STUDY OF THE STABILITY OF PARTICLE MOTION IN STORAGE RINGS

Final Report

(05/15/2003 – 02/14/2009)

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I. INTRODUCTION

This is the Final Report of Grant DE-FG02-04ER41288 for the period of 05/15/2003–02/14/2009. During that period, the group had four Ph.D. graduate students (Lihui Jin, Ben Anhalt, Jin Xu and Wade Rush), in additional to principle investigate. Lihui Jin, Ben Anhalt, and Jin Xu were supported as RA through this grant and Wade Rush was supported as a TA by the Department of Physics & Astronomy of the University of Kansas. During this period, our research was concentrated on the study of beam-beam effects in large storage-ring colliders and coherent synchrotron radiation (CSR) effect in light sources. Our group was involved in and made significant contribution to several international accelerator projects such as the US-LHC project for the design of the LHC interaction regions, the luminosity upgrade of Tevatron and HERA, the design of eRHIC, and the U.S. LHC Accelerator Research Program (LARP) for the future LHC luminosity upgrade. Our research was in collaborations with beam physics groups at Fermilab, DESY, Cornell University, JLab, and BNL/MIT-Bates.

II. Publications during this period

[1] L. Jin and J. Shi, "Importance of beam-beam tune spread to collective beam-beam instability in hadron colliders", Phys. Rev. E**69**, 036503 (2004).

[2] J. Shi, L. Jin, and O. Kheawpum, "Multipole compensation of long-range beam-beam interactions with minimization of nonlinearities in Poincaré map of a storage-ring collider", Phys. Rev. E**69**, 036502 (2004).

[3] F. Wang, et al., "Progress of the eRHIC Electron Ring Design", *Proc. of the 9th European Particle Accelerator Conference*, Lucerne, July 2004.

[4] L. Jin and J. Shi, "Strong-Strong Simulation Study for the Wire Compensation of Long-Range Beam-Beam Effect in LHC", Nucl. Instr. & Meth. A**550**, pp. 6-13, (2005).

[5] L. Jin, J. Shi, and G.H. Hoffstaetter, "Coherent Beam-Beam Tune Shift of Unsymmetrical Beam-Beam Interactions with Large Beam-Beam Parameter", Phys. Rev. E**71**, 036501 (2005).

[6] J. Shi and B. Anhalt, "Importance of the Linear Coupling and Multipole Compensation of Long-Range Beam-Beam Interactions in Tevatron", in *Proc. of the 2005 IEEE Particle Accelerator Conference*, Knoxville, (2005).

[7] J. Shi, L. Jin, F. Wang, and D. Wang, "Beam-Beam Effects in the Ring-Ring Version of eRHIC", in *Proc. of the 2005 IEEE Particle Accelerator Conference*, Knoxville, (2005).

[8] J. Shi, L. Jin, and F. Wang, "Study of Beam-Beam Effects in eRHIC with Self-Consistent Beam-Beam Simulation", Nucl. Instr. & Meth. A**555**, 6 (2005).

[9]J. Shi and L. Jin, "Coherent Instability of Weak-Strong Beam-Beam Interactions", Nucl. Instr. & Meth. A**568**, 566 (2006).

[10] W. Fischer, et al., "Observation of Long-Range Beam-Beam Effect in RHIC and Plans for Compensation", *Proc. of the 10th European Particle Accelerator Conference*, Edinburgh, June 2006.

[11] J. Shi and G.H. Hoffstetter, "Method of Perturbative-PIC Simulation for Interactions between a Bunch and Its Synchrotron Radiation", in Proc. of the 2007 U.S. Particle Accelerator Conference, Albuquerque, (2007).

III. Summary of Research Activities

1. Study of the Coherent Beam-Beam Tune Shift of Unsymmetrical Beam-Beam Interactions

Coherent beam-beam tune shift of unsymmetrical beam-beam interactions was studied experimentally and numerically in HERA where the lepton beam has such a large beam-beam parameter (up to $\xi_y = 0.272$) that the single-particle motion is locally unstable at the closed orbit. Unlike the symmetrical case of beam-beam interactions, the ratio of the coherent beam-beam tune shift and the beam-beam parameter in this unsymmetrical case of beam-beam interactions was found to decrease monotonically with increase of the beam-beam parameter. The results of self-consistent beam-beam simulation, the linearized Vlasov equation, and the rigid-beam model were compared with the experimental measurement. It was found that the coherent beam-beam tune shifts measured in the experiment and calculated in the simulation agree remarkably well but they are much smaller than those calculated by the linearized Vlasov equation with the single-mode approximation or the rigid-beam model. The study indicated that the single-mode approximation in the linearization of Vlasov equation is not valid in the case of unsymmetrical beam-beam interactions. The rigid-beam model is found to be valid only with a small beam-beam parameter in the case of unsymmetrical beam-beam interactions.

2. Study of Beam-Beam Effects in eRHIC

During this period, we have completed a zeroth-order design study of the beam-beam effect of eRHIC. In the proposed ring-ring version of eRHIC, an electron ring will be constructed on the side of the RHIC rings that will provide collisions between a polarized electron beam and an ion beam from one of RHIC rings. In order to achieve a high luminosity, large bunch intensity and small beta-functions at the IP have to be employed. The beam-beam effect, especially the coherent beam-beam effect, is one of important issues to the viability of eRHIC. The beam-beam effect of eRHIC was therefore studied with a self-consistent simulation of the electron-proton collision. Beam-beam limits of the two beams were examined as the thresholds of the onset of coherent beam-beam instability. With the proposed machine parameters, no beam-beam instability has been observed in the simulation and the beam-beam interaction results in a limited ($< 15\%$) luminosity reduction. The study, however, found that the proposed beam-beam parameter of the electron beam is uncomfortably too close to the threshold of the onset of coherent beam-beam instability. In consideration of the beam-beam effect, on the other hand, there is still a possibility to further increase the bunch intensity of the electron beam for the dedicated single IP mode of eRHIC since the proposed beam-beam parameter of the proton beam is relatively far away from its threshold of the beam-beam instability as compared with the electron beam. In order to have a comfortable margin to the beam-beam limit and to meet or exceed the proposed luminosity, one option is reducing the bunch intensity of the proton beam slightly and increasing the bunch intensity of the electron beam accordingly. Our recommendation of the design bunch currents has been adapted by the eRHIC collaboration. With a higher electron bunch current, we were able to further increase the design luminosity of eRHIC.

A new beam-beam phenomenon observed in this study is the high-order coherent beam-beam instability of PACMAN bunches. In order to have more flexibility and cost savings in the design, the circumference of the electron ring is chosen to be one third of that of the RHIC rings. During collision, each electron bunch will circulate three revolutions and collide with three different ion bunches during each revolution of the ion beam. In the case that there are missing bunches in the ion beam, however, an electron bunch could miss one or two head-on collisions at the IP during every three revolutions of the electron beam. Such a PACMAN bunch experiences a head-on beam-beam perturbation with an additional periodicity of three turns. Our study showed that this additional modulation in the beam-beam interaction can induce a high-order coherent beam-beam instability with a spontaneous oscillation of transverse beam size and a chaotic oscillation of beam centroid of PACMAN bunches. This coherent beam-beam instability can result in a severe emittance blowup on both the beams. To avoid an increase of background due to large number of tail particles of the PACMAN bunches, in the case of missing proton bunches, the number of electron bunches has to be reduced accordingly in eRHIC.

3. Collective Instability of Weak-Strong Beam-Beam Interactions

In both HERA and eRHIC, the beam-beam interaction at IPs is considered as weak-strong or very unsymmetrical since the beam-beam parameter of lepton beam is much larger than that of proton beam and the two beams have a very different working point. For the weak-strong case of beam-beam interactions, traditional thinking is that the collective (coherent) beam-beam effect is not important because of the existence of strong Landau damping and a very weak beam-beam perturbation on the strong proton beam. Our studies, however, showed that the collective beam-beam instability could also occur with weak-strong or very unsymmetrical beam-beam interactions. It was found that when the beam-beam parameter of the weak lepton beam exceeds a threshold that corresponds to an overlap of the weak beam with low-order single-particle beam-beam resonances, the onset of the collective beam-beam instability results in a significant emittance growth of the strong proton beam. After the onset of this collective beam-beam instability, both the beams could develop a spontaneous coherent oscillation and the quasi-equilibrium distribution of the lepton beam deviates from a Gaussian with the formation of a beam halo. The collective beam-beam instability has been observed in a beam experiment on HERA. The phenomena observed in the experiment agree with our numerical calculation. In the nonlinear regime of beam-beam interactions, the traditional boundary between the strong-strong and weak-strong beam-beam interactions is therefore no longer valid and the beam-beam effect has to be studied self-consistently in both the situations.

4. Method of Self-Consistent Beam Simulation for CSR Effects

We have developed a new method for a self-consistent simulation of coherent synchrotron radiation (CSR) effects based on a perturbation expansion of retarded radiation field. The perturbation expansion of the radiation field is possible because the time dependence of a bunch particle distribution has typically two significantly different time scales, a fast time scale related to the linear dynamics and a slow time scale of the beam-size growth due to nonlinear perturbations. Since the retardation of the radiation field is usually much shorter than the slow time scale of the particle distribution, the retardation on the slow time scale of the particle distribution can be treated perturbatively while the retardation on the fast time scale can be removed by transformations associated the linear lattice. The perturbation expansion of the retarded radiation field can then be calculated numerically by using the particle-in-cell (PIC) method during a beam simulation. With this method, the CSR effects of a high-intensity ultra-short bunch can be simulated self-consistently without the need for the history of a bunch particle distribution. A detail of this method is in Section II.1 (Proposed Work). We are currently writing a paper on this work.

5. Compensation of Long-Range Beam-Beam Effects in Storage-Ring Colliders

(a) Multipole Compensation of Long-Range Beam-Beam Interactions In Tevatron

In cases of multi-bunches operation in storage-ring colliders, serious long-range beam-beam effects are due to many parasitic collisions that are localized inside interaction regions such as in LHC or/and distributed around the ring such as in Tevatron. To reduce long-range beam-beam effects especially in the non-localized case of long-range beam-beam interactions, we have proposed a multipole compensation scheme by using magnetic multipole correctors. The principle of the multipole compensation of long-range beam-beam interactions is based on the fact that the nonlinear beam dynamics in a storage ring can usually be described by a one-turn map that contains all global information of nonlinearities in the system. By minimizing nonlinear terms of one-turn or/and sectional maps order-by-order with a few groups of multipole correctors, one could reduce the nonlinearity of the system globally or/and locally. To include long-range beam-beam interactions into the maps, one should recognize that a large beam separation is typical at parasitic crossings. In Tevatron, the beam separation at most parasitic crossings is between 6 to 12 σ, where σ is the nominal beam size. There are a few "bad spots" where the separations are $\sim 5\sigma$. In the phase-space region ($< 4\sigma$) that is relevant to the beam, the long-range beam-beam interactions (not at those "bad spots") can be expanded into a Taylor series around the beam separation and be included into the maps for a global or/and local compensation of the linear coupling and nonlinearities due to the beam-beam interactions. To refine the compensation, the beam-beam interactions at those "bad spots" could be further compensated locally with the wire or multipole compensation. In order to construct the maps, the long-range beam-beam interactions have to be expanded in the direction of the beam separation to ensure a clean order-by-order expansion. In Tevatron, since the direction of the separation rotates along the helix around the ring, the transverse coordinate for the expansion has to be rotated accordingly. We have thus developed a differential-algebra code that expands the long-range beam-beam interactions in a rotating coordinate along the helix. The map obtained with this expansion technique was found to converge very well. To examine the effect of the multipole compensation, the emittance growth of both the proton and anti-proton beam at injection or collision were studied with a beam-beam simulation. For Tevatron during collision, headon collisions at two interaction points were also included in the simulation and the effect of the headon beam-beam interactions were studied self-consistently by using the PIC method. The result of the simulation showed that the multipole compensation, especially a correction of the linear coupling due to the long-range beam-beam interactions and helix orbits, can significantly reduces the emittance growth of the anti-proton beam.

(b) Wire Compensation of Long-Range Beam-Beam Interactions In LHC

Many studies have shown that the long-range beam-beam interactions in LHC due to parasitic collisions inside interaction regions have significant adverse effects on the beam stability. The long-range beam-beam effect has also been recognized as one of the most important issues for the future LHC luminosity upgrade. A wire compensation scheme has therefore been proposed (J.-P. Koutchouk) for the current LHC as well as its future luminosity upgrade. The basic principle of the wire compensation of long-range beam-beam interactions is that at large distance the electric field of a current wire is the same as the field produced by a bunch. To examine the direct effect of the wire compensation on the dynamics of beams, we have studied the emittance growth in LHC with a beam-beam simulation. In the simulation, head-on beam-beam interactions were calculated self-consistently with the particle-in-cell method and long-range beam-beam interactions calculated with the soft-Gaussian approximation. Multipole field errors in the LHC lattice were also included in the simulation. The study showed that the wire compensation significantly improves the linearity of the phase space region near the closed orbit and is very effective in reducing the beam-size growth due to long-range beam-beam interactions. The robustness of the compensation to static and random errors in the electric currents on the wires was also investigated. It was found that the compensation is not sensitive to static current errors but is sensitive to current fluctuations. A power ripple of 0.5% or more can heat up the beams and make the situation worse than that without the compensation. The use of the wire compensation thus requires a strict tolerance on the power ripple (it is not technically difficult.)

To test the compensation of long-range beam-beam interactions with colliding beams, LARP (U.S. LHC Accelerator Research Program) has been coordinating a series of beam experiments in RHIC. In order to test the compensation in RHIC, there must be an observable effect from long-range beam-beam interactions. The beam lifetime has therefore been measured in RHIC with a single long-range parasitic crossing and without headon beam-beam interaction. To better understand the experimental observation, we have coordinated with LARP on a beam-beam simulation of the RHIC experiment. This is a relatively difficult case for numerical simulation since the emittance growth or particle loss is very small in the time scale that is accessible for the simulation. To ensure a correct calculation of beam-beam interactions and to reduce computational noise in the emittance growth, a large number of macroparticles (several millions) had to be used. Consistent results have been obtained from different beam-beam simulation codes (ours and LBNL) and the simulation result agrees qualitatively with the experimental observation.

6. Study the Nonlinear Coherent Beam Oscillation in Storage-Ring Colliders

With our PIC beam-beam tracking code, we have studied the coherent beam-beam effect in LHC, HERA, and the proposed eRHIC. It was found that when the beam-beam parameter exceeds a threshold (ξ_c), a collective (coherent) beam-beam instability characterized by an enhanced emittance growth that is due to the dynamics of the counter-rotating beam occurs. Such a collective beam-beam instability has been observed in HERA. It was observed that if the beam-beam tune shift of HERA is such that the positron beam overlaps with the 4th-order resonance, the emittance of the proton beam increases more than 30% in both horizontal and vertical directions. The phenomena observed in the experiment agree with our predictions based on the beam-beam simulation for HERA. Note that the beam-beam interaction in the HERA experiment is considered as a typical strong-weak or very unsymmetrical case as the beam-beam parameter of the positron beam is over 10 and 100 times larger than that of the proton beam in the horizontal and vertical direction, respectively, and the two rings have a very different working point. In the traditional consideration of the incoherent beam-beam effect for the weak-strong beam-beam interaction, the dynamics of the strong proton beam in HERA should not be such strongly affected by the dynamics of the weak lepton beam. During this project period, we have also developed two new computational components in our PIC beam tracking code for studying the nonlinear beam oscillation.

(a) Liapunov exponents of the Vlasov equation of Beam-Beam Interactions

In the study of the collective beam-beam instability, we also observed that after the onset of the coherent beam-beam instability, the phase-space region nearby origin (closed orbit) could become unstable for beam centroids (center of mass) and the two initially centered beams could develop a spontaneous chaotic coherent oscillation. Noted that the chaotic coherent oscillation of hadron beams is very different from the coherent oscillation (π mode) of lepton beams. Consider an example of symmetrical case in which two proton beams have the same beam-beam parameter and betatron tune such as in LHC. Without nonlinear field errors in lattice, the Landau damping of the coherent oscillation is absent. When the beam-beam parameter is much smaller than ξ_c, the phase space of the beam centroid consists mostly of invariant (KAM) tori and motion of the beam centroid is stable. When the beam-beam parameter is above ξ_c, all the invariant tori in the phase-space region nearby origin (close orbit) are disappeared and the motion of the beam centroid becomes chaotic and locally unstable. In the case of lepton beams, the balance (or competition) between the dissipation due to synchrotron radiation and the nonlinear force of beam-beam interactions could result in a stable coherent beam oscillation. To study the nonlinear coherent beam oscillation, we have developed a code to calculate the Liapunov exponents of the Vlasov equation during the PIC beam-beam simulation. The study of the calculated Liapunov exponent has confirmed the chaotic beam oscillation when the beam-beam parameter exceeds its threshold.

(b) Calculation of Coherent Beam-Beam Tune Shift Dynamically During the Beam-Beam Simulation

To study the coherent beam oscillation, one important quantity that can be measured experimentally is the coherent beam-beam tune shift. Two theoretical models, the linearized Vlasov equation (A. Chao and R. Ruth, 1985) and the rigid-beam model (K. Hirata, 1988), have been studied extensively for cases of weak beam-beam perturbation in which the beam-beam parameter is relatively small. When two beams have the same or very close lattice tunes, the calculation of the coherent beam-beam tune shift based on the linearized Vlasov equation with single-mode approximation agrees with beam measurements and computer simulations. The rigid-beam model is, however, inconsistent with the experiments and simulations. When the two beams have very different lattice tunes, on the other hand, the calculation based on the rigid-beam model provides a good agreement with the measurements. The situation of strong beam-beam perturbation is much more complicated and less understood. When the beam-beam parameter is below the beam-beam threshold, the coherent beam oscillation is stable. An understanding of the coherent beam-beam tune shift in this regime not only is necessary for the interpretation of the tune measurement during operation of colliders with high-intensity beams but also could shed light on the onset of the coherent beam-beam instability. During this project period, we have developed a computational method based on the linearized Vlasov equation for calculating the coherent beam-beam tune shift during the PIC beam-beam simulation, where the charge densities of colliding beams could be significantly non-Gaussian and vary with time.

To explore the beam-beam effect with a large beam-beam parameter we have been collaborating with DESY to study the beam-beam tune shift in HERA. One phenomenon observed in the HERA experiment is that the measured coherent beam-beam tune shifts of the lepton beam are much smaller than those calculated from the rigid-beam model. Our tracking study has confirmed the experimental result. It was found that the ratio of the coherent beam-beam tune shift and the beam-beam parameter decreases monotonically with the increase of the beam-beam parameter in the case of unsymmetrical beam-beam interactions. On the contrary, in the symmetrical case of beam-beam interactions, this ratio maintains approximately a constant of 1.2 for a round beam or 1.3 for a flat beam in a large range of beam-beam parameter. The reason for this different characteristic of the coherent beam-beam tune shift is the intrinsic beam-size mismatch between two unsymmetrical colliding beams due to the difference in the equilibrium (or quasi-equilibrium) distributions of the two beams. This intrinsic mismatch in the beam distributions becomes more pronounced as the strength of beam-beam perturbations increases. The ratio of the coherent beam-beam tune shift and the beam-beam parameter therefore decreases, in general, with the increase of beam-beam parameters in the unsymmetrical case of beam-beam interactions.

7. Development of A New Simulation Code For CSR Effect in Storage Rings

In study of collective particle-field interactions, the chief of difficulties is that the calculation of the field requires a knowledge of particle distribution while the distribution itself is perturbed by the field. Currently, the most effective numerical method for a self-consistent treatment of such a collective interaction is the particle-in-cell (PIC) method in which the time evolution of the particle distribution is calculated on a mesh in configuration space by using a large number of macroparticles. For the interaction between an ultra short bunch and its coherent synchrotron radiation (CSR), however, one additional difficulty for a self-consistent treatment is the retardation of the radiation field as the particle-radiation interaction involves a retarded bunch particle distribution. A direct use of the PIC method is very inefficient, if not impossible, because with a reasonable density of the mesh and a reasonable number of macroparticles, keeping the history of the particle distribution in computer memories during a beam tracking in storage rings is technically impractical (expect maybe in the case of a very short beam path.)

To overcome the difficulty of the retardation in a self-consistent treatment of the CSR problem, we have developed a perturbative-PIC method based on a perturbation expansion of the retarded radiation field. For the perturbation expansion of the radiation field, we utilize the fact that the time dependence of a bunch particle distribution has two significantly different time scales. A fast time scale of the distribution is related to the linear dynamics of a bunch as its centroid moves along the lattice and its beam sizes vary with lattice functions while a slow time scale of the distribution is of the slow beam-size growth due to nonlinear perturbations. The retardation on the fast time scale can be eliminated from the distribution analytically since the linear dynamics is known. As the retardation of the radiation is usually much shorter than the slow time scale of the distribution, the retardation on the slow time scale in the distribution can be approximated by an expansion in terms of the retardation. The use of the PIC method provides a smooth distribution constructed on a mesh in configuration space so that the expansion of the retarded radiation field can be calculated numerically on the mesh with a desired accuracy. With this method, the retarded radiation field can be calculated self-consistently without a need of memorizing the history of the distribution during a beam tracking. Due to the unexpected departures of two graduate students (Ben Anhalt and Jin Xu), we still cannot finish this code.