LM-02K044
June 24, 2002

Effects of Equipment Loading on the Vibrations of Edge-Stiffened Plates and Associated Modeling Issues

R.L. Campbell, S.A. Hambric

Proceedings of
2002 ASME International Mechanical Engineering Congress and Exposition
November 17-22, 2002, New Orleans, LA

IMECE 2002/NCA-32692

EFFECTS OF EQUIPMENT LOADING ON THE VIBRATIONS OF EDGE-STIFFENED PLATES AND ASSOCIATED MODELING ISSUES

Robert L. Campbell
The Pennsylvania State University
Applied Research Laboratory
PO Box 30
State College, PA 16804
Email: rlc@wt.arl.psu.edu

Stephen A. Hambric
The Pennsylvania State University
Applied Research Laboratory
PO Box 30
State College, PA 16804
Email: sah@wt.arl.psu.edu

ABSTRACT

Predicting structural radiated noise is a process that involves several steps, often including the development of a finite element (FE) model to provide structural response predictions. Limitations of these FE models often govern the success of overall noise predictions. The purpose of the present investigation is to identify the effects of real world attachments on edge-stiffened plates and identify advanced modeling methods to facilitate vibroacoustic analyses of such complex structures. A combination of experimental and numerical methods is used in the evaluation. The results show the effects of adding attachments to the edge-stiffened plate in terms of mode shape mass loading, creation of new mode shapes, modifications to original mode shapes, and variations in damping levels. A finite element model of the edge-stiffened plate with simplified attachments has been developed and is used in conjunction with experimental data to aid in the developments. The investigation presented here represents a necessary first step toward implementing an advanced modeling technique.

INTRODUCTION

Finite element (FE) modeling is often used to model structural acoustic behavior of complex systems. Limitations of the FE results are generally caused by time and cost constraints when modeling such structures. The focus of the present work is to investigate the effects of complicated, 'real world' attachments on an edge-stiffened plate, ultimately leading to the development of an advanced modeling technique. The edge-stiffened plate with attachments, Figure 1, was chosen for the testing because similar structures are commonly found in industry (e.g., computer housings, equipment racks, etc.) and it is often very difficult to adequately predict the structural response. An important aspect of this type of structure is that the attachments are often electrical components, such as AC or DC fans, which provide complicated forcing functions that excite the structure. The addition of these components has a mass loading and damping effect on the base structure.

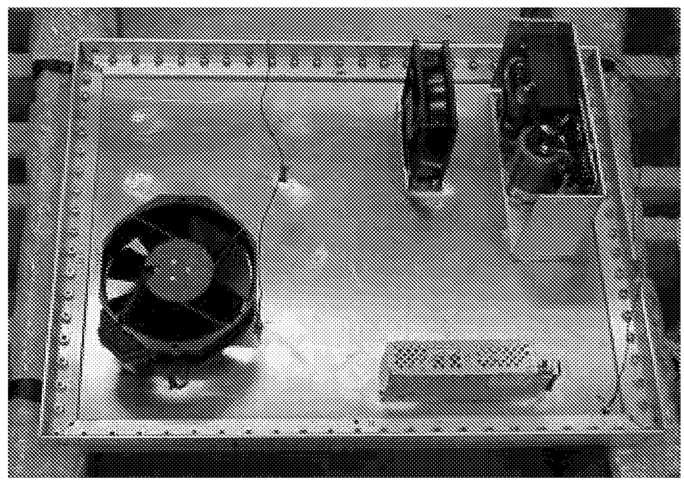

Figure 1: Edge-stiffened plate with real attachments.

As a first approximation to a FE model of the overall structure, simplified models of the attachments could be developed and attached to the base plate model. Notionally, the model accuracy would depend on the amount of simplification involved for the attachments. For complicated attachments, the amount of simplification necessary to avoid having a tedious model could result in a model with extreme accuracy limits, ultimately resulting in inaccurate noise predictions. The overall goal of the present investigation is to augment the traditional FE modeling approach to bypass such problems associated with simplified FE models. The augmented models should provide accurate responses of these complicated structures without resorting to overwhelming model sizes.

APPROACH

The approach taken for this investigation involves both experimental and modeling aspects. Experimentally, modal analyses are performed for different combinations of the edge-stiffened plate and attachments: edge-stiffened plate with no attachments (referred to as the “bare plate”), plate with real attachments, and the plate with simplified attachments. Comparisons between results from each of these tests show the effect of attachments on the plate in terms of mass loading, changing mode shapes, creating new modes, and changing damping levels.

Four different commercial off-the-shelf (COTS) components are used in this study to represent real attachments that might be used in the field. These components include a linear power supply, switching power supply, DC cooling fan, and an AC cooling fan. Each of these is commonly found in industrial settings and have a variety of different sizes and dynamic characteristics.

The simplified attachments, or ‘dummy’ masses, are simply metal (mass/inertia) blocks that have been designed to have approximately the same masses and inertias as the real components. The idea behind the dummy masses is to 1) attempt to separate the effects of the mass loading of the modes and changes in the structural damping, and 2) provide a simpler structure for initial FE model investigations. It is expected that attaching the real components to the plate will cause a change in both the structural damping and the mode shapes, while the dummy masses should only alter the mode shapes with a negligible effect on the damping.

The experimental modal analyses provide frequency response functions (FRFs), which can subsequently be used to obtain damping levels and mode shapes. The damping levels were extracted from the FRFs using the commercially available modal analysis software, StarModal [1]. Comparisons between the damping levels for each configuration show how well the dummy masses do at separating the effects of mass loading and the structural damping.

Other structural response comparisons are made using a combination of two different methods. The first method is the surface averaged velocity, calculated as shown in Eq. (1), with units of dB relative to $1\,\frac{\text{m/s}}{\text{N}}$.

$$\left\langle \frac{v}{F} \right\rangle_{dB} = 20 \cdot \log_{10}\left(\frac{1}{\sum_{i=1}^{N} A_i} \cdot \sum_{i=1}^{N} \left| \frac{v}{F} \right|_i A_i \right) \qquad (1)$$

where A_i is the area for mesh element i, $\left| \frac{v}{F} \right|_i$ is the average mobility magnitude for element i, and N is the number of experimental mesh elements. This metric allows comparisons of the overall response of the structure in terms of the velocity magnitude and resonance locations, which appear as peaks in the averaged response. What this method does not show is how well individual mode shapes agree with one another. For this comparison, a modal assurance criteria (MAC) analysis is used.

In simple terms, a MAC analysis is used to determine how much two vibration distributions resemble each other. The calculations are very similar to those used to evaluate the coherence between two signals, only now two spatial vibration distributions are compared rather than two time signals. The basic formula, as described in Ewins [2], for the modal assurance criteria with respect to mode μ is given as

$$MAC(m, l) = \frac{\left| \sum_{j=1}^{N} \mathbf{f}_m(\mathbf{x}_j)\, \mathbf{f}_l^*(\mathbf{x}_j) \right|^2}{\left(\sum_{j=1}^{N} \mathbf{f}_m(\mathbf{x}_j)\, \mathbf{f}_m^*(\mathbf{x}_j) \right) \cdot \left(\sum_{j=1}^{N} \mathbf{f}_l(\mathbf{x}_j)\, \mathbf{f}_l^*(\mathbf{x}_j) \right)} \qquad (2)$$

where $\mathbf{f}_m(\mathbf{x}_j)$ is the displacement for mode μ at position $\mathbf{x}_j$, and $\mathbf{f}_l(\mathbf{x}_j)$ is the displacement for mode λ at position $\mathbf{x}_j$. The resulting MAC values approach unity for two mode shapes (or displacements) that closely resemble each other and approach zero for dissimilar responses. Generally, MAC values in excess of 0.9 should be attained for correlated modes and values less than 0.05 for uncorrelated modes. However, these ranges must consider the intended use of the results, as some may be much more demanding than others. MAC analyses will be used to compare measured or predicted mode shapes to other measured or predicted mode shapes for different plate configurations.

Finite element models of the bare plate and the plate with dummy masses have been developed and the results are compared to corresponding experimental data. Limitations of these models are discussed along with different methods for enhancing the model to provide better predictions.

EXPERIMENTAL SETUP

The base structure to which the attachments are mounted, pictured in Figure 2, consists of a 61.0 cm x 48.3 cm x 0.48 cm aluminum plate with 3.18 cm x 3.18 cm x 0.48 cm aluminum angles as stiffeners. The stiffeners are attached to the base plate using 23 equally spaced stainless steel screws along the lengthwise edges and 19 equally spaced stainless steel screws along the widthwise edges. This assembly represents the bare, edge-stiffened plate used in each of the tests. J-B® WELD Kwik Weld adhesive is used to join the attachments to the base structure to avoid drilling multiple holes in the plate with a minimal effect on the structural damping. Small flat washers are used between the attachments and the plate to reduce the attachment interface from a large region to a point-like area, thus greatly simplifying the FE modeling procedure. Note that the attachments were bonded to the underside of the plate in each case to avoid interfering with the modal testing. Also, the attachments were not operating during the testing.

All tests were performed with the plate edges resting on bubble-wrap to simulate free boundary conditions. In addition, the tests consistently used a stationary reference point located at approximately x = 20.3 cm, y = 35.6 cm (see the accelerometer location in Figure 3). Numerous lines were scribed onto the plate surface to generate a grid of 256 points. Grid points were also located on the attachment surfaces such that the response of these structures could also be obtained. The experimental grid for the plate and attachments is shown in Figure 4. The drive point varied across the structure’s surface according to the prescribed mesh. Data was acquired up to 2000 Hz with a frequency resolution of 1.25 Hz using ten averages per data point. No windowing was applied to the data.

The dummy masses, shown attached to the plate in Figure 2, were constructed to approximate the linear and rotational inertias of the real attachments. In addition, the base size of each dummy mass was chosen so the attachment points are the same as the attachment points for the corresponding COTS component.

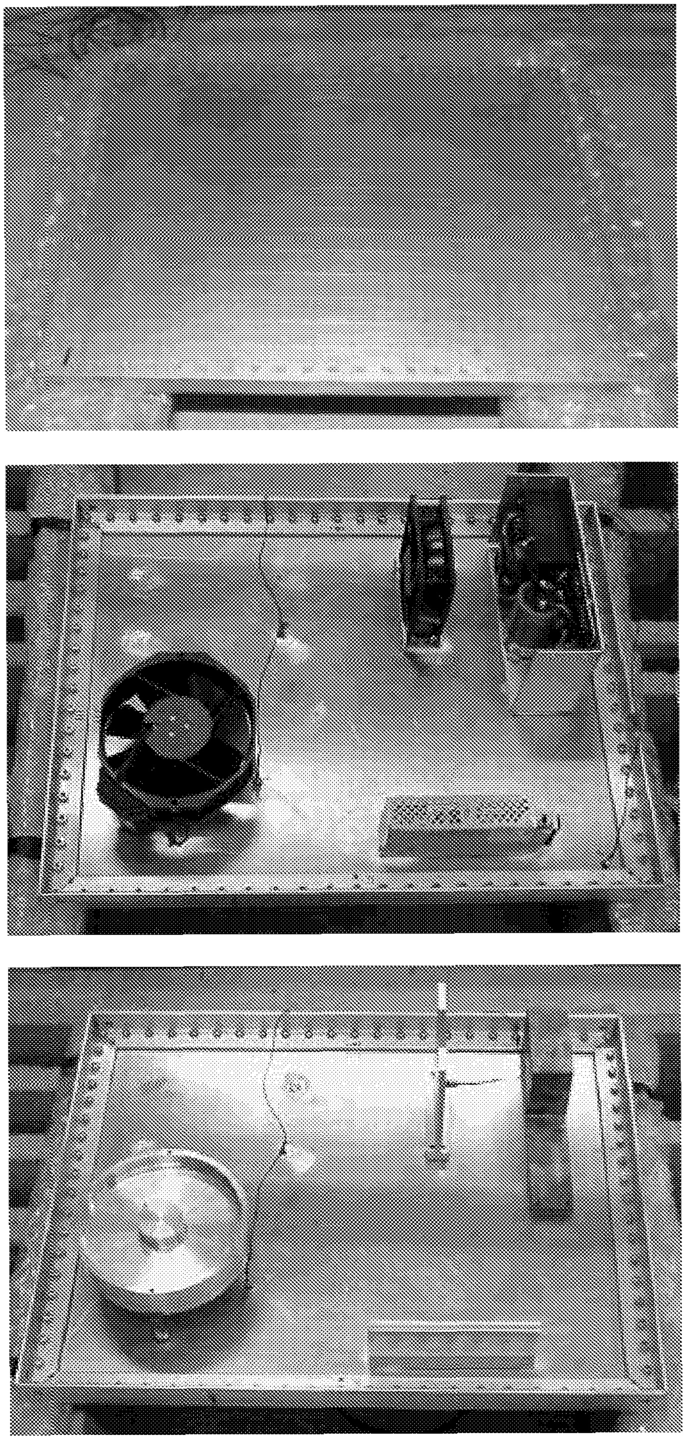

Figure 2: Plate configurations: top – edge-stiffened plate with no attachments; middle – plate with real attachments; bottom – plate with dummy masses.

EXPERIMENTAL RESULTS

Plots of the surface averaged normal velocity for each test configuration are shown in Figure 5. As indicated by these plots, the dynamic responses of the plate with attachments differ significantly from the response of the bare plate in terms of resonance frequencies and modal density. The responses of the plate with real masses and the plate with dummy masses compare reasonably well only up to slightly less than 200 Hz. The range over which these results are comparable is somewhat less than the anticipated range, but is not surprising when the complexity of the real attachments is considered. Note that the first flexible mode of each test specimen occurs at around 50 Hz. The apparent resonances below this frequency are actually rigid body modes of the test structure and occur at finite frequencies due to the bubble wrap. These modes are not included in any of the following comparisons for mode shapes and loss factors.

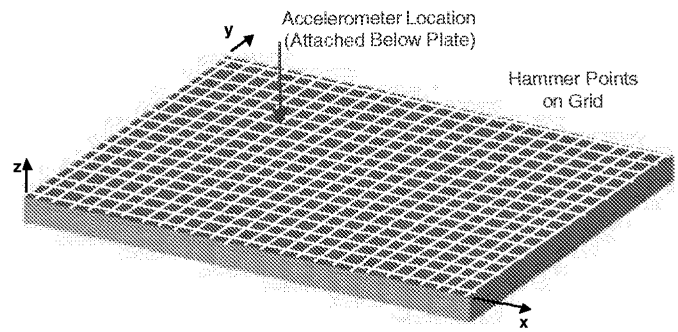

Figure 3: Schematic of impact hammer testing setup.

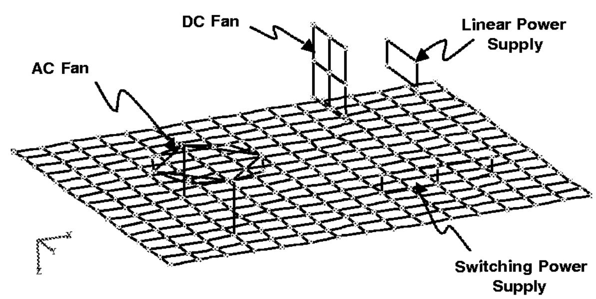

Figure 4: Experimental mesh for edge-stiffened plate with attachments.

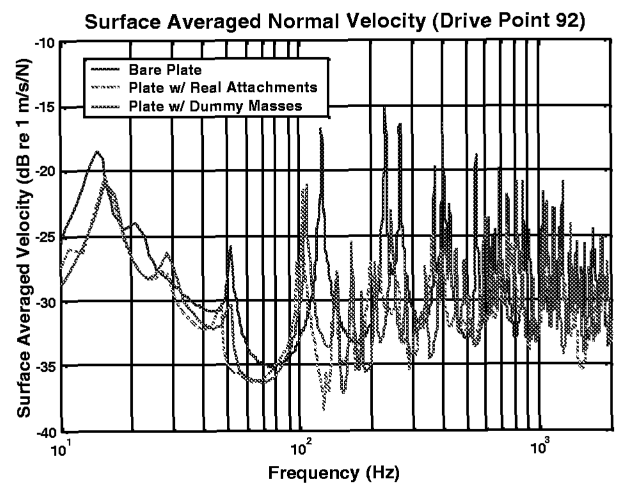

Figure 5: Surface averaged velocity for the edge-stiffened plate with and without attachments.

Loss factors were extracted from the FRFs for all test cases using StarModal. The results, shown in Figure 6, indicate that the bare plate and the plate with dummy masses have nearly the same loss factors (generally between 0.005 and 0.010, not including the first mode). The difference in the number of data points for each test structure is due to differing modal densities, as each loss factor corresponds to a particular mode. Based on this, it appears that the attempt to separate damping and mass loading effects through use of the dummy masses has been successful.

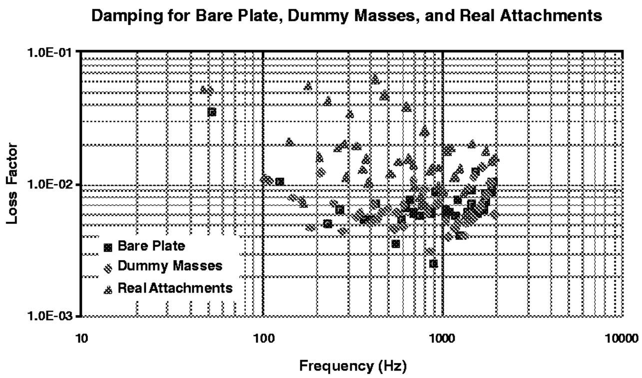

Figure 6: Loss factors extracted from measured data for edge-stiffened plate with and without attachments.

Comparison of the loss factors for the bare plate and the plate with real attachments shows an increase in damping for the entire frequency range except for a few distinct modes. The similarity in loss factor for these modes is most likely a result of the attachments moving rigidly with the structure. As shown in Figure 6, at some frequencies the loss factors increase by a factor of two or more to between 0.010 and 0.060.

Some selected mode shapes for the plate with real attachments are provided in Figure 7. This figure shows the involvement of the attachments in the dynamic response, thus explaining the increase in the modal density as witnessed by the plot in Figure 5. The upper two modes in Figure 7 show the masses moving rigidly with the base structure. This situation results in mode shapes similar to those of the bare plate, but with a mass loading effect (see the shift in the corresponding peaks in Figure 5) and relatively no effect on damping. The bottom two mode shapes represent cases where the attachments have created a mode that does not exist in the bare plate. For these two modes, it is the presence of the AC and DC fans and their connection to the plate that causes the modes. Flexure within the attachments themselves result in the increased damping shown in Figure 6.

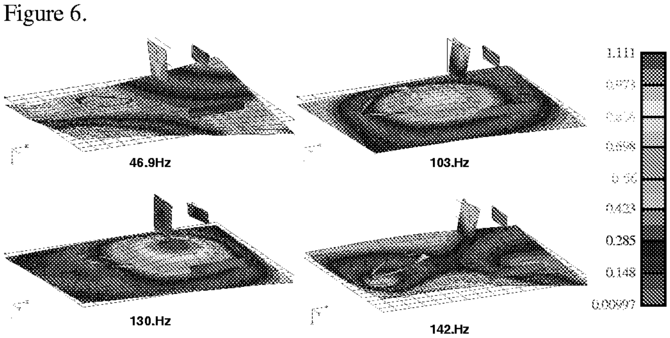

Figure 7: Sample measured mode shapes for edge-stiffened plate with real attachments (representative color scale shown).

MAC analyses were used to quantitatively compare these and other measured mode shapes for each of the three test structures. Figure 8 shows the MAC results for the bare plate and the plate with real attachments. The intent of this MAC analysis is to search for bare plate mode shapes in the response of the plate with real attachments. The results are presented in terms of the MAC peak value, the frequency ratio between the two responses where the peak value occurs (f_{Bare}/f_{Real}), and the resonance frequency for the bare plate response. The two horizontal lines shown on the frequency ratio plot are at ±10% to aid comparisons between each of the plots. As discussed previously, the limits placed on MAC values for determining correlated modes can be somewhat fuzzy, generally depending on the intended use of the results. Based on comparisons between numerous MAC values and mode shapes for these structures, modes with MAC values of 0.6 or higher tend to be reasonably correlated with only slight changes in the mode shape relative to mode order and node/antinode locations. Modes with MAC values lower than 0.4 tend to be substantially different.

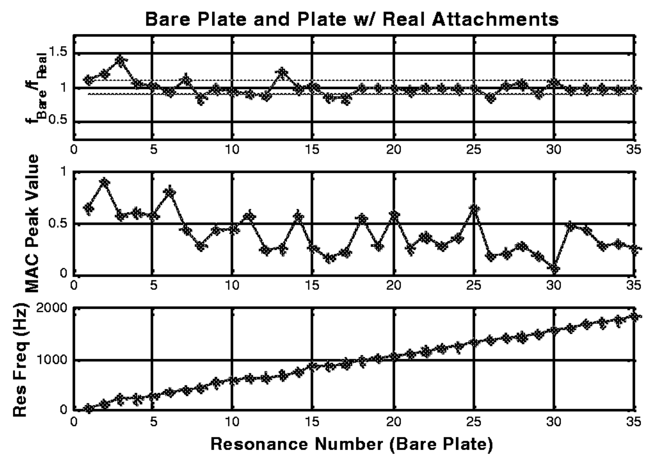

Figure 8: MAC results for bare plate and plate with real attachments.

As indicated by the MAC peak values in Figure 8, bare plate modes two and six correlate reasonably well with modes of the more complicated test structure, as these modes have MAC values larger than 0.8. Several modes are on the borderline with MAC values around 0.6. However, the point to be made from these results is that the addition of the attachments to the bare plate changes the modes considerably. This result is in agreement with what was expected for these structures. Also, we see that for the second bare plate mode, the mode shapes correlate well, but the frequency differs by nearly 20% indicating a significant mass loading effect on the mode shape of the more complicated structure. Similar results are observed from the MAC analysis between the bare plate and the plate with dummy masses, as shown in Figure 9. For this case, the MAC values are even less, indicating less correlation between the mode shapes. Also, many of the frequency ratios are somewhat larger than, but comparable to the frequency ratios shown in Figure 8. These results indicate significant differences between the effects of the real attachments and dummy masses on the base structure. A comparison between the response with real attachments and the response with dummy masses will help quantify these differences.

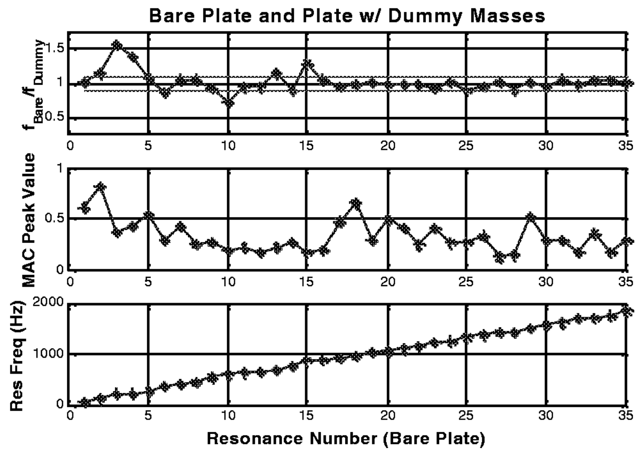

Figure 9: MAC results for bare plate and plate with dummy masses.

Surprisingly, the MAC results for the plate with dummy masses and the plate with real attachments show only a few modes correlating well, see Figure 10. Because the dummy masses were designed to match the bulk masses and inertias of the real attachments, it was expected that several modes would correlate between these two structures. The results indicate frequencies generally within ±10% but poor mode shape agreement. Based on the results, it can be speculated that the dynamic nature of the real attachments plays an important rule in the response of the structure even at frequencies as low as 150 Hz for this test structure. Therefore, in order to accurately model the real structures it will be necessary to use more detail than a simple mass/inertia representation of the attachments.

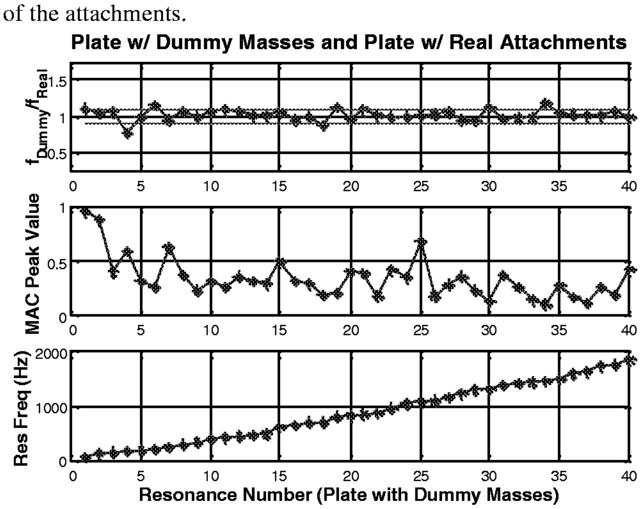

Figure 10: MAC results for edge-stiffened plate with dummy masses and edge-stiffened plate with real attachments.

FINITE ELEMENT MODELING

Comparisons between each set of experimental data highlight the drastic changes the attachments have on the base structure and also show the large differences between responses for the plate with dummy masses and the plate with the real masses. As a first order approximation, the approach that would be used to model the real attachments would be to develop simple plate models, similar to models envisioned for the dummy masses. It is expected that these models would capture the gross dynamic nature of the real attachments. As is shown experimentally, this is not a good approach and thus warrants an improved modeling procedure to enhance the model's fidelity.

Finite element models of the bare plate and the plate with dummy masses have been developed and results compared to corresponding experimental data. While both of these structures are relatively simple in design, there are complicating factors that are not straightforward to model and consequently result in differences between the model and experimental results. One complicating factor is the presence of the stiffeners. As discussed above, the stiffeners are attached to the plate with stainless steel screws and are not an integral part of the plate. Attaching the stiffeners in this manner requires some level of model simplification, which ultimately yields model errors. Another complicating factor is the presence of the attachments. For the dummy mass attachments, fairly accurate models of the attachments themselves can be developed using two-dimensional plate elements. However, the process of joining the dummy mass models to the base structure requires some amount of simplification.

The purpose of the structural finite element modeling is to investigate limitations related to FE modeling of the edge-stiffened plate and attachments. These limitations will help define the requirements for improving the structural response predictions, leading to increased accuracy for acoustic predictions. As discussed above, there are complicating factors related to modeling the plate stiffeners and the attachments even though these are relatively simple structures. The presence of these factors will ultimately cause discrepancies between the FE and experimental results.

The FE models used in this investigation were developed from a combination of plate, bar, and rigid elements (see Figure 11). Plate elements are used to model the plate and attachments while bar elements model the stiffeners. The stainless steel screws are modeled as rigid connections between the plate and stiffeners, and the attachment meshes are joined to the plate mesh using rigid elements. Mode shapes were obtained from the model via a normal modes analysis. The first four mode shapes are shown in Figure 12. The first three of these correspond to the first three experimental mode shapes shown in Figure 7.

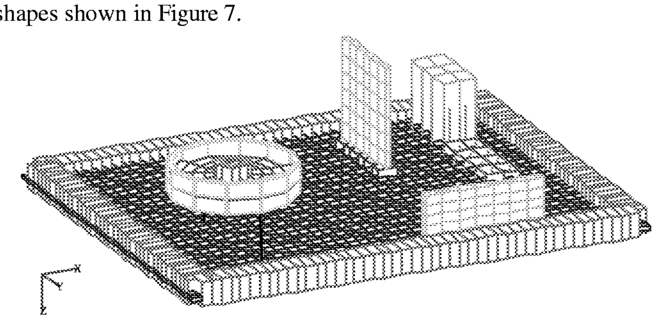

Figure 11: Finite element model of plate with dummy masses.

In all cases, the FE model accuracy is tested by comparing the FE mode shapes to experimental FRF data using a MAC analysis. Because the experimental and numerical meshes are different, the vibration data was interpolated from the coarser experimental mesh onto the finer FE mesh. The interpolation method involved the use of a nonlinear optimization routine for finding mesh locations and a linear interpolation scheme for interpolating the measured nodal values to each numerical grid point. A MAC analysis was then performed using the interpolated experimental data and numerical vibration predictions as input. The results of the MAC analysis for each of the plates are discussed below.

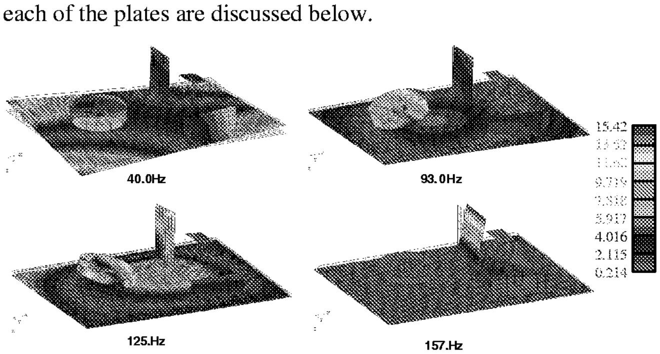

Figure 12: First four mode shapes for model of plate with dummy masses (representative color scale shown).

The first comparison, shown in Figure 13, is between the bare plate model results and the bare plate measured FRFs. The bare plate model is identical to the model shown in Figure 11, but without the attachments. The correspondence between the numerically-predicted and experimentally-measured mode shapes is good, as can be judged by the MAC values near unity at many of the resonance peaks and the corresponding near unity frequency ratios. The occurrence of low MAC values for some of the modes (e.g., modes 4 and 8) is a result of the drive point being close to a resonance node (point of very low vibration) for these modes, thus resulting in low response of the structure at the associated frequencies. The presence of decreased MAC values for modes 17 and above is attributed to the simplifications used to model the stiffeners. These discrepancies could be a result of several factors not accounted for in the model, including the surface contact between the stiffeners and the plate, the stiffness of the attachment method, and differences in attachment locations. While it is feasible to incorporate some of these factors into the model, other factors, such as the surface contact, would make the problem non-linear and hence substantially increase the overall modeling effort.

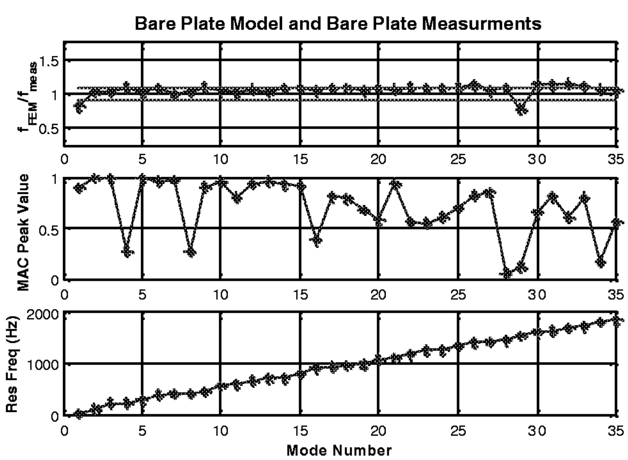

Figure 13: MAC results for FE predictions and measured data for the bare plate.

Figure 14 shows MAC results for the finite element model of the bare plate with the dummy masses, Figure 11, and the corresponding experimental results. While only a few mode shapes correlate well throughout the frequency range, several modes are correlated with MAC values in the 0.6 to 0.8 range indicating a moderate level of correlation. The resonance frequencies match considerably well throughout the range with differences generally less than 10%. Discrepancies in the mode shapes are attributed to a combination of the stiffener and attachment model limitations. Because many modes involving the plate edges are created with the addition of the attachments, the stiffener model limitations are more pronounced in these results than for the bare plate. Limitations associated with the attachments are related to the attachment method and location. In addition, discrepancies exist between the model and predictions due to the non-point-like attachment of the dummy masses to the plate. The model approximates this connection as a point connection when in reality the attachment has finite area. The location of the attachments is also a potential source of error. Because attachments were added and removed multiple times, the location of the attachments may have changed slightly relative to the model, thus causing a change in the mode shapes. Each of these model shortcomings could be improved if sufficient resources were available. However, if no experimental data were available for validation, it would be difficult to judge the level of detail necessary for the model.

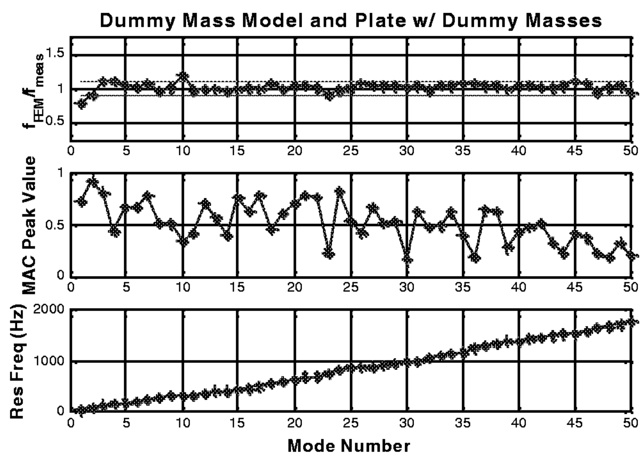

Figure 14: MAC results for dummy mass model and plate with dummy masses.

When the same model is compared to the structure with the real attachments, as expected the model deviates significantly from the actual structure as shown in Figure 15. Here, both low MAC values and large frequency errors are observed. The resonance frequency differences are not necessarily due to a shifting of the resonances, but more likely a result of comparing two completely different modes that happen to have the largest MAC value for the data set. Based on this and previous results, it is apparent that modeling the real attachments using plate elements and rigid connections may not be adequate for structural vibration predictions. Therefore, a more robust method for modeling these structures is necessary if accurate acoustic predictions are to be made.

Two methods are currently being considered as enhancements to the modeling process. The first method involves the use of fuzzy structures analysis (FSA). The idea behind FSA is to have a well characterized master structure for which a conventional FE model would be developed, and one or more fuzzy substructures to represent things such as missing degrees of freedom, fine scale structural details, or attached equipment, to name a few. Details of this method can be found in the literature [3-6]. This method lends itself to the structures considered here because a reasonable model of the bare plate is feasible, and the attachments and stiffeners could be represented by the fuzzy components in the model.

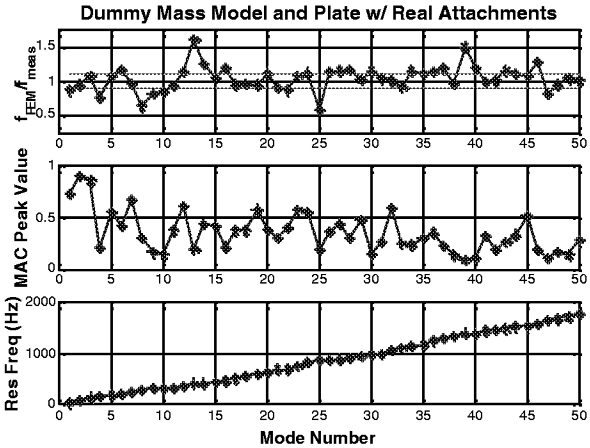

Figure 15: MAC results for dummy mass model and plate with real attachments.

Another potential method for augmenting the base FE model involves a combination of the FE model and experimental data. The idea behind this method is to experimentally characterize drive point impedances of the attachments and directly incorporate this data into the model at the attachment locations. If successful, this method will require a small amount of testing but will potentially yield drastic reductions in the model size from the size of an equivalently accurate model. Also, vibration predictions for the attachments themselves will be lost, which may have a significant impact on a subsequent acoustic boundary element analysis. However, these potential shortcomings also exist in the FSA approach.

It is expected that the required measurements for a general case include translational and rotational drive point impedances of the attachments. While translational drive point impedances are easy to obtain using a force gauge and accelerometer or an impedance head, rotational impedances are difficult to measure [7-8]. However, as shown below, reasonable predictions may be possible with only translational impedances for some attachments.

Replacing a plate model of an attachment with point masses at the attachment points allows the importance of translational and rotational dynamics of the attachment to be determined, see Figure 16. As a simple test case, results for the plate model with two dummy masses were compared to results for the same model with the attachments replaced by point masses, as shown in Figure 17. The point masses in this model have translational inertias but do not include offsets or rotational inertias, which is representative of what would be obtained from a translational drive point impedance measurement. MAC comparisons for these FE models are shown in Figure 18. As indicated by the MAC values and frequency ratios in this figure, the model results compare reasonably well with the exception of a few modes. The modes that do not compare well are mainly associated with the attachment modes, which do not appear in the simplified model. For instance, the third mode at around 180 Hz is the first bending mode of one of the simplified masses. This mode cannot exist in the simplified model as there is no stiffness associated with the point masses to allow energy exchange necessary for a resonance. Other modes with low MAC values are a result of neglecting the rotational inertias. However, the number of these modes is minimal and the MAC values indicate at least some level of correlation between the modes.

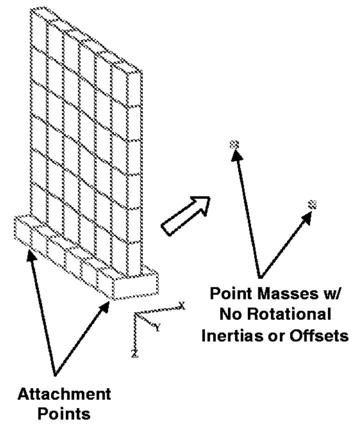

Figure 16: Model simplification by using point masses at the attachment location.

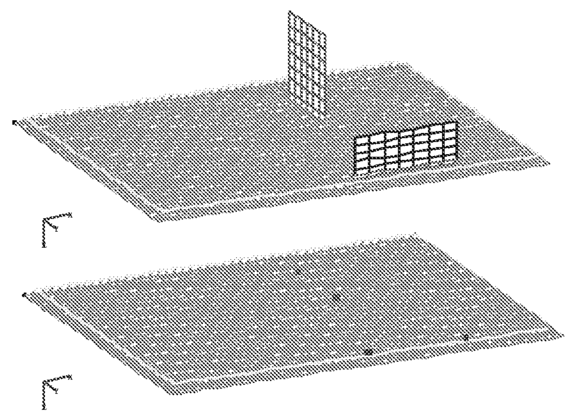

Figure 17: Simplified edge-stiffened plate with two dummy masses (top) and with dummy masses replaced by point masses at the attachment points (bottom).

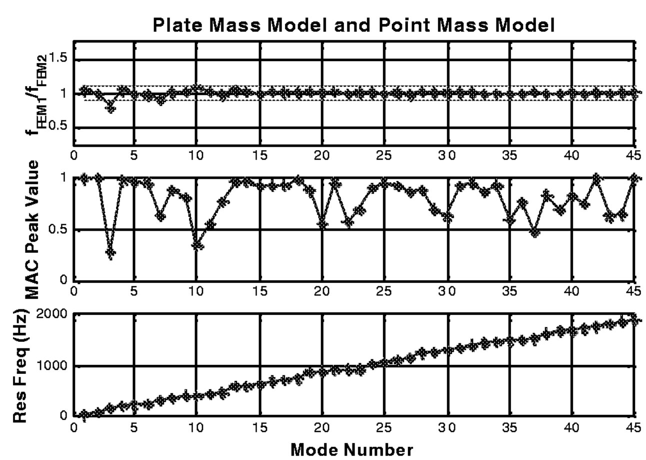

Figure 18: MAC results for FE model of edge-stiffened plate with plate element masses and with point masses.

Structures such as these where the rotational dynamics are not that important to the structural response are potential targets for using empirical drive point translation impedance data to augment the model. Otherwise, this type of model enhancement may not be feasible until improvements are made in the technology for measuring rotational drive point impedances. Several methods are available for incorporating these measurements into a commercial finite element code.

SUMMARY AND CONCLUSIONS

Experimental modal analyses of an edge-stiffened plate with and without attachments have been performed. Results from these analyses suggest that the complicated nature of attachments on the edge-stiffened plate does not lend itself to simple finite element representations. The presence of attachments on the structure has been shown to cause the following:

- The additional mass and inertia from the attachments results in significant mass loading and modifications to the base mode shapes.
- Additional modes appear in the response as a result of plate/attachment resonances, thus increasing the structure's modal density.
- The presence of the dummy masses modifies the mode shapes and resonance frequencies, but does not significantly change the structural damping.
- Damping levels of the plate increase with the presence of the real attachments.

The attempt to separate the mass and damping effects using dummy masses was successful as witnessed by the extracted loss factors and the MAC results. However, the response of the plate with dummy masses did not compare well to the plate with real attachments, as the responses diverged at a frequency near 150 Hz. Based on these results, it is clear that even simplified models that capture the salient aspects of the attachments, such as inertias and physical size, may not provide accurate predictions for the corresponding complicated structure. In light of this, other methods for modeling such structures are being investigated. These methods include the fuzzy structures approach and a method that combines experimental results into a base finite element model. Limitations are anticipated for both of these approaches related to the lack of attachment modes and attachment response. In either case, the predictions must be reasonably accurate and must provide some level of confidence in the results if the method is to be used in a design process where limited or no experimental data is available.

REFERENCES

[1] Anonymous, "The STAR System User Manual – Part 4 Theory and Applications," 1994, pp. 1-37.

[2] Ewins, D. J., 1995, *Modal Testing: Theory and Practice*, John Wiley & Sons Incorporated, New York, NY, pp. 256-259.

[3] Sparrow, V. W., Buehrle, R. D., 2001, "Fuzzy Structures Analysis of Aircraft Panels in Nastran," AIAA Paper No. 2001-1320.

[4] Sparrow, V. W., Russell, D. A. and Rochat, J. L., 1994, "Implementation of Discrete Fuzzy Structure Models in Mathematica," Int. J. Num. Meth. Engr., **37**(17), pp. 3005-3014.

[5] Soize, C., 1995, "Vibration Damping in Low-Frequency Range due to Structural Complexity. A Model Based on the Theory of Fuzzy Structures and Model Parameters Estimation," Computers & Structures, **58**(5), pp. 901-915.

[6] Soize, C., 1993, "A Model and Numerical Method in the Medium Frequency Range for Vibroacoustic Predictions using the Theory of Structural Fuzzy," J. Acoust. Soc. Am. **94**(2), pp. 849-865.

[7] Sanderson, M. A., and Fredo, C. R., 1995, "Direct Measurement of Moment Mobility, Part I: A Theoretical Study," J. Sound and Vib. **179**(4), pp. 669-684.

[8] Helderweirt, S. *et al*, 2001, "Application of Accelerometer-Based Rotational Degree of Freedom Measurements for Engine Subframe Modeling," Proceedings, International Modal Analysis Conference, IMAC-XIX: A Conference on Structural Dynamics, Kissimmee, FL, vol. 2, pp. 1298-1304.