



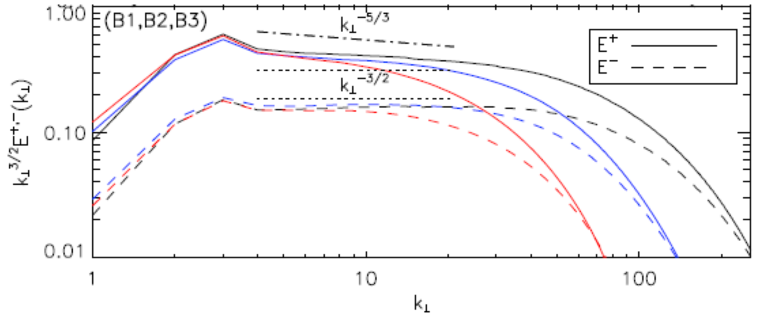
The spectrum of (kinetic + magnetic) energy in numerical simulations strong MHD turbulence is fond to be close to $E(k_\perp) \propto k_\perp^{-3/2}$, where $k_\perp$ is the wavenumber normal to the background magnetic field. From *Mason, Cattaneo & Boldyrev Phys. Rev. E 77, 036403, 2008.*

It has been proposed that the observed spectral scaling reflects the novel process of self-organization in driven, steady-state MHD turbulence – the scale dependent alignment between the magnetic and velocity fluctuations. This process was predicted bases on phenomenological arguments in *Boldyrev, Phys. Rev. Lett. 96, 115002, 2006*. The alignment progressively depletes the nonlinear interaction at small scales, changing the energy scaling from the Kolmogorov -5/3 spectrum to the observed -3/2 spectrum.



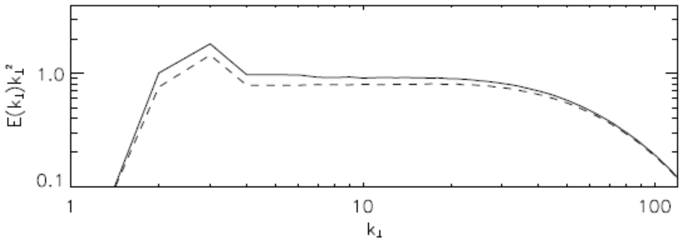
A correlated region of (counter-aligned) magnetic and velocity fluctuations (red and blue vectors) in a plane perpendicular to the background magnetic field. The fluctuations are aligned predominantly in the x direction while their directions and amplitudes change predominantly in the y direction. From *Perez & Boldyrev, Phys. Rev. Lett. 102, 025003, 2009.*

The regions of alignment or imbalance are spontaneously formed in MHD turbulence, even if it is balanced overall. It has been established therefore that there is no fundamental difference between the overall balanced and imbalanced turbulence. Imbalanced regime means that there are more Alfven wave energy moving in one direction along the background magnetic field than in the other. Such turbulence is apparently more common in nature, where sources of energy can be localized (e.g., the solar wind, where more Alfven waves move away from the sun than toward the sun). Imbalanced MHD turbulence has been studies as a part of this project. The energy spectra of oppositely propagating Alfven packets are found to scale in similar way.



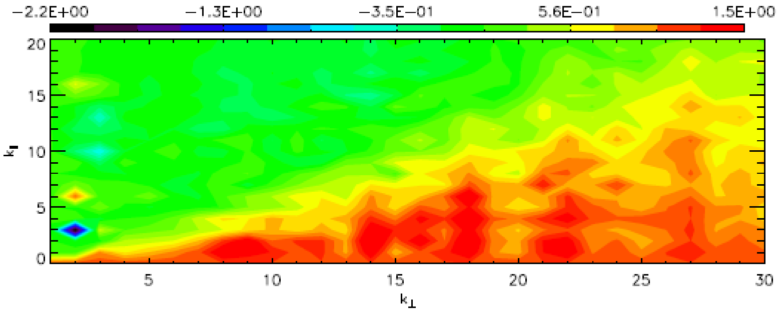
Spectra of counter-propagating Elsasser fields in numerical studies of imbalanced MHD turbulence, at three different Reynolds numbers. The scaling of both spectra is close to -3/2, reflecting similarity between balanced and imbalanced turbulence. From *Perez & Boldyrev, Astrophys. J. Lett. 710, L63, 2010.*

When MHD turbulence consists of waves weakly interacting with each other, such turbulence is in weak regime. As the spectral width of the external force in the field-parallel direction was increased in our simulations, we were able to observe a transition into the *weak* turbulence regime, where the energy spectrum was observed to be $E(k_\perp) \propto k_\perp^{-2}$. The recovered spectrum has been predicted based on analytic considerations (e.g., Ng & Bhattacharjee (1996); Galtier et al (2000)). However, prior to our numerical results, this spectrum has not been observed in numerical simulations. Our developed numerical method provided an effective way to investigate the regimes of both strong and weak MHD turbulence in the same unifying setting.



Spectrum of weak MHD turbulence in numerical simulations. The spectrum is compensated by k^2. Good agreement with phenomenological prediction is obtained. From *Boldyrev & Perez Phys. Rev. Lett. 103, 225001, 2009*

In our studies of weak MHD turbulence we discovered an important previously unknown effect that has to be taken into account when considering imbalanced turbulence. In particular, it has been found both analytically and numerically that weak MHD turbulence spontaneously generates a “condensate”, that is, concentration of magnetic and kinetic energy at small $k_{||}$. The condensate is a fully nonlinear phenomenon. MHD waves with large $k_{||}$ scatter from the low frequency “diffraction grating” provided by the condensate. The spectrum of weak turbulence is thus defined by the scaling of the condensate.

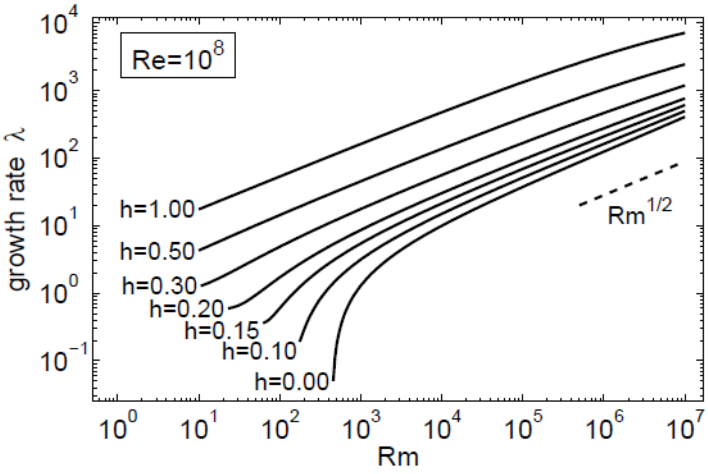


Condensation of energy (red) at small field-parallel wavenumbers in weak MHD turbulence. From *Wang, Boldyrev & Perez, Astrophys. J. Lett. 740, L36, 2011*

A related topic that was addressed in the project is turbulent dynamo action, that is, generation of magnetic field in a turbulent flow. We were specifically concentrated on the generation of large-scale magnetic field compared to the scales of the turbulent velocity field. We investigate magnetic field amplification in a turbulent velocity field with nonzero helicity, in the framework of the kinematic Kazantsev-Kraichnan model. The advantage of our approach is the possibility to consider arbitrarily large Reynolds and magnetic Reynolds numbers, close to those relevant for astrophysics, which is impossible to do in numerical simulations.

We present the numerical solution of the model for the practically important case of Kolmogorov distribution of velocity fluctuations, with a large magnetic Reynolds number. We find that in contrast to the nonhelical case where growing magnetic fields are described by a few bound eigenmodes concentrated inside the inertial interval of the velocity field, in the helical case the number of bound eigenmodes considerably increases; moreover, new unbound eigenmodes appear. Both bound and unbound eigenmodes contribute to the large-scale magnetic field. This indicates a limited applicability of the conventional alpha model of a large-scale dynamo action, which captures only unbound modes. The results are published in *Malyshkin & Boldyrev, Astrophys. J. 697, 1433, 2009.*

Special attention was given to the case of small magnetic Prandtl number, Such a regime is relevant for planets and stars interiors, as well as for liquid metal laboratory experiments. A comprehensive analysis based on the Kazantsev-Kraichnan model was developed, which establishes the dynamo threshold and the dynamo growth rates for varying kinetic helicity of turbulent fluctuations. It was proposed that in contrast with the case of large magnetic Prandtl numbers, the kinematic dynamo action at small magnetic Prandtl numbersis significantly affected by kinetic helicity, and it can be made quite efficient with an appropriate choice of the helicity spectrum.



Scaling of the dynamo growth rate vs the magnetic Reynolds number Rm, obtained for various levels of kinetic helicity h. in numerical solution of the Kazantsev-Kraichnan model. From *Malyshkin & Boldyrev, Phys. Rev. Lett. 105, 215002, 2010.*

In conjunction with this project our group was awarded several allocations of computing time through the Texas Advances Computing Center (TACC), and through the 2010 DOE INCITE Award. The research supported by this grant lead to numerous presentations at the conferences, workshops, and student schools, and to the following refereed publications:

1. Spectral Scaling Laws in Magnetohydrodynamic Turbulence Simulations and in the Solar Wind,
   Boldyrev, S., Perez, J. C., Borovsky, J. E., Podesta, J. J., Astrophys J. Lett 741, L19 (2011), arXiv:1106.0700 (2011)
2. Residual Energy in Magnetohydrodynamic Turbulence,
   Wang, Y. Boldyrev, S. Perez, J. C., Astrophys. J. Lett., 740, L36 (2011) , arXiv:1106.2238 (2011)
3. Residual energy in in magnetohydrodynamic turbulence and in the solar wind,
   Boldyrev, S, Perez, J. C., Zhdankin, V., In "Physics of the Heliosphere: a 10-year Retrospective", Jacob Heerikhuisen, Gang Li, and Gary P. Zank, Eds., arXiv:1108.6072 (2011)
4. Extended Scaling Laws in Numerical Simulations of Magnetohydrodynamic Turbulence,Mason, Joanne; Perez, Jean Carlos; Cattaneo, Fausto; Boldyrev, Stanislav, Astrophys. J. Lett. 735, L26 (2011), arXiv:1104.1437
5. Magnetic Dynamo Action in Random Flows with Zero and Finite Correlation Times,
   Mason, J.; Malyshkin, L.; Boldyrev, S.; Cattaneo, F, Astrophys. J. 730, 86 (2011) , arXiv:1101.5181 (2011)
6. MHD Dynamos and Turbulence,
   Tobias, Steven M.; Cattaneo, Fausto; Boldyrev, Stanislav.
   A book chapter in 'The Nature of Turbulence,' P. Davidson, Y. Kaneda, and K.R. Sreenivasan, eds., arXiv:1103.3138 (2011)
7. Magnetic dynamo action at low magnetic Prandtl numbers,
   Malyshkin, L., Boldyrev, S. Phys. Rev. Lett. 105, 215002 (2010), arXiv:1011.0202 (2010)
8. Strong magnetohydrodynamic turbulence with cross helicity, Perez, J.C., Boldyrev, S., Phys. Plasmas 17, 055903 (2010), arXiv:1004.3798 (2010).

9. Numerical Simulations of Imbalanced Strong Magnetohydrodynamic Turbulence, Perez, J.C., Boldyrev, S., Astrophys. J. 710, L63 (2010); arXiv:0912.0901 (2010)

10. Analog of astrophysical magnetorotational instability in a Couette-Taylor flow of polymer fluids, Boldyrev, S., Huynh, D., Pariev, V., Phys. Rev. E, 80, 066310 (2009) ; arXiv:0810.5073 (2008)

11. Spectrum of Weak Magnetohydrodynamic Turbulence, Boldyrev, S., Perez, J.C., Phys. Rev. Lett., 103, 225001 (2009) ; arXiv:0907.4475 (2009)

12. Magnetic Dynamo Action in Astrophysical Turbulence, Malyshkin, L., Boldyrev, S., Astrophys. J., 697, 1433 (2009) ; arXiv:0809.0720 (2009)

13. Role of Cross-Helicity in Magnetohydrodynamic Turbulence, Perez, J.C., Boldyrev, S., Phys. Rev. Lett., 102, 025003 (2009); arXiv:0807.2635 (2009)

14. Amplification of magnetic fields by dynamo action in Gaussian-correlated helical turbulence, Malyshkin, L., Boldyrev, S., Physica Scripta, 132, 014028 (2008); arXiv:0810.2950 (2008)

15. On Weak and Strong Magnetohydrodynamic Turbulence. Perez, J.C., Boldyrev, S. Astrophys. J. 672, L61 (2008); arXiv:0712.2086 (2008)

16. Numerical Measurements of the Spectrum in Magnetohydrodynamic Turbulence, Mason, J., Cattaneo, F., Boldyrev, S., Physical Review E, 77, 036403 (2008), arXiv:0706.2003 (2007)

17. Magnetic Dynamo Action in Helical Turbulence, Malyshkin, L., Boldyrev, S., Astrophys. J. 671, L185 (2007); arXiv:0711.0973 (2007)

18. Dynamic Alignment and Exact Scaling Laws in MHD Turbulence, Boldyrev, S., Mason, J., Cattaneo, F., Astrophysical J. 699, L39-L42 (2009); arXiv:astro-ph/0605233